Robust $\mathcal{H}_\infty$ Control for Model-Based Networked Control Systems with Uncertainties and Packet Dropouts

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***Abstract*— A general class of model-based networked control systems is investigated where the plant has time-varying norm-bounded parameter uncertainties and both the sensor-to-controller and controller-to-actuator channels experience random packet dropouts. Sufficient conditions for guaranteeing the robust stochastic stability and synthesis of stabilisation controller as well as the design of $\mathcal{H}_\infty$ controller are derived in the form of linear matrix inequalities. An illustrative example is provided to demonstrate the effectiveness of our proposed design approach.**

Keywords – Model-based networked control systems, packet dropouts, robust stability, $\mathcal{H}_\infty$ control, norm-bounded uncertainty

I. Introduction

A networked control system (NCS) [1]–[3] is a control system in which a control loop is closed via a shared communication network. Compared to the conventional point-to-point system connection, the use of an NCS has advantages of low installation cost, reducing system wiring, simple system diagnosis and easy maintenance. However, some inherent shortcomings of NCSs, such as packet dropouts, packet delays and bandwidth constraints, will degrade performance of NCSs or even cause instability. Packet dropouts, which can randomly occur due to node failures or network congestion, is one of the most important issues in NCSs. Stochastic approaches are typically adopted to deal with packet dropouts in the literature, and these approaches attempt to establish the stability in terms of mean square stability [4], [5]. Under such a stochastic approach, the packet dropout process is usually modeled as an independently and identically distributed Bernoulli process [6], [7] or a Markov chain [8]–[10], and the system is considered as a special case of the jump linear system. In some works [10], [11], the NCSs with arbitrary packet dropouts are modelled as switched systems.

When the system has parameter uncertainties, the standard $\mathcal{H}_\infty$ control [12] cannot provide guaranteed $\mathcal{H}_\infty$ performance and stability. Robust $\mathcal{H}_\infty$ control has been investigated for both continuous-time and discrete-time systems [8], [13]–[15]. All these references only consider the systems with delays, such as state or network packet delays. To the best of our knowledge, robust $\mathcal{H}_\infty$ control has not been studied for NCSs with packet dropouts. Most of the works in NCS research utilise fixed controllers. Some exceptions are model-based NCSs (MB-NCSs) [16], [17], which utilise more flexible controllers. For the MB-NCSs considered in [16], [17], only the sensor and the controller is separated by the network, and the underlying idea of MB-NCSs is as follows. When packet dropouts occur in the sensor-to-controller (S/C) channel, a nominal plant model is employed in the controller to estimate the plant behaviour, which replaces the real plant behaviour information in computing the control signal, while when the controller can access sensor data, the controller performs the same feedback control as standard closed-loop control system without network. All the plant parameters are assumed to be known in the works [16], [17] but this assumption is not met in most control practice. It is significant to remove this strict assumption on the plant and to study MB-NCSs with robustness considerations. It is also important to consider more general MB-NCSs, where not only the network is located between the sensor and the controller but also the controller and the actuator is separated by the network.

The novel contribution of this paper is that we study robust stochastic stability and synthesis of robust stabilisation control as well as design of robust $\mathcal{H}_\infty$ control for the generic MB-NCS where the plant has time-varying norm-bounded parameter uncertainties and packet dropouts occur in both the S/C and controller-to-actuator (C/A) channels. We formulate this class of NCSs as the Markovian jump linear system by using a Markov process to model packet dropouts occurring randomly in both the S/C and C/A channels. Sufficient conditions are derived for ensuring robust stochastic stability and for synthesising robust stabilisation controller as well as for designing robust $\mathcal{H}_\infty$ controller. These conditions are formulated in terms of linear matrix inequalities (LMIs) that can be solved by the existing numerical techniques [18].

The remainder of this contribution is organised as follows. The NCS problem is formulated in Section II while Section III addresses the robust stochastic stability and synthesis of robust stabilisation control. Section IV considers the robust $\mathcal{H}_\infty$ control design and present the LMI solution for the control law that stabilises the uncertain Markovian jump linear system with a prescribed disturbance attenuation level. A numerical example is provided in Section V to illustrate the proposed method, and our conclusion is given in Section VI.

Throughout this contribution we adopt the following notational conventions. $\mathbb{R}$ stands for real numbers and $\mathbb{N}$ for nonnegative integers. $\mathbf{W} > 0$ indicates that $\mathbf{W}$ is a positive-definite matrix, while $\mathbf{I}$ and $\mathbf{0}$ represent the identity and zero matrices of appropriate dimensions, respectively. The notation $*$ within a matrix denotes symmetric entries. For a discrete-time signal $\mathbf{w} = \{\mathbf{w}(k)\}_{k\in\mathbb{N}}$ with $\mathbf{w}(k) \in \mathbb{R}^p$, ℓ_2^p denotes the set of $\mathbf{w}$s with $\sum_{k=0}^{\infty} \mathbf{w}^{\mathsf{T}}(k)\mathbf{w}(k) < \infty$. $\mathsf{E}[\cdot]$ denotes the expectation.

II. Problem formulation

The NCS $\hat{P}_K$ of Fig. 1 contains a generalised discrete-time plant $\hat{P}$ and a discrete-time controller $\hat{K}$ with the control loop closed via a shared communication network. The plant $\hat{P}$ with parameter uncertainties is described by

$$\left\{ \begin{array}{l} \mathbf{x}(k+1) = [\mathbf{A} + \Delta\mathbf{A}(k)]\mathbf{x}(k) \\ \qquad +[\mathbf{B} + \Delta\mathbf{B}(k)]\mathbf{u}(k) + \mathbf{B}_w\mathbf{w}(k), \quad \forall k \in \mathbb{N} \\ \mathbf{z}(k) = \mathbf{C}\mathbf{x}(k) + \mathbf{D}\mathbf{u}(k), \end{array} \right. \tag{1}$$

where $\mathbf{x}(k) \in \mathbb{R}^n$, $\mathbf{u}(k) \in \mathbb{R}^m$ and $\mathbf{z}(k) \in \mathbb{R}^q$ are the state, input and controlled output vectors, respectively, $\mathbf{w}(k) \in \mathbb{R}^p$ is the disturbance input and $\mathbf{w} \in \ell_2^p$. $\mathbf{A}$, $\mathbf{B}$, $\mathbf{B}_w$, $\mathbf{C}$ and $\mathbf{D}$ are the known constant matrices of appropriate dimensions, while $\Delta\mathbf{A}(k)$ and $\Delta\mathbf{B}(k)$ are the unknown matrices representing the time-varying parameter uncertainties which satisfy the following condition

$$[\Delta\mathbf{A}(k)\ \Delta\mathbf{B}(k)] = \mathbf{M}\ \mathbf{F}(k)\ [\mathbf{N}_a\ \mathbf{N}_b]. \tag{2}$$

Here $\mathbf{M}$, $\mathbf{N}_a$ and $\mathbf{N}_b$ are the known constant matrices of appropriate dimensions and $\mathbf{F}(k)$ is an unknown time-varying matrix function with

$$\mathbf{F}^{\mathsf{T}}(k)\mathbf{F}(k) \leq \mathbf{I}. \tag{3}$$

Network packet dropouts occur in both the S/C and C/A channels. Define $\theta_{k+1}^s \in \{0,1\}$ and $\theta_k^a \in \{0,1\}$ as the indicators of the packet dropouts in the S/C and C/A channels, respectively, where a value 0 indicates that the packet is

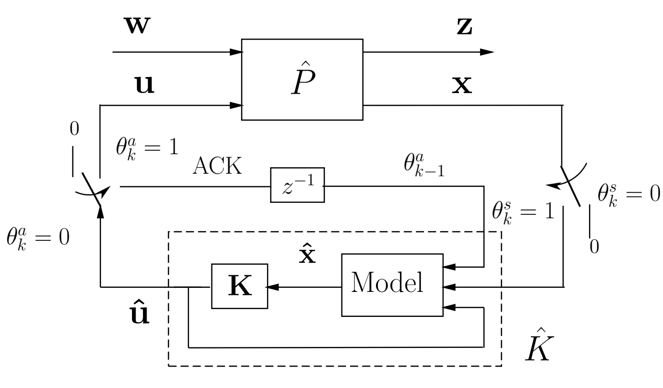

Fig. 1. Networked control system $\hat{P}_K$.

dropped while a value 1 indicates that the packet is transmitted successfully. Define the set

$$\mathcal{S} \triangleq \{(\theta_{k+1}^s, \theta_k^a) :\ \theta_{k+1}^s \in \{0,1\}, \theta_k^a \in \{0,1\}\}. \tag{4}$$

Further define $r_k \in \mathcal{N} \triangleq \{1,2,3,4\}$ and the one to one mapping $f : \mathcal{S} \to \mathcal{N}$

$$r_k = f(\theta_{k+1}^s, \theta_k^a) = \left\{ \begin{array}{ll} 1, & (\theta_{k+1}^s, \theta_k^a) = (0,0), \\ 2, & (\theta_{k+1}^s, \theta_k^a) = (1,0), \\ 3, & (\theta_{k+1}^s, \theta_k^a) = (0,1), \\ 4, & (\theta_{k+1}^s, \theta_k^a) = (1,1). \end{array} \right. \tag{5}$$

Assumption 1: r_k is driven by a discrete-time Markov chain and takes values in $\mathcal{N}$ with the transition probability matrix $\mathbf{\Upsilon} \triangleq [p_{ij}] \in \mathbb{R}^{4\times4}$, where $p_{ij} = \mathrm{Prob}(r_{k+1} = j | r_k = i)$ with $p_{ij} \geq 0$ and $\sum_{j=1}^{4} p_{ij} = 1$, $\forall i, j \in \mathcal{N}$.

Assumption 2: TCP-like protocol is assumed, in which there is acknowledgement for a received packet. Thus at each instant k the network sends an ACK signal to the controller to indicate whether the current control input is received or not by the actuator.

The controller $\hat{K}$ consists of the state feedback gain matrices $\mathbf{K}_{r_k} \in \mathbb{R}^{m\times n}$ and the plant model. The controller output is given by

$$\hat{\mathbf{u}}(k) = \mathbf{K}_{r_k}\hat{\mathbf{x}}(k),\ r_k \in \mathcal{N}, \tag{6}$$

where $\hat{\mathbf{x}}(t) \in \mathbb{R}^n$ denotes the model state. Referring to Fig. 1, if the packet is transmitted successfully in the C/A channel, $\mathbf{u}(k) = \hat{\mathbf{u}}(k)$. If the packet is lost, the actuator does nothing, i.e. $\mathbf{u}(k) = \mathbf{0}$. Thus we have

$$\mathbf{u}(k) = \theta_k^a \hat{\mathbf{u}}(k). \tag{7}$$

The plant model is given by

$$\hat{\mathbf{x}}(k+1) = \mathbf{A}\hat{\mathbf{x}}(k) + \mathbf{B}\mathbf{u}(k). \tag{8}$$

If the sensor data $\mathbf{x}(k)$ is transmitted successfully in the S/C channel, the controller $\hat{K}$ uses $\mathbf{x}(k)$ to update the model state $\hat{\mathbf{x}}(k)$ as $\hat{\mathbf{x}}(k) = \mathbf{x}(k)$. If the data is lost, $\hat{K}$ uses the plant model (8) to derive $\hat{\mathbf{x}}(k)$. Thus

$$\begin{array}{rcl} \hat{\mathbf{x}}(k+1) & = & \theta_{k+1}^s \mathbf{x}(k+1) \\ & & +(1-\theta_{k+1}^s)(\mathbf{A}\hat{\mathbf{x}}(k) + \mathbf{B}\mathbf{u}(k)) \\ & = & \left\{ \begin{array}{ll} \mathbf{x}(k+1), & \theta_{k+1}^s = 1, \\ \mathbf{A}\hat{\mathbf{x}}(k) + \mathbf{B}\mathbf{u}(k), & \theta_{k+1}^s = 0. \end{array} \right. \end{array} \tag{9}$$

Define $\mathbf{e}(k) = \mathbf{x}(k) - \hat{\mathbf{x}}(k)$ and $\overline{\mathbf{x}}(k) \triangleq [\mathbf{x}^{\mathsf{T}}(k)\ \mathbf{e}^{\mathsf{T}}(k)]^{\mathsf{T}}$. From (1), (6) and (9), the NCS $\hat{P}_K$ can be described by

$$\left[\begin{array}{c} \overline{\mathbf{x}}(k+1) \\ \mathbf{z}(k) \end{array} \right] = \left[\begin{array}{cc} \overline{\mathbf{A}}_{r_k}(k) & \overline{\mathbf{B}}_{r_k} \\ \overline{\mathbf{C}}_{r_k} & \mathbf{0} \end{array} \right] \left[\begin{array}{c} \overline{\mathbf{x}}(k) \\ \mathbf{w}(k) \end{array} \right], r_k \in \mathcal{N} \tag{10}$$

where

$$\begin{array}{rcl} \overline{\mathbf{A}}_{r_k}(k) & = & \left[\begin{array}{c} \mathbf{A} + \Delta\mathbf{A}(k) + \theta_k^a(\mathbf{B} + \Delta\mathbf{B}(k))\mathbf{K}_{r_k} \\ (1-\theta_{k+1}^s)(\Delta\mathbf{A}(k) + \theta_k^a \Delta\mathbf{B}(k)\mathbf{K}_{r_k}) \end{array} \right. \\ & & \left. \begin{array}{c} -\theta_k^a(\mathbf{B} + \Delta\mathbf{B}(k))\mathbf{K}_{r_k} \\ (1-\theta_{k+1}^s)(\mathbf{A} - \theta_k^a \Delta\mathbf{B}(k)\mathbf{K}_{r_k}) \end{array} \right], \end{array} \tag{11}$$

$$\overline{\mathbf{B}}_{r_k} = \left[\begin{array}{c} \mathbf{B}_w \\ (1-\theta_{k+1}^s)\mathbf{B}_w \end{array} \right], \tag{12}$$

$$\overline{\mathbf{C}}_{r_k} = \left[\begin{array}{cc} \mathbf{C} + \theta_k^a \mathbf{D}\mathbf{K}_{r_k} & -\theta_k^a \mathbf{D}\mathbf{K}_{r_k} \end{array}\right]. \quad (13)$$

From (2), (3) and (5), $\overline{\mathbf{A}}_{r_k}(k)$ can be written as $\overline{\mathbf{A}}_i(k) = \mathbf{\Phi}_i + \overline{\mathbf{M}}\ \overline{\mathbf{F}}(k)\mathbf{\Gamma}_i$, $i \in \mathcal{N}$, where

$$\mathbf{\Phi}_i = \left[\begin{array}{cc} \mathbf{A} + \theta_k^a \mathbf{B}\mathbf{K}_i & -\theta_k^a \mathbf{B}\mathbf{K}_i \\ \mathbf{0} & (1-\theta_{k+1}^s)\mathbf{A} \end{array}\right], \quad (14)$$

$$\mathbf{\Gamma}_i = \left[\begin{array}{cc} \mathbf{N}_a + \theta_k^a \mathbf{N}_b \mathbf{K}_i & -\theta_k^a \mathbf{N}_b \mathbf{K}_i \\ (1-\theta_{k+1}^s)(\mathbf{N}_a + \theta_k^a \mathbf{N}_b \mathbf{K}_i) & -(1-\theta_{k+1}^s)\theta_k^a \mathbf{N}_b \mathbf{K}_i \end{array}\right], \quad (15)$$

$$\overline{\mathbf{M}} = \mathrm{diag}\{\mathbf{M}, \mathbf{M}\},\ \overline{\mathbf{F}}(k) = \mathrm{diag}\{\mathbf{F}(k), \mathbf{F}(k)\}. \quad (16)$$

It is easy to see $\overline{\mathbf{F}}^{\mathsf{T}}(k)\overline{\mathbf{F}}(k) \leq \mathbf{I}$. Specifically, the four modes of $\mathbf{\Phi}_i$ and $\mathbf{\Gamma}_i$ for $i \in \mathcal{N}$ are

$$\mathbf{\Phi}_1 = \left[\begin{array}{cc} \mathbf{A} & \mathbf{0} \\ \mathbf{0} & \mathbf{A} \end{array}\right],\ \mathbf{\Gamma}_1 = \left[\begin{array}{cc} \mathbf{N}_a & \mathbf{0} \\ \mathbf{N}_a & \mathbf{0} \end{array}\right]; \quad (17)$$

$$\mathbf{\Phi}_2 = \left[\begin{array}{cc} \mathbf{A} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{array}\right],\ \mathbf{\Gamma}_2 = \left[\begin{array}{cc} \mathbf{N}_a & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{array}\right]; \quad (18)$$

$$\begin{array}{l} \mathbf{\Phi}_3 = \left[\begin{array}{cc} \mathbf{A} + \mathbf{B}\mathbf{K}_3 & -\mathbf{B}\mathbf{K}_3 \\ \mathbf{0} & \mathbf{A} \end{array}\right], \\ \mathbf{\Gamma}_3 = \left[\begin{array}{cc} \mathbf{N}_a + \mathbf{N}_b\mathbf{K}_3 & -\mathbf{N}_b\mathbf{K}_3 \\ \mathbf{N}_a + \mathbf{N}_b\mathbf{K}_3 & -\mathbf{N}_b\mathbf{K}_3 \end{array}\right]; \end{array} \quad (19)$$

$$\begin{array}{l} \mathbf{\Phi}_4 = \left[\begin{array}{cc} \mathbf{A} + \mathbf{B}\mathbf{K}_4 & -\mathbf{B}\mathbf{K}_4 \\ \mathbf{0} & \mathbf{0} \end{array}\right], \\ \mathbf{\Gamma}_4 = \left[\begin{array}{cc} \mathbf{N}_a + \mathbf{N}_b\mathbf{K}_4 & -\mathbf{N}_b\mathbf{K}_4 \\ \mathbf{0} & \mathbf{0} \end{array}\right]. \end{array} \quad (20)$$

$\overline{\mathbf{B}}_{r_k}$ and $\overline{\mathbf{C}}_{r_k}$ can be expressed respectively as

$$\overline{\mathbf{B}}_1 = \overline{\mathbf{B}}_3 = \left[\begin{array}{c} \mathbf{B}_w \\ \mathbf{B}_w \end{array}\right],\ \overline{\mathbf{B}}_2 = \overline{\mathbf{B}}_4 = \left[\begin{array}{c} \mathbf{B}_w \\ \mathbf{0} \end{array}\right]; \quad (21)$$

$$\begin{array}{l} \overline{\mathbf{C}}_1 = \overline{\mathbf{C}}_2 = \left[\begin{array}{cc} \mathbf{C} & \mathbf{0} \end{array}\right], \\ \overline{\mathbf{C}}_3 = \left[\begin{array}{cc} \mathbf{C} + \mathbf{D}\mathbf{K}_3 & -\mathbf{D}\mathbf{K}_3 \end{array}\right], \\ \overline{\mathbf{C}}_4 = \left[\begin{array}{cc} \mathbf{C} + \mathbf{D}\mathbf{K}_4 & -\mathbf{D}\mathbf{K}_4 \end{array}\right]. \end{array} \quad (22)$$

Definition 1: (See [8], [15]) The NCS $\hat{P}_K$ (10) with $\mathbf{w}(k) \equiv \mathbf{0}$ is said to be robustly stochastically stable if for any initial state $\overline{\mathbf{x}}(0) \in \mathbb{R}^{2n}$ and any initial mode $r_0 \in \mathcal{N}$,

$$\sum_{k=0}^{\infty} \mathsf{E}\left[\|\overline{\mathbf{x}}(k)|\overline{\mathbf{x}}(0), r_0\|^2\right] < \infty \quad (23)$$

holds for all the admissible uncertainties $\Delta\mathbf{A}(k)$ and $\Delta\mathbf{B}(k)$.

Definition 2: (See [8], [15]) The NCS $\hat{P}_K$ (10) is said to be robustly stochastically stable with disturbance attenuation level $\gamma > 0$ if $\hat{P}_K$ with $\mathbf{w}(k) \equiv 0$ is robustly stochastically stable and, for any nonzero $\mathbf{w}(k) \in \ell_2^p$, the response $\{\mathbf{z}(k)\}$ under the zero initial condition $\overline{\mathbf{x}}(0) = \mathbf{0}$ satisfies

$$\sum_{k=0}^{\infty} \mathsf{E}\Big[\mathbf{z}^{\mathsf{T}}(k)\mathbf{z}(k)\big|\overline{\mathbf{x}}(0) = \mathbf{0}, r_0\Big] < \gamma^2 \Big[\sum_{k=0}^{\infty} \mathbf{w}^{\mathsf{T}}(k)\mathbf{w}(k)\Big]. \quad (24)$$

Our objective is to establish criteria for robust stochastic stability and synthesis of robust stabilisation control as well as to design appropriate robust $\mathcal{H}_\infty$ state feedback controllers that guarantee robust stochastic stability of the NCS $\hat{P}_K$.

III. Robust Stability and Stabilisation Control

The following lemma from [19] is useful for the proofs of our main results.

Lemma 1: Let $\mathbf{S}$, $\mathbf{U}$, $\mathbf{H}$, $\mathbf{G}$ and $\tilde{\mathbf{F}}$ be the real matrices of appropriate dimensions such that $\mathbf{G} > 0$ and $\tilde{\mathbf{F}}^{\mathsf{T}}\tilde{\mathbf{F}} \leq \mathbf{I}$. Then, for any scalar $\epsilon > 0$ such that $\mathbf{G} - \epsilon\mathbf{U}\mathbf{U}^{\mathsf{T}} > 0$, we have

$$(\mathbf{S} + \mathbf{U}\tilde{\mathbf{F}}\mathbf{H})^{\mathsf{T}}\mathbf{G}^{-1}(\mathbf{S} + \mathbf{U}\tilde{\mathbf{F}}\mathbf{H}) \leq \mathbf{S}^{\mathsf{T}}(\mathbf{G} - \epsilon\mathbf{U}\mathbf{U}^{\mathsf{T}})^{-1}\mathbf{S} + \epsilon^{-1}\mathbf{H}^{\mathsf{T}}\mathbf{H}. \quad (25)$$

We present our robust stochastic stability result in the following theorem.

Theorem 1: The NCS $\hat{P}_K$ with $\mathbf{w}(k) \equiv \mathbf{0}$ and driven by the Markov chain as specified in **Assumption 1** is robustly stochastically stable if there exist scalars $\epsilon_i > 0$ and matrices $\mathbf{X}_i > 0$ for $i \in \mathcal{N}$ such that $\forall i \in \mathcal{N}$ the following LMIs are satisfied

$$\left[\begin{array}{ccc} -\mathbf{X}_i & \mathbf{X}_i\mathbf{\Phi}_i^{\mathsf{T}}\mathbf{W}_i & \mathbf{X}_i\mathbf{\Gamma}_i^{\mathsf{T}} \\ * & \epsilon_i\overline{\mathbf{W}}_i - \mathbf{X} & \mathbf{0} \\ * & * & -\epsilon_i\mathbf{I} \end{array}\right] < 0, \quad (26)$$

where $\mathbf{\Phi}_i$ and $\mathbf{\Gamma}_i$ are given in (14) and (15), while

$$\mathbf{W}_i = \left[\sqrt{p_{i1}}\ \mathbf{I}\ \sqrt{p_{i2}}\ \mathbf{I}\ \sqrt{p_{i3}}\ \mathbf{I}\ \sqrt{p_{i4}}\ \mathbf{I}\right], \quad (27)$$

$$\overline{\mathbf{W}}_i = \mathbf{W}_i^{\mathsf{T}}\overline{\mathbf{M}}\ \overline{\mathbf{M}}^{\mathsf{T}}\mathbf{W}_i, \quad (28)$$

$$\mathbf{X} = \mathrm{diag}\{\mathbf{X}_1, \mathbf{X}_2, \mathbf{X}_3, \mathbf{X}_4\}, \quad (29)$$

and $\overline{\mathbf{M}}$ is defined in (16).

Proof Define $\mathcal{F}_n \triangleq \sigma\{\overline{\mathbf{x}}(0), r_0, \cdots, \overline{\mathbf{x}}(n), r_n\}$, $\forall n \geq 0$, as the σ-algebra generated by $\{(\overline{\mathbf{x}}(k), r_k), 0 \leq k \leq n\}$. Let $\mathbf{P}_i = \mathbf{X}_i^{-1}$, $i \in \mathcal{N}$, and $\mathbf{P} = \mathbf{X}^{-1}$. Then, from (26), it is easy to show that

$$\mathbf{\Psi}_i \triangleq \mathbf{P}^{-1} - \epsilon_i\overline{\mathbf{W}}_i > 0,\ i \in \mathcal{N}. \quad (30)$$

Now for the NCS $\hat{P}_K$, construct the Lyapunov function

$$V(k, r_k) \triangleq \overline{\mathbf{x}}^{\mathsf{T}}(k)\mathbf{P}_{r_k}\overline{\mathbf{x}}(k). \quad (31)$$

Noticing $\epsilon_i > 0$, (16) and (30) as well as using **Lemma 1**, we have

$$\begin{array}{l} \mathsf{E}[V(k+1, r_{k+1})\big|\mathcal{F}_k] - V(k, r_k) \\ = \overline{\mathbf{x}}^{\mathsf{T}}(k)\Big[\overline{\mathbf{A}}_i^{\mathsf{T}}(k)\big(\sum_{j=1}^{4} p_{ij}\mathbf{P}_j\big)\overline{\mathbf{A}}_i(k) - \mathbf{P}_i\Big]\overline{\mathbf{x}}(k) \\ = \overline{\mathbf{x}}^{\mathsf{T}}(k)\Big[\overline{\mathbf{A}}_i^{\mathsf{T}}(k)\mathbf{W}_i\mathbf{P}\mathbf{W}_i^{\mathsf{T}}\overline{\mathbf{A}}_i(k) - \mathbf{P}_i\Big]\overline{\mathbf{x}}(k) \\ = \overline{\mathbf{x}}^{\mathsf{T}}(k)\Big[(\mathbf{W}_i^{\mathsf{T}}\mathbf{\Phi}_i + \mathbf{W}_i^{\mathsf{T}}\overline{\mathbf{M}}\ \overline{\mathbf{F}}(k)\mathbf{\Gamma}_i)^{\mathsf{T}}\mathbf{P} \\ \times(\mathbf{W}_i^{\mathsf{T}}\mathbf{\Phi}_i + \mathbf{W}_i^{\mathsf{T}}\overline{\mathbf{M}}\ \overline{\mathbf{F}}(k)\mathbf{\Gamma}_i) - \mathbf{P}_i\Big]\overline{\mathbf{x}}(k) \\ \leq \overline{\mathbf{x}}^{\mathsf{T}}(k)\ \hat{\mathbf{\Theta}}_i\overline{\mathbf{x}}(k), \end{array} \quad (32)$$

where

$$\hat{\mathbf{\Theta}}_i = \mathbf{\Phi}_i^{\mathsf{T}}\mathbf{W}_i\mathbf{\Psi}_i^{-1}\mathbf{W}_i^{\mathsf{T}}\mathbf{\Phi}_i + \epsilon_i^{-1}\mathbf{\Gamma}_i^{\mathsf{T}}\mathbf{\Gamma}_i - \mathbf{P}_i, \quad (33)$$

and $\mathbf{\Psi}_i$ is defined in (30). On the other hand, pre- and post-multiplying (26) by diag$(\mathbf{P}_i, \mathbf{I})$ yields

$$\begin{bmatrix} -\mathbf{P}_i & \mathbf{\Phi}_i^{\mathsf{T}}\mathbf{W}_i & \mathbf{\Gamma}_i^{\mathsf{T}} \\ * & \epsilon_i\overline{\mathbf{W}}_i - \mathbf{P}^{-1} & \mathbf{0} \\ * & * & -\epsilon_i\mathbf{I} \end{bmatrix} < 0. \tag{34}$$

By Schur complement, (34) implies that $\hat{\mathbf{\Theta}}_i < 0$. This together with (32) leads to

$$\begin{aligned} &\mathsf{E}[V(k+1, r_{k+1})\big|\mathcal{F}_k] - V(k, r_k) \\ &\leq -\lambda_{\min}(-\hat{\mathbf{\Theta}}_i)\overline{\mathbf{x}}^{\mathsf{T}}(k)\overline{\mathbf{x}}(k) \leq -\tau\ \overline{\mathbf{x}}^{\mathsf{T}}(k)\overline{\mathbf{x}}(k) \end{aligned} \tag{35}$$

where $\lambda_{\min}(-\hat{\mathbf{\Theta}}_i)$ denotes the minimal eigenvalue of $-\hat{\mathbf{\Theta}}_i$ and $\tau = \inf\{\lambda_{\min}(-\hat{\mathbf{\Theta}}_i), i \in \mathcal{N}\}$. From (35), we obtain

$$\begin{aligned} &\mathsf{E}[V(T+1, r_{T+1})\big|\mathcal{F}_T] - \mathsf{E}[V(0, r_0)] \\ &= \sum_{k=0}^{T}\Big(\mathsf{E}[V(k+1, r_{k+1})\big|\mathcal{F}_k] - V(k, r_k)\Big) \\ &\leq -\tau\sum_{k=0}^{T}\mathsf{E}[\overline{\mathbf{x}}^{\mathsf{T}}(k)\overline{\mathbf{x}}(k)] \end{aligned} \tag{36}$$

for any $T \geq 1$, which implies

$$\begin{aligned} \sum_{k=0}^{T}\mathsf{E}[\overline{\mathbf{x}}^{\mathsf{T}}(k)\overline{\mathbf{x}}(k)] &\leq \frac{1}{\tau}\big[\mathsf{E}[V(0, r_0)] - \mathsf{E}[V(T+1, r_{T+1})]\big] \\ &\leq \frac{1}{\tau}\mathsf{E}[V(0, r_0)]. \end{aligned} \tag{37}$$

Finally, from (37) we can directly obtain

$$\sum_{k=0}^{\infty}\mathsf{E}[\overline{\mathbf{x}}^{\mathsf{T}}(k)\overline{\mathbf{x}}(k)] \leq \frac{1}{\tau}\mathsf{E}[V(0, r_0)] < \infty. \tag{38}$$

According to **Definition 1**, the NCS $\hat{P}_K$ is robustly stochastically stable. ■

Using **Theorem 1**, we obtain the following theorem for synthesis of robust stabilisation control.

Theorem 2: The NCS $\hat{P}_K$ with $\mathbf{w}(k) \equiv 0$ and driven by the Markov chain as specified in **Assumption 1** is robustly stochastically stable if there exist scalars $\epsilon_i > 0$, matrices $\mathbf{Q}_i > 0$ and $\mathbf{Y}_i$ for $i \in \mathcal{N}$ such that $\forall i \in \mathcal{N}$ the following LMIs are satisfied

$$\begin{bmatrix} -\tilde{\mathbf{Q}}_i & \tilde{\mathbf{\Phi}}_i^{\mathsf{T}}\mathbf{W}_i & \tilde{\mathbf{\Gamma}}_i^{\mathsf{T}} \\ * & \epsilon_i\overline{\mathbf{W}}_i - \tilde{\mathbf{Q}} & \mathbf{0} \\ * & * & -\epsilon_i\mathbf{I} \end{bmatrix} \triangleq \mathbf{\Theta}_i < 0, \tag{39}$$

where $\mathbf{W}_i$ and $\overline{\mathbf{W}}_i$ are given in (27) and (28), respectively,

$$\tilde{\mathbf{Q}}_i = \text{diag}\{\mathbf{Q}_i, \mathbf{Q}_i\}, \tag{40}$$

$$\tilde{\mathbf{Q}} = \text{diag}\{\tilde{\mathbf{Q}}_1, \tilde{\mathbf{Q}}_2, \tilde{\mathbf{Q}}_3, \tilde{\mathbf{Q}}_4\}, \tag{41}$$

$$\tilde{\mathbf{\Phi}}_i = \begin{bmatrix} \mathbf{A}\mathbf{Q}_i + \theta_k^a\mathbf{B}\mathbf{Y}_i & -\theta_k^a\mathbf{B}\mathbf{Y}_i \\ \mathbf{0} & (1-\theta_{k+1}^s)\mathbf{A}\mathbf{Q}_i \end{bmatrix}, \tag{42}$$

$$\tilde{\mathbf{\Gamma}}_i = \begin{bmatrix} \mathbf{N}_a\mathbf{Q}_i + \theta_k^a\mathbf{N}_b\mathbf{Y}_i & -\theta_k^a\mathbf{N}_b\mathbf{Y}_i \\ (1-\theta_{k+1}^s)(\mathbf{N}_a\mathbf{Q}_i + \theta_k^a\mathbf{N}_b\mathbf{Y}_i) & -(1-\theta_{k+1}^s)\theta_k^a\mathbf{N}_b\mathbf{Y}_i \end{bmatrix}. \tag{43}$$

In this case, the state feedback gain matrices can be chosen as $\mathbf{K}_i = \mathbf{Y}_i\mathbf{Q}_i^{-1}$.

Proof Given the state feedback gain matrices $\mathbf{K}_i$, let $\mathbf{Y}_i = \mathbf{K}_i\mathbf{Q}_i$ in (42) and (43). Then (39) can be expressed in a form of (26) and the proof is as given in **Theorem 1**. ■

Remark 1: From (5), we have $\theta_k^a = 0$ for $r_k = 1, 2$. Thus $\mathbf{K}_1$ and $\mathbf{K}_2$ have no impact on $\hat{P}_K$, and they do not need to be chosen. In fact they do not appear in the problem formulation, see (17) and (18).

IV. Robust $\mathcal{H}_\infty$ Control

A sufficient condition is proposed in this section for designing robust $\mathcal{H}_\infty$ controller, and our main result is given in the following theorem.

Theorem 3: Given a scalar $\gamma > 0$, the NCS $\hat{P}_K$ driven by the Markov chain as specified in **Assumption 1** is robustly stochastically stable with disturbance attenuation level γ, if there exist scalars $\epsilon_i > 0$, matrices $\mathbf{Q}_i > 0$ and $\mathbf{Y}_i$ for $i \in \mathcal{N}$ such that $\forall i \in \mathcal{N}$ the following LMIs are satisfied

$$\begin{bmatrix} -\tilde{\mathbf{Q}}_i & \mathbf{0} & \tilde{\mathbf{\Phi}}_i^{\mathsf{T}}\mathbf{W}_i & \tilde{\mathbf{\Gamma}}_i^{\mathsf{T}} & \tilde{\mathbf{C}}_i^{\mathsf{T}} \\ * & -\gamma^2\mathbf{I} & \overline{\mathbf{B}}_i^{\mathsf{T}}\mathbf{W}_i & \mathbf{0} & \mathbf{0} \\ * & * & \epsilon_i\overline{\mathbf{W}}_i - \tilde{\mathbf{Q}} & \mathbf{0} & \mathbf{0} \\ * & * & * & -\epsilon_i\mathbf{I} & \mathbf{0} \\ * & * & * & * & -\mathbf{I} \end{bmatrix} < 0, \tag{44}$$

where

$$\tilde{\mathbf{C}}_i = \begin{bmatrix} \mathbf{C}\mathbf{Q}_i + \theta_k^a\mathbf{D}\mathbf{Y}_i & -\theta_k^a\mathbf{D}\mathbf{Y}_i \end{bmatrix}, \tag{45}$$

$\mathbf{W}_i$ and $\overline{\mathbf{W}}_i$ are defined in (27) and (28), respectively, while $\tilde{\mathbf{Q}}_i$, $\tilde{\mathbf{Q}}$, $\tilde{\mathbf{\Phi}}_i$ and $\tilde{\mathbf{\Gamma}}_i$ are given in (40) to (43).

In this case, the state feedback gain matrices are given by $\mathbf{K}_i = \mathbf{Y}_i\mathbf{Q}_i^{-1}$.

Proof From (44), we can directly obtain

$$\begin{aligned} \mathbf{\Theta}_i \ &\leq\ \mathbf{\Theta}_i + \begin{bmatrix} \tilde{\mathbf{C}}_i^{\mathsf{T}} \\ \mathbf{0} \\ \mathbf{0} \end{bmatrix}\begin{bmatrix} \tilde{\mathbf{C}}_i & \mathbf{0} & \mathbf{0} \end{bmatrix} \\ &+\frac{1}{\gamma^2}\begin{bmatrix} \mathbf{0} \\ \mathbf{W}_i^{\mathsf{T}}\overline{\mathbf{B}}_i \\ \mathbf{0} \end{bmatrix}\begin{bmatrix} \mathbf{0} & \overline{\mathbf{B}}_i^{\mathsf{T}}\mathbf{W}_i & \mathbf{0} \end{bmatrix} < 0, \end{aligned}$$

where $\mathbf{\Theta}_i$ is defined in (39). Therefore, it follows from **Theorem 2** that the NCS $\hat{P}_K$ with $\mathbf{w}(k) \equiv 0$ is robustly stochastically stable.

Next, we proceed to prove that $\hat{P}_K$ has the required noise attenuation level γ for all the $\mathbf{w}(k) \in \ell_2^p$. Let $\tilde{\mathbf{P}}_i = \tilde{\mathbf{Q}}_i^{-1}$ and $\tilde{\mathbf{P}} = \tilde{\mathbf{Q}}^{-1}$. Consider the Lyapunov function $\tilde{V}(k, r_k) = \overline{\mathbf{x}}^{\mathsf{T}}(k)\tilde{\mathbf{P}}_{r_k}\overline{\mathbf{x}}(k)$ with the zero initial condition, $\overline{\mathbf{x}}(0) = 0$ and $\tilde{V}(0, r_0) = 0$. It follows from (36) that for any $T \geq 1$

$$\begin{aligned} &\sum_{k=0}^{T}\Big(\mathsf{E}[\tilde{V}(k+1, r_{k+1})\big|\mathcal{F}_k] - \tilde{V}(k, r_k)\Big) \\ &=\ \mathsf{E}[\tilde{V}(T+1, r_{T+1})\big|\mathcal{F}_T] \geq 0. \end{aligned} \tag{46}$$

Since $\epsilon_i > 0$ for $i \in \mathcal{N}$ and

$$\tilde{\mathbf{\Psi}}_i \triangleq \tilde{\mathbf{P}}^{-1} - \epsilon_i \overline{\mathbf{W}}_i > 0, \qquad (47)$$

according to **Lemma 1**, we have

$$\begin{aligned}
&\mathsf{E}\big[\tilde{V}(k+1, r_{k+1})\big|\mathcal{F}_k\big] \\
&\quad = \big[\overline{\mathbf{x}}^{\mathsf{T}}(k)\ \mathbf{w}^{\mathsf{T}}(k)\big]\mathbf{\Xi}_i\big[\overline{\mathbf{x}}^{\mathsf{T}}(k)\ \mathbf{w}^{\mathsf{T}}(k)\big]^{\mathsf{T}} \\
&\quad \leq \big[\overline{\mathbf{x}}^{\mathsf{T}}(k)\ \ \mathbf{w}^{\mathsf{T}}(k)\big]\tilde{\mathbf{\Xi}}_i\big[\overline{\mathbf{x}}^{\mathsf{T}}(k)\ \ \mathbf{w}^{\mathsf{T}}(k)\big]^{\mathsf{T}}, \qquad (48)
\end{aligned}$$

where

$$\begin{aligned}
\mathbf{\Xi}_i &= \Big[\overline{\mathbf{A}}_i(k)\ \ \overline{\mathbf{B}}_i\Big]^{\mathsf{T}} \mathbf{W}_i \tilde{\mathbf{P}} \mathbf{W}_i^{\mathsf{T}} \Big[\overline{\mathbf{A}}_i(k)\ \ \overline{\mathbf{B}}_i\Big] \\
&= \Big(\mathbf{W}_i^{\mathsf{T}}\big[\mathbf{\Phi}_i\ \ \overline{\mathbf{B}}_i\big] + \mathbf{W}_i^{\mathsf{T}}\overline{\mathbf{M}}\ \overline{\mathbf{F}}(k)\big[\mathbf{\Gamma}_i\ \ \mathbf{0}\big]\Big)^{\mathsf{T}}\tilde{\mathbf{P}} \\
&\quad \times\Big(\mathbf{W}_i^{\mathsf{T}}\big[\mathbf{\Phi}_i\ \ \overline{\mathbf{B}}_i\big] + \mathbf{W}_i^{\mathsf{T}}\overline{\mathbf{M}}\ \overline{\mathbf{F}}(k)\big[\mathbf{\Gamma}_i\ \ \mathbf{0}\big]\Big), \\
\tilde{\mathbf{\Xi}}_i &= \begin{bmatrix} \mathbf{\Phi}_i^{\mathsf{T}}\mathbf{W}_i \\ \overline{\mathbf{B}}_i^{\mathsf{T}}\mathbf{W}_i \end{bmatrix} \tilde{\mathbf{\Psi}}_i^{-1} \begin{bmatrix} \mathbf{W}_i^{\mathsf{T}}\mathbf{\Phi}_i & \mathbf{W}_i^{\mathsf{T}}\overline{\mathbf{B}}_i \end{bmatrix} \\
&\quad + \epsilon_i^{-1}\begin{bmatrix} \mathbf{\Gamma}_i^{\mathsf{T}} \\ \mathbf{0} \end{bmatrix}\begin{bmatrix} \mathbf{\Gamma}_i & \mathbf{0} \end{bmatrix}.
\end{aligned}$$

Combining (10) and (48) yields

$$\begin{aligned}
&\mathsf{E}\big[\tilde{V}(k+1, r_{k+1})\big|\mathcal{F}_k\big] - \tilde{V}(k, r_k) \\
&+\mathbf{z}^{\mathsf{T}}(k)\mathbf{z}(k) - \gamma^2\mathbf{w}^{\mathsf{T}}(k)\mathbf{w}(k) \\
&\quad \leq \big[\overline{\mathbf{x}}^{\mathsf{T}}(k)\ \ \mathbf{w}^{\mathsf{T}}(k)\big]\tilde{\mathbf{\Theta}}_i\big[\overline{\mathbf{x}}^{\mathsf{T}}(k)\ \ \mathbf{w}^{\mathsf{T}}(k)\big]^{\mathsf{T}}, \qquad (49)
\end{aligned}$$

where

$$\begin{aligned}
\tilde{\mathbf{\Theta}}_i &= \tilde{\mathbf{\Xi}}_i + \begin{bmatrix} \overline{\mathbf{C}}_i^{\mathsf{T}} \\ \mathbf{0} \end{bmatrix}\begin{bmatrix} \overline{\mathbf{C}}_i & \mathbf{0} \end{bmatrix} - \begin{bmatrix} \tilde{\mathbf{P}}_i & \mathbf{0} \\ \mathbf{0} & \gamma^2\mathbf{I} \end{bmatrix} \\
&= \begin{bmatrix} \mathbf{W}_i^{\mathsf{T}}\mathbf{\Phi}_i & \mathbf{W}_i^{\mathsf{T}}\overline{\mathbf{B}}_i \\ \mathbf{\Gamma}_i & \mathbf{0} \\ \overline{\mathbf{C}}_i & \mathbf{0} \end{bmatrix}^{\mathsf{T}} \begin{bmatrix} \tilde{\mathbf{\Psi}}_i^{-1} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \epsilon_i^{-1}\mathbf{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{I} \end{bmatrix} \\
&\quad \times \begin{bmatrix} \mathbf{W}_i^{\mathsf{T}}\mathbf{\Phi}_i & \mathbf{W}_i^{\mathsf{T}}\overline{\mathbf{B}}_i \\ \mathbf{\Gamma}_i & \mathbf{0} \\ \overline{\mathbf{C}}_i & \mathbf{0} \end{bmatrix} - \begin{bmatrix} \tilde{\mathbf{P}}_i & \mathbf{0} \\ \mathbf{0} & \gamma^2\mathbf{I} \end{bmatrix}, \qquad (50)
\end{aligned}$$

where $\tilde{\mathbf{\Psi}}_i$ is defined in (47). On the other hand, pre- and post-multiplying (44) by diag$(\tilde{\mathbf{P}}_i, \mathbf{I})$ as well as applying Schur complement yield

$$\tilde{\mathbf{\Theta}}_i < 0. \qquad (51)$$

Let us define the performance function

$$J(T) = \sum_{k=0}^{T} \mathsf{E}\Big[[\mathbf{z}^{\mathsf{T}}(k)\mathbf{z}(k) - \gamma^2\mathbf{w}^{\mathsf{T}}(k)\mathbf{w}(k)]\Big|\mathcal{F}_k\Big]. \qquad (52)$$

Then from (46), (49) and (52), we derive

$$\begin{aligned}
J(T) &= \sum_{k=0}^{T} \mathsf{E}\Big[\big(\mathbf{z}^{\mathsf{T}}(k)\mathbf{z}(k) - \gamma^2\mathbf{w}^{\mathsf{T}}(k)\mathbf{w}(k) \\
&\qquad + \tilde{V}(k+1, r_{k+1}) - \tilde{V}(k, r_k)\big) \\
&\qquad - \big(\tilde{V}(k+1, r_{k+1}) - \tilde{V}(k, r_k)\big)\Big|\mathcal{F}_k\Big] \\
&\leq \sum_{k=0}^{T} \mathsf{E}\Big[\big[\tilde{\mathbf{x}}^{\mathsf{T}}(k)\ \ \mathbf{w}^{\mathsf{T}}(k)\big]\ \tilde{\mathbf{\Theta}}_i\ \big[\tilde{\mathbf{x}}^{\mathsf{T}}(k)\ \ \mathbf{w}^{\mathsf{T}}(k)\big]^{\mathsf{T}}\Big] \\
&\qquad - \mathsf{E}[\tilde{V}(T+1, r_{T+1})\big|\mathcal{F}_T]. \qquad (53)
\end{aligned}$$

For all the $\mathbf{w}(k) \neq \mathbf{0}$, (51) and (53) lead to

$$J(\infty) < 0. \qquad (54)$$

This completes the proof of **Theorem 3**. ■

V. A Numerical Example

To illustrate the effectiveness of the proposed approach, we considered the following uncertain NCS $\hat{P}_K$ of $\mathbf{x}(k) \in \mathbb{R}^3$, $\mathbf{u}(k) \in \mathbb{R}^2$, $\mathbf{z}(k) \in \mathbb{R}$ and $\mathbf{w}(k) \in \mathbb{R}$, with the following parameters

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 0 \\ 0.4 & 0.6 & 0.2 \\ 1 & 0.2 & -1.1 \end{bmatrix}, \ \mathbf{B} = \begin{bmatrix} 0.5 & 1 \\ 0.5 & 0.2 \\ 1 & 0.4 \end{bmatrix},$$

$$\mathbf{B}_w = \begin{bmatrix} 0.1 \\ 0.1 \\ -0.2 \end{bmatrix}, \ \mathbf{M} = \begin{bmatrix} 0.1 \\ -0.1 \\ 0.2 \end{bmatrix},$$

$$\mathbf{C} = \begin{bmatrix} 0.2 & 0.3 & 0.3 \end{bmatrix}, \ \mathbf{D} = \begin{bmatrix} 0.7 & 0.9 \end{bmatrix},$$

$$\mathbf{N}_a = \begin{bmatrix} 0.5 & 0.2 & 0.3 \end{bmatrix}, \ \mathbf{N}_b = \begin{bmatrix} 0.1 & 0.2 \end{bmatrix}.$$

The plant's eigenvalues: $1.0757,\ -0.6274,\ -0.9483$. The system was driven by the Markov chain with the following transition probability matrix

$$\mathbf{\Upsilon} = \begin{bmatrix} 0.2 & 0.1 & 0.1 & 0.6 \\ 0.1 & 0.2 & 0.1 & 0.6 \\ 0.1 & 0.1 & 0.2 & 0.6 \\ 0 & 0.1 & 0.1 & 0.8 \end{bmatrix}.$$

Our objective was to design the state feedback gain matrices $\mathbf{K}_3$ and $\mathbf{K}_4$ such that, for all the admissible uncertainties, the NCS $\hat{P}_K$ was robustly stochastically stable with the specified disturbance attenuation level γ.

Assuming $\gamma = 0.45$, we applied the Matlab LMI Control Toolbox to solve the LMIs (44) and obtained the following solution

$$\mathbf{Q}_1 = \begin{bmatrix} 2.5613 & -1.1880 & 2.5657 \\ -1.1880 & 1.4562 & -1.8515 \\ 2.5657 & -1.8515 & 4.6658 \end{bmatrix},$$

$$\mathbf{Q}_2 = \begin{bmatrix} 3.3872 & -1.7206 & 3.3227 \\ -1.7206 & 1.8596 & -2.3236 \\ 3.3227 & -2.3236 & 5.5412 \end{bmatrix},$$

$$\mathbf{Q}_3 = \begin{bmatrix} 3.1346 & -1.3269 & 3.3523 \\ -1.3269 & 1.4702 & -2.0870 \\ 3.3523 & -2.0870 & 5.8350 \end{bmatrix},$$

$$\mathbf{Q}_4 = \begin{bmatrix} 4.0753 & -1.1078 & 3.0745 \\ -1.1078 & 2.2382 & -2.0678 \\ 3.0745 & -2.0678 & 7.6812 \end{bmatrix},$$

$$\mathbf{Y}_3 = \begin{bmatrix} 0.0895 & -0.0445 & 0.1588 \\ -0.4495 & 0.1736 & -0.6552 \end{bmatrix},$$

$$\mathbf{Y}_4 = \begin{bmatrix} -0.8924 & -0.5845 & 1.9063 \\ 0.0233 & 0.0424 & -2.5885 \end{bmatrix},$$

$$\epsilon_1 = 15.7628,\ \epsilon_2 = 14.4109,$$

$$\epsilon_3 = 14.3356,\ \epsilon_4 = 16.5168.$$

It followed from **Theorem 3** that the robust $\mathcal{H}_\infty$ control problem was solvable with the state feedback gain matrices given by

$$\mathbf{K}_3 = \mathbf{Y}_3\mathbf{Q}_3^{-1} = \begin{bmatrix} 0.0004 & 0.0170 & 0.0331 \\ -0.0707 & -0.0964 & -0.1062 \end{bmatrix},$$

$$\mathbf{K}_4 = \mathbf{Y}_4\mathbf{Q}_4^{-1} = \begin{bmatrix} -0.5959 & -0.1417 & 0.4485 \\ 0.3396 & -0.3326 & -0.5625 \end{bmatrix}.$$

VI. CONCLUSIONS

In this contribution we have investigated a general class of model-based networked control systems where the plant model has time-varying norm-bounded parameter uncertainties and both the sensor-to-controller and controller-to-actuator channels experience random packet dropouts. Firstly we have established sufficient conditions in the form of linear matrix inequalities for guaranteeing the robust stochastic stability and synthesising robust stabilisation controller. Secondly we have considered the robust $\mathcal{H}_\infty$ controller design and have presented the linear matrix inequality solution for robust $\mathcal{H}_\infty$ control law that stabilises this class of model-based networked control systems with a prescribed disturbance attenuation level. A numerical example has been included to illustrate our proposed design approach.

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