Semi-Blind Joint Channel Estimation and Data Detection for Space-Time Shift Keying Systems

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Abstract—A low-complexity semi-blind joint channel estimation and data detection scheme is proposed for space-time shift keying (STSK) based multiple-input multiple-output systems. The minimum number of STSK training blocks, which is related to the number of transmitter antennas, is first utilized to provide a rough initial least square channel estimate (LSCE). Then low-complexity single-stream maximum likelihood (ML) data detection is carried out based on the initial LSCE and the detected data are employed to refine the decision-directed LSCE. It is demonstrated that a few iterations are sufficient to approach the optimal ML detection performance obtained with the perfect channel state information.

Index Terms—Least square estimation, maximum likelihood detection, multiple-input multiple-output, space-time shift keying.

I. INTRODUCTION

ULTIPLE-INPUT multiple-output (MIMO) techniques exploit the space and/or time dimensions to achieve multiplexing and/or diversity gains. The vertical Bell Lab layered space-time (V-BLAST) scheme [1], for example, offers a high multiplexing gain at a high decoding complexity, which is imposed by the mitigation of the associated inter-channel interference (ICI). Orthogonal space-time block codes (OS-TBCs) [2] on the other hand achieve the maximum diversity gain at the expense of a reduced bandwidth efficiency. Linear dispersion codes (LDCs) [3] are capable of providing a more flexible tradeoff between the attainable diversity and multiplexing gains, but also require sophisticated high-complexity detection. The spatial modulation (SM) and space-shift keying (SSK) schemes [4]–[6] do offer an increased transmission rate, but only achieve receive diversity, no transmit diversity. The main advantage of SM and SSK is that they are free from the effects of ICI and therefore facilitate a low-complexity single-antenna-based ML detection [5]. The recently proposed space-time shift keying (STSK) [7], [8] offers a unified MIMO architecture, which includes V-BLAST, OSTBCs, LDCs, SM and SSK as its special cases. In contrast to the SM and SSK schemes [4]–[6], which only exploit the spatial dimension, the STSK scheme utilizes both the space and time dimensions. Specifically, the STSK system is based on the activation of the appropriately indexed space-time dispersion matrices within each STSK block duration, instead of the indexed antennas at each symbol duration, as in the SM and SSK systems of [4]-[6]. Owing to its high degree of design freedom, the STSK scheme

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is capable of striking a flexible diversity versus multiplexing gain tradeoff, which is achieved by optimizing both the number and size of the dispersion matrices as well as the number of transmit and receive antennas. In particular, the STSK system is capable of exploiting both transmit and receive diversity gains, unlike the SM and SSK schemes, which can only attain a receive diversity gain. Moreover, like the SM system, the STSK scheme does not impose ICI. As a consequence, the single-antenna based ML detector of [5] can readily be employed in the STSK system to attain optimal ML detection at a low complexity.

In general, a MIMO system's ability to approach its capacity heavily relies on the accuracy of the channel state information (CSI). Training based adaptive schemes are capable of accurately estimating a MIMO channel at the expense of considerable reduction in system throughput, since a large training overhead is required to obtain a reasonably accurate CSI estimate. Blind methods not only impose high complexity and slow convergence, but also suffer from unavoidable estimation and decision ambiguities [9]. Semi-blind methods offer attractive practical means of implementing adaptive MIMO systems. In the semi-blind methods of [10]–[13], a few training symbols are used to provide an initial MIMO channel estimate. Then the channel estimator as well as the data detector iteratively exchange their information, where the channel estimator relies on decision-directed adaptation. In the scheme of [14] and [15], aided by an initial training-based MIMO channel estimate, blind joint ML data detection and channel estimation is carried out by a computational intelligence based optimization algorithm. In these studies, however, the MIMO systems induce ICI, hence potentially complex multi-antenna-based ML data detection has to be carried out. Their complexity may be reduced for example, with the aid of sphere-decoding based algorithms [16], such as the K-best sphere-decoding algorithm [17], which may still remain computationally expensive. Hence high-complexity ML data detection coupled with a large number of iterations to achieve convergence imposes considerable computational requirements in these previous semi-blind methods.

Against this back-cloth, our novel contribution is that we exploit the inherent low-complexity of the single-stream ML data detection in STSK systems and propose a semi-blind joint channel estimation and data detection scheme. In order to maintain a high system throughput, the minimum number of STSK training blocks, which is determined by the number of transmitter antennas, is utilized to provide an initial least square channel estimate (LSCE). Then low-complexity single-antenna based ML data detection is carried out based on the initial LSCE, and the detected data are then remodulated and used for the decision-directed LSCE update. Our study demonstrates that a few iterations, typically no more than five, are sufficient to approach the optimal detection performance obtained with the aid of perfect CSI.

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II. SPACE-TIME SHIFT KEYING SYSTEM MODEL

Consider the coherent STSK based MIMO system [8], which employs N_T antennas at the transmitter and N_R antennas at the receiver for communication in a frequency-flat Rayleigh fading environment. Let \mathbb{C} denote the field of complex numbers, T_n be the number of time slots, and *i* indicate the STSK block index. Then the received signal matrix $\mathbf{Y}(i) \in \mathbb{C}^{N_R \times T_n}$ is expressed by the following model:

$$\mathbf{Y}(i) = \mathbf{H}\mathbf{S}(i) + \mathbf{V}(i) \tag{1}$$

where $\mathbf{H} \in \mathbb{C}^{N_R \times N_T}$ and $\mathbf{V}(i) \in \mathbb{C}^{N_R \times T_n}$ denote the channel matrix and the *i*th noise matrix, respectively, while $\mathbf{S}(i) \in \mathbb{C}^{N_T \times T_n}$ denotes the *i*th transmitted space-time signal matrix and the *m*th row's elements of $\mathbf{S}(i)$ are transmitted from the *m*th transmitter antenna in T_n time slots. We assume that each element of \mathbf{H} obeys the complex-valued Gaussian distribution with zero mean and a variance of 0.5 per dimension, which is denoted as $\mathcal{CN}(0, 1)$. Each element of $\mathbf{V}(i)$ obeys the complex-valued zero-mean Gaussian distribution of $\mathcal{CN}(0, N_o)$. Furthermore, the channel matrix \mathbf{H} is assumed to remain constant for at least T_n time slots.

Each STSK block $\mathbf{S}(i) \in \mathbb{C}^{N_T \times T_n}$ is given by [7], [8] as $\mathbf{S}(i) = s(i)\mathbf{A}(i)$, where $s(i) \in \mathcal{S} = \{s_l \in \mathbb{C}, 1 \le l \le L\}$ is the complex-valued symbol of the conventional modulation scheme employed, such as L-PSK or L-QAM, which is associated with $\log_2(L)$ input bits, while $\mathbf{A}(i) \in \mathcal{A} = \{\mathbf{A}_q \in \mathbb{C}^{N_T \times T_n}, 1 \leq$ $q \leq Q$ is selected from the Q pre-assigned dispersion matrices \mathbf{A}_q , $1 \leq q \leq Q$, according to $\log_2(Q)$ input bits. Thus a total of $\log_2(L \cdot Q)$ source bits are mapped to each STSK block, and the normalised throughput R per time-slot, of this STSK scheme can be expressed as $R = \log_2(Q \cdot L)/T_n$ [bits/symbol]. The design of dispersion matrices is an important research subject entirely in itself [18]. To maintain a unity average transmission power for each STSK block, each of the Q dispersion matrices must meet the power constraint of $\operatorname{tr} [\mathbf{A}_q^H \mathbf{A}_q] = T_n$, $1 \leq q \leq Q$, where $\operatorname{tr}[\cdot]$ denotes the matrix trace operator. Given the STSK parameters of (N_T, N_R, T_n, Q) as well as the constellation size L, a numerical search is adopted in [8] to obtain the set of Q dispersion matrices \mathcal{A} by maximizing the discrete-input continuous-output memoryless channel's capacity [19] subject to the constraints of $tr[\mathbf{A}_q^H \mathbf{A}_q] = T_n$. The signal to noise ratio (SNR) of the system is defined as $SNR = E_s/N_o$, where E_s is the average symbol energy of the modulated symbol s(i).

Let us introduce the following notations now:

$$\overline{\mathbf{y}}(i) = \operatorname{vec}[\mathbf{Y}(i)] \in \mathbb{C}^{N_R T_n \times 1}$$
(2)

$$\overline{\mathbf{H}} = \mathbf{I}_{T_{n}} \otimes \mathbf{H} \in \mathbb{C}^{N_{R}T_{n} \times N_{T}T_{n}}$$
(3)

$$\overline{\mathbf{v}}(i) = \operatorname{vec}[\mathbf{V}(i)] \in \mathbb{C}^{N_R T_n \times 1}$$
(4)

$$\boldsymbol{\Theta} = \left[\operatorname{vec}[\mathbf{A}_1] \cdots \operatorname{vec}[\mathbf{A}_Q] \right] \in \mathbb{C}^{N_T T_n \times Q}$$
(5)

where $\operatorname{vec}[\cdot]$ denotes the vector stacking operator, \mathbf{I}_M is the $M \times M$ identity matrix and \otimes the Kronecker product. Define furthermore the equivalent transmitted signal vector $\mathbf{k}(i) \in \mathbb{C}^{Q \times 1}$ as

$$\mathbf{k}(i) = [\underbrace{\mathbf{0} \cdots \mathbf{0}}_{q-1} s(i) \underbrace{\mathbf{0} \cdots \mathbf{0}}_{Q-q}]^T \tag{6}$$

where the modulated symbol s(i) is situated in the *q*th element. Note that the index *q* corresponds to the index of the dispersion matrix \mathbf{A}_q activated during the *i*th STSK block and, therefore, the transmitted signal vector $\mathbf{k}(i)$ takes its value from the set $\mathcal{K} = \{\mathbf{k}_{q,l} \in \mathbb{C}^{Q \times 1}, 1 \leq q \leq Q, 1 \leq l \leq L\}$, which contains the $Q \cdot L$ legitimate transmitted signal vectors,

$$\mathbf{k}_{q,l} = [\underbrace{0 \cdots 0}_{q-1} s_l \underbrace{0 \cdots 0}_{Q-q}]^T, \ 1 \le q \le Q, \ 1 \le l \le L \quad (7)$$

where s_l is the *l*th symbol in the *L*-point constellation S. Given the notations (2) to (6), the signal model equivalent to (1) becomes

$$\overline{\mathbf{y}}(i) = \overline{\mathbf{H}} \mathbf{\Theta} \mathbf{k}(i) + \overline{\mathbf{v}}(i). \tag{8}$$

When **H** is known at the receiver, data detection can be carried out very efficiently [8]. This is because the equivalent system model (8) is free from the effects of ICI, and the low-complexity single-antenna based ML detector of [5] may readily be applied to achieve optimal data detection. Let (q, l)correspond to the specific inputs bits of a STSK block, which are mapped to the *l*th symbol s_l and the *q*th dispersion matrix \mathbf{A}_q . Then the ML estimates (\hat{q}, \hat{l}) are given by

$$\begin{aligned} (\hat{q}, \hat{l}) &= \arg \min_{\substack{1 \le q \le Q \\ 1 \le l \le L}} \|\overline{\mathbf{y}}(i) - \overline{\mathbf{H}} \mathbf{\Theta} \mathbf{k}_{q, l}\|^2 \\ &= \arg \min_{\substack{1 \le q \le Q \\ 1 \le l \le L}} \|\overline{\mathbf{y}}(i) - s_l \big(\overline{\mathbf{H}} \mathbf{\Theta}\big)_q \|^2 \tag{9}$$

where $(\overline{\mathbf{H}} \Theta)_q$ denotes the *q*th column of the matrix $\overline{\mathbf{H}} \Theta$. The calculation of $(\overline{\mathbf{H}} \Theta)_q$, $1 \leq q \leq Q$, requires $QT_nN_R4N_T$ real-valued multiplications and $QT_nN_R(4N_T-2)$ real-valued additions. In a slow-fading environment, this calculation can be reused during the channel's coherent time. The detection of a STSK block or $\log_2(Q \cdot L)$ bits using (9) requires $6LQN_RT_n$ real-valued multiplications and $LQ(6N_RT_n-1)$ real-valued additions. Let the channel's coherence time be the duration of τ STSK blocks. Then the total complexity of detecting $\tau \log_2(Q \cdot L)$ bits is summarized as $[8]C_{\rm ML} \approx 4QT_nN_R(3\tau L + 2N_T)$ [Flops].

III. SEMI-BLIND ITERATIVE JOINT CHANNEL ESTIMATION AND DATA DETECTION

Let the number of available training blocks be M and arrange the training data as $\mathbf{Y}_{tM} = [\mathbf{Y}(1) \cdots \mathbf{Y}(M)]$ and $\mathbf{S}_{tM} = [\mathbf{S}(1) \cdots \mathbf{S}(M)]$. Then the LSCE based on $(\mathbf{Y}_{tM}, \mathbf{S}_{tM})$ is given by

$$\hat{\mathbf{H}}_{\text{LSCE}} = \mathbf{Y}_{\text{t}M} \mathbf{S}_{\text{t}M}^{H} \left(\mathbf{S}_{\text{t}M} \mathbf{S}_{\text{t}M}^{H} \right)^{-1}.$$
 (10)

To maintain a high system throughput, we should only use the minimum number of STSK training blocks. In order for $\mathbf{S}_{tM}\mathbf{S}_{tM}^H$ to have the full rank of N_T , it is necessary that $M \cdot T_n \geq N_T$ and this requires a minimum of $M = \left\lceil N_T/T_n \right\rceil$ training blocks, where $\lceil x \rceil$ denotes the integer ceiling that is larger than or equal to x. Thus we will choose the number of initial training blocks according to $M = \left\lceil N_T/T_n \right\rceil$. For example, if $N_T = 4$ and $T_n = 2$, then the minimum number of STSK training blocks is M = 2. Given such a small training data set, the accuracy of the LSCE (10) will be poor and the achievable bit error ratio (BER) of the ML detector (9) based on this rough CSI estimate will also be poor. We propose to use the following iterative joint channel estimation and data detection scheme to improve the detection and estimation performance. Let the observation data for the ML detector be denoted as $\mathbf{Y}_{d\tau} = [\mathbf{Y}(1) \mathbf{Y}(2) \cdots \mathbf{Y}(\tau)]$ and fix the number of iterations to I_{max} .

Semi-Blind Iterative Algorithm

- 1) Set the iteration index to t = 0 and the channel estimate to $\tilde{\mathbf{H}}^{(t)} = \hat{\mathbf{H}}_{\text{LSCE}}$.
- 2) Given $\tilde{\mathbf{H}}^{(t)}$, perform ML data detection on $\mathbf{Y}_{d\tau}$ and remodulate the detected data, yielding $\hat{\mathbf{S}}_{e\tau}^{(t)} = [\hat{\mathbf{S}}^{(t)}(1) \ \hat{\mathbf{S}}^{(t)}(2) \ \cdots \ \hat{\mathbf{S}}^{(t)}(\tau)].$
- 3) Update the channel estimate with $\hat{\mathbf{H}}^{(t+1)} = \mathbf{Y}_{\mathrm{d}\tau} (\hat{\mathbf{S}}_{\mathrm{e}\tau}^{(t)})^H (\hat{\mathbf{S}}_{\mathrm{e}\tau}^{(t)} (\hat{\mathbf{S}}_{\mathrm{e}\tau}^{(t)})^H)^{-1}$. 4) Set t = t + 1: If $t < I_{\mathrm{max}}$, go to Step 2); otherwise, stop.

The total complexity of this semi-blind iterative joint channel estimation and data detection process is proportional to I_{\max} . $C_{\rm ML}$. Our empirical results show that the number of iterations required for the iterative procedure to converge is low, typically, $I_{\rm max} \leq 5$. We will demonstrate that for medium to high SNR values this iterative procedure is capable of converging to the optimal ML detection performance obtained under perfect CSI. In fact, if the initial channel estimate $\tilde{\mathbf{H}}^{(0)}$ is capable of yielding a BER below 0.1, the decision-directed channel estimator of Step 3) is capable of improving the accuracy of the channel estimate. This in turn significantly enhances the BER of the ML data detection in Step 2). Therefore, a few iterations are sufficient to approach the optimal ML solution. For low SNR values, however, some degradation may be expected with respect to the optimal ML performance, particularly when the initial BER is higher than 0.1. These observations will be further discussed in the following simulation study.

It may readily be seen that this semi-blind scheme designed for STSK based MIMO systems imposes a dramatically lower complexity than semi-blind ML schemes designed for joint channel estimation and data detection in conventional MIMO systems [10]–[13]. This is simply because in STSK system, the optimal ML detection of $\tau \times \log_2(L \cdot Q)$ bits only requires us to search for a total of $\tau \times (L \cdot Q)$ points, as shown in (9). For a conventional MIMO system of the same rate R, the full ML detection of $\tau \times \log_2(L \cdot Q)$ bits would require us to search for $\tau \times N_R^{L \cdot Q}$ points, which may become prohibitive. The efficient K-best sphere decoding algorithm [17] of conventional MIMO systems, which is capable of approximating the ML performance with K set to $K = L \cdot Q$, will require us to search for a total of $\tau \times (L \cdot Q + (N_R - 1)(L \cdot Q)^2)$ constellation points, while imposing some additional complexity necessitated by the associated Cholesky factorization.

IV. SIMULATION STUDY

We considered a coherent STSK scheme having the parameters of $(N_T = 4, N_R = 4, T_n = 2, Q = 4)$ and the QPSK constellation of size L = 4. The achievable performance was assessed in our simulation study using three metrics, namely the estimated mean square error (MSE) defined by $J_{\text{MSE}}(\hat{\mathbf{H}}) = (1/(\tau \cdot N_R \cdot T_n)) \sum_{i=1}^{\tau} \|\mathbf{Y}(i) - \hat{\mathbf{H}}\hat{\mathbf{S}}(i)\|^2,$ the mean channel estimation error (MCE) given by $J_{\text{MCE}}(\tilde{\mathbf{H}}) = (1/(N_R \cdot N_T)) ||\mathbf{H} - \tilde{\mathbf{H}}||^2$, and the achievable BER, where $\hat{\mathbf{H}}$ is the channel estimate, $\hat{\mathbf{S}}(i)$ is the ML-detected and remodulated data, while H denotes the true



Fig. 1. Bit error rate of the proposed semi-blind scheme with two initial training STSK blocks, in comparison with the training-based cases using M = 2 and 30 training STSK blocks as well as the case of perfect channel state information.



Fig. 2. Convergence of the estimated mean square error for the proposed semiblind scheme with two initial training STSK blocks, given different values of $E_{\rm s}/N_{\rm o}$.

MIMO channel matrix. All the results were averaged over 100 channel realizations. The length of data blocks used for performing ML detection was $\tau = 200$, which corresponded to a block of 800 source bits.

The achievable BER performance associated with assuming perfect CSI is given in Fig. 1 as the benchmark. The trainingbased ML detection performance using M = 2 and 30, respectively, is also shown in Fig. 1 for comparison. It can be seen that the LSCE obtained using only M = 2 STSK training blocks was inadequate and, in order to approximate the true ML detection performance, more than 30 training STSK blocks were required. The performance of the proposed semi-blind scheme using M = 2 initial STSK training blocks was then investigated. Figs. 2 and 3 characterize the convergence performance of the semi-blind iterative scheme in terms of the estimated MSE and MCE, respectively, for three different SNR values. The results shown in Figs. 2 and 3 indicate that reliable convergence required no more than five iterations. Furthermore, it can be seen from Fig. 2 that the estimated MSE converged to the noise floor $N_{\rm o}$.

The BERs of the semi-blind iterative scheme are also shown in Fig. 1. For this MIMO system, there were $N_R \cdot N_T = 16$ complex-valued channel taps. Two training STSK blocks corresponded to eight training bits, and this represented a training overhead of 0.5 bit per channel. The semi-blind iterative scheme operating with such a low training overhead was capable of approaching the optimal ML performance for SNR values of 5 dB or higher, as seen in Fig. 1. For SNR < 5 dB, some degradation



Fig. 3. Convergence of the mean channel error for the proposed semi-blind scheme with two initial training STSK blocks, given different values of E_s/N_o .



Fig. 4. Bit error rate of the proposed semi-blind scheme with three initial training STSK blocks, in comparison with the training-based cases using M = 2, 3, and 30 training STSK blocks as well as the case of perfect channel state information.

was observed with respect to the optimal BER performance¹. This was not surprising, since the BER achieved by the rough initial LSCE was higher than 0.1 for SNR < 5 dB. Having a better initial LSCE should be able to improve the performance. We also employed M = 3 initial STSK training blocks for the iterative semi-blind scheme, which still represented a low training overhead of less than 1 bit per channel. The results obtained are shown in Fig. 4, where the BER performance of the semi-blind scheme now closely approximated the optimal BER even at low SNRs.

V. CONCLUSIONS

A semi-blind iterative scheme of joint channel estimation and ML data detection has been proposed for STSK based MIMO systems. The scheme is semi-blind, since it utilizes the minimum number of training STSK blocks to provide a rough initial LSCE for aiding the joint iterative procedure. This semi-blind joint channel estimation and single-stream ML data detection scheme has an inherently low-complexity. It has been shown that the iterative procedure converges rapidly, typically in no more than five iterations, to the optimal ML data detection performance obtained for perfect CSI.

REFERENCES

- P. W. Wolniansky, G. J. Foschini, G. D. Golden, and R. A. Valenzuela, "V-BLAST: An architecture for realizing very high data rates over the rich-scattering wireless channel," in *Proc. Int. Symp. Signals, Systems,* and Electronics, Pisa, Italy, Sep.-Oct. 29–2, 1998, pp. 295–300.
- [2] V. Tarokh, N. Seshadri, and A. R. Calderbank, "Space-time codes for high data rate wireless communication: Performance criterion and code construction," *IEEE Trans. Inf. Theory*, vol. 44, no. 2, pp. 744–765, Mar. 1998.
- [3] B. Hassibi and B. M. Hochwald, "High-rate codes that are linear in space and time," *IEEE Trans. Inf. Theory*, vol. 48, no. 7, pp. 1804–1824, Jul. 2002.
- [4] R. Y. Mesleh, H. Haas, S. Sinanović, C. W. Ahn, and S. Yun, "Spatial modulation," *IEEE Trans. Veh. Technol.*, vol. 57, pp. 2228–2241, Jul. 2008.
- [5] J. Jeganathan, A. Ghrayeb, and L. Szczecinski, "Spatial modulation: Optimal detection and performance analysis," *IEEE Commun. Lett.*, vol. 12, pp. 545–547, Aug. 2008.
- [6] J. Jeganathan, A. Ghrayeb, L. Szczecinski, and A. Ceron, "Space shift keying modulation for MIMO channels," *IEEE Trans. Wireless Commun.*, vol. 8, no. 7, pp. 3692–3703, Jul. 2009.
- [7] S. Sugiura, S. Chen, and L. Hanzo, "A unified MIMO architecture subsuming space shift keying, OSTBC, BLAST and LDC," in *Proc. VTC* 2010-Fall, Ottawa, Canada, Sep. 6–9, 2010, p. 5.
- [8] S. Sugiura, S. Chen, and L. Hanzo, "Coherent and differential space-time shift keying: A dispersion matrix approach," *IEEE Trans. Commun.*, Nov. 2010, to be published.
- [9] L. Tang, R. W. Liu, V. C. Soon, and Y. F. Huang, "Indeterminacy and identifiability of blind identification," *IEEE Trans. Circuits Syst.*, vol. 38, no. 5, pp. 499–509, May 1991.
- [10] A. Medles and D. T. M. Slock, "Semiblind channel estimation for MIMO spatial multiplexing systems," in *Proc. VTC2001-Fall*, Atlantic City, NJ, Oct. 7–11, 2001, vol. 2, pp. 1240–1244.
- [11] C. Cozzo and B. L. Hughes, "Joint channel estimation and data detection in space-time communications," *IEEE Trans. Commun.*, vol. 51, pp. 1266–1270, Aug. 2003.
- [12] S. Buzzi, M. Lops, and S. Sardellitti, "Performance of iterative data detection and channel estimation for single-antenna and multiple-antennas wireless communications," *IEEE Trans. Veh. Technol.*, vol. 53, pp. 1085–1104, Jul. 2004.
- [13] T. Wo, P. A. Hoeher, A. Scherb, and K. D. Kammeyer, "Performance analysis of maximum-likelihood semiblind estimation of MIMO channels," in *Proc. VTC2006-Spring*, Melbourne, Australia, May 7–10, 2006, vol. 4, pp. 1738–1742.
- [14] M. Abuthinien, S. Chen, and L. Hanzo, "Semi-blind joint maximum likelihood channel estimation and data detection for MIMO systems," *IEEE Signal Processing Lett.*, vol. 15, pp. 202–205, 2008.
- [15] H. R. Palally, S. Chen, W. Yao, and L. Hanzo, "Particle swarm optimisation aided semi-blind joint maximum likelihood channel estimation and data detection for MIMO systems," in *Proc. SSP09*, Cardiff, UK, Aug.-Sep. 31–3, 2009, pp. 309–312.
- [16] E. Viterbo and J. Boutros, "A universal lattice code decoder for fading channels," *IEEE Trans. Inf. Theory*, vol. 45, no. 5, pp. 1639–1642, Jul. 1999.
- [17] L. Wang, L. Xu, S. Chen, and L. Hanzo, "Three-stage irregular convolutional coded iterative centre-shifting K-best sphere decoding for soft-decision SDMA-OFDM," *IEEE Trans. Veh. Technol.*, vol. 58, pp. 2013–2109, May 2009.
- [18] M. Jiang and L. Hanzo, "Unitary linear dispersion code design and optimisation for MIMO communication systems," *IEEE Signal Process. Lett.*, vol. 17, pp. 497–500, May 2010.
- [19] S. Ng and L. Hanzo, "On the MIMO channel capacity of multidimensional signal sets," *IEEE Trans. Veh. Technol.*, vol. 55, pp. 528–536, Mar. 2006.

¹This implies that at low SNR values the proposed semi-blind scheme with the minimum number of training blocks may only achieve a BER performance which is slightly higher than those of other training based schemes, such as the minimum mean square error (MMSE) criterion based channel estimator and the robust decision-directed channel prediction scheme. Nevertheless, the proposed semi-blind scheme attains a much higher bandwidth efficiency as a benefit of its reduced pilot overhead in comparison to training based schemes.