ELEC6014 RCNSs: Additional Topic Notes

Single-Carrier Block Transmission With Frequency-Domain Equalisation

Professor Sheng Chen

School of Electronics and Computer Science

University of Southampton

Southampton SO17 1BJ, UK

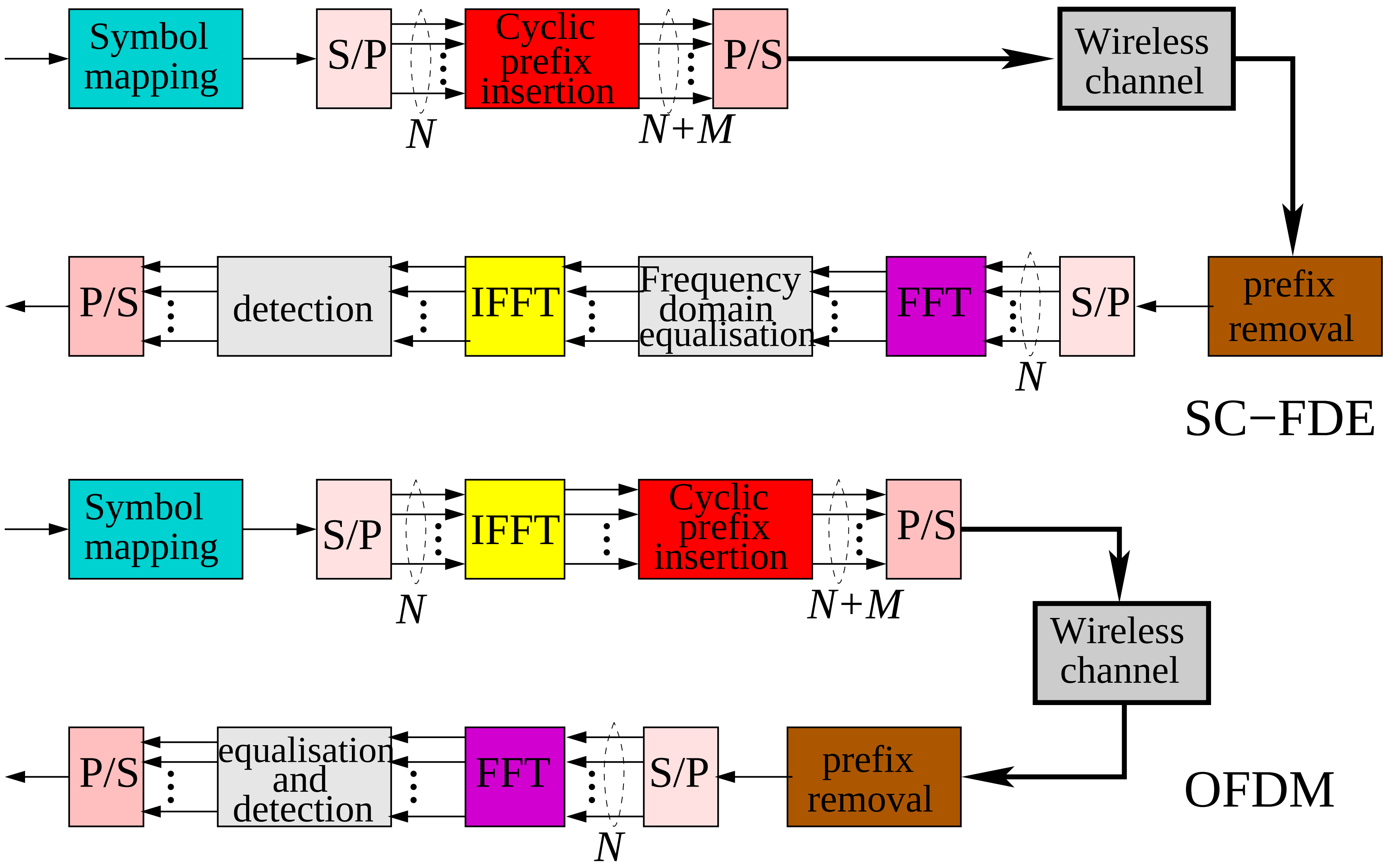
E-mail: sqc@ecs.soton.ac.uk

http://www.ecs.soton.ac.uk/∼sqc/EZ412-612/

Motivations

- For 4G and B4G **high-speed broadband applications**, data rate of tens Mbps over wireless channel of typical delay spread in microseconds
  - ISI spanning tens or even hundreds of symbols
  - Nightmare for time-domain equalisation: impractically long equaliser, excessively slow convergence ⇒ poor performance
- **Orthogonal frequency division multiplexing**, a multi-carrier technique, offers a viable low-complexity high-performance solution for ISI mitigation
  - High peak-to-average power ratio, intolerance to amplifier nonlinearity, and high sensitivity to carrier frequency offsets
- Alternative solution for ISI mitigation is **single-carrier** modulation with **frequency-domain equalisation**
  - Similar low-complexity and performance, but avoiding OFDM's drawbacks
  - Not as flexible as OFDM in managing bandwidth and energy resources

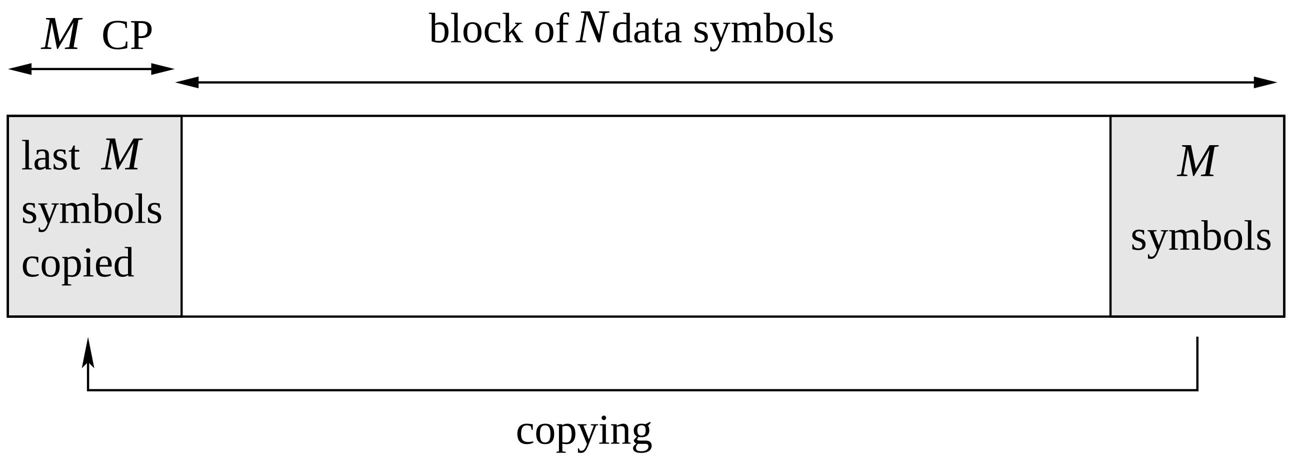
Quick Comparison



Both transceivers have similar implementation complexity, but SC-FDE transmitter is simpler and hence better for uplink handset

Block Transmission

- Data symbols $\{s_n\}$ are transmitted in **blocks** of N symbols with **cyclic prefix** of length M $\Rightarrow$ block length of $N + M$
  - M is chosen to be larger than channel impulse response length
- Cyclic prefix insertion



  - Last M symbols may be **training** symbols, e.g. known PN sequence
- Spot "**difference**"
  - SC-FDE: data symbols are "**time-domain**" quantity and transmitted directly
  - OFDM: data symbols are "**frequency-domain**" quantity (and are IDFT into "time-domain" for transmission)

OFDM

- **Cyclic prefix** at beginning of each block has two main functions
  - Prevent contamination of a block by ISI from previous block, by simply dropping first M time-domain samples of received block of length $N + M$
  - Make received block **periodic with period** N, essential for DFT to lead single-tap equalisation in frequency domain
- Received block has **cyclicity property**

$$r_m = \sum_{k=0}^{N_h - 1} h_k s_{m-(k \mod N)}, \ 0 \le m \le N - 1$$

where $N_h \le M$ is length of CIR, and h_k, $0 \le k \le N_h - 1$, CIR taps

- Process received block $\{r_m\}_{m=0}^{N-1}$ by DFT:

$$R_l = \sum_{m=0}^{N-1} r_m e^{-j\frac{2\pi l m}{N}} = H_l \cdot S_l + V_l, \ 0 \le l \le N - 1$$

CFR $\{H_l\}_{l=0}^{N-1}$ is N-point DFT of CIR $\{h_k\}_{k=0}^{N_h-1}$, V_l is noise term

- One-tap equalisation

$$Y_l = W_l \cdot R_l, \ 0 \le l \le N - 1$$

$\{Y_l\}_{l=0}^{N-1}$ provides sufficient statistics for **detection** of transmitted data symbols $\{S_l\}_{l=0}^{N-1}$

Cyclic Prefix and Cyclicity

- Transmitted time-domain OFDM signal: $\mathbf{s}_{N+M} = \left[s_{N-1}\ s_{N-2} \cdots s_{N-M} \cdots s_1\ s_0 | s_{-1}\ s_{-2} \cdots s_{-M} \right]^{\mathrm{T}}$
  - N data symbols $\mathbf{s}_N = \left[s_{N-1} \cdots s_1\ s_0 \right]^{\mathrm{T}}$, and
  - M-length **cyclic prefix** $\left[s_{-1}\ s_{-2} \cdots s_{-M} \right]^{\mathrm{T}} = \left[s_{N-1}\ s_{N-2} \cdots s_{N-M} \right]^{T}$
  - $M \geq$ CIR length, and for convenience, let CIR be: $\left[h_0\ h_1 \cdots h_M \right]^{\mathrm{T}}$
- Received block of length $N+M$: $\mathbf{r}_{N+M} = \left[r_{N-1} \cdots r_1\ r_0 | r_{-1}\ r_{-2} \cdots r_{-M} \right]^{\mathrm{T}}$
  - Dropping $\left[r_{-1}\ r_{-2} \cdots r_{-M} \right]^{\mathrm{T}}$ removes ISI from **previous block**
  - N-length received block $\mathbf{r}_N = \left[r_{N-1} \cdots r_1\ r_0 \right]^{\mathrm{T}}$ has **cyclicity property**
- **Linear convolution** $\mathbf{r}_N = \mathbf{H}_{\mathrm{L}} \mathbf{s}_{N+M},\ \mathbf{H}_{\mathrm{L}} : N \times (N+M)$

$$
\begin{aligned}
r_{N-1} &= h_0 s_{N-1} + h_1 s_{N-2} + \cdots + h_M s_{N-M+1} \\
&\vdots \\
r_M &= h_0 s_M + h_1 s_{M-1} + \cdots + h_M s_0 \\
r_{M-1} &= h_0 s_{M-1} + h_1 s_{M-2} + \cdots + h_{M-1} s_0 + s_M s_{-1} \\
&\vdots \\
r_1 &= h_0 s_1 + h_1 s_0 + h_2 s_{-1} + \cdots + h_M s_{-M+1} \\
r_0 &= h_0 s_0 + s_1 s_{-1} + \cdots + h_M s_{-M}
\end{aligned}
$$

Cyclicity (continue)

$$\begin{bmatrix} r_{N-1} \\ \vdots \\ r_M \\ r_{M-1} \\ \vdots \\ r_1 \\ r_0 \end{bmatrix} = \begin{bmatrix} h_0 & h_1 & \cdots & h_{M-1} & h_M & & & & & & \\ & \ddots & \ddots & \cdots & \ddots & \ddots & & & & & \\ & & h_0 & h_1 & \cdots & h_{M-1} & h_M & & & & \\ & & & h_0 & h_1 & \cdots & h_{M-1} & h_M & & & \\ & & & & \ddots & \ddots & \cdots & \ddots & \ddots & & \\ & & & & & h_0 & h_1 & \cdots & h_{M-1} & h_M & \\ & & & & & & h_0 & h_1 & \cdots & h_{M-1} & h_M \end{bmatrix} \begin{bmatrix} s_{N-1} \\ s_{N-2} \\ \vdots \\ s_{N-M} \\ \vdots \\ s_0 \\ s_{-1} \\ s_{-2} \\ \vdots \\ s_{-M} \end{bmatrix}$$

- **Circular convolution** $\mathbf{r}_N = \mathbf{H}_{\text{C}} \mathbf{s}_N, \ \mathbf{H}_{\text{C}} : N \times N$

$$\begin{bmatrix} r_{N-1} \\ \vdots \\ r_M \\ r_{M-1} \\ \vdots \\ r_1 \\ r_0 \end{bmatrix} = \begin{bmatrix} h_0 & h_1 & \cdots & h_{M-1} & h_M & & \\ & \ddots & \ddots & \cdots & \ddots & \ddots & \\ & & h_0 & h_1 & \cdots & h_{M-1} & h_M \\ h_M & & & h_0 & h_1 & \cdots & h_{M-1} \\ \vdots & \ddots & & & \ddots & \ddots & \vdots \\ h_2 & \cdots & h_M & & & h_0 & h_1 \\ h_1 & \cdots & h_{M-1} & h_M & & & h_0 \end{bmatrix} \begin{bmatrix} s_{N-1} \\ s_{N-2} \\ \vdots \\ s_0 \end{bmatrix}$$

Block Processing in SC-FDE

- **Cyclic prefix** at beginning of each block has two main functions
  - Prevent contamination of a block by intersymbol interference from previous block, by simply dropping first M samples of received block of length $N+M$
  - Make received block **periodic with period** N, essential for DFT to lead single-tap equalisation in frequency domain

- Received block has **cyclicity property**

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- Process received block $\{r_m\}_{m=0}^{N-1}$ by DFT:

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CFR $\{H_l\}_{l=0}^{N-1}$ is N-point DFT of CIR $\{h_k\}_{k=0}^{N_h-1}$, V_l is noise term

One-Tap Equalisation

- In **frequency-domain**, equalisation can be achieved by **one-tap** linear equaliser

$$Y_l = W_l \cdot R_l, \quad 0 \le l \le N-1$$

  – **Zero-forcing**:

$$W_l = \frac{H_l^*}{|H_l|^2}, \quad 0 \le l \le N-1$$

  – **Minimum mean square error**:

$$W_l = \frac{H_l^*}{|H_l|^2 + \frac{\sigma_v^2}{\sigma_s^2}}, \quad 0 \le l \le N-1$$

  where σ_v^2 is noise power and $\sigma_s^2 = E[|s_n|^2]$ signal power

  – SC-FDE with decision feedback equaliser

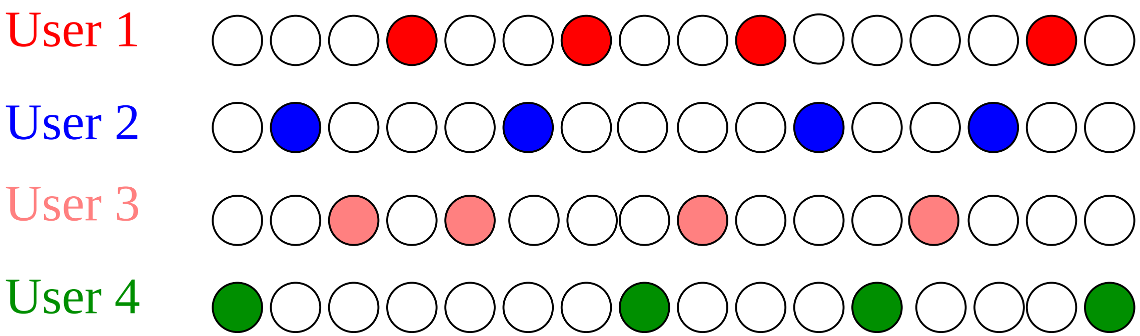
- $\{y_n\}_{n=0}^{N-1}$, IDFT of $\{Y_l\}_{l=0}^{N-1}$

$$y_n = \frac{1}{N}\sum_{l=0}^{N-1} Y_l e^{j\frac{2\pi ln}{N}}, \; 0 \le n \le N-1$$

  provides sufficient statistics for **detection** of transmitted data symbols $\{s_n\}_{n=0}^{N-1}$

OFDMA / SC-FDMA

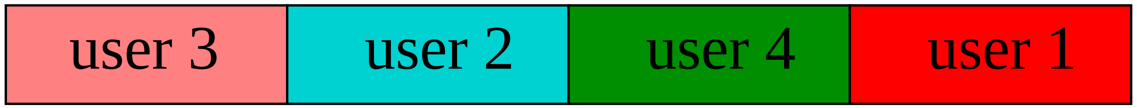
- Multi-carrier system to support multi users: orthogonal frequency division multiple access **OFDMA**
  - N subcarriers to support K users
  - Carrier assignment scheme **CAS** assigns N/K subcarriers to each user



4 users 16 subcarriers

  - Each user assigns its data symbols to subcarriers it occupies, and assigns zero to unoccupied subcarriers

- Single-carrier system to support multi users: frequency division multiple access **SC-FDMA**



Total bandwidth

- For MC systems, such as OFDMA, flexible for resource allocation, such as **power allocation** to subcarriers

References

1. D. Falconer, S.L. Ariyavisitakul, A. Benyamin-Seeyar and B. Eidson, "Frequency domain equalization for single carrier broadband wireless systems," *IEEE Communications Magazine*, vol.40, no.4, pp.58–66, April 2002

2. F. Pancaldi, G.M. Vitetta, R. Kalbasi, N. Al-Dhahir, M. Uysal and H. Mheidat, "Single-carrier frequency domain equalization," *IEEE Signal Processing Magazine*, vol.25, no.5, pp.37–56, Sept. 2008

3. N. Benvenuto and S. Tomasin, "Iterative design and detection of a DFE in the frequency domain," *IEEE Trans. Communications*, vol.53, no.11, pp.1867–1875, Nov. 2005

4. C. Zhang, Z. Wang, C. Pan, S. Chen and L. Hanzo, "Low-complexity iterative frequency domain decision feedback equalization," submitted to *IEEE Trans. Vehicular Technology*, 2010