Revision of Lecture Two

- **Power lost** (Budget):

  Propagation path loss

  Slow (large-scale) fading

  Fast (small-scale) fading

- Two killer factors in mobile medium:
  - **Doppler spread**: time-varying nature of channel causes frequency dispersion

    Physical dimension/quantity – Doppler frequency f_D

  - **Multipath**: which causes time dispersion

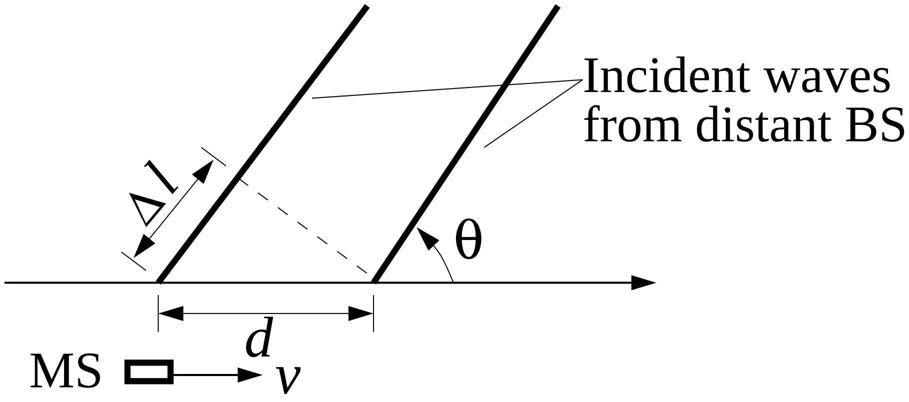
    Physical dimension/quantity – excess delay τ

- We will have indepth look into these two phenomena

Doppler Frequency

- Consider MS moving at speed v:

  Moving "changes" frequency $\rightarrow$ Doppler shift



- Difference in path lengths from BS to MS is $\Delta l = d\cos\theta = v\Delta t\cos\theta$

- Let λ be wavelength, then phase change in received signal due to difference in path lengths is:

$$\Delta\phi = \frac{2\pi\Delta l}{\lambda} = \frac{2\pi v\Delta t\cos\theta}{\lambda}$$

- Doppler frequency is defined as rate of phase change due to moving:

$$f_D = \frac{1}{2\pi}\frac{\Delta\phi}{\Delta t} = \frac{v}{\lambda}\cos\theta = f_m\cos\theta$$

  where f_m is the maximum Doppler frequency (unit: Hz)

Doppler Spectrum

- The arrival angle θ can be viewed as uniformly distributed

$$\mathrm{PDF}(\theta) = \frac{1}{2\pi}, \quad 0 \le \theta \le 2\pi$$

  The Doppler frequency $f_D = f_m \cos\theta$ is then cosine distributed

- Received power in $d\theta$ around θ is proportional to $\frac{|d\theta|}{2\pi}$ (Use of absolute operation is due to fact that power is always nonnegative). Using

$$\frac{d\cos^{-1} x}{dx} = -\frac{1}{\sqrt{1-x^2}}, \quad -1 \le x \le 1$$

- **Doppler power spectrum density**: (absolute operation because power ≥ 0)

$$S(f_D) \propto \frac{1}{2\pi}\left|\frac{d\theta}{df_D}\right| = \frac{1}{2\pi}\left|\frac{d(\cos^{-1}(f_D/f_m))}{df_D}\right| \quad \text{or} \quad S(f_D) = \frac{C}{\sqrt{1-(f_D/f_m)^2}}$$

- Implications: frequency dispersion
  - Single frequency f_c broadened to a spectrum $(f_c - f_m, f_c + f_m)$
  - Signal with bandwidth $2B$ centre at f_c broadened to a bandwidth approximately $2B + 2f_m$

Doppler Spread

- **Doppler spread** B_D is defined as the "bandwidth" of Doppler spectrum. It is a measure of spectral broadening caused by the time varying nature of the channel

- **Coherence time** $T_C \propto \frac{1}{B_D}$ is used to characterise the time varying nature of the frequency dispersion of the channel in time domain

- **Fading** effects due to Doppler spread: determined by mobile speed and signal bandwidth. Let baseband signal bandwidth be B_S and symbol period T_S, then
  - "Slow fading" channel: $T_S \ll T_C$ or $B_S \gg B_D$, signal bandwidth is much greater than Doppler spread, and effects of Doppler spread are negligible
  - "Fast fading" channel: $T_S > T_C$ or $B_S < B_D$, channel changes rapidly in during one symbol period T_S

Do not confuse with slow (large-scale) and fast (small-scale) fadings in propagation pathloss model

Here slow and fast fading are used to describe relationship between **time rate of change** in the **channel** and the transmitted **signal**

Normalised Doppler Frequency

- Velocity of mobile and signal bandwidth determine whether a signal undergoes fast or slow fading
- Fading rate describes the relationship between rate of change in channel and rate of change in signal
  - Rate of change in channel is specified by velocity of mobile v and carrier frequency f_c, as characterised in the (maximum) Doppler frequency

$$f_m = \frac{v}{\lambda} = \frac{v \cdot f_c}{c}, \qquad \lambda \text{ being wavelength}, \; c \text{ being speed of light}$$

  - As signal bandwidth is much smaller than f_c, Doppler spread is approximately f_m
  - Rate of change in signal is specified by symbol rate or symbol period T_s
- Often **normalised Doppler frequency** is used to specify fading rate

$$\bar{f}_m = f_m \cdot T_s$$

  - $\bar{f}_m = 10^{-6}$ is considered very slow fading, $\bar{f}_m = 10^{-4}$ quite fast
  - Example: Carrier frequency of 1 GHz $\longrightarrow$ wavelength $\lambda = c/f_c = 3 \cdot 10^8/10^9 = 0.3$ m

    User velocity of 10 m/s (36 km/h) and $\lambda = 0.3$ m $\longrightarrow$ Doppler frequency $f_m = v/\lambda \approx 33$ Hz

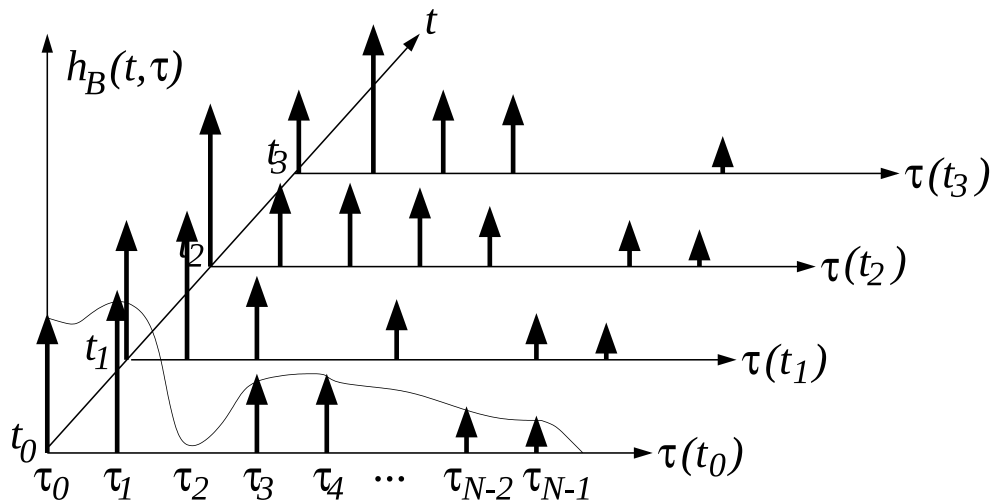
    At symbol rate of 3.3 Msymbols/s, the normalised Doppler frequency becomes $\bar{f}_m = f_m \cdot T_s = 33/(3.3 \cdot 10^6) = 10^{-5}$

Impulse Response of Multipath Channels

- Multipath causes time dispersion, as described by bandpass CIR $h(t,\tau)$
  - As channel can be time-varying, time t is needed, and τ is multipath delay
  - Generally, $h(t,\tau)$ is a function of two inputs t and τ
- Let equivalent baseband **complex-envelope** channel impulse response be $h_B(t,\tau)$

$$h_B(t,\tau) = \sum_{i=0}^{N-1} a_i(t,\tau)\exp(-j\theta_i(t,\tau))\delta(\tau-\tau_i(t))$$

- Useful to discrete τ into delay bins, each bin represents a multipath component
- $a_i(t,\tau)$, $\theta_i(t,\tau)$ and $\tau_i(t)$ are amplitude, phase shift and excess delay of ith multipath component, respectively



Channel Impulse Response (continue)

- Interpretation of $h_B(t, \tau)$: there are N multipaths, i.e. there are N copies of transmitted signal arriving at the receiver

- At time t, each copy arrives at the receiver with a different amplitude $a_i(t, \tau)$, goes through a different phase shift $\theta_i(t, \tau)$ and has a different excess delay $\tau_i(t)$

- Excess delay τ is function of t, amplitude is function of t and τ, phase shift is function of t and τ, and they are stochastic processes

- A special case is the time invariant channel, where

$$h_B(\tau) = \sum_{i=0}^{N-1} a_i \exp(-j\theta_i)\delta(\tau - \tau_i)$$

  θ_i: uniformly distributed, a_i: Rayleigh distributed, τ_i: Poisson distributed

- **Multipath** causes **time dispersion** and results in **intersymbol interference**

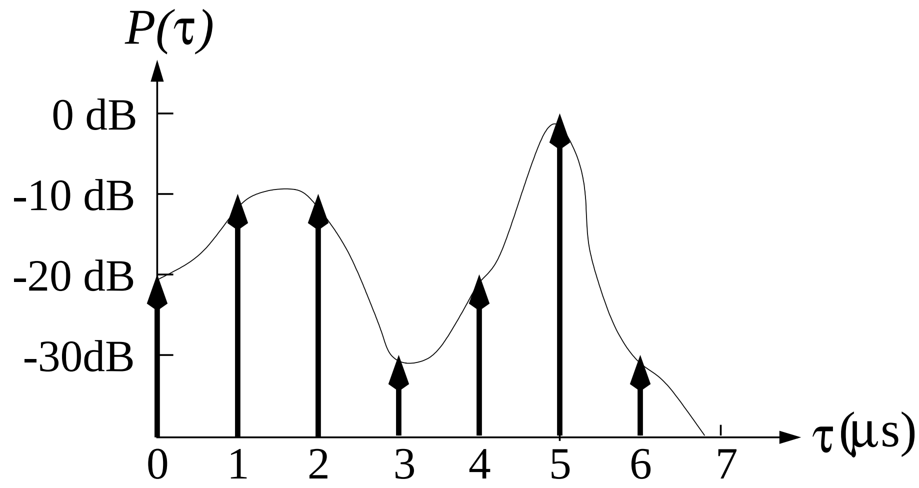
Power Delay Profile

- **Power delay profile** $P(\tau)$: the channel power spectral density as a function of delay, i.e. how "channel power" is distributed along **dimension excess delay** τ

- Consider a local area around a spatial position, averaging $|h_B(t,\tau)|^2$ over time gives rise to $P(\tau)$

- Specifically, $P(\tau)$ is Fourier transform of autocorrelation function of $h_B(t,\tau)$

- Again, it is useful to discrete τ into bins



- **Mean excess delay** is defined as the first moment of power delay profile

$$\bar{\tau} = \frac{\sum_i P(\tau_i)\tau_i}{\sum_i P(\tau_i)}$$

Power delay profile or power spectral density has "properties" of probability density function, so one can talks about moments of the underlying "stochastic process"

Power Delay Profile (continue)

- **Root mean square** (RMS) **delay spread** is defined as the square root of the second central moment of power delay profile:

$$\sigma_\tau = \sqrt{\bar{\tau^2} - (\bar{\tau})^2}$$

where the second moment is given by

$$\bar{\tau^2} = \frac{\sum_i P(\tau_i)\tau_i^2}{\sum_i P(\tau_i)}$$

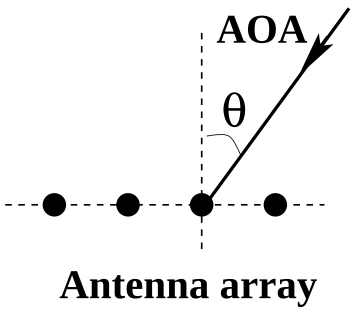
- **Coherence bandwidth** is a measure of the range of frequencies over which the channel is "flat" (i.e. passing spectral components with approximately equal gain and linear phase)

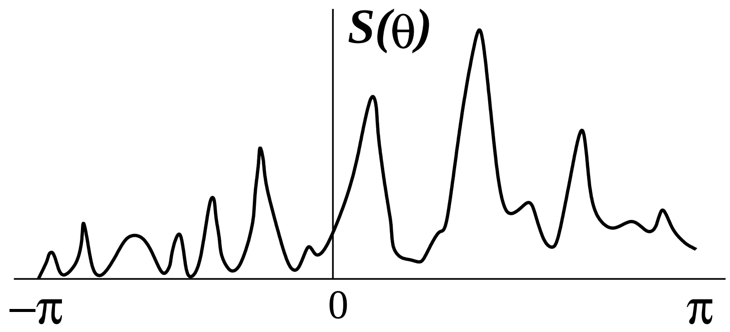
- 50% coherence bandwidth is defined as:

$$B_C \approx \frac{1}{5\sigma_\tau}$$

Angle Power Spectrum

- **Angle power spectrum** defines average power as a function of θ (angle-of-arrival for receive antenna and angle-of-departure for transmit antenna)





- Similar to delay power profile, one can define Mean angle $\bar{\theta}$ and RMS angle spread σ_θ

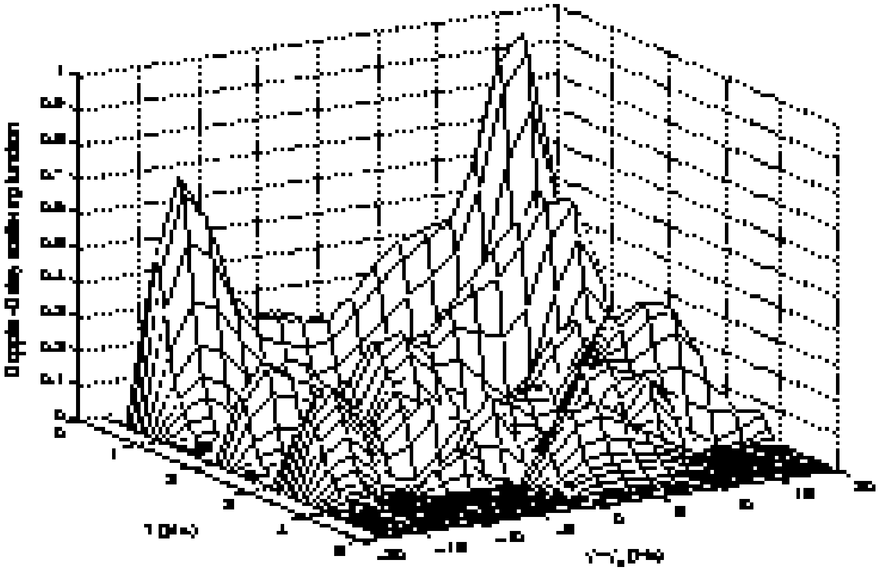
- **Angle spread** causes **space selective fading** $\rightarrow$ signal amplitude depends on spatial location of antenna/signals

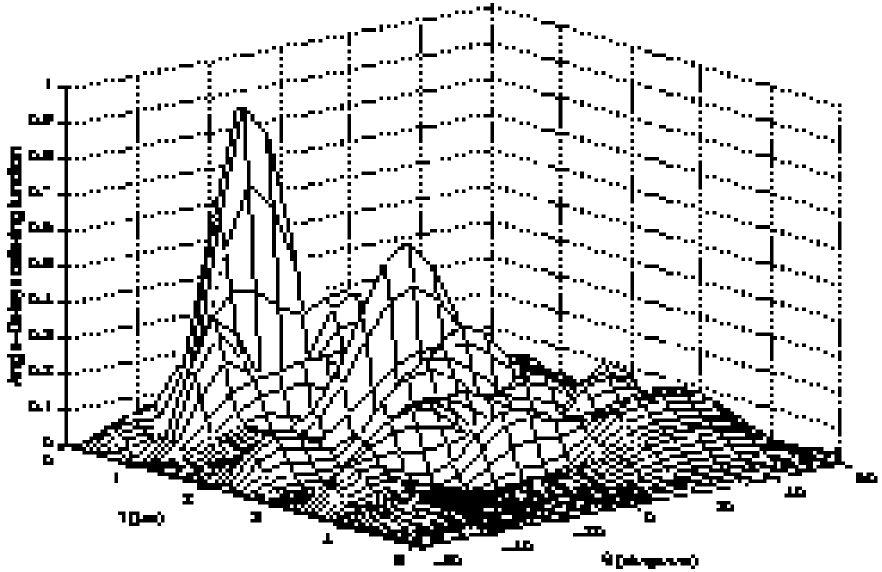
- **Coherence distance** D_C is spatial separation for which autocorrelation coefficient of spatial fading drops to 0.7

$$D_c \propto \frac{1}{\sigma_\theta}$$

Scattering Functions

- Doppler-delay and angle-delay **scattering functions** $S(f_D, \tau)$ and $S(\theta, \tau)$





- Complete channel statistics are captured in a triple scattering function: **Doppler-angle-delay scattering function** $S(f_D, \theta, \tau)$

- Doppler, delay and angle power spectra $S(f_D)$, $P(\tau)$ and $S(\theta)$ are **marginal spectra** related to scattering function $S(f_D, \theta, \tau)$

Summary

- Mobile channels are hostile due to:
  - Doppler spread which causes frequency dispersion
  - Multipath which causes time dispersion
- Doppler spectrum (note speed broadens signal spectrum):
  - Doppler PSD, Doppler spread, coherence time
  - What are **slow** and **fast** fading channels
  - Normalised Doppler frequency
- Multipath: channel impulse response and power delay profile
  - mean excess delay, RMS delay spread, coherence bandwidth
- Doppler-angle-delay scattering function, and marginal spectra
  - angle power spectrum, RMS angle spread, coherence distance