Revision of Lecture Twelve

- Previous lecture is about channel coding introduction

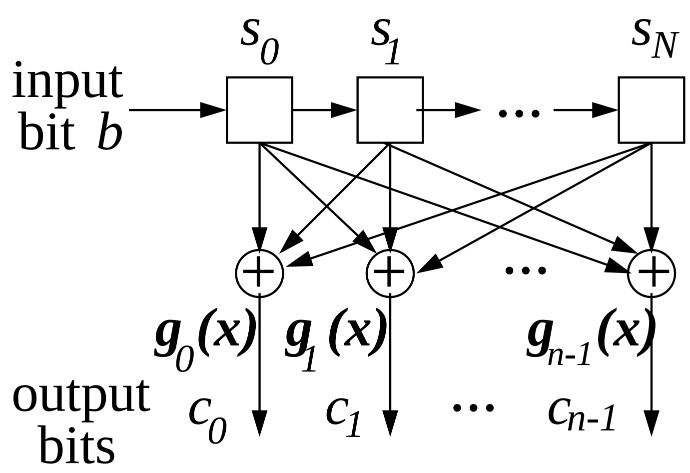
  Basic concepts introduced in this lecture are important and you should have a deep understand of them

- In the next two lectures, we will discuss two classes of practical channel coding schemes, namely,

  Convolutional codes and a particular class of linear block codes known as BCH

Convolutional Coding

- **Convolutional codes** can be systematic or non-systematic, non-systematic ones are more powerful
- $CC(n, k, N)$: code rate $R = k/n$, N is **constraint length** (or **memory** $N + 1$ stages), usually n, k and N are small, and in particular, $k = 1$ is often used
- $CC(n, k = 1, N)$ encoder circuit:
  - During each bit interval, the register is shifted one stage: s_N is shifted out, $s_i \to s_{i+1}$, and the data bit enters s_0
  - After this shift, the values of $s_1 \cdots s_N$ define the current **state** of the register, which is used to produce code bits
  - Modulo-2 adders produce code bits c_i, $0 \le i \le n-1$, which are specified by the n polynomials



$$g_i(x) = g_{i,0} + g_{i,1}x + \cdots + g_{i,N}x^N, \quad 0 \le i \le n-1$$

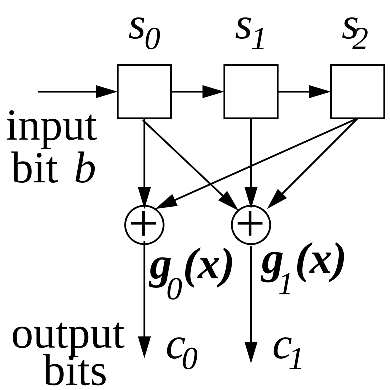
  - Systematic CC: $g_0(x) = 1$ and $c_0 = b$ (data bit), while non-systematic CC: some $g_{0,l} = 1$, $l > 0$
  - Output (code) bits depend on the state of $s_1 \cdots s_N$ after shift and the input bit

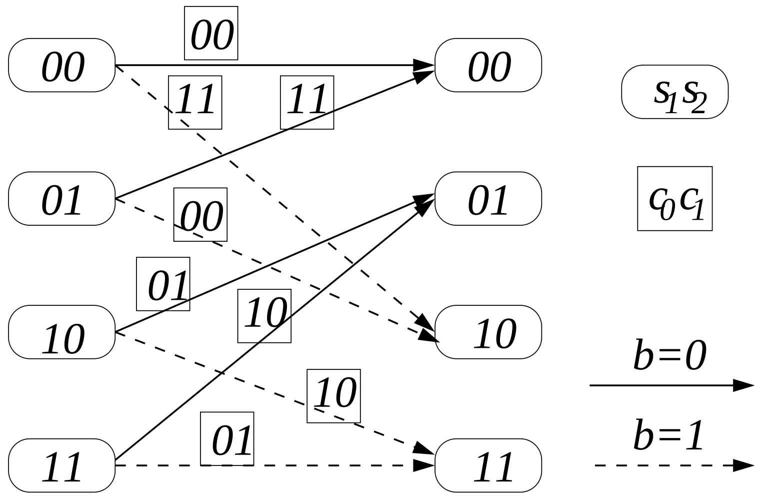
CC Encoder: State-Transition Diagram

- State-transition diagram describes the encoder of a CC code, showing all the 2^N states and all the state transitions together with the output bits

- Example: half-rate constraint length $N = 2$, $CC(2, 1, 2)$, defined by $g_0(x) = 1 + x^2$ and $g_1(x) = 1 + x + x^2$, table of state transition, encoder circuit, state-transition diagram

input b	current $s_1 s_2$	next $s_1 s_2$	output $c_0 c_1$
0	0 0	0 0	0 0
0	0 1	0 0	1 1
0	1 0	0 1	0 1
0	1 1	0 1	1 0
1	0 0	1 0	1 1
1	0 1	1 0	0 0
1	1 0	1 1	1 0
1	1 1	1 1	0 1



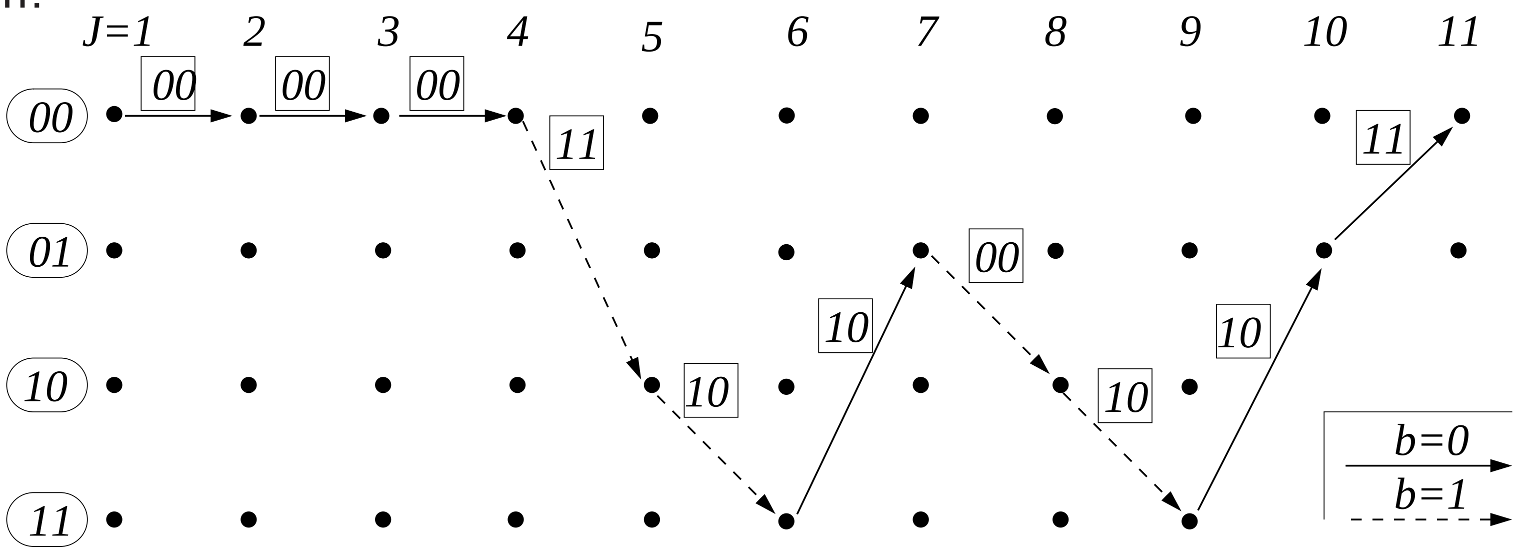


- Only **two legitimate state transitions** for each state, depending on the input bit b; similarly, each state has two merging paths; output bits are shown in the box

CC Encoder: Trellis Diagram

- An alternative way to describe a CC encoder is **trellis diagram**
- Same example, $CC(2,1,2)$, with $g_0(x) = 1 + x^2$ and $g_1 = 1 + x + x^2$

  Information bit sequence $\cdots 0011011000$ (rightmost enters encoder first). Trellis diagram:



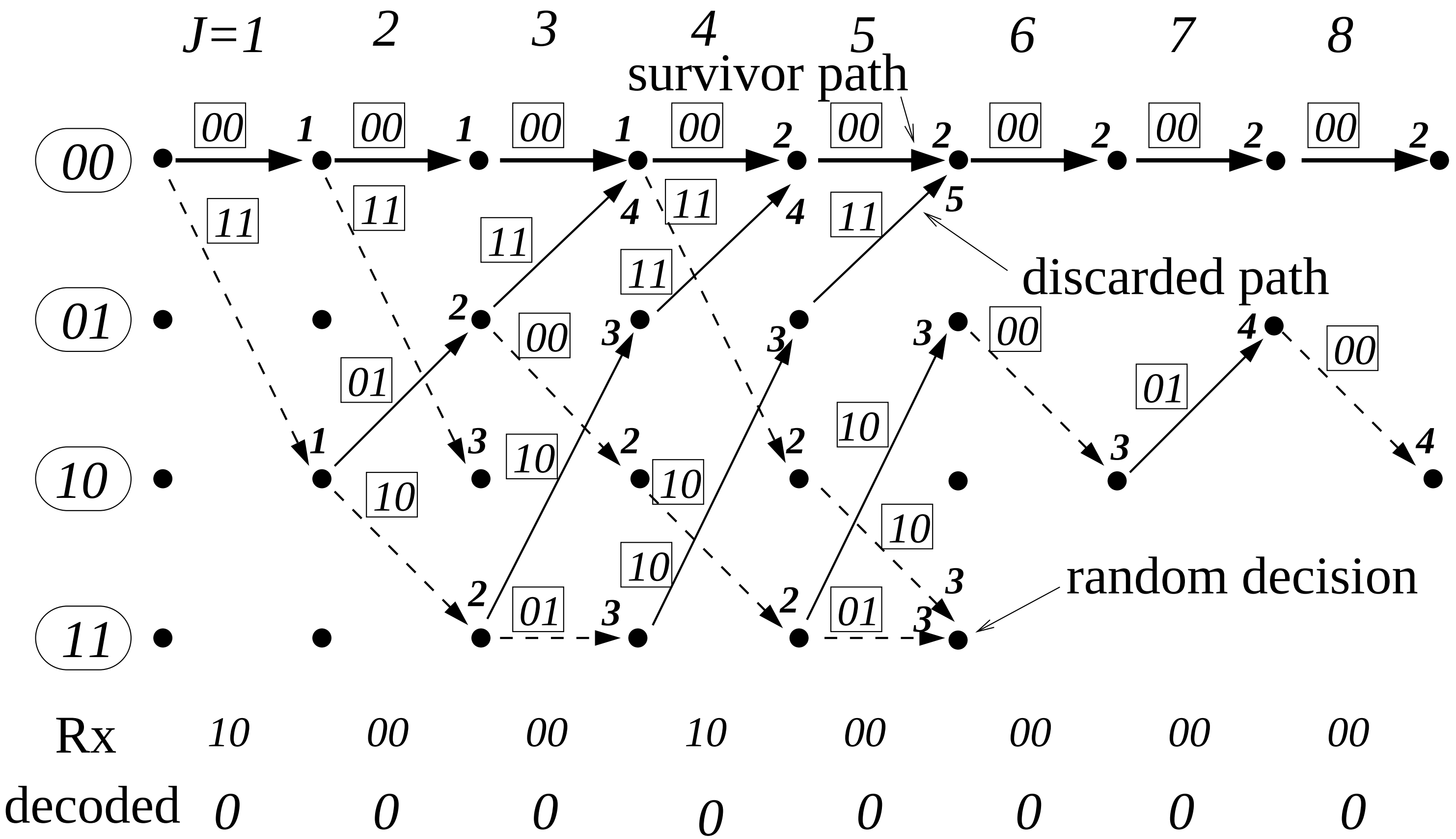
The state is initialised at zero and as the data bit sequence enters, trellis diagram shows the history of state transitions with output bits on it

CC Decoding

- In CC encoder: bits enter encoder register and has to travel through it
  - Thus the register sequence of state transitions is not arbitrary, and modulo-2 gates impose additional constraints on output bits
  - Result is only certain output bit sequences are **legitimate** and transmitted sequences are restricted to these legitimate sequences
- If a non-legitimate received sequence is encountered in the decoder, it must be due to channel errors, as such a sequence cannot be transmitted
  - In this case, the decoder can choose a legitimate sequence that is most resemblant to the received sequence (e.g. in the sense of smallest Hamming distance)
- Such a decoding strategy is called **maximum likelihood** sequence decoding, as it finds the most likely transmitted sequence, given the received sequence
  - MLSD can be implemented efficiently using the **Viterbi algorithm**, which can be hard-input hard-output, soft-input hard-output, soft-input soft-output

Hard-Input Hard-Output Viterbi Algorithm

- **Hard-input** decoding is for hard-decision demodulation where demodulator has made binary decision concerning received bits, and **hard-output** decoding makes hard decision concerning decoded bits
- Example: $CC(2,1,2)$ with $g_0(x) = 1 + x^2$ and $g_1 = 1 + x + x^2$, all zero sequence is transmitted and received sequence is $10000010000000 \cdots$ (the leftmost bit at leftmost position of trellis)

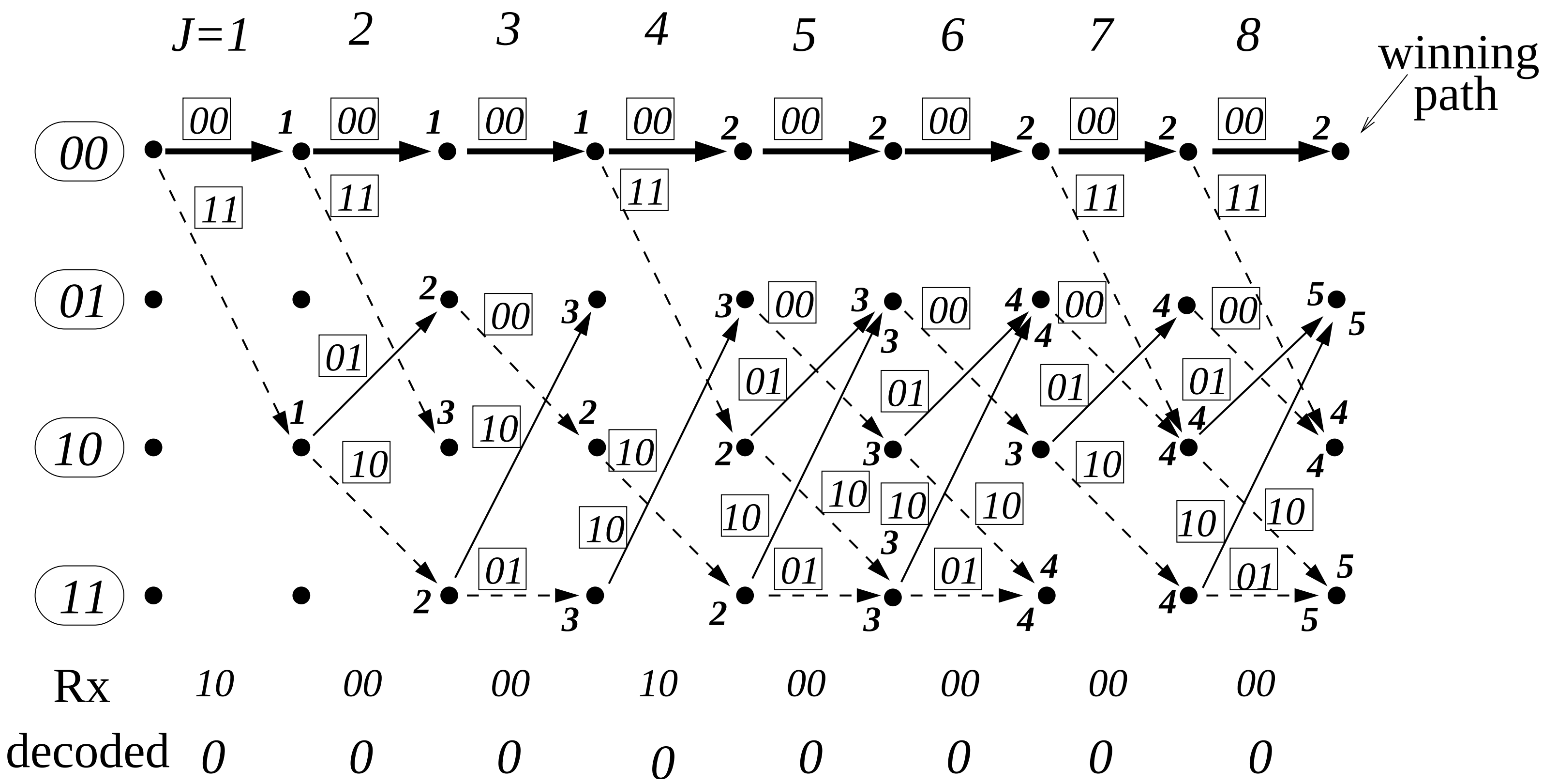


HIHO Viterbi Algorithm Decoding Rules

- Branch metric is the Hamming distance between the legitimate encoded (output) bits of a trellis branch at a stage and the received bits at the same stage
- Path metric is the accumulated branch metrics of a path (bold number shown on trellis diagram)
- For two merging paths at a stage, the one with a larger path metric is discarded and the other, called survivor path, is kept; if two metrics are equal, a random decision is made to keep one path
- A final decision is made after a sufficiently large number of stages (8 for this example) to choose a winning path with the smallest path metric
- If decoding decision is correct, the winning path metric is the number of transmission errors inflicted by the channel

Full Trellis for Previous VA decoding Example

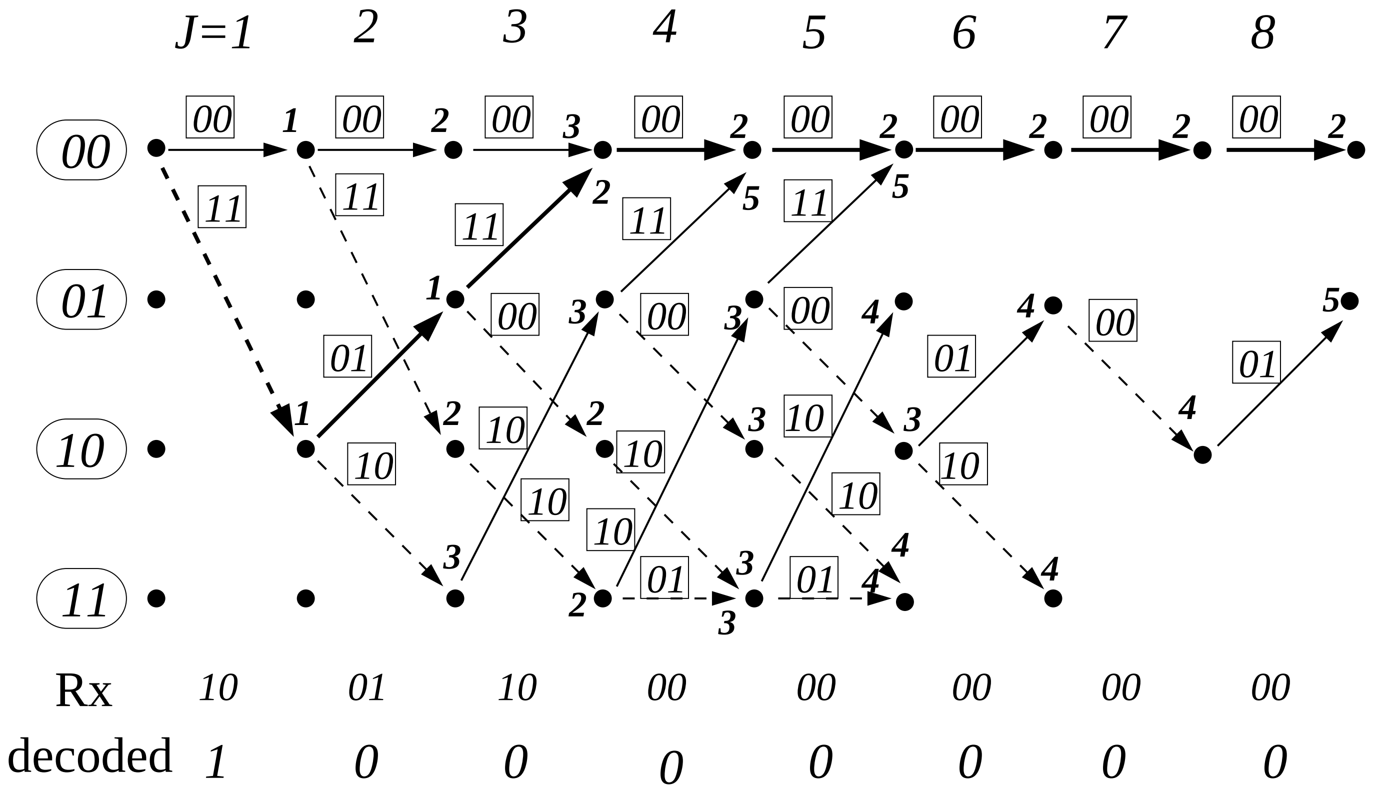
- $CC(2,1,2)$ with $g_0(x) = 1 + x^2$ and $g_1 = 1 + x + x^2$, all zero sequence is transmitted and received sequence is $10000010000000\cdots$ (the leftmost bit at leftmost position of trellis)



- Since this example has 4 states, from stage 3 onward, there are 4 survivor paths (only survivor paths are kept but random decisions were not made in the diagram)

Another Example of Hard-Input Hard-Output VA

- Example: $CC(2,1,2)$ with $g_0(x) = 1 + x^2$ and $g_1 = 1 + x + x^2$, all zero sequence is transmitted but received sequence is $10011000000000 \cdots$ (the leftmost bit at leftmost position of trellis). Note a burst error of two bits in demodulated sequence



- This is an erroneous decoding. If decoding decision is incorrect, the winning path metric is not the number of transmission errors caused by the channel

Soft-Input Hard-Output Viterbi Algorithm

- **Soft-input** decoding is for soft-decision demodulation where demodulator does not make hard binary decision but gives confidence measure concerning probability of a binary bit being 1 or 0

  As the decoder has more information, it does better than hard-input decoding

- **Confidence measure** is a real number such as $+1.5$ or -0.3, etc. In the following example, for simplicity, we use $\pm 4, \pm 3, \pm 2, \pm 1$ 8-level (or 3-bit) confidence measure with interpretation:

  $+4$: extremely like to be 1, $+3$: strongly like to be 1, $+2$: like to be 1, $+1$: weakly like to be 1
  -4: extremely like to be 0, -3: strongly like to be 0, -2: like to be 0, -1: weakly like to be 0

- More generally, input to a soft-input decoder is a sequence of **log likelihood ratios**, but here we will use a sequence of confidence measures and one can interpret confidence measure as likelihood ratio

  Hard-output decoder outputs hard decoded bits, but **soft-output decoder** outputs sequence of log likelihood ratios or confidence measures and it is for iterative decoding

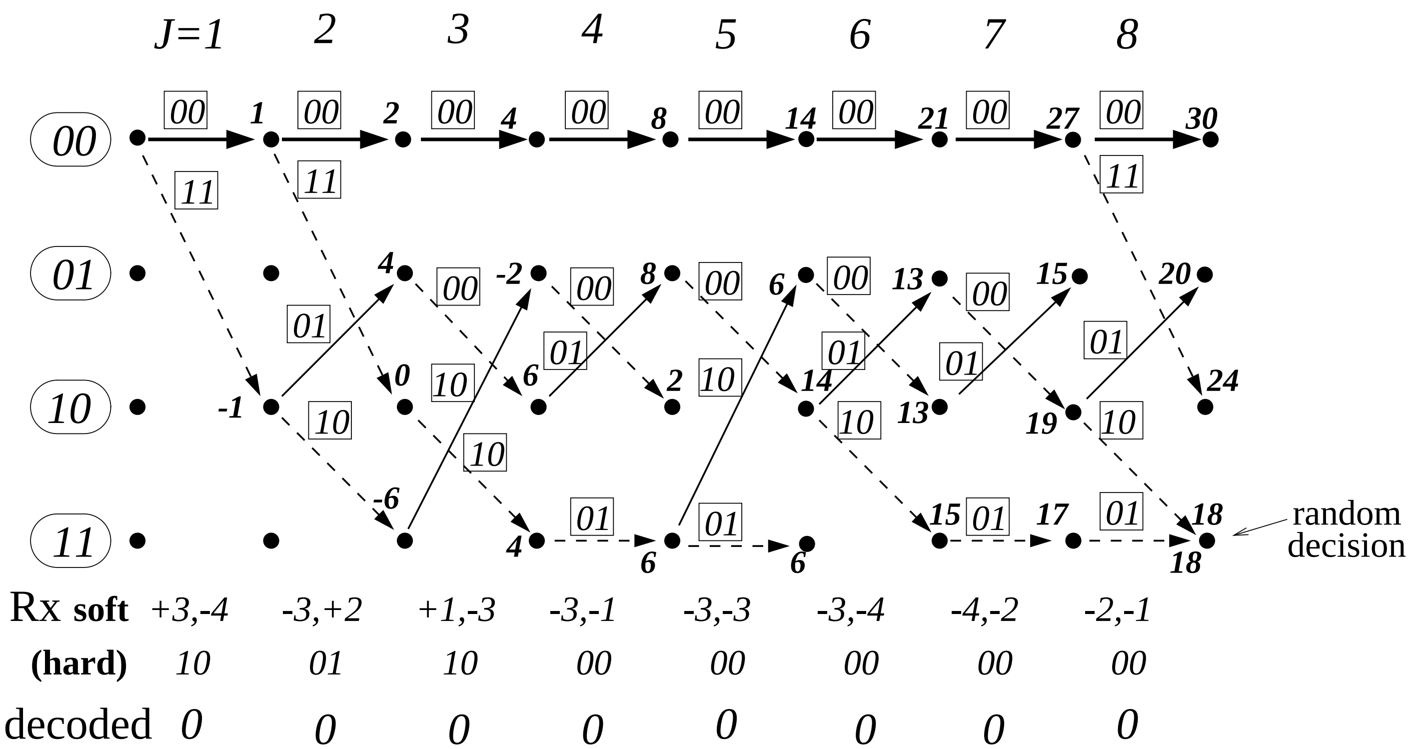
- Example: $CC(2,1,2)$ with $g_0(x) = 1 + x^2$ and $g_1 = 1 + x + x^2$

  An all zero sequence is transmitted and the received soft decision sequence is $+3, -4, -3, +2, +1, -3, -3, -1, -3, -3, -3, -4, -4, -2, -2, -1, \cdots$ (the hard decisions would be $10011000000000 \cdots$ with the leftmost bit at leftmost position of trellis)

SIHO Viterbi Algorithm (continue)

- Trellis of SIHO VA decoding:

  Previously, HIHO VA gave erroneous decoding but SIHO VA produces correct decoding



- In soft-decision decoding, the meaning of metric has changed

  If the trellis branch output bits are 01 and the receive soft decisions are +3,+1, it has a penalty -3 for the 1st bit and a credit $+1$ for the 2nd bit, so that the branch metric is $(-3) + (+1) = -2$

  If the trellis branch output bits are 00 and the receive soft decisions are $+3$, -4, it has a penalty -3 for the 1st bit and a credit $+4$ for the 2nd bit, so that the branch metric is $(-3)+(+4) = +1$

- The winning path is the survivor path with the largest path metric

Summary

- Convolutional code $CC(n, k, N)$: code rate $R = k/n$, constraint length N (memory length $N + 1$), encoder states, state transitions, generator polynomials
- $CC(n, k = 1, N)$ encoding: encoder circuit, table of state transitions and output bits, state-transition diagram, trellis diagram
- $CC(n, k = 1, N)$ decoding: maximum likelihood sequence decoding, trellis diagram based Viterbi decoding, hard-input and hard-output decoding, soft-input and hard-output decoding