Adaptive Minimum Symbol Error Rate Beamforming Assisted Receiver for Quadrature Amplitude Modulation Systems

S. Chen, H.-Q. Du and L. Hanzo

School of Electronics and Computer Science
University of Southampton, Southampton SO17 1BJ, U.K.
E-mails: {sqc,lh,hd404}@ecs.soton.ac.uk

ABSTRACT

The paper considers adaptive beamforming assisted receiver for multiple antenna aided multiuser systems that employ the bandwidth efficient quadrature amplitude modulation scheme. A minimum symbol error rate (MSER) design is proposed for the beamforming assisted receiver, and it is shown that this MSER approach provides significant performance enhancement, in terms of achievable symbol error rate, over the standard minimum mean square error design. A sample-by-sample adaptive algorithm, referred to as the least symbol error rate, is derived for adaptive implementation of the MSER beamforming solution.

I. INTRODUCTION

The ever-increasing demand for mobile communication capacity has motivated the development of adaptive antenna array assisted spatial processing techniques [1]–[10] in order to further improve the achievable spectral efficiency. A particular technique that has shown real promise in achieving substantial capacity enhancements is the use of adaptive beamforming with antenna arrays. Through appropriately combining the signals received by the different elements of an antenna array to form a single output, adaptive beamforming is capable of separating signals transmitted on the same carrier frequency, and thus provides a practical means of supporting multiusers in a space division multiple access scenario. Classically, the beamforming process is carried out by minimizing the mean square error (MSE) between the desired output and the actual array output. For a communication system, however, it is the bit error rate (BER) or symbol error rate (SER) that really matters. Adaptive beamforming based on directly minimizing the system's BER has been proposed for binary phase shift keying and quadrature phase shift keying modulation schemes [11],[12].

For the sake of improving the achievable bandwidth efficiency, high-throughput quadrature amplitude modulation (QAM) schemes [13] have become popular in numerous wireless network standards. Adaptive minimum SER (MSER) equalization has been investigated for the single-antenna single-user system with the pulse-amplitude modulation scheme [14] and with the QAM scheme [15]. In this paper, we derive the MSER beamforming design for the multiple antenna assisted multiuser system with QAM signalling. We show that the MSER design can provide significant performance gains, in terms of smaller SER, over the traditional minimum MSE (MMSE) design. An adaptive implementation of the MSER beamforming solution is proposed based on a stochastic gradient algorithm, which we refer to as the least symbol error rate (LSER). Our proposed technique is very different to the method proposed in [15]. The adaptive LSER algorithm has its root in the Parzen

window density estimation [16]-[18]. In this sense, the proposed adaptive MSER technique is an extension of the method proposed in [14] to the interference-limited multiuser communication system employing the QAM scheme.

II. SYSTEM MODEL

The system consists of S users, and each user transmits an M-QAM signal on the same carrier frequency $\omega = 2\pi f$. The receiver is equipped with a linear antenna array consisting of L uniformly spaced elements. Assume that the channel is narrow-band which does not induce intersymbol interference. Then the symbol-rate received signal samples can be expressed as

$$x_l(k) = \sum_{i=1}^{S} A_i b_i(k) e^{j\omega t_l(\theta_i)} + n_l(k) = \bar{x}_l(k) + n_l(k), \quad (1)$$

for $1 \leq l \leq L$, where $t_l(\theta_i)$ is the relative time delay at element l for source i with θ_i being the direction of arrival for source i, $n_l(k)$ is a complex-valued Gaussian white noise with $E[|n_l(k)|^2] = 2\sigma_n^2$, A_i is the channel coefficient for user i, and $b_i(k)$ is the kth symbol of user i which takes the value from the M-QAM symbol set

$$\mathcal{B} \stackrel{\triangle}{=} \{b_{l,q} = u_l + ju_q, \ 1 \leq l, q \leq \sqrt{M}\} \quad (2)$$

with $u_l = 2l - \sqrt{M} - 1$ and $u_q = 2q - \sqrt{M} - 1$. Source 1 is the desired user and the rest of the sources are interfering users. The desired-user signal to noise ratio is SNR= $|A_1|^2 \sigma_b^2 / 2\sigma_n^2$ and the desired signal to interferer i ratio is SIR$_i = A_1^2 / A_i^2$, for $2 \leq i \leq S$, where σ_b^2 denotes the M-QAM symbol energy. The received signal vector $\mathbf{x}(k) = [x_1(k) \ x_2(k) \cdots x_L(k)]^T$ is given by

$$\mathbf{x}(k) = \mathbf{P}\mathbf{b}(k) + \mathbf{n}(k) = \bar{\mathbf{x}}(k) + \mathbf{n}(k), \quad (3)$$

where $\mathbf{n}(k) = [n_1(k) \ n_2(k) \cdots n_L(k)]^T$, the system matrix $\mathbf{P} = [A_1 \mathbf{s}_1 \ A_2 \mathbf{s}_2 \cdots A_S \mathbf{s}_S]$ with the steering vector for source i $\mathbf{s}_i = [e^{j\omega t_1(\theta_i)} \ e^{j\omega t_2(\theta_i)} \cdots e^{j\omega t_L(\theta_i)}]^T$, and the transmitted QAM symbol vector $\mathbf{b}(k) = [b_1(k) \ b_2(k) \cdots b_S(k)]^T$.

A linear beamformer is employed, whose soft output is given by

$$y(k) = \mathbf{w}^H \mathbf{x}(k) = \mathbf{w}^H(\bar{\mathbf{x}}(k) + \mathbf{n}(k)) = \bar{y}(k) + e(k) \quad (4)$$

where $\mathbf{w} = [w_1 \ w_2 \cdots w_L]^T$ is the beamformer weight vector and $e(k)$ is Gaussian distributed with zero mean and $E[|e(k)|^2] = 2\sigma_n^2 \mathbf{w}^H \mathbf{w}$. Define the combined impulse response of the beamformer and system as $\mathbf{w}^H \mathbf{P} = \mathbf{w}^H [\mathbf{p}_1 \ \mathbf{p}_2 \cdots \mathbf{p}_S] = [c_1 \ c_2 \cdots c_S]$. The beamformer's output can alternatively be expressed as

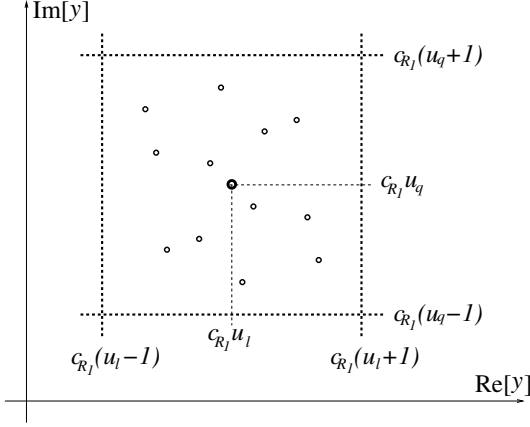$$y(k) = c_1 b_1(k) + \sum_{i=2}^{S} c_i b_i(k) + e(k). \quad (5)$$

Fig. 1. Decision thresholds associated with point $c_1 b_{l,q}$ assuming $c_{R_1} > 0$ and $c_{I_1} = 0$, and illustrations of symmetric distribution of $\mathcal{Y}_{l,q}$ around $c_1 b_{l,q}$.

Provided that $c_1 = c_{R_1} + jc_{I_1}$ satisfies $c_{R_1} > 0$ and $c_{I_1} = 0$, the symbol decision $\hat{b}_1(k) = \hat{b}_{R_1}(k) + j\hat{b}_{I_1}(k)$ can be decoupled into

$$\hat{b}_{R_1}(k) = \begin{cases} u_1, & \text{if } y_R(k) \le c_{R_1}(u_1 + 1) \\ u_l, & \text{if } c_{R_1}(u_l - 1) < y_R(k) \le c_{R_1}(u_l + 1) \\ & \text{for } 2 \le l \le \sqrt{M} - 1 \\ u_{\sqrt{M}}, & \text{if } y_R(k) > c_{R_1}(u_{\sqrt{M}} - 1) \end{cases} \tag{6}$$

$$\hat{b}_{I_1}(k) = \begin{cases} u_1, & \text{if } y_I(k) \le c_{R_1}(u_1 + 1) \\ u_q, & \text{if } c_{R_1}(u_q - 1) < y_I(k) \le c_{R_1}(u_q + 1) \\ & \text{for } 2 \le q \le \sqrt{M} - 1 \\ u_{\sqrt{M}}, & \text{if } y_I(k) > c_{R_1}(u_{\sqrt{M}} - 1) \end{cases} \tag{7}$$

where $y(k) = y_R(k) + jy_I(k)$ and $\hat{b}_1(k)$ is the estimate for $b_1(k) = b_{R_1}(k) + jb_{I_1}(k)$. Fig. 1 depicts the decision thresholds associated with the decision $\hat{b}_1(k) = b_{l,q}$. In general, $c_1 = \mathbf{w}^H \mathbf{p}_1$ is complex-valued and the rotating operation

$$\mathbf{w}^{\text{new}} = \frac{c_d^{\text{old}}}{\left| c_d^{\text{old}} \right|} \mathbf{w}^{\text{old}} \tag{8}$$

can be used to make c_1 real and positive. This rotation is a linear transformation and does not alter the system's SER. Thus the desired user's channel A_1 and steering vector $\mathbf{s}_1$ are required at the receiver in order to apply the decision rules (6) and (7).

III. MINIMUM SYMBOL ERROR RATE BEAMFORMING

The classical MMSE solution for the beamformer (4) is given by

$$\mathbf{w}_{\text{MMSE}} = \left(\mathbf{P}\mathbf{P}^H + \frac{2\sigma_n^2}{\sigma_b^2} \mathbf{I}_L \right)^{-1} \mathbf{p}_1, \tag{9}$$

Since the SER is the true performance indicator, it is desired to consider the optimal MSER Beamforming solution. Denote the $N_b = M^S$ number of possible sequences of $\mathbf{b}(k)$ as $\mathbf{b}_i$, $1 \le i \le N_b$. Then $\bar{\mathbf{x}}(k)$ can only take values from the finite signal set defined by $\mathcal{X} \triangleq \{\bar{\mathbf{x}}_i = \mathbf{P}\mathbf{b}_i, 1 \le i \le N_b\}$. The set $\mathcal{X}$ can be partitioned into M subsets, depending on the value of $b_1(k)$

$$\mathcal{X}_{l,q} \triangleq \{\bar{\mathbf{x}}_i \in \mathcal{X} : b_1(k) = b_{l,q}\}, \ 1 \le l, q \le \sqrt{M}. \tag{10}$$

The noise-free part of the beamformer's output $\bar{y}(k)$ only takes values from the scalar set $\mathcal{Y} \triangleq \{\bar{y}_i = \mathbf{w}^H \bar{\mathbf{x}}_i, 1 \le i \le N_b\}$, and $\mathcal{Y}$ can be divided into the M subsets conditioned on the value of $b_1(k)$

$$\mathcal{Y}_{l,q} \triangleq \{\bar{y}_i \in \mathcal{Y} : b_1(k) = b_{l,q}\}, \ 1 \le l, q \le \sqrt{M}. \tag{11}$$

Lemma 1: The subsets $\mathcal{Y}_{l,q}, 1 \le l, q \le \sqrt{M}$, satisfy the shifting properties

$$\mathcal{Y}_{l+1,q} = \mathcal{Y}_{l,q} + 2c_1, \ 1 \le l \le \sqrt{M} - 1, \tag{12}$$

$$\mathcal{Y}_{l,q+1} = \mathcal{Y}_{l,q} + j2c_1, \ 1 \le q \le \sqrt{M} - 1, \tag{13}$$

$$\mathcal{Y}_{l+1,q+1} = \mathcal{Y}_{l,q} + (2 + j2)c_1, \ 1 \le l, q \le \sqrt{M} - 1. \tag{14}$$

The proof of Lemma 1 is straightforward.

Lemma 2: The points of $\mathcal{Y}_{l,q}$ are distributed symmetrically around the symbol point $c_1 b_{l,q}$.

Lemma 2 is a direct consequence of symmetric distribution of the symbol constellation (2). This symmetric property is also illustrated in Fig. 1. Note that the distribution of $\mathcal{Y}_{l,q}$ is symmetric with respect to the two vertical decision thresholds $c_{R_1}(u_l \pm 1)$ and with respect to the two horizontal decision threshold $c_{R_1}(u_q \pm 1)$.

For the beamformer with weight vector $\mathbf{w}$, denote

$$P_E(\mathbf{w}) = \text{Prob}\{\hat{b}_1(k) \ne b_1(k)\}, \tag{15}$$

$$P_{E_R}(\mathbf{w}) = \text{Prob}\{\hat{b}_{R_1}(k) \ne b_{R_1}(k)\}, \tag{16}$$

$$P_{E_I}(\mathbf{w}) = \text{Prob}\{\hat{b}_{I_1}(k) \ne b_{I_1}(k)\}. \tag{17}$$

It is then easy to see that the SER is given by

$$P_E(\mathbf{w}) = P_{E_R}(\mathbf{w}) + P_{E_I}(\mathbf{w}) - P_{E_R}(\mathbf{w})P_{E_I}(\mathbf{w}). \tag{18}$$

The conditional probability density function (PDF) of $y(k)$ given $b_1(k) = b_{l,q}$ is a Gaussian mixture defined by

$$p(y|b_{l,q}) = \frac{1}{N_{sb} 2\pi\sigma_n^2 \mathbf{w}^H \mathbf{w}} \sum_{i=1}^{N_{sb}} e^{-\frac{|y - \bar{y}_i^{(l,q)}|^2}{2\sigma_n^2 \mathbf{w}^H \mathbf{w}}}, \tag{19}$$

where $N_{sb} = N_b/M$ is the size of $\mathcal{Y}_{l,q}$, $\bar{y}_i^{(l,q)} = \bar{y}_{R_i}^{(l,q)} + j\bar{y}_{I_i}^{(l,q)} \in \mathcal{Y}_{l,q}$, and $y = y_R + jy_I$. Noting that c_1 is real-valued and positive and taking into account the symmetric distribution of $\mathcal{Y}_{l,q}$ (lemma 2), for $2 \le l \le \sqrt{M} - 1$, the conditional error probability of $\hat{b}_{R_1}(k) \ne u_l$ given $b_{R_1}(k) = u_l$ can be shown to be

$$P_{E_R,l}(\mathbf{w}) = \frac{2}{N_{sb}} \sum_{i=1}^{N_{sb}} Q(g_{R_i}^{(l,q)}(\mathbf{w})), \tag{20}$$

where

$$Q(u) = \frac{1}{\sqrt{2\pi}} \int_u^\infty e^{-\frac{z^2}{2}} dz, \tag{21}$$

$$g_{R_i}^{(l,q)}(\mathbf{w}) = \frac{\bar{y}_{R_i}^{(l,q)} - c_{R_1}(u_l - 1)}{\sigma_n \sqrt{\mathbf{w}^H \mathbf{w}}}. \tag{22}$$

Further taking into account the shifting property (lemma 1), it is straightforward to show that

$$P_{E_R}(\mathbf{w}) = \gamma \frac{1}{N_{sb}} \sum_{i=1}^{N_{sb}} Q(g_{R_i}^{(l,q)}(\mathbf{w})), \tag{23}$$

where $\gamma = \frac{2\sqrt{M}-2}{\sqrt{M}}$. It is seen that P_{E_R} can be evaluated using (real part of) any single subset $\mathcal{Y}_{l,q}$. Similarly, P_{E_I} can be evaluated using (imaginary part of) any single subset $\mathcal{Y}_{l,q}$ as

$$P_{E_I}(\mathbf{w}) = \gamma \frac{1}{N_{sb}} \sum_{i=1}^{N_{sb}} Q(g_{I_i}^{(l,q)}(\mathbf{w})) \qquad (24)$$

with

$$g_{I_i}^{(l,q)}(\mathbf{w}) = \frac{\bar{y}_{I_i}^{(l,q)} - c_{R_1}(u_q - 1)}{\sigma_n \sqrt{\mathbf{w}^H \mathbf{w}}}. \qquad (25)$$

Note that the SER is invariant to a positive scaling of $\mathbf{w}$.

The MSER solution $\mathbf{w}_{\text{MSER}}$ is defined as the one that minimizes the upper bound of the SER given by

$$P_{E_B}(\mathbf{w}) = P_{E_R}(\mathbf{w}) + P_{E_I}(\mathbf{w}), \qquad (26)$$

that is,

$$\mathbf{w}_{\text{MSER}} = \arg \min_{\mathbf{w}} P_{E_B}(\mathbf{w}). \qquad (27)$$

The upper bound $P_{E_B}(\mathbf{w})$ is very tight, i.e. very close to the true SER $P_E(\mathbf{w})$. The gradients of $P_{E_R}(\mathbf{w})$ and $P_{E_I}(\mathbf{w})$ with respect to $\mathbf{w}$ can be shown to be respectively

$$\nabla P_{E_R}(\mathbf{w}) = \frac{\gamma}{2N_{sb}\sqrt{2\pi}\sigma_n \sqrt{\mathbf{w}^H \mathbf{w}}} \sum_{i=1}^{N_{sb}} e^{-\frac{\left(\bar{y}_{R_i}^{(l,q)} - c_{R_1}(u_l-1)\right)^2}{2\sigma_n^2 \mathbf{w}^H \mathbf{w}}}$$

$$\times \left(\frac{\bar{y}_{R_i}^{(l,q)} - c_{R_1}(u_l - 1)}{\mathbf{w}^H \mathbf{w}} \mathbf{w} - \bar{\mathbf{x}}_i^{(l,q)} + (u_l - 1)\mathbf{p}_1 \right), \qquad (28)$$

$$\nabla P_{E_I}(\mathbf{w}) = \frac{\gamma}{2N_{sb}\sqrt{2\pi}\sigma_n \sqrt{\mathbf{w}^H \mathbf{w}}} \sum_{i=1}^{N_{sb}} e^{-\frac{\left(\bar{y}_{I_i}^{(l,q)} - c_{R_1}(u_q-1)\right)^2}{2\sigma_n^2 \mathbf{w}^H \mathbf{w}}}$$

$$\times \left(\frac{\bar{y}_{I_i}^{(l,q)} - c_{R_1}(u_q - 1)}{\mathbf{w}^H \mathbf{w}} \mathbf{w} + j\bar{\mathbf{x}}_i^{(l,q)} + (u_q - 1)\mathbf{p}_1 \right), \qquad (29)$$

where $\bar{\mathbf{x}}_i^{(l,q)} \in \mathcal{X}_{l,q}$. With the gradient $\nabla P_{E_B}(\mathbf{w}) = \nabla P_{E_R}(\mathbf{w}) + \nabla P_{E_I}(\mathbf{w})$, the optimization problem (27) can be solved iteratively using a gradient optimization algorithm, such as the simplified conjugate gradient algorithm [11]. The rotating operation (8) should be applied after each iteration, to ensure a real and positive c_1.

The PDF $p(y)$ of $y(k)$ can be estimated using the Parzen window estimate based on a block of training data. This leads to an estimated SER for the beamformer. Minimizing this estimated SER based on a gradient optimization yields an approximated MSER solution. To derive a sample-by-sample adaptive algorithm, consider a single-sample "estimate" of $p(y)$

$$\tilde{p}(y, k) = \frac{1}{2\pi\rho_n^2} e^{-\frac{|y-y(k)|^2}{2\rho_n^2}} \qquad (30)$$

and the corresponding one-sample SER "estimate" $\tilde{P}_{E_B}(\mathbf{w}, k)$. Using the instantaneous stochastic gradient of $\nabla \tilde{P}_{E_B}(\mathbf{w}, k) = \nabla \tilde{P}_{E_R}(\mathbf{w}, k) + \nabla \tilde{P}_{E_I}(\mathbf{w}, k)$ with

$$\nabla \tilde{P}_{E_R}(\mathbf{w}, k) = \frac{\gamma}{2\sqrt{2\pi}\rho_n} e^{-\frac{\left(y_R(k) - \hat{c}_{R_1}(k)(b_{R_1}(k)-1)\right)^2}{2\rho_n^2}}$$

$$\times (-\mathbf{x}(k) + (b_{R_1}(k) - 1)\hat{\mathbf{p}}_1) \qquad (31)$$

and

$$\nabla \tilde{P}_{E_I}(\mathbf{w}, k) = \frac{\gamma}{2\sqrt{2\pi}\rho_n} e^{-\frac{\left(y_I(k) - \hat{c}_{R_1}(k)(b_{I_1}(k)-1)\right)^2}{2\rho_n^2}}$$

$$\times (j\mathbf{x}(k) + (b_{I_1}(k) - 1)\hat{\mathbf{p}}_1) \qquad (32)$$

gives rise to a stochastic gradient adaptive algorithm, which we refer to as the LSER algorithm

$$\mathbf{w}(k+1) = \mathbf{w}(k) + \mu \left(-\nabla \tilde{P}_{E_B}(\mathbf{w}(k), k)\right), \qquad (33)$$

$$\hat{c}_1(k+1) = \mathbf{w}^H(k+1)\hat{\mathbf{p}}_1, \qquad (34)$$

$$\mathbf{w}(k+1) = \frac{\hat{c}_1(k+1)}{|\hat{c}_1(k+1)|} \mathbf{w}(k+1), \qquad (35)$$

where $\hat{\mathbf{p}}_1$ is an estimated $\mathbf{p}_1$. The step size μ and the kernel width ρ_n are the two algorithmic parameters that should be set appropriately in order to ensure an adequate performance in terms of convergence rate and steady-state SER misadjustment.

IV. SIMULATION STUDY

Stationary system. The example consisted of four sources and a three-element antenna array. Fig. 2 shows the locations of the desired source and the interfering sources graphically. The simulated channel conditions were $A_i = 1 + j0$, $1 \leq i \leq 4$. Thus $\text{SIR}_i = 0$ dB for $2 \leq i \leq 4$. The modulation scheme was 16-QAM. Fig. 3 compares the SER performance of the MSER solution with that of the MMSE solution under three different conditions: **(a)** the minimum spatial separation between the desired user 1 and the interfering user 4 $\theta = 32°$ **(b)** $\theta = 30°$, and **(c)** $\theta = 28°$. For this example, the MSER beamformer achieved significantly better performance than the MMSE beamformer.

Performance of the adaptive LSER algorithm was investigated using the system with $\theta = 30°$ and SNR= 26 dB. Given $\mathbf{w}(0) = \mathbf{w}_{\text{MMSE}}$ and with the step size $\mu = 0.001$ and the kernel width $\rho_n = \sigma_n$, Fig. 4 (a) depicts the learning curves of the LSER algorithm, where DD denotes the decision-directed adaptation with $\hat{b}_1(k)$ substituting for $b_1(k)$. Fig. 4 (b) shows the learning curves of the LSER algorithm under the same conditions except that $\mathbf{w}(0) = [0.1 + j0.1 \ 0.1 - j0.01 \ 0.1 - j0.1]^T$. It can be seen from Fig. 4 that the LSER beamformer had a reasonable convergence speed. It can also be seen that the initial condition $\mathbf{w}(0)$ had some influence on convergence rate.
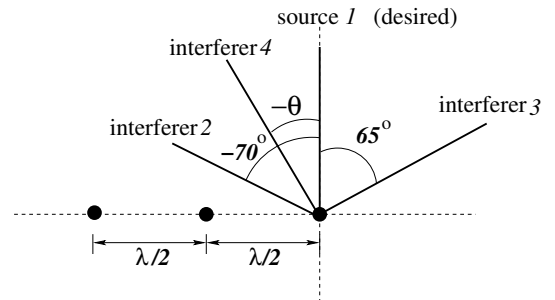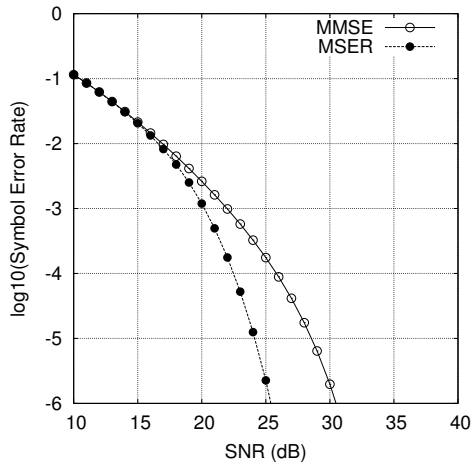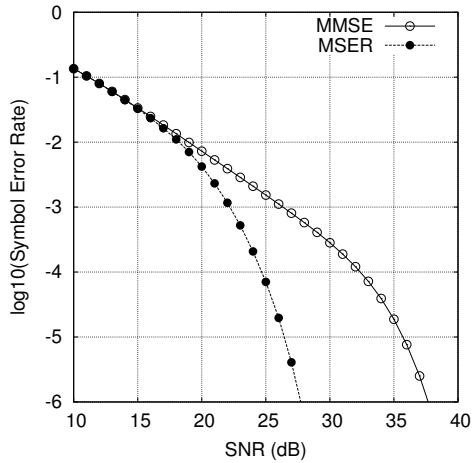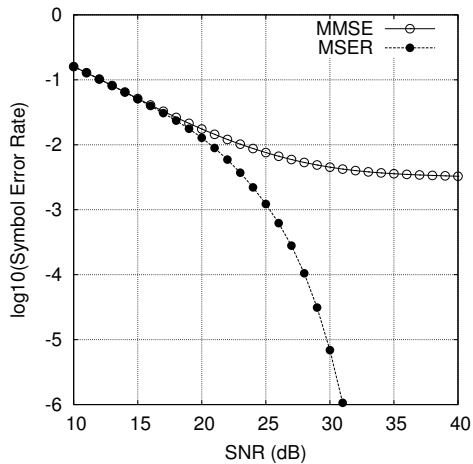

Fig. 2. Locations of the desired source and the interfering sources with respect to the three-element linear array with $\lambda/2$ element spacing, λ being the wavelength.

(a) $\theta = 32°$

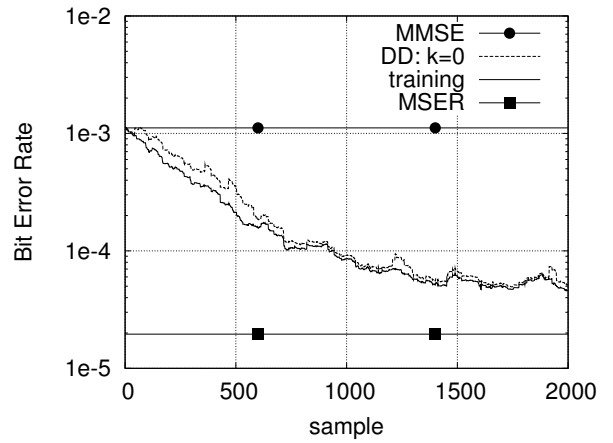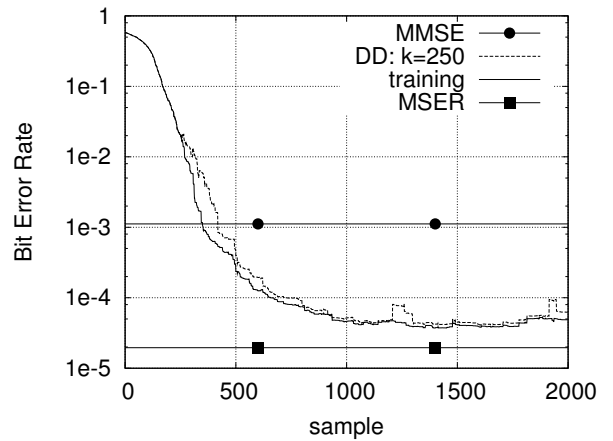

(b) $\theta = 30°$

(c) $\theta = 28°$

Fig. 3. Comparison of symbol error rate performance for the stationary system.

(a) $\mathbf{w}(0) = \mathbf{w}_{\mathrm{MMSE}}$

(b) $\mathbf{w}(0) = [0.1 + j0.1\ 0.1 - j0.01\ 0.1 - j0.1]^T$

Fig. 4. Learning curves of the stochastic gradient adaptive LSER algorithm for the stationary system averaged over 20 runs, given $\theta = 30°$ and SNR$= 26$ dB, where DD denotes decision-directed adaptation with $\hat{b}_1(k)$ substituting for $b_1(k)$. The step size $\mu = 0.001$ and kernel width $\rho_n = \sigma_n$.

V. CONCLUSIONS

An adaptive MSER beamforming technique has been proposed for multiple antenna aided multiuser wireless communication systems with QAM signalling. It has been demonstrated that the MSER beamforming design can provide significant performance enhancement, in terms of the system SER, over the standard MMSE beamforming design. An adaptive implementation of the MSER beamforming solution has been realized using the stocastic gradient adaptive algorithm known as the LSER technique.

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