ASIA-PACIFIC JOURNAL OF CHEMICAL ENGINEERING
Asia-Pac. J. Chem. Eng. 2008; **3**: 688–694
Published online in Wiley InterScience
(www.interscience.wiley.com) DOI:10.1002/apj.199

Curtin University of Technology

Special Theme Research Article

Single-input and single-output (SISO) controller reduction based on the L_1-norm

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Received 25 April 2007; Revised 20 July 2008; Accepted 23 July 2008

ABSTRACT: This paper proposes a new method to solve the controller-reduction problem based on the L_1-norm. This method uses a reduced-order closed-loop system to deduce reduced-order controllers. The problem of obtaining the required lower-order closed-loop system was formulated as an L_1-norm optimization, and the conditions were provided for guaranteeing the internal stability and the existence of lower-order controllers from the obtained reduced-order closed-loop system. In addition, the particle swarm optimization and sequence linear programming were adopted to solve the resultant L_1-norm optimization. Two numerical examples demonstrated the effectiveness of the proposed method. © 2008 Curtin University of Technology and John Wiley & Sons, Ltd.

KEYWORDS: L_1-norm; controller reduction; particle swarm optimization; sequence linear programming

INTRODUCTION

The problem of L_1 control was first formulated by Vidyasagar,[1] which aims to minimize the maximum of the peak–peak gain of a closed-loop system driven by the bounded disturbance. The complete solutions to general one-block L_1 problems are given by Dahleh and Pearson[2] for discrete-time systems by using Youla parameterization and convex optimization. Although L_1 controllers can be used in the areas of disturbance rejection or tracking of bounded input, they have certain weaknesses that both the denominator and numerator of the controller are irrational, which means infinite dimensions. Yoshito and Kodama[3] found a way to obtain the rational controller to approximate the optimal controller. However, the resultant controller had a very high order, which cannot be realized in practice. There exist two approaches to solve this problem. The first approach designs the low-order suboptimal controller directly. Nagpal *et al*.[4] provided a method, which optimizes the upper bound of the introduced L_1-norm by the linear matrix inequality (LMI) and obtained a controller mostly with the same order of plant. The second approach designs a high-order controller first, and then reduces the controller order to obtain a low-order one. Caponetto *et al*.[5] proposed a controller-reduction method based on genetic algorithms. Gugercin *et al*.[6] introduced a Krylov-based controller-reduction technique for large-scale systems. In this paper, we focus on the second approach.

In fact, controller reduction is a significant topic that is discussed frequently. In Zhou *et al*.,[7] the H_∞ controller reduction based on the weighted balanced truncation method, which directly truncates the controller's state-space matrix, is discussed. Although the stability of the closed-loop system is maintained, this method does not guarantee the minimization of the error between the new closed-loop system and the original one. Furthermore, the H_∞ - and L_1-norms are defined on different norm spaces. Therefore, techniques developed for the H_∞ controller reduction cannot be applied to the L_1-norm reduction. Sarkar *et al*.[8] developed a controller-reduction method based on the delta domain, but yet this cannot be used for the L_1 controller. In this paper, we focus on the SISO controller because although multiple-input and multiple-output (MIMO) controllers are more useful and well developed,[9] many of them can actually be decoupled and treated as multi-SISO controllers.[10] The method proposed by us first obtained an approximate and stable closed-loop system from which the corresponding reduced-order controller was derived. This led to an L_1 -norm minimization problem with constraints. Because the L_1 -norm is the integral of absolute value, techniques based on the differential theory cannot be used directly to solve the resultant L_1 -norm optimization problem. In this paper,

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we have presented and compared two methods to optimize the L_1 -norm. The first one is an intellectual search called *particle swarm optimization* (PSO).[11] In fact, intellectual search techniques have been used in controller design[12] and reduction,[5] which have shown their power. The other method is based on sequence linear programming (SLP),[13,14] which is a conventional optimization approach. Solving the L_1 -norm optimization problem based on the PSO or SLP can result in a lower-order closed-loop system. Next, we derived the required lower-order controller from this system. However, not all the lower-order closed-loop systems can deduce a low-order controller by using this method. The main contribution of this paper is to propose a new framework to guarantee the existence of the low-order controllers.

This paper is organized as follows: Section 2 provides the mathematical preliminaries for the computation of the L_1 -norm optimization. Section 3 describes the conditions for guaranteeing the existence of lower-order controllers, and it contains the main result of the paper. In Section 4, we discuss the two optimization methods for solving the L_1 -norm optimization. In Section 5, the complete procedure of the proposed controller reduction based on the L_1 -norm is summarized. Two numerical examples are shown in Section 6, and the paper concludes in Section 7.

MATHEMATICAL PRELIMINARIES

In this section, some mathematical preliminaries about the L_1 -norm are given.

Definition 1: $L_1(R_+)$ is defined as the set of the Lebesgue intergratable functions on R_+

$$\|F(s)\|_1 = \int_0^{\infty} |f(t)|\mathrm{d}t \qquad (1)$$

In order to solve the L_1 control problem, we should consider the computation of the L_1 -norm first. Since the L_1 -norm is an integral from zero to infinite, it is difficult to get the precise value. Many approximate methods were introduced to calculate it. In this paper, we have used the method given in Ref. [15] By this approach, the error of the L_1 -norm computation can be made arbitrarily small.

For the transfer function

$$F(s) = \frac{b_n s^n + b_{n-1}s^{n-1} + \cdots + b_1 s + b_0}{s^n + a_{n-1}s^{n-1} + \cdots + a_1 s + a_0}$$
$$b_0, b_1, \ldots, b_n \in R, a_0, a_1, \ldots, a_{n-1} \in R \qquad (2)$$

This can be transformed to the partial fraction expansion form

$$F(s) = \sum_{j=1}^{q}\sum_{k=1}^{m_j} \frac{r_{jk}}{(s-p_j)^k} + b_n \qquad (3)$$

where $p_j = \alpha_j + \mathrm{i}\beta_j$, $r_{jk} = |r_{jk}|e^{\mathrm{i}\theta_{jk}}$;

p_j is the pole of the function, q is the number of the different poles, and m_j is the number of multiple pole p_j. The computation of the L_1 -norm is shown below.

Lemma 1: The L_1 norm of the $F(s)$ is approximated to

$$\|F(s)\|_1 = \int_0^N \left| \sum_{j=1}^{q}\sum_{k=1}^{m_j} \frac{1}{(k-1)!}|r_{jk}|t^{k-1}e^{\alpha_j t}\cos(\theta_{jk} + \beta_j t) \right| \mathrm{d}t + |b_n| \qquad (4)$$

where N is the number that ensures only a small amount of errors.

The calculation of the value N and the proof of this lemma can be found in Ref. [15].

MAIN RESULTS

The problem

Denote $G(s)$ as the plant, $K(s)$ the controller, and $\Phi(s)$ the closed-loop transfer function. Let $\hat{K}(s)$ be the low-order controller and $\hat{\Phi}(s)$ the new closed-loop system, which is stable. The following equation is obtained.

$$\Phi(s) = \frac{G(s)}{1+KG(s)}, \quad \hat{\Phi}(s) = \frac{G(s)}{1+\hat{K}G(s)} \qquad (5)$$

Our aim is to search for a controller such that

$$\hat{K}_{OP}(s) = \{\hat{K}(s)|\min \|\Phi(s) - \hat{\Phi}(s)\|_1\} \qquad (6)$$

Internal stability

Let x and $\hat{x}$ be the state vectors for G and K, respectively.

Definition 2: If the system is asymptotically stable at $(x, \hat{x}) = (0, 0)$, it is internally stable.

To the feedback system, the internal stability is the basic requirement which guarantees that, if the input signal is bounded, all the output signals in this system are bounded.

Let us define that n_k is the number of poles in right of poles in the right half-plane for controller $K(s)$ and n_p is the number of poles in right of poles in the right half-plane for plant $G(s)$.

Lemma 2[7]: The system is internally stable if and only if

(1) the number of the poles in the right half-plane of $G(s)K(s)$ is $n_k + n_p$ and
(2) $(I + G(s)K(s))^{-1}$ is stable.

DOI: 10.1002/apj

Simple case (relative degree is zero)

Let us consider a special system

$$G(s) = K\frac{(s-q_1)(s-q_2)\cdots(s-q_n)}{(s-p_1)(s-p_2)\cdots(s-p_n)} \tag{7}$$

where $q_1, q_2, \cdots, q_n < 0$ and $p_1, p_2, \cdots, p_n$ represent the zeros and poles of the transfer function, respectively; K is the gain and n is the degree of the denominator and the numerator. Equation (7) is a system whose relative degree is zero. Let us divide the zeros of Eqn (7) into two parts: the first part is $q_{i1}, \cdots, q_{im}$ and the other is $q_{t1}, \cdots, q_{t(n-m)}$. Then, we have the following theorem:

Theorem 1: For system (7), suppose we have the closed-loop system

$$\hat{\Phi}(s) = a\frac{(s-q_{i1})\cdots(s-q_{im})(s-q_{c1})\cdots(s-q_{cj})}{(s-p_{c1})(s-p_{c2})\cdots(s-p_{c(m+j)})} \tag{8}$$

where a is the gain of the old closed-loop system, $q_{i1}\ldots q_{im}$ are part of the zeros of system (7) $p_{c1}, p_{c2}, \ldots p_{c(m+j)} \in RH_\infty$, and j is determined by the requirement of the controller's degree. Then, we can obtain a controller $\hat{K}(s)$ whose degree is $j+n$. Furthermore, the system is internally stable.

Proof: From Eqn (5), we can obtain $\hat{K}(s)$

$$\begin{aligned}\hat{K}(s) &= \frac{1}{\Phi(s)} - \frac{1}{G(s)} \\ &= \frac{\prod_{k=1}^{m+j}(s-p_{ck})}{a\prod_{y=1}^{m}(s-q_{iy})\times\prod_{x=1}^{j}(s-q_{cx})} - \frac{\prod_{i=1}^{n}(s-p_i)}{K\prod_{i=1}^{n}(s-q_i)} \\ &= \frac{K\prod_{k=1}^{m+j}(s-p_{ck})\times\prod_{d=1}^{n-m}(s-q_{td}) - a\prod_{i=1}^{n}(s-p_i)\times\prod_{x=1}^{j}(s-q_{cx})}{a\times K\prod_{i=1}^{n}(s-q_i)\times\prod_{x=1}^{j}(s-q_{cx})}\end{aligned} \tag{9}$$

Since the degrees of the denominator and the numerator are equal, the controller is realizable and its order is $j+\ n$.

For the SISO system, we have

$$(I+GK)^{-1} = I + (I+GK)^{-1}GK = I + \Phi K \tag{10}$$

The poles of $I+\Phi K$ are $p_{c1}, p_{c2}, \ldots p_{c(m+j)}$ and $q_{t1}, \cdots, q_{t(n-m)}$, which are in the left half-plane. Accordingly, $(I+GK)^{-1}$ is stable.

Let $q_{k1}, q_{k2}, \ldots, q_{k(n+j)}$ denote the zeros of the controller $K(s)$. Because $q_i < 0$, $i = 1, 2, \ldots, n$, the poles of $K(s)$ in the right half-plane only exist in q_{cx}, $x = 1, 2, \ldots, j$. The unstable poles of $G(s)$ exist in p_i, $i = 1, 2, \ldots, n$. Because the poles and zeros of $K(s)$ are derived from the optimization method, it is easy to find the suitable value

$$q_{cx} \neq q_i, \quad x = 1, 2, \ldots, j, i = 1, 2, \ldots,\ n$$

and

$$q_{ks} \neq p_i, \quad s = 1, 2, \ldots, n+j, i = 1, 2, \ldots,\ n$$

Then, the system satisfies Lemma 2 and it is internally stable.

The reason we did not change the gain of the closed-loop system is that, from Eqn (4), the system whose relative degree is zero would have an extra item in the L_1 -norm computation. To minimize this value, the relative degree of the error transform function should not be zero. Apparently, when these two systems with zero relative degrees have the same gain, their error functions meet the above requirement.

Usually, the controller designed by the method given in Ref. [3] has a degree much larger than that of the plant. The above theorem, however, shows that we can obtain lower-order controllers.

General case (relative degree is not zero)

In the special case, we assumed that the relative degree of the system is zero. However, in reality, many systems have nonzero relative degrees. The generic transfer function of the plant can be expressed as

DOI: 10.1002/apj

$$G(s) = K\frac{(s-q_1)\cdots(s-q_{n-r})}{(s-p_1)(s-p_2)\cdots(s-p_n)} \tag{11}$$

where $q_1, q_2, \ldots, q_{n-r} < 0$ and $p_1, p_2, \ldots, p_n$ represent the zeros and poles of the transfer function, respectively. K is the gain and the relative degree $r > 0$.

Theorem 2: For the system (11), assume that the closed-loop system is

$$\hat{\Phi}(s) = K\frac{(s-q_{i1})\cdots(s-q_{im})(s-q_{c1})\cdots(s-q_{cj})}{(s-p_{c1})(s-p_{c2})\cdots(s-p_{cl})} \tag{12}$$

where $q_{i1}\ldots q_{im}$ are the first part of the zeros of system (11), as similarly defined for system (7), and $p_{c1}, p_{c2}, \ldots p_{cl} \in RH_\infty$, which keeps the stability of the closed-loop system. Moreover, we have $l = r + m + j$. That is, the relative degree of Eqn (12) must be equal to that of Eqn (11). Furthermore, for notational convenience, if we use $k_1, k_2 \ldots k_{n+j}$ to represent $q_{t1}\ldots q_{t(n-r-m)}, p_{c1}\ldots p_{cl}$, and use $x_1, x_2 \ldots x_{n+j}$ to denote $q_{c1}, q_{c2}\ldots q_{cj}, p_1, \ldots p_n$, the following equations are obtained:

$$\begin{cases} \sum_{i=1}^{n+j} k_i = \sum_{i=1}^{n+j} x_i \\ \sum_{i1\neq i2} k_{i1}k_{i2} = \sum_{i1\neq i2} x_{i1}x_{i2} \\ \vdots \\ \sum_{i1\neq i2\neq\cdots i(r-1)} k_{i1}k_{i2}\cdots k_{i(r-1)} \\ = \sum_{i1\neq i2\neq\cdots i(r-1)} x_{i1}x_{i2}\cdots x_{i(r-1)} \end{cases} \tag{13}$$

Then, the lower-degree controller $\hat{K}(s)$ can be obtained, whose order is $n - r + j$.

Proof: Similar to the proof of Theorem 1, we have

$$\hat{K}(s) = \frac{1}{\Phi(s)} - \frac{1}{G(s)} = \frac{\prod_{k=1}^{l}(s-p_{ck}) \times \prod_{d=1}^{n-r-m}(s-q_{td}) - \prod_{i=1}^{n}(s-p_i) \times \prod_{x=1}^{j}(s-q_{cx})}{K\prod_{i=1}^{n-r}(s-q_i) \times \prod_{x=1}^{j}(s-q_{cx})} \tag{14}$$

The order of the denominator is $n - r + j$. If this controller can be realized, its denominator must have an order higher than or equal to its numerator's order. Accordingly, all the coefficients of the items of the numerator, whose orders are higher than $n - r + j$, must be zero. We arrive at Eqn (13).

The proof of the internal stability is the same to that of Theorem 1.

In practice, the gain of $\hat{\Phi}(s)$ is not always equal to the old closed-loop system. For convenience, we consider the gain of the plant instead. Thus, if a closed-loop system $\hat{\Phi}(s)$, which satisfies Theorem 2 can be found, the lower-order controller $\hat{K}(s)$ can be obtained. In the next section, we use PSO or SLP to find a closed-loop system that minimizes the L_1 -norm error and satisfies Theorem 2.

OPTIMIZATION

Using particle swarm optimization (PSO) to solve the L_1 -norm minimization problem

PSO is a guided random search optimization algorithm, which was introduced by Kennedy and Eberhart.[11] Let $X_i = (X_{i1}, X_{i2}, X_{i3}\ldots, X_{in})$ represent the current position of particle i, $V_i = (V_{i1}, V_{i2}, V_{i3}\ldots V_{in})$ represent the velocity and $P_i = (P_{i1}, P_{i2}, P_{i3}\ldots P_{in})$ represent the past optimal position of each particle. Furthermore, denote $P_g = (P_{g1}, P_{g2}, P_{g3}\ldots P_{gn})$ as the global optimal position. The following equations can be used to renew the current position.

$$V_i(t+1) = C_0V_i(t) + C_1r_1[X_i(t) - P_i(t)] + C_2r_2[X_i(t) - P_g(t)] \tag{15}$$

$$X_i(t+1) = X_i(t) + V_i(t) \tag{16}$$

where C_1 and C_2 are constants in the range of [0, 2], r_1 and r_2 are the random numbers in the range of [0,1], and C_0 is the weight that keeps the momentum of the particles. As X converges to the optimal position, C_0 should be made smaller. We consider the poles and unknown zeros in Eqn (12) as the vector of the position. Then, we must cycle this process to renew the position of the particles until we can find a vector that minimizes the L_1 -norm error as well as meets the constraints of Eqn (13). If the vector of the position does not meet the constraints of Eqn (13), it is discarded.

The main merit of PSO is that it can be realized easily. But its convergence usually cannot be guaranteed. The algorithmic parameters affect the result dramatically, and it is difficult to get the feasible vectors of the position when there are many constraints.

Using sequence linear programming (SLP) to solve the L_1 -norm minimization problem

The L_1 -norm of the transform function usually have the following form:

$$\int_0^\infty |g(t)|\mathrm{d}t = \int_0^\infty |\sum_{i=1}^{n} r_i e^{-p_i t}|\mathrm{d}t \tag{17}$$

where p_i are the poles of the transfer function and r_i is the numerator corresponding to p_i in the partial fraction expansion form.

This is a nonlinear function with absolute value and integral. To solve its optimization problem, we have the following methods:

DOI: 10.1002/apj

Step 1: Discretize (17), then we can obtain

$$\int_0^{\infty} |g(t,p,r)| \approx \sum_{k=1}^{N} |g(t_k,p,r)| = \sum_{k=1}^{N} |\sum_{i=1}^{n} r_i e^{-p_i t_k}| \quad (18)$$

where t_k is the sample point of the function and N is same as that in Lemma 1. Because the function has the term e^{-pt}, it is believed that N is not big. The size of t_k is determined by N and the sample frequency.

Step 2: The above optimization problem is equal to the following optimization problem:

$$Min \quad \sum_{k=1}^{N}(m_k + n_k)$$
$$s.t. \begin{cases} g(t_k,p) = m_k - n_k \\ (8) \end{cases} \quad (19)$$

The proof of this step is shown in.[14] In this step, the absolute value is reduced successfully at the cost of increasing $2N$ variables in this optimization problem.

Step 3: Define $\delta > 0$ so that it is small enough to meet our requirement. Choose the start values as p_0 and r_0, then make the Taylor's expansion to the constraints.

$$g(t_k,p,r) \approx gl(t_k,p_0,r_0) = g(t_k,p_0,r_0) + \nabla g(t_k,p_0,r_0)^T(\Delta p) + \nabla g(t_k,p_0,r_0)^T(\Delta r) \quad (20)$$

In order to keep the accuracy, Δp should be limited in a small range $|\Delta p| \le s$. Then, the problem changes into a linear programming problem. But Taylor's expansion can only promise the small error that occurred near the expansion point. Hence, we should iterate this step frequently until the error meets our demand. At the same time, we could adjust the bound of Δp to keep the approximation to the real value. Let P_i and r_i be the solution of step 3. It is obvious that

$$prek_i = gl(t,p_o,r_0) - gl(t,p_i,r_i) \ge 0 \quad (21)$$

We can also obtain the real value

$$arek_i = g(t,p_o,r_0) - g(t,p_i,r_i) \quad (22)$$

If $arek \ge 0$, the ratio can be computed as follows:

$$ratio_i = \frac{arek_i}{prek_i} \quad (23)$$

and the change of the boundary determined by it.

The detailed steps are shown as follows:

(1) Compute the Taylor's expansion in the position (p_0, r_0).
(2) Use linear programming to find the optimal value.
(3) If $arek < 0$, we deny the new position; let $s = s/2$, and then return to step 1. If $\delta > arek \ge 0$, stop this procedure, else let $p_0 = p_i$, $r_0 = r_i$ and go to the next step.
(4) Compare $arek$ and $prek$ by computing the *ratio*.
If $ratio \ge 0.75$, let $s = s \times 2$
If $0.75 > ratio > 0.25$, s does not change.
If $ratio \le 0.25$, let $s = s/2$
(5) Return to the step 1.

This method is faster than PSO. Its convergence has been proved by Zhang *et al.*[14] Compared with PSO, its efficiency is not affected by constraints. But our problem is nonconvex; it may converge to the local optimal value.

For details and proof of this method, please see Ref. [13,14].

SOLUTION STEPS

In this section, the summary of our solution steps is given below:

Step 1: Design $\hat{\Phi}(s)$ such that it satisfies the form of Theorem 1 or 2, and determine m,j according to the requirements of the order.
Step 2: Use PSO or SLP, choose the poles and zeros of $\hat{\Phi}(s)$ and minimize the error.
Step 3:

$$\hat{K}(s) = \frac{1}{\Phi(s)} - \frac{1}{G(s)} \quad (24)$$

NUMERICAL EXAMPLE

Example 1

Let us consider a system[3]

$$G(s) = \frac{s+1}{s-2} \quad (25)$$

There is an L_1 optimal controller

$$K(s) = -1.3186 \frac{s^5 + 24.1s^4 + 226.6s^3 + 1002s^2 + 1753s + 2439}{s^5 + 24.2s^4 + 227.8s^3 + 1000s^2 + 1596s + 2106}. \quad (26)$$

The gain of the closed-loop system is −3.1387. Apparently, this controller's degree is much higher than the degree of the plant; and it should be reduced. The system's relative degree is zero. We could use Theorem 1 to obtain a three-degree controller.

DOI: 10.1002/apj

Assume that

$$\hat{\Phi}(s) = -3.1387\frac{(s+1)(s-q_{c1})(s-q_{c2})}{(s-p_1)(s-p_2)(s-p_3)} \quad (27)$$

Then, by using PSO, 10 particles and recycling 30 times, the result is

$$q_{c1} = -0.3038\ q_{c2} = -2.3519\ p_1 = -6.6247$$
$$p_2 = -5.3176\ p_3 = -0.2984\ \|\Phi(s) - \hat{\Phi}(s)\|_1 = 0.2216 \quad (28)$$

According to the Theorem 1, the three-degree controller is

$$\hat{K}(s) = -1.3186\frac{(s+0.2937)(s^2+3.161s+4.958)}{(s+1)(s+2.352)(s+0.3038)} \quad (29)$$

Example 2

Let us consider a new example and try to optimize the L_1 -norm by two methods.

$$G(s) = \frac{1}{(s+1)(s+2)} \quad (30)$$

Its relative degree is two and the controller is

$$K(s) = \frac{1.2(s-1.2)(s^2-4s+13.61)}{(s+3.2)(s+4.25)(s+1.8)(s^2+4s+32.09)} \quad (31)$$

To reduce it to a one-degree controller, we should assume that

$$\hat{\Phi}(s) = \frac{(s-q_{c1})}{(s-p_{c1})(s-p_{c2})(s-p_{c3})} \quad (32)$$

and the bound is

$$q_{c1} - 1 - 2 = p_{c1} + p_{c2} + p_{c3} \quad (33)$$

By using PSO first, 10 particles and recycling 30 times, the result is

$$p_{c1} = -1.66 + 0.0142i \quad p_{c2} = -1.66 - 0.0142i$$
$$p_{c3} = -0.8157 \quad \mathrm{q}_{c1} = -1.1357$$

The new controller is

$$\hat{K}(s) = \frac{0.056826(s-0.4134)}{(s+1.136)} \quad (34)$$

$$\|\Phi(s) - \hat{\Phi}(s)\|_1 = 0.0022 \quad (35)$$

Then, we also use SLP. We assumed that in the sample, the period is 1 and the initial poles are −5, −8, and −7. There are 17 zeros and the initial value of s is 1. We can calculate parameters by SLP:

$$p_{c1} = -0.6783 \quad p_{c2} = -1.9361 \quad p_{c3} = -1.139$$
$$q_{c1} = -0.7534$$

The new controller is

$$\frac{0.030815(s-0.3581)}{(s+0.7534)}$$
$$\|\Phi(s) - \hat{\Phi}(s)\|_1 = 0.0029 \quad (36)$$

Comparing the errors of these two methods, it is indicated that the result of SLP is approximated to PSO; however, PSO consumes much more time than does SLP. Thus, SLP is a better optimization method.

CONCLUSION

A new approach has been proposed to solve the L_1 -norm-based controller-reduction problem. On the basis of the idea of obtaining the controller through the closed-loop system, a computational framework has been developed to reduce the order of the controller. Conditions for guaranteeing the existence of reduced-order controllers have been established, and two efficient optimization algorithms, namely the PSO and SLP, have been adopted to solve the resulting L_1 -norm optimization. The major advantage of the proposed method is its simplicity, and the resultant reduced-order controller is not confined by the original controller. Two numerical examples have demonstrated the efficiency of this method.

Acknowledgements

This work is supported by the National Natural Science Foundation of China (Grants No. 60774003, No. 60736201 and No. 60721062), the 973 program of China (Grant No. 2009 CB320603), the 863 program of China (Grant No. 2008 AA042602), the 111 project of China (Grant No. B07031), and the Qian Jiang expert program of Hangzhou.

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DOI: 10.1002/apj