Blind Equalisation of High-Order QAM Channels Using a Fuzzy-Logic Tuned Constant Modulus Algorithm and Soft Decision-Directed Scheme

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Outline

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State-of-the-Art

For high-throughput QAM systems, the combined CMA and SDD blind equaliser offers state-of-the-art.

- Low complexity, only requires slightly more computational requirements than the very simple CMA.
- High performance, capable of achieving an equalisation performance close to the optimal MMSE solution.

For high-order QAM systems, even this state-of-the-art requires tens of thousands samples of adaptation.

- The choice of step size for the CMA is critical to convergence performance.

Existing Works

- Variable step size mechanisms, in particular, fuzzy-logic tuned step size, have been developed for the training based adaptive LMS algorithm.
- There existing some works on variable step size CMA techniques.
- No published work considers fuzzy-logic tuning of the CMA's step size for blind equalisation of QAM systems.

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Our Novelty

- Develop a fuzzy-logic unit for adjusting the CMA's step size.
- This fuzzy step-size CMA is combined with the SDD scheme to obtain the fuzzy-logic assisted CMA and SDD blind equaliser.
  - It requires several thousands fewer samples to converge to the same steady-state solution achieved by the constant step size CMA and SDD scheme.

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Signal Model

- Let the CIR of length n_{ch} be

$$\mathbf{c}_{\text{CIR}} = [c_0 \; c_1 \cdots c_{n_{\text{ch}}-1}]^T,$$

the equaliser input of length n_{eq}

$$\mathbf{x}(k) = [x(k) \; x(k-1) \cdots x(k-n_{\text{eq}}+1)]^T,$$

and the AWGN vector

$$\mathbf{e}(k) = [e(k) \; e(k-1) \cdots e(k-n_{\text{eq}}+1)]^T.$$

- Then the equalisation signal model is given by

$$\mathbf{x}(k) = \mathbf{C} \, \mathbf{s}(k) + \mathbf{e}(k)$$

Signal Model

- The $n_{\text{eq}} \times (\tau_{\max} + 1)$ Toeplitz CIR matrix

$$\mathbf{C} = \begin{bmatrix} \mathbf{c}_{\text{CIR}}^T & 0 & \cdots & 0 \\ 0 & \mathbf{c}_{\text{CIR}}^T & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & \mathbf{c}_{\text{CIR}}^T \end{bmatrix} = [\mathbf{c}_0 \ \mathbf{c}_1 \cdots \mathbf{c}_{\tau_{\max}}]$$

  with $\tau_{\max} = n_{\text{eq}} + n_{\text{ch}} - 2$.

- The noise power is $E[|e(k)|^2] = 2\sigma_e^2$ and data symbol $s(k)$ takes value from M-QAM set

$$\mathcal{S} = \{s_{i,l} = u_i + ju_l,\ 1 \le i, l \le \sqrt{M}\}$$

  with $\Re[s_{i,l}] = u_i = 2i - \sqrt{M} - 1$, $\Im[s_{i,l}] = u_l = 2l - \sqrt{M} - 1$, $j^2 = -1$ and the symbol energy $E[|s(k)|^2] = \sigma_s^2$.

Equaliser

- The equaliser output is given by

$$y(k) = \mathbf{w}^H \mathbf{x}(k),$$

  with the equaliser's weight vector $\mathbf{w} = [w_0 \; w_1 \cdots w_{n_{\mathrm{eq}}-1}]^T$.

- $y(k)$ is used to produce an estimate $\hat{s}(k-\tau)$ of the transmitted symbol $s(k-\tau)$, where τ is decision delay.
  - $0 \leq \tau \leq \tau_{\mathrm{max}}$, and is unknown in blind equalisation.

- When $\mathbf{C}$ is known, the optimal MMSE solution that minimises $J_{\mathrm{MSE}}(\mathbf{w}) = E[|s(k-\tau) - y(k)|^2]$ is given by

$$\mathbf{w}_{\mathrm{MMSE}} = \left(\mathbf{C}\mathbf{C}^H + \frac{2\sigma_e^2}{\sigma_s^2} \mathbf{I}_{n_{\mathrm{eq}}} \right)^{-1} \mathbf{c}_\tau .$$

Performance Metrics

- Mean square error (MSE)

$$J_{\mathrm{MSE}}(\mathbf{w}) = \sigma_s^2\Big(\big(1 - \mathbf{w}^H\mathbf{c}_\tau - \mathbf{w}^T\mathbf{c}_\tau^*\big) + \mathbf{w}^H\big(\mathbf{C}\mathbf{C}^H + \frac{2\sigma_e^2}{\sigma_s^2}\mathbf{I}_{n_{\mathrm{eq}}}\big)\mathbf{w}\Big).$$

- The maximum distortion (MD) measure

$$\mathrm{MD}(\mathbf{w}) \triangleq \Big(\sum_{i=0}^{\tau_{\max}} |f_i| - |f_{i_{\max}}|\Big)/|f_{i_{\max}}|$$

  with the combined equaliser and channel response

$$\mathbf{f}^T = [f_0\ f_1 \cdots f_{\tau_{\max}}] \triangleq \mathbf{w}^H\mathbf{C},$$

  and the unknown decision delay $\tau = i_{\max}$ with

$$i_{\max} = \arg \max_{0\le i\le \tau_{\max}} |f_i|.$$

- The symbol error rate is the ultimate performance metric.

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CMA and SDD

- Set $\mathbf{w} = \mathbf{w}_c + \mathbf{w}_d$, and give $y(k) = \mathbf{w}^H(k-1)\mathbf{x}(k)$.
- The constant modulus algorithm

$$\begin{cases} \varepsilon(k) = y(k)\left(\Delta - |y(k)|^2\right), \\ \mathbf{w}_c(k) = \mathbf{w}_c(k-1) + \mu_{\mathrm{CMA}}\varepsilon^*(k)\mathbf{x}(k), \end{cases}$$

  $\Delta = E\left[|s(k)|^4\right] / E\left[|s(k)|^2\right]$ and μ_{CMA} is the CMA step size.
- The soft decision-directed adaptation

$$\mathbf{w}_d(k) = \mathbf{w}_d(k-1) + \mu_{\mathrm{SDD}} \frac{\partial J_{\mathrm{LMAP}}(\mathbf{w}(k-1), y(k))}{\partial \mathbf{w}_d},$$

  where the local marginal PDF criterion is defined as

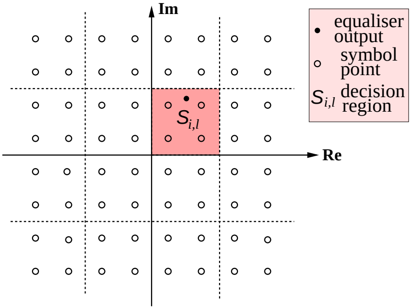
$$J_{\mathrm{LMAP}}(\mathbf{w}, y(k)) = \rho \log\left(\sum_{r=2i-1}^{2i} \sum_{m=2l-1}^{2l} \frac{1}{8\pi\rho} e^{-\frac{|y(k)-s_{r,m}|^2}{2\rho}} \right),$$

  μ_{SDD} is the SDD step size, and ρ is the cluster width.

Soft Decision

- The complex plan is divided into $M/4$ regions

$$\mathcal{S}_{i,l} = \{s_{r,m},\ r = 2i-1, 2i, m = 2l-1, 2l\},\ 1 \le i, l \le \sqrt{M}/2,$$



- If $y(k) \in \mathcal{S}_{i,l}$, the PDF of $y(k)$: $\hat{p}(\mathbf{w}, y(k)) \approx$

$$\sum_{r=2i-1}^{2i} \sum_{m=2l-1}^{2l} \frac{1}{8\pi\rho} e^{-\frac{|y(k)-s_{r,m}|^2}{2\rho}}.$$

- It is not committed to a single hard decision;
- but considers the tentative decisions, including the hard one, in $\mathcal{S}_{i,l}$.

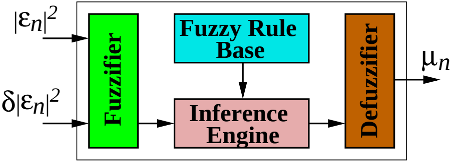
Outline

Fuzzy Inference System

- The FIS maps two input variables, defined by

$$|\varepsilon_n|^2 = \frac{1}{N_{\mathrm{sm}}} \sum_{l=0}^{N_{\mathrm{sm}}-1} |\varepsilon(k-l)|^2, \quad \delta|\varepsilon_n|^2 = |\varepsilon_n|^2 - |\varepsilon_{n-1}|^2,$$

into an appropriate step size μ_n, where



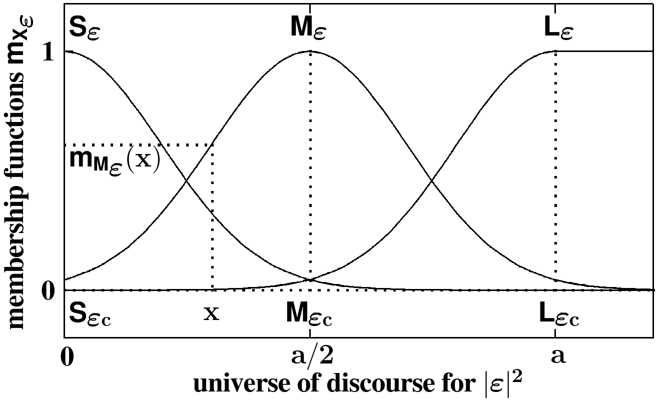
- N_{sm} is the short-term average length,
- $n = \lfloor k/N_{\mathrm{sm}} \rfloor$ with $\lfloor \bullet \rfloor$ denoting the integer floor operator.

- It operates once every N_{sm} samples, and μ_n is used as the CMA step size for the subsequent N_{sm} samples

$$\mu_{\mathrm{CMA}} = \mu_n, \quad nN_{\mathrm{sm}} \le k < (n+1)N_{\mathrm{sm}}.$$

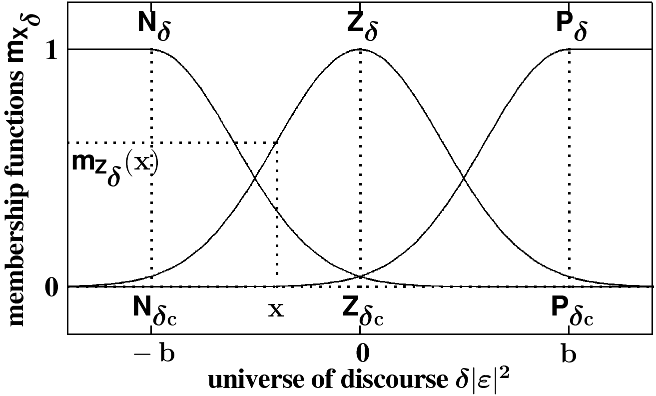
Fuzzifier for Error Power

- The fuzzy sets for $|\varepsilon_n|^2$: small S_ε, medium M_ε and large L_ε.
- Membership functions for $\mathrm{X}_\varepsilon = \mathrm{S}_\varepsilon$, M_ε or L_ε:



Fuzzifier for Error Power Variation

- Fuzzy sets for $\delta|\varepsilon_n|^2$: negative N_δ, zero Z_δ and positive P_δ.
- Membership functions for $\mathrm{X}_\delta = \mathrm{N}_\delta$, Z_δ or P_δ:



Inference Engine

- Fuzzy sets for crisp μ_n:

Fuzzy set	S_μ	M_μ	L_μ
centroid	$\mu_{\min}$	$2\mu_{\min}$	$\mu_{\max} = 4\mu_{\min}$
universe of discourse	$[\mu_{\min}, \quad \mu_{\max}]$		

μ_n		$\delta\lvert\varepsilon_n\rvert^2$		
		N_δ	Z_δ	P_δ
$\lvert\varepsilon_n\rvert^2$	S_ε	**1** L_μ	**2** M_μ	**3** S_μ
	M_ε	**4** M_μ	**5** S_μ	**6** S_μ
	L_ε	**7** M_μ	**8** S_μ	**9** S_μ

- There are nine fuzzy `IF-THEN` rules.
- The min operator is applied to truncate the output fuzzy set for each rule.
- The centroid calculation is used:

$$\mu_n = \frac{\sum_{i=1}^{9} \mu_n[i] \cdot \mathrm{m}_{X_\mu}(\mu_n[i])}{\sum_{i=1}^{9} \mathrm{m}_{X_\mu}(\mu_n[i])}.$$

Outline

Combining FL CMA with SDD

- The fuzzy logic step size CMA can be combined with the SDD to form a novel blind equalisation scheme.
- No need for a variable step size SDD,
  - as the "error" or stochastic gradient for the SDD weight updating is well "normalised" by the normalisation factor Z_N.
- To achieve the maximum benefit in convergence speed without sacrificing the steady-state performance,
  - Use fuzzy step size in the initial stage of blind adaptation, then switch to a small constant step size $\mu_{\text{CMA}} = \mu_{\min}$.

Computational Complexity

algorithm	multiplications	additions	$e^{\{\bullet\}}$ evaluations
CMA	$8 \times n_{\mathrm{eq}} + 6$	$8 \times n_{\mathrm{eq}}$	–
CMA+SDD	$12 \times n_{\mathrm{eq}} + 29$	$14 \times n_{\mathrm{eq}} + 21$	4
FIS	$2 + 22/N_{\mathrm{sm}}$	$2 + 22/N_{\mathrm{sm}}$	$6/N_{\mathrm{sm}}$

The symbol rate is N_{sm} times faster than the operational rate of the fuzzy inference system, and n_{eq} is the equaliser length.

Outline

Channel and Equaliser

- Modulation was 64-QAM and the CIR was given by

$$\mathbf{c}_{\text{CIR}} = [-0.2+j0.3 \;\; -0.5+j0.4 \;\; 0.7-j0.6 \;\; 0.4+j0.3 \;\; 0.2+j0.1]^T.$$

- The equaliser length was $n_{\text{eq}} = 23$ and with the middle tap initialisation, the blind equalise had decision delay $\tau = 13$.
- The CMA step size was $\mu_{\text{CMA}} = 2 \times 10^{-7}$, while the SDD step size was $\mu_{\text{SDD}} = 2 \times 10^{-4}$ with $\rho = 0.6$.
- FIS parameters: $a = 10^5$, $b = 10^3$; GMBF variances $\rho_\varepsilon = (0.01a)^2$, $\rho_\delta = (0.2b)^2$; $N_{\text{sm}} = 20$, while $\mu_{\min} = 2 \times 10^{-7}$.

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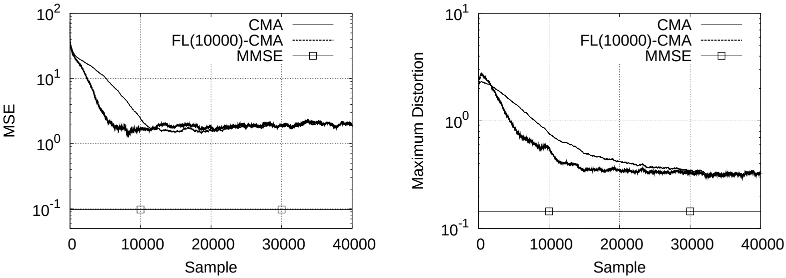
Convergence Performance

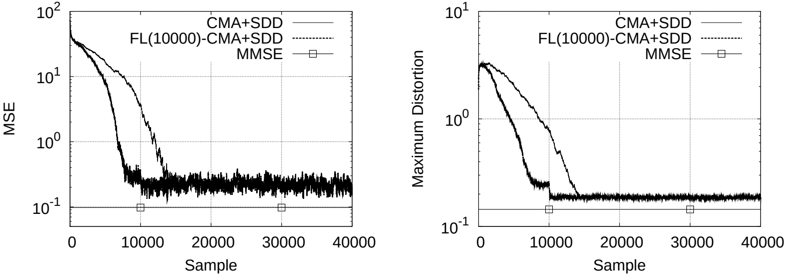
Convergence comparison of the CMA and FL(10000) aided CMA, in terms of the MSE and MD, given SNR$=$ 38 dB.



The FL(10000)-CMA uses the fuzzy step size for the first 10000 samples and then switches to a constant step size.

Convergence Performance

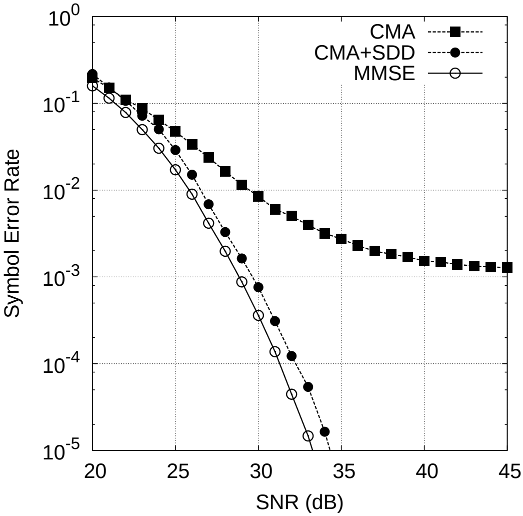
Convergence comparison of the CMA+SDD and FL(10000) aided CMA+SDD, in terms of the MSE and MD, given SNR= 38 dB.



The FL(10000)-CMA uses the fuzzy step size for the first 10000 samples and then switches to a constant step size.

Symbol Error Rate

The FL(10000)-CMA+SDD and CMA+SDD had identical SER.



Conclusions

- Low-complexity high-performance blind equalisation of high-order QAM channels has been studied.
- We have designed a fuzzy-logic unit for tuning the CMA's step size
  - which costs a very small extra computation, and
  - has a few parameters that requires to choose empirically.
- The fuzzy logic assisted CMA and SDD blind equaliser significantly improves the convergence speed
  - over the previous state-of-the-art, the constant step size CMA and SDD blind equaliser.