Guaranteed cost control of linear uncertain time-delay switched singular systems based on an LMI approach

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Abstract: This contribution considers the guaranteed cost control for a class of linear uncertain time-delay switched singular systems under arbitrary switching laws with a given quadratic performance index. Based on a Lyapunov function approach and a linear matrix inequality (LMI) technique, a sufficient condition on the existence of guaranteed cost state feedback controllers is derived, which ensures that the uncertain time-delay switched singular system is admissible and a desired level of cost index can be guaranteed. The design problem of guaranteed cost controller is turned into the solvable problem of a set of LMIs. A numerical example is given to show the effectiveness and feasibility of the presented method.

Keywords: switched singular system; uncertainty; time-delay; guaranteed cost control; linear matrix inequality.

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1 Introduction

Singular systems, also referred to as implicit systems, descriptor systems or generalised state-space systems, have extensive applications in many practical systems, such as circuit boundary control systems, chemical processes, electrical networks, economy systems and other areas (Cobb, 1983; Lewis, 1986; Dai, 1989). In recent years, much attention has been focused on singular systems (Xu et al., 2001, 2002; Ishihara and Terra, 2002; Liu and Yang, 2010), and a number of fundamental notions and results in control and system theory, originally developed for standard state-space systems, have been extended successfully to singular systems. On the other hand, there has been an increasing interest recently in stability analysis and controller design for switched systems, see the survey papers (Branicky, 1998; Liberzon and Morse, 1999; Skafiads et al., 1999; Decarlo et al., 2000; Daafouz et al., 2002; Sun and Ge, 2005a; Lin and Antsaklis, 2009), the books (Liberzon, 2003; Sun and Ge, 2005b), and the references cited therein. One motivation for studying switched systems is that many practical systems are inherently multi-modal in the sense that several dynamical subsystems are required to describe their behaviours which may depend on various environmental factors (Koumboulis et al., 2011). Another important motivation is that switching among a set of controllers for a specified system can be regarded as a switched system. Switching has been used in adaptive control to ensure stability in the situations where stability otherwise cannot be proved, or to improve transient response of adaptive control systems. Many methods of intelligent control design may also be regarded as based on the idea of switching among different controllers (Xiong and Li, 2010).

It can be observed from the above discussion that switched descriptor systems belong to an important class of systems that are interesting in both theoretic and practical sense. However, since stability, regularity, impulse elimination and state consistence must be considered at the same time, switched singular systems are difficult to analyse, and a few available results include Meng and Zhang (2006a, 2006b, 2006c), Xie and Wang (2004), Fu and Fei (2008), Narendra and Balakrishnan (1994), Zhai et al. (2009a, 2009b) and Zhai and Xu (2010). More specifically, the reachability for continue-time and discrete-time switched singular systems were considered in Meng and Zhang (2006a, 2006b), respectively, while the work (Meng and Zhang, 2006c) studied output feedback stabilisation for discrete-time switched singular systems based on linear matrix inequality (LMI) techniques. The study (Xie and Wang, 2004) analysed the stability and stabilisation of switched singular systems in discrete-time domain. Based on common Lyapunov function approaches and convex combination techniques, Fu and Fei (2008) investigated the output feedback control problem for a class of uncertain switched singular systems. Zhai and Xu (2010) proposed a new commutation condition for stability analysis of switched linear singular systems, which is a natural extension to the commutation conditions discussed in Zhai et al. (2009a, 2009b).

Guaranteed cost control approach (Chang and Peng, 1972) is a practical way of designing a control system to achieve a desired-level of robust performance. In the work (Yu and Chu, 1999), this approach was applied to study linear time-delay systems based on an LMI approach, while the approach was investigated for linear repetitive systems in Paszke et al. (2006). Guaranteed cost control for linear descriptor systems was also studied by Fu et al. (2006), while Wang and Zhao (2007) considered guaranteed cost control for a class of linear switched delay systems. The approach was also found its application in networked control systems (Wang et al., 2010) and other control systems (Meng et al., 2010). Against this background and motivated by the existing results of guaranteed cost control as well as linear switched singular systems, this contribution investigates the guaranteed cost control for a class of linear time-delay switched singular systems with time-varying norm-bounded parameter uncertainty based on an LMI approach. A sufficient condition for the existence of memoryless state feedback guaranteed cost controllers is derived. It is shown that the design problem of guaranteed cost controller for the uncertain time-delay switched singular system is equivalent to solving a set of LMIs, whose solutions provide parametrised representations of guaranteed cost controllers. This design process is computationally simple.

The rest of this contribution is organised as follows. In Section 2, we provide notation definitions and the problem formulation, while our main results are presented and proved in Section 3. A numerical example is given in Section 4 to demonstrate our approach, and some concluding remarks are offered in Section 5.

2 Problem formulation

Consider a class of linear uncertain time-delay switched singular systems governed by the following state-space equation:

$$\begin{cases} \boldsymbol{E}\dot{\boldsymbol{x}}(t) = \big(\boldsymbol{A}_{\sigma(t)} + \Delta\boldsymbol{A}_{\sigma(t)}\big)\boldsymbol{x}(t) \\ \qquad\quad + \big(\boldsymbol{A}_{d\sigma(t)} + \Delta\boldsymbol{A}_{d\sigma(t)}\big)\boldsymbol{x}(t-d) \\ \qquad\quad + \big(\boldsymbol{B}_{\sigma(t)} + \Delta\boldsymbol{B}_{\sigma(t)}\big)\boldsymbol{u}(t) \\ \qquad\quad + \big(\boldsymbol{B}_{h\sigma(t)} + \Delta\boldsymbol{B}_{h\sigma(t)}\big)\boldsymbol{u}(t-h), \\ \boldsymbol{x}(t) = \boldsymbol{\phi}(t),\ t \in [-t_0,\ 0],\ t_0 \triangleq \max\{d, h\}. \end{cases} \tag{1}$$

Here, $\boldsymbol{x}(t) \in \mathbb{R}^{n_x}$ and $\boldsymbol{u}(t) \in \mathbb{R}^{n_u}$ are the state and control input vectors, respectively, while $d > 0$ and $h > 0$ are their respective delay constants. $\boldsymbol{\phi}(t)$ in (1) is a given continuous vector-valued initial function. The right continuous function

$$\sigma(t) : [0,\ \infty) \to \mathbb{N} \triangleq \{1, 2, \cdots, m\} \tag{2}$$

is the switching rule to be designed, where m is the number of subsystems. Thus, $\sigma(t) = i$ indicates that the i^{th} subsystem is active. $\boldsymbol{A}_i$, $\boldsymbol{B}_i$, $\boldsymbol{A}_{di}$ and $\boldsymbol{B}_{hi}$, $i \in \mathbb{N}$, are the known constant real matrices of appropriate dimensions, while $\Delta\boldsymbol{A}_i$, $\Delta\boldsymbol{B}_i$, $\Delta\boldsymbol{A}_{di}$ and $\Delta\boldsymbol{B}_{hi}$, $i \in \mathbb{N}$, are the matrix-valued functions representing the time-varying parameter uncertainties in the system model. The matrix $\boldsymbol{E} \in \mathbb{R}^{n_x \times n_x}$ may be singular with $\text{rank}\big(\boldsymbol{E}\big) \le n_x$.

The parameter uncertainties are assumed to be norm bounded, which take the form

$$\begin{aligned}&\big[\Delta\boldsymbol{A}_i\ \Delta\boldsymbol{A}_{di}\ \Delta\boldsymbol{B}_i\ \Delta\boldsymbol{B}_{hi}\big]\\&=\boldsymbol{D}_i\boldsymbol{\Sigma}_i(t)\big[\boldsymbol{F}_{1i}\ \boldsymbol{F}_{di}\ \boldsymbol{F}_{2i}\ \boldsymbol{F}_{hi}\big],\end{aligned} \tag{3}$$

$\forall i \in \mathbb{N}$, where $\boldsymbol{D}_i$, $\boldsymbol{F}_{1i}$, $\boldsymbol{F}_{di}$, $\boldsymbol{F}_{2i}$ and $\boldsymbol{F}_{hi}$ are the known constant real matrices of appropriate dimensions, which represent the structure of uncertainties, while $\boldsymbol{\Sigma}_i(t)$ is an unknown matrix function of appropriate dimension with Lebesgue measurable elements and satisfies

$$\boldsymbol{\Sigma}_i^{\mathrm{T}}(t)\boldsymbol{\Sigma}_i(t) \le \boldsymbol{I} \tag{4}$$

in which $\boldsymbol{I}$ denotes the identity matrix of appropriate dimension. The notation in (4) means that the matrix $\boldsymbol{I} - \boldsymbol{\Sigma}_i^{\mathrm{T}}(t)\boldsymbol{\Sigma}_i(t)$ is semi-positive definite.

The autonomous linear switched singular system associated with the system (1) can be written as

$$\boldsymbol{E}\dot{\boldsymbol{x}}(t) = \boldsymbol{A}_i\boldsymbol{x}(t),\ i \in \mathbb{N}. \tag{5}$$

Definition 1: The switched singular system (5) is said to be regular if $\det\big(s\boldsymbol{E} - \boldsymbol{A}_i\big)$ is not identically zero, $i \in \mathbb{N}$.

Definition 2: The switched singular system (5) is said to be impulse free if it is regular and
$\deg\big(\det\big(s\boldsymbol{E} - \boldsymbol{A}_i\big)\big) = \text{rank}(\boldsymbol{E})$, $i \in \mathbb{N}$.

All the switched singular systems discussed in this contribution are assumed to be impulse free.

Let the quadratic cost function associated with the system (1) be defined by

$$J = \int_0^{\infty} \big(\boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{Q}\boldsymbol{x}(t) + \boldsymbol{u}^{\mathrm{T}}(t)\boldsymbol{R}\boldsymbol{u}(t)\big)dt, \tag{6}$$

where $\boldsymbol{Q}$ and $\boldsymbol{R}$ are the given symmetric positive-definite weighting matrices.

Definition 3: For the uncertain system (1), if there exist a control law $\boldsymbol{u}(t)$ and a positive J^* such that, for all the admissible uncertainties, the closed-loop system is stable and the value of the cost function (6) associated with the closed-loop system satisfies $J \le J^*$, then $\boldsymbol{u}(t)$ is said to be a guaranteed cost control law for the uncertain system (1) with the guaranteed cost J^*.

Our objective is to develop a procedure to design a memoryless state feedback guaranteed cost control law $\boldsymbol{u}(t) = \boldsymbol{K}_{\sigma(t)}\boldsymbol{x}(t)$ for the linear uncertain time-delay switched singular system (1). Before presenting our results, we have the following two lemmas.

Lemma 1 (see Petersen, 1987): Give a symmetric matrix $\boldsymbol{Y}$ as well as the two matrices $\boldsymbol{D}$ and $\boldsymbol{F}$ with appropriate dimensions. Then

$$\boldsymbol{Y} + \boldsymbol{D}\boldsymbol{\Sigma}\boldsymbol{F} + \boldsymbol{F}^{\mathrm{T}}\boldsymbol{\Sigma}^{\mathrm{T}}\boldsymbol{D}^{\mathrm{T}} < \boldsymbol{0}$$

for all $\boldsymbol{\Sigma}$ satisfying $\boldsymbol{\Sigma}^{\mathrm{T}}\boldsymbol{\Sigma} \le \boldsymbol{I}$, if and only if there exists $\lambda > 0$ such that

$$\boldsymbol{Y} + \lambda\boldsymbol{F}^{\mathrm{T}}\boldsymbol{F} + \lambda^{-1}\boldsymbol{D}\boldsymbol{D}^{\mathrm{T}} < \boldsymbol{0},$$

where $\boldsymbol{0}$ denotes the zero matrix of appropriate dimension, and the notation $\boldsymbol{Y} > \boldsymbol{0}$ ($< \boldsymbol{0}$) indicates that $\boldsymbol{Y}$ is positive (negative) definite.

Lemma 2 (Schur complement): For a given symmetric matrix with the partition

$$\boldsymbol{S} = \begin{bmatrix}\boldsymbol{S}_{11} & \boldsymbol{S}_{12}\\ \boldsymbol{S}_{21} & \boldsymbol{S}_{22}\end{bmatrix},$$

where $\boldsymbol{S}_{11}$ is a square matrix and $\boldsymbol{S}_{12}^{\mathrm{T}} = \boldsymbol{S}_{21}$, the following three conditions are equivalent:

1. $\boldsymbol{S} < \boldsymbol{0}$;
2. $\boldsymbol{S}_{11} < \boldsymbol{0}$ and $\boldsymbol{S}_{22} - \boldsymbol{S}_{12}^{\mathrm{T}}\boldsymbol{S}_{11}^{-1}\boldsymbol{S}_{12} < \boldsymbol{0}$;
3. $\boldsymbol{S}_{22} < \boldsymbol{0}$ and $\boldsymbol{S}_{11} - \boldsymbol{S}_{12}\boldsymbol{S}_{22}^{-1}\boldsymbol{S}_{12}^{\mathrm{T}} < \boldsymbol{0}$.

3 Main results

We first present a sufficient condition for the existence of memoryless state feedback guaranteed cost control laws for the system (1) with the parameter uncertainties (3).

Theorem 1: For the system (1) with the cost function (6), if there exist an invertible matrix $\boldsymbol{P}$, two symmetric positive-definite matrices $\boldsymbol{S}$ and $\boldsymbol{W}$, and matrices $\boldsymbol{K}_i$, $i \in \mathbb{N}$, of appropriate dimensions, such that the following matrix inequalities are satisfied

$$\boldsymbol{E}^{\mathrm{T}}\boldsymbol{P} = \boldsymbol{P}^{\mathrm{T}}\boldsymbol{E} \ge \boldsymbol{0}, \tag{7}$$

$$\begin{bmatrix}\boldsymbol{\Theta}_i + \boldsymbol{Q} + \boldsymbol{K}_i^{\mathrm{T}}\boldsymbol{R}\boldsymbol{K}_i & \boldsymbol{V}_1 & \boldsymbol{V}_2\\ \boldsymbol{V}_1^{\mathrm{T}} & -\boldsymbol{S} & \boldsymbol{0}\\ \boldsymbol{V}_2^{\mathrm{T}} & \boldsymbol{0} & -\boldsymbol{W}\end{bmatrix} < \boldsymbol{0}, \tag{8}$$

where

$$\boldsymbol{V}_1 = \boldsymbol{P}^{\mathrm{T}}\big(\boldsymbol{A}_{di} + \Delta\boldsymbol{A}_{di}\big), \tag{9}$$

$$\boldsymbol{V}_2 = \boldsymbol{P}^{\mathrm{T}}\big(\boldsymbol{B}_{di} + \Delta\boldsymbol{B}_{di}\big), \tag{10}$$

and

$$\boldsymbol{\Theta}_i = \boldsymbol{\Xi}_i^{\mathrm{T}}\boldsymbol{P} + \boldsymbol{P}^{\mathrm{T}}\boldsymbol{\Xi}_i + \boldsymbol{S} + \boldsymbol{K}_i^{\mathrm{T}}\boldsymbol{W}\boldsymbol{K}_i \tag{11}$$

with

$$\boldsymbol{\Xi}_i = \boldsymbol{A}_i + \Delta\boldsymbol{A}_i + \boldsymbol{B}_i\boldsymbol{K}_i + \Delta\boldsymbol{B}_i\boldsymbol{K}_i, \tag{12}$$

then the state feedback control law

$$\boldsymbol{u}(t) = \boldsymbol{K}_i\boldsymbol{x}(t),\ i \in \mathbb{N}, \tag{13}$$

is a guaranteed cost control law, and

$$\begin{aligned}J^* = {}&\boldsymbol{\phi}^{\mathrm{T}}(0)\boldsymbol{E}^{\mathrm{T}}\boldsymbol{P}\boldsymbol{\phi}(0) + \int_{-d}^{0}\boldsymbol{\phi}^{\mathrm{T}}(\tau)\boldsymbol{S}\boldsymbol{\phi}(\tau)d\tau\\&+ \max_{i\in\mathbb{N}}\int_{-h}^{0}\boldsymbol{\phi}^{\mathrm{T}}(\tau)\boldsymbol{K}_i^{\mathrm{T}}\boldsymbol{W}\boldsymbol{K}_i\boldsymbol{\phi}(\tau)d\tau\end{aligned} \tag{14}$$

is a guaranteed cost for the uncertain system (1).

Proof: By substituting $\boldsymbol{u}(t)=\boldsymbol{K}_i\boldsymbol{x}(t),\ i\in\mathbb{N}$, in (1), the resulting closed-loop system is

$$\begin{aligned}\boldsymbol{E}\dot{\boldsymbol{x}}(t)=&\big(\boldsymbol{A}_i+\Delta\boldsymbol{A}_i+\boldsymbol{B}_i\boldsymbol{K}_i+\Delta\boldsymbol{B}_i\boldsymbol{K}_i\big)\boldsymbol{x}(t)\\&+\big(\boldsymbol{A}_{di}+\Delta\boldsymbol{A}_{di}\big)\boldsymbol{x}(t-d)\\&+\big(\boldsymbol{B}_{hi}+\Delta\boldsymbol{B}_{hi}\big)\boldsymbol{K}_i\boldsymbol{x}(t-h).\end{aligned}\tag{15}$$

Suppose that there exist an invertible matrix $\boldsymbol{P}$, and the two symmetric matrices $\boldsymbol{S}>\boldsymbol{0}$ and $\boldsymbol{W}>\boldsymbol{0}$, as well as $\boldsymbol{K}_i$, $i\in\mathbb{N}$, such that the matrix inequalities (7) and (8) hold for all the admissible uncertainties. Now define the Lyapunov function

$$\begin{aligned}V(\boldsymbol{x}(t))=&\boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{E}^{\mathrm{T}}\boldsymbol{P}\boldsymbol{x}(t)+\textstyle\int_{t-d}^{t}\boldsymbol{x}^{\mathrm{T}}(\tau)\boldsymbol{S}\boldsymbol{x}(\tau)d\tau\\&+\textstyle\int_{t-h}^{t}\boldsymbol{x}^{\mathrm{T}}(\tau)\boldsymbol{K}_{\sigma(\tau)}^{\mathrm{T}}\boldsymbol{W}\boldsymbol{K}_{\sigma(\tau)}\boldsymbol{x}(\tau)d\tau.\end{aligned}\tag{16}$$

Then the time derivative of $V(\cdot)$ along any trajectory of the closed-loop system (15) is given by

$$\begin{aligned}&L(\boldsymbol{x}(t);t)\\&=\boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{\Theta}_i\boldsymbol{x}(t)+\boldsymbol{x}^{\mathrm{T}}(t-d)\boldsymbol{V}_1^{\mathrm{T}}\boldsymbol{x}(t)\\&\quad+\boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{V}_1\boldsymbol{x}(t-d)+\boldsymbol{x}^{\mathrm{T}}(t-h)\boldsymbol{K}_i^{\mathrm{T}}\boldsymbol{V}_2^{\mathrm{T}}\boldsymbol{x}(t)\\&\quad+\boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{V}_2\boldsymbol{K}_i\boldsymbol{x}(t-h)-\boldsymbol{x}^{\mathrm{T}}(t-d)\boldsymbol{S}\boldsymbol{x}(t-d)\\&\quad-\boldsymbol{x}^{\mathrm{T}}(t-h)\boldsymbol{K}_i^{\mathrm{T}}\boldsymbol{W}\boldsymbol{K}_i\boldsymbol{x}(t-h)\\&=\begin{bmatrix}\boldsymbol{x}(t)\\\boldsymbol{x}(t-d)\\\boldsymbol{K}_i\boldsymbol{x}(t-h)\end{bmatrix}^{\mathrm{T}}\begin{bmatrix}\boldsymbol{\Theta}_i&\boldsymbol{V}_1&\boldsymbol{V}_2\\\boldsymbol{V}_1^{\mathrm{T}}&-\boldsymbol{S}&\boldsymbol{0}\\\boldsymbol{V}_2^{\mathrm{T}}&\boldsymbol{0}&-\boldsymbol{W}\end{bmatrix}\begin{bmatrix}\boldsymbol{x}(t)\\\boldsymbol{x}(t-d)\\\boldsymbol{K}_i\boldsymbol{x}(t-h)\end{bmatrix}\end{aligned}\tag{17}$$

where $\boldsymbol{V}_1$, $\boldsymbol{V}_2$ and $\boldsymbol{\Theta}_i$ are defined in (9), (10) and (11), respectively. The matrix inequality (8) implies that

$$L(\boldsymbol{x}(t);t)<\boldsymbol{x}^{\mathrm{T}}(t)\big(-\boldsymbol{Q}-\boldsymbol{K}_i^{\mathrm{T}}\boldsymbol{R}\boldsymbol{K}_i\big)\boldsymbol{x}(t)<0.\tag{18}$$

Therefore, the closed-loop system (15) is asymptotically stable. Furthermore, by integrating the both sides of the inequality (18) from 0 to T_t and using the initial condition, we obtain

$$\begin{aligned}&-\int_0^{T_t}\boldsymbol{x}^{\mathrm{T}}(t)\big(\boldsymbol{Q}+\boldsymbol{K}_i^{\mathrm{T}}\boldsymbol{R}\boldsymbol{K}_i\big)\boldsymbol{x}(t)dt\\&>\boldsymbol{x}^{\mathrm{T}}(T_t)\boldsymbol{E}^{\mathrm{T}}\boldsymbol{P}\boldsymbol{x}(T_t)-\boldsymbol{x}^{\mathrm{T}}(0)\boldsymbol{E}^{\mathrm{T}}\boldsymbol{P}\boldsymbol{x}(0)\\&+\int_{T_t-d}^{T_t}\boldsymbol{x}^{\mathrm{T}}(\tau)\boldsymbol{S}\boldsymbol{x}(\tau)d\tau-\int_{-d}^{0}\boldsymbol{x}^{\mathrm{T}}(\tau)\boldsymbol{S}\mathbf{x}(\tau)d\tau\\&+\int_{T_t-h}^{T_t}\boldsymbol{x}^{\mathrm{T}}(\tau)\boldsymbol{K}_{\sigma(\tau)}^{\mathrm{T}}\boldsymbol{W}\boldsymbol{K}_{\sigma(\tau)}\boldsymbol{x}(\tau)d\tau\\&-\int_{-h}^{0}\boldsymbol{x}^{\mathrm{T}}(\tau)\boldsymbol{K}_{\sigma(\tau)}^{\mathrm{T}}\boldsymbol{W}\boldsymbol{K}_{\sigma(\tau)}\boldsymbol{x}(\tau)d\tau.\end{aligned}\tag{19}$$

As the closed-loop system (15) is asymptotically stable,

$$\lim_{T_t\to\infty}\boldsymbol{x}^{\mathrm{T}}(T_t)\boldsymbol{E}^{\mathrm{T}}\boldsymbol{P}\boldsymbol{x}(T_t)=0,$$

$$\lim_{T_t\to\infty}\int_{T_t-d}^{T_t}\boldsymbol{x}^{\mathrm{T}}(\tau)\boldsymbol{S}\boldsymbol{x}(\tau)d\tau=0,$$

$$\lim_{T_t\to\infty}\int_{T_t-h}^{T_t}\boldsymbol{x}^{\mathrm{T}}(\tau)\boldsymbol{K}_{\sigma(\tau)}^{\mathrm{T}}\boldsymbol{W}\boldsymbol{K}_{\sigma(\tau)}\boldsymbol{x}(\tau)d\tau=0.$$

Hence, we obtain

$$\begin{aligned}&\int_0^{\infty}\big(\boldsymbol{x}^{\mathrm{T}}(t)\boldsymbol{Q}\boldsymbol{x}(t)+\boldsymbol{u}^{\mathrm{T}}(t)\boldsymbol{R}\boldsymbol{u}(t)\big)dt\le\boldsymbol{\phi}^{\mathrm{T}}(0)\boldsymbol{E}^{\mathrm{T}}\boldsymbol{P}\boldsymbol{\phi}(0)\\&\quad+\int_{-d}^{0}\boldsymbol{\phi}^{\mathrm{T}}(\tau)\boldsymbol{S}\boldsymbol{\phi}(\tau)d\tau\\&\quad+\int_{-h}^{0}\boldsymbol{\phi}^{\mathrm{T}}(\tau)\boldsymbol{K}_{\sigma(\tau)}^{\mathrm{T}}\boldsymbol{W}\boldsymbol{K}_{\sigma(\tau)}\boldsymbol{\phi}(\tau)d\tau\\&\le\boldsymbol{\phi}^{\mathrm{T}}(0)\boldsymbol{E}^{\mathrm{T}}\boldsymbol{P}\boldsymbol{\phi}(0)+\int_{-d}^{0}\boldsymbol{\phi}^{\mathrm{T}}(\tau)\boldsymbol{S}\boldsymbol{\phi}(\tau)d\tau\\&\quad+\max_{i\in\mathbb{N}}\int_{-h}^{0}\boldsymbol{\phi}^{\mathrm{T}}(\tau)\boldsymbol{K}_i^{\mathrm{T}}\boldsymbol{W}\boldsymbol{K}_i\boldsymbol{\phi}(\tau)d\tau.\end{aligned}\tag{20}$$

It follows from Definition 3 that the result of the theorem is true. This completes the proof. □

Next we prove that the above sufficient condition for the existence of guaranteed cost controllers is equivalent to a solvable system of LMIs.

Theorem 2: For the system (1) with the parameter uncertainties (3), if there exist scalars $\lambda_i>0,\ i\in\mathbb{N}$, an invertible matrix $\boldsymbol{X}$, two symmetric positive-definite matrices $\boldsymbol{G}$ and $\boldsymbol{H}$, and matrices $\boldsymbol{M}_i$, $i\in\mathbb{N}$, such that the following LMIs are satisfied

$$\boldsymbol{E}^{\mathrm{T}}\boldsymbol{X}^{-1}=\boldsymbol{X}^{-\mathrm{T}}\boldsymbol{E}\ge\boldsymbol{0},\tag{21}$$

$$\begin{bmatrix}\boldsymbol{\Psi}_i&\boldsymbol{A}_{di}\boldsymbol{G}&\boldsymbol{B}_{hi}\boldsymbol{H}&\boldsymbol{X}^{\mathrm{T}}&\boldsymbol{M}_i^{\mathrm{T}}&\boldsymbol{X}^{\mathrm{T}}&\boldsymbol{M}_i^{\mathrm{T}}&\boldsymbol{C}_i^{\mathrm{T}}\\\boldsymbol{G}\boldsymbol{A}_{di}^{\mathrm{T}}&-\boldsymbol{G}&0&0&0&0&0&\boldsymbol{G}\boldsymbol{F}_{di}^{\mathrm{T}}\\\boldsymbol{H}\boldsymbol{B}_{hi}^{\mathrm{T}}&0&-\boldsymbol{H}&0&0&0&0&\boldsymbol{H}\boldsymbol{F}_{hi}^{\mathrm{T}}\\\boldsymbol{X}&0&0&-\boldsymbol{G}&0&0&0&0\\\boldsymbol{M}_i&0&0&0&-\boldsymbol{H}&0&0&0\\\boldsymbol{X}&0&0&0&0&-\boldsymbol{Q}^{-1}&0&0\\\boldsymbol{M}_i&0&0&0&0&0&-\boldsymbol{R}^{-1}&0\\\boldsymbol{C}_i&\boldsymbol{F}_{di}\boldsymbol{G}&\boldsymbol{F}_{hi}\boldsymbol{H}&0&0&0&0&-\lambda_i^{-1}\boldsymbol{I}\end{bmatrix}<\boldsymbol{0},\tag{22}$$

where all the matrices concerned have appropriate dimensions and

$$\boldsymbol{C}_i=\boldsymbol{F}_{1i}\boldsymbol{X}+\boldsymbol{F}_{2i}\boldsymbol{M}_i,\tag{23}$$

$$\begin{aligned}\boldsymbol{\Psi}_i=&\big(\boldsymbol{A}_i\boldsymbol{X}+\boldsymbol{B}_i\boldsymbol{M}_i\big)^{\mathrm{T}}+\boldsymbol{A}_i\boldsymbol{X}\\&+\boldsymbol{B}_i\boldsymbol{M}_i+\lambda_i^{-1}\boldsymbol{D}_i\boldsymbol{D}_i^{\mathrm{T}},\end{aligned}\tag{24}$$

then the state feedback control law

$$\boldsymbol{u}(t)=\boldsymbol{M}_i\boldsymbol{X}^{-1}\boldsymbol{x}(t),\ i\in\mathbb{N},\tag{25}$$

is a guaranteed cost control law, and

$$\begin{aligned}J^*=&\boldsymbol{\phi}^{\mathrm{T}}(0)\boldsymbol{E}^{\mathrm{T}}\boldsymbol{X}^{-1}\boldsymbol{\phi}(0)\\&+\int_{-d}^{0}\boldsymbol{\phi}^{\mathrm{T}}(\tau)\boldsymbol{G}^{-1}\boldsymbol{\phi}(\tau)d\tau\\&+\max_{i\in\mathbb{N}}\int_{-h}^{0}\boldsymbol{\phi}^{\mathrm{T}}(\tau)\boldsymbol{K}_i^{\mathrm{T}}\boldsymbol{H}^{-1}\boldsymbol{K}_i\boldsymbol{\phi}(\tau)d\tau\end{aligned}\tag{26}$$

is a guaranteed cost for the uncertain system (1), where $\boldsymbol{K}_i=\boldsymbol{M}_i\boldsymbol{X}^{-1}$, $i\in\mathbb{N}$.

Proof: Define

$$\boldsymbol{Y}=\begin{bmatrix} \overline{\boldsymbol{\Omega}}_i & \boldsymbol{P}^{\mathrm{T}}\boldsymbol{A}_{di} & \boldsymbol{P}^{\mathrm{T}}\boldsymbol{B}_{hi} & \boldsymbol{I} & \boldsymbol{K}_i^{\mathrm{T}} & \boldsymbol{I} & \boldsymbol{K}_i^{\mathrm{T}} \\ \boldsymbol{A}_{di}^{\mathrm{T}}\boldsymbol{P} & -\boldsymbol{S} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{B}_{hi}^{\mathrm{T}}\boldsymbol{P} & \boldsymbol{0} & -\boldsymbol{W} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{I} & \boldsymbol{0} & \boldsymbol{0} & -\boldsymbol{S}^{-1} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{K}_i & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & -\boldsymbol{W}^{-1} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{I} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & -\boldsymbol{Q}^{-1} & \boldsymbol{0} \\ \boldsymbol{K}_i & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & -\boldsymbol{R}^{-1} \end{bmatrix}, \quad (27)$$

where

$$\overline{\boldsymbol{\Omega}}_i=\left(\boldsymbol{A}_i+\boldsymbol{B}_i\boldsymbol{K}_i\right)^{\mathrm{T}}\boldsymbol{P}+\boldsymbol{P}^{\mathrm{T}}\left(\boldsymbol{A}_i+\boldsymbol{B}_i\boldsymbol{K}_i\right). \quad (28)$$

By the Schur complement, the matrix inequality (8) is equivalent to

$$\begin{aligned}&\mathbf{Y}+\left[\overline{\boldsymbol{C}}_i\ \boldsymbol{F}_{di}\ \boldsymbol{F}_{hi}\ \boldsymbol{0}\ \boldsymbol{0}\ \boldsymbol{0}\ \boldsymbol{0}\right]^{\mathrm{T}}\boldsymbol{\Sigma}_i^{\mathrm{T}}(t)\left[\boldsymbol{D}_i^{\mathrm{T}}\boldsymbol{P}\ \boldsymbol{0}\ \boldsymbol{0}\ \boldsymbol{0}\ \boldsymbol{0}\ \boldsymbol{0}\ \boldsymbol{0}\right]\\ &+\left[\boldsymbol{D}_i^{\mathrm{T}}\boldsymbol{P}\ \boldsymbol{0}\ \boldsymbol{0}\ \boldsymbol{0}\ \boldsymbol{0}\ \boldsymbol{0}\ \boldsymbol{0}\right]^{\mathrm{T}} \quad (29)\\ &\quad\boldsymbol{\Sigma}_i(t)\left[\overline{\boldsymbol{C}}_i\ \boldsymbol{F}_{di}\ \boldsymbol{F}_{hi}\ \boldsymbol{0}\ \boldsymbol{0}\ \boldsymbol{0}\ \boldsymbol{0}\right]<\boldsymbol{0},\end{aligned}$$

with

$$\overline{\boldsymbol{C}}_i=\boldsymbol{F}_{1i}+\boldsymbol{F}_{2i}\boldsymbol{K}_i. \quad (30)$$

By applying Lemma 1, the above matrix inequality holds for all the $\boldsymbol{\Sigma}_i(t)$ satisfying $\boldsymbol{\Sigma}_i^{\mathrm{T}}(t)\boldsymbol{\Sigma}_i(t)\le\boldsymbol{I},\ i\in\mathbb{N}$, if and only if there exist constants $\lambda_i>0,\ i\in\mathbb{N}$, such that

$$\begin{aligned}&\boldsymbol{Y}+\lambda_i\left[\overline{\boldsymbol{C}}_i\ \boldsymbol{F}_{di}\ \boldsymbol{F}_{hi}\ \boldsymbol{0}\ \boldsymbol{0}\ \boldsymbol{0}\ \boldsymbol{0}\right]^{\mathrm{T}}\left[\overline{\boldsymbol{C}}_i\ \boldsymbol{F}_{di}\ \boldsymbol{F}_{hi}\ \boldsymbol{0}\ \boldsymbol{0}\ \boldsymbol{0}\ \boldsymbol{0}\right]\\ &+\lambda_i^{-1}\left[\boldsymbol{D}_i^{\mathrm{T}}\boldsymbol{P}\ \boldsymbol{0}\ \boldsymbol{0}\ \boldsymbol{0}\ \boldsymbol{0}\ \boldsymbol{0}\ \boldsymbol{0}\right]^{\mathrm{T}} \quad (31)\\ &\quad\left[\boldsymbol{D}_i^{\mathrm{T}}\boldsymbol{P}\ \boldsymbol{0}\ \boldsymbol{0}\ \boldsymbol{0}\ \boldsymbol{0}\ \boldsymbol{0}\ \boldsymbol{0}\right]<\boldsymbol{0}.\end{aligned}$$

It follows from the Schur complement that the inequality (31) is equivalent to

$$\begin{bmatrix} \boldsymbol{\Omega}_i & \boldsymbol{P}^{\mathrm{T}}\boldsymbol{A}_{di} & \boldsymbol{P}^{\mathrm{T}}\boldsymbol{B}_{hi} & \boldsymbol{I} & \boldsymbol{K}_i^{\mathrm{T}} & \boldsymbol{I} & \boldsymbol{K}_i^{\mathrm{T}} & \overline{\boldsymbol{C}}_i^{\mathrm{T}} \\ \boldsymbol{A}_{di}^{\mathrm{T}}\boldsymbol{P} & -\boldsymbol{S} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{F}_{di}^{\mathrm{T}} \\ \boldsymbol{B}_{hi}^{\mathrm{T}}\boldsymbol{P} & \boldsymbol{0} & -\boldsymbol{W} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{F}_{hi}^{\mathrm{T}} \\ \boldsymbol{I} & \boldsymbol{0} & \boldsymbol{0} & -\boldsymbol{S}^{-1} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{K}_i & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & -\boldsymbol{W}^{-1} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{I} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & -\boldsymbol{Q}^{-1} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{K}_i & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & -\boldsymbol{R}^{-1} & \boldsymbol{0} \\ \overline{\boldsymbol{C}}_i & \boldsymbol{F}_{di} & \boldsymbol{F}_{hi} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & -\lambda_i^{-1}\boldsymbol{I} \end{bmatrix} \quad (32)$$
$$<\boldsymbol{0},$$

where $\overline{\boldsymbol{C}}_i$ is defined in (30), and

$$\boldsymbol{\Omega}_i=\overline{\boldsymbol{\Omega}}_i+\lambda_i^{-1}\boldsymbol{P}^{\mathrm{T}}\boldsymbol{D}_i\boldsymbol{D}_i^{\mathrm{T}}\boldsymbol{P}. \quad (33)$$

Pre- and post-multiplying the both sides of the matrix inequality (32) by $\mathrm{diag}\{\boldsymbol{P}^{-\mathrm{T}},\boldsymbol{S}^{-1},\boldsymbol{W}^{-1},\boldsymbol{I},\boldsymbol{I},\boldsymbol{I},\boldsymbol{I},\boldsymbol{I}\}$ and $\mathrm{diag}\{\boldsymbol{P}^{-1},\boldsymbol{S}^{-1},\boldsymbol{W}^{-1},\boldsymbol{I},\boldsymbol{I},\boldsymbol{I},\boldsymbol{I},\boldsymbol{I}\}$, respectively, as well as denoting $\boldsymbol{X}=\boldsymbol{P}^{-1}$, $\boldsymbol{M}_i=\boldsymbol{K}_i\boldsymbol{X}$, $\boldsymbol{G}=\boldsymbol{S}^{-1}$ and $\boldsymbol{H}=\boldsymbol{W}^{-1}$ yield the LIMs (21) and (22).

Following the proof of Theorem 1, it can then be concluded that this theorem is true. This completes the proof. □

Remark 1: Theorem 2 shows that when the LMIs defined in (21) and (22) have a solution, the guaranteed cost control law, as specified in (25), can be obtained with the corresponding guaranteed cost of (26). Therefore, the design of a guaranteed cost controller for the linear uncertain time-delay switched singular system specified by (1) and (3) is equivalent to solving a set of the LMIs. Convex optimisation involved in solving LMIs (Boyd et al., 1994) makes this design process practical and computationally attractive.

4 A numerical example

An example was given to illustrate the proposed design method. The linear uncertain time-delay switched singular system considered was given by

$$\begin{cases}\boldsymbol{E}\dot{\boldsymbol{x}}(t)=\left(\boldsymbol{A}_i+\Delta\boldsymbol{A}_i\right)\boldsymbol{x}(t)+\left(\boldsymbol{A}_{di}+\Delta\boldsymbol{A}_{di}\right)\boldsymbol{x}(t-d)\\ \qquad+\left(\boldsymbol{B}_i+\Delta\boldsymbol{B}_i\right)\boldsymbol{u}(t)+\left(\boldsymbol{B}_{hi}+\Delta\boldsymbol{B}_{hi}\right)\boldsymbol{u}(t-h),\\ \boldsymbol{x}(t)=\boldsymbol{\phi}(t),\ t\in[-t_0,\ 0],\ t_0\triangleq\max\{d,h\},\end{cases}$$

for $i=1,2$, with $n_x=n_u=2$ and the parameters

$$\boldsymbol{E}=\begin{bmatrix}0&0\\0&1\end{bmatrix},\ \boldsymbol{A}_1=\begin{bmatrix}2&0\\0&-3\end{bmatrix},\ \boldsymbol{B}_1=\begin{bmatrix}2&0\\0&2\end{bmatrix},$$

$$\boldsymbol{A}_{d1}=\begin{bmatrix}0&0.2\\0.2&0\end{bmatrix},\boldsymbol{B}_{h1}=\begin{bmatrix}0.3&0.2\\0.2&0.4\end{bmatrix},\boldsymbol{A}_2=\begin{bmatrix}-3&0\\0&2\end{bmatrix},$$

$$\boldsymbol{B}_2=\begin{bmatrix}0&2\\2&0\end{bmatrix},\ \boldsymbol{A}_{d2}=\begin{bmatrix}0.2&0\\0&0.2\end{bmatrix},\ \boldsymbol{B}_{h2}=\begin{bmatrix}0.4&0.3\\0.3&0.5\end{bmatrix},$$

$$\boldsymbol{\Sigma}_1(t)=\begin{bmatrix}\cos t&0\\0&\sin t\end{bmatrix},\ \boldsymbol{D}_1=\begin{bmatrix}0.4&0.3\\0.3&0.6\end{bmatrix},$$

$$\boldsymbol{F}_{11}=\begin{bmatrix}0.6&0.2\\0.3&0.6\end{bmatrix},\ \boldsymbol{F}_{d1}=\begin{bmatrix}0.2&0\\0&0.2\end{bmatrix},$$

$$\boldsymbol{F}_{21}=\begin{bmatrix}0&0.4\\0.4&0\end{bmatrix},\ \boldsymbol{F}_{h1}=\begin{bmatrix}0.5&0.4\\0.2&0.3\end{bmatrix},$$

$$\boldsymbol{\Sigma}_2(t)=\begin{bmatrix}\sin t&0\\0&\cos t\end{bmatrix},\ \boldsymbol{D}_2=\begin{bmatrix}0.2&0.7\\0.6&0.3\end{bmatrix},$$

$$\boldsymbol{F}_{12}=\begin{bmatrix}0.4&0.2\\0.6&0.4\end{bmatrix},\ \boldsymbol{F}_{d2}=\begin{bmatrix}0.3&0\\0&0.3\end{bmatrix},$$

$$\boldsymbol{F}_{22}=\begin{bmatrix}0&0.2\\0.2&0\end{bmatrix},\ \boldsymbol{F}_{h2}=\begin{bmatrix}0.3&0.2\\0.5&0.4\end{bmatrix},$$

$d=2$ and $h=1$, as well as the initial condition

$$\boldsymbol{\phi}(t)=\left[e^t\ \ -e^t\right]^{\mathrm{T}},\ t\in[-2,\ 0].$$

The weighting matrices in the cost function (6) were chosen to be

$$\boldsymbol{Q}=\boldsymbol{R}=\begin{bmatrix}1&0\\0&1\end{bmatrix}.$$

The LMIs (21) and (22) were formed for this system. Solving these LMIs with the Matlab LMI Control Toolbox yielded a solution with $\lambda_1 = \lambda_2 = 1$,

$$\boldsymbol{X} = \begin{bmatrix} 4.2826 & 0 \\ -3.6754 & 6.7437 \end{bmatrix}, \ \boldsymbol{G} = \begin{bmatrix} 0.6853 & 0.0096 \\ 0.0096 & 0.4679 \end{bmatrix},$$

$$\boldsymbol{H} = \begin{bmatrix} 0.4651 & 0.0098 \\ 0.0098 & 0.4784 \end{bmatrix}, \boldsymbol{M}_1 = \begin{bmatrix} -20.2918 & -4.3713 \\ 45.0516 & -53.2193 \end{bmatrix},$$

$$\boldsymbol{M}_2 = \begin{bmatrix} 39.8495 & -44.1254 \\ -13.9619 & -4.7584 \end{bmatrix}.$$

By applying Theorem 2, we obtained the memoryless state feedback guaranteed cost control law

$$\boldsymbol{K}_1 = \begin{bmatrix} -5.2945 & -0.6482 \\ 3.7469 & -7.8917 \end{bmatrix}, \boldsymbol{K}_2 = \begin{bmatrix} 3.6895 & -6.5432 \\ -3.8657 & -0.7056 \end{bmatrix}.$$

The corresponding guaranteed cost of the uncertain closed-loop system was $J^* = 16.8794$.

5 Conclusions

This contribution has studied the guaranteed cost control problem for a class of linear uncertain time-delay switched singular systems with a given quadratic performance index. By adopting a Lyapunov function approach, we have deduced a sufficient condition for the existence of guaranteed cost memoryless state feedback controllers in the form of LMIs. We have therefore turned the design problem of guaranteed cost controller for a linear uncertain time-delay switched singular system into the convex optimisation of solving a set of LMIs, the solution of which provides a parametric representation of the guaranteed cost control law. This design procedure is computationally simple and practically attractive.

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