Minimum Bit Error Rate Beamforming Receiver for Space-Division Multiple-Access Based Quadrature Amplitude Modulation Systems

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2012 IEEE Wireless Communications and Networking Conference
Paris, France, April 1-4, 2012

Outline

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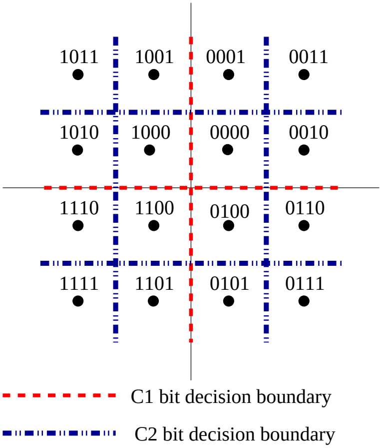
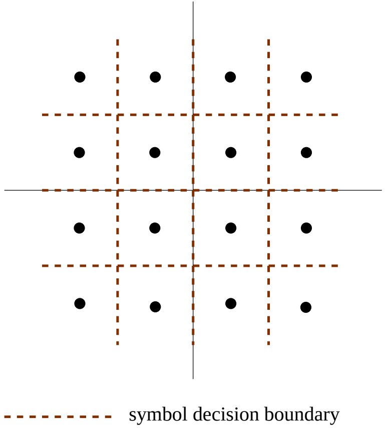
Background

- **Bit error rate** is ultimate performance metric, but making **bit** decision is more complicated than making **symbol** decision





Our Contributions

- Many previous works, including ours, have focused on **minimum symbol error rate** designs for QAM systems
- It was generally believed that
  1. A **minimum bit error rate** design is too complicated, and complexity may be much higher than MSER design **?**
  2. MSER design may be as good as MBER design **?**
- It would be nice at least intellectually to know the answers
- In this work, we specifically look into **MBER design for QAM systems**, and our findings are
  1. MBER design has similar complexity as MSER design, at least for 16QAM
  2. MSER design indeed achieves the same performance of MBER design, in terms of BER

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MIMO Model

- SDMA with L-element receive antenna array to support M QAM users, where receive signal vector $\mathbf{x}(k) = [x_1(k)\ x_2(k) \cdots x_L(k)]^{\mathrm{T}}$

$$\mathbf{x}(k) = \mathbf{H}\,\mathbf{b}(k) + \mathbf{n}(k) = \bar{\mathbf{x}}(k) + \mathbf{n}(k)$$

- Complex-valued AWGN vector $\mathbf{n}(k) = [n_1(k)\ n_2(k) \cdots\ n_L(k)]^{\mathrm{T}}$ with covariance matrix $E[\mathbf{n}(k)\mathbf{n}^H(k)] = 2\sigma_n^2 \mathbf{I}_L$
- Channel matrix $\mathbf{H} = [A_1\mathbf{s}_1\ A_2\mathbf{s}_2 \cdots A_M\mathbf{s}_M] = [\mathbf{h}_1\ \mathbf{h}_2 \cdots \mathbf{h}_M]$ with ith channel coefficient A_i and steering vector for user i

$$\mathbf{s}_i = \left[e^{\mathrm{j}\omega_c t_1(\theta_i)}\ e^{\mathrm{j}\omega_c t_2(\theta_i)} \ldots e^{\mathrm{j}\omega_c t_L(\theta_i)} \right]^{\mathrm{T}}$$

  $t_l(\theta_i)$: relative time delay at array element l for user i, θ_i: direction of arrival for user i, $\omega_c = 2\pi f_c$: angular carrier frequency
- Transmitted symbol vector of M users $\mathbf{b}(k) = [b_1(k) \cdots b_M(k)]^{\mathrm{T}}$

Beamforming Receiver

- Assume **user 1** is desired user, beamformer output

$$y(k) = \mathbf{w}^{\mathrm{H}}\mathbf{x}(k) = \bar{y}(k) + e(k) = c_1 b_1(k) + \sum_{i=2}^{M} c_i b_i(k) + e(k)$$

$c_1 b_1(k)$: **desired signal**, summation term: **residual interfering signal**, $e(k)$: zero-mean **Gaussian** with $E[|e(k)|^2] = 2\sigma_n^2 \mathbf{w}^{\mathrm{H}}\mathbf{w}$

- Weight vector $\mathbf{w} = [w_1 \ w_2 \cdots w_L]^{\mathrm{T}}$, c_1 is made real and positive
- 16-QAM modulation, 4 bits per complex-valued symbol:

$$b_i(k) = b_{R_i}(k) + \mathrm{j} b_{I_i}(k) \in \{\pm 1 \pm \mathrm{j}, \ \pm 1 \pm 3\mathrm{j}, \ \pm 3 \pm \mathrm{j}, \ \pm 3 \pm 3\mathrm{j}\}$$

- Two bits per in-phase / quadrature symbol mapping:

$$\mathbf{11}, \mathbf{10}, \mathbf{00}, \mathbf{01} \leftrightarrow -\mathbf{3}, -\mathbf{1}, +\mathbf{1}, +\mathbf{3}$$

Notice the class 1 (**C1**) bit and the class 2 (**C2**) bit

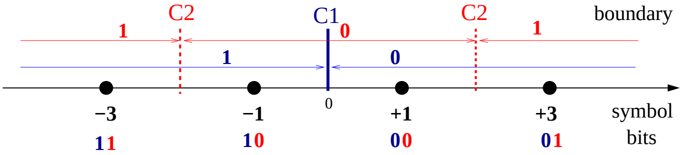
Detection of Bits

- $y(k) = y_R(k) + \mathrm{j}y_I(k)$ used to detect four bits of $b_1(k)$
- Decision for in-phase C1 bit is given by

$$\begin{cases} \text{C1 bit} = 0, & \text{if } y_R(k) > 0 \\ \text{C1 bit} = 1, & \text{if } y_R(k) \le 0 \end{cases}$$

and decision for in-phase C2 bit is given by

$$\begin{cases} \text{C2 bit} = 0, & \text{if } -2c_1 < y_R(k) < 2c_1 \\ \text{C2 bit} = 1, & \text{if } y_R(k) \le -2c_1 \text{ or } y_R(k) \ge 2c_1 \end{cases}$$



- Decisions for quadrature C1 and C2 bits are given similarly based on $y_I(k)$

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Bit Error Rate

- BER of 16-QAM beamformer with weight vector **w** is defined by

$$P_E(\mathbf{w}) = \frac{1}{4}\Big(P_{E_R,C1}(\mathbf{w}) + P_{E_I,C1}(\mathbf{w}) + P_{E_R,C2}(\mathbf{w}) + P_{E_I,C2}(\mathbf{w})\Big)$$

- Let $\mathbf{b}^{(q)}$, $1 \le q \le N_b = 16^M$, be legitimate sequences of $\mathbf{b}(k)$:

$$\bar{\mathbf{x}}(k) \in \mathbb{X} \triangleq \{\bar{\mathbf{x}}^{(q)} = \mathbf{H}\,\mathbf{b}^{(q)}, 1 \le q \le N_b\}$$

- Set of beamformer scalar states

$$\bar{y}(k) \in \mathbb{Y} \triangleq \{\bar{y}^{(q)} = \mathbf{w}^{\mathrm{H}}\bar{\mathbf{x}}^{(q)}, 1 \le q \le N_b\} = \mathbb{Y}_R + \mathrm{j}\mathbb{Y}_I$$

- 16 subsets of beamformer scalar states

$$\begin{aligned}\mathbb{Y}^{(l,i)} &\triangleq \{\bar{y}^{(q)} \in \mathbb{Y} : b_{R_1}(k) = l, b_{I_1}(k) = i\} \\ &= \mathbb{Y}_R^{(l,i)} + \mathrm{j}\,\mathbb{Y}_I^{(l,i)}, \ \ l, i = \pm 1, \pm 3\end{aligned}$$

C1 Bit Error Rate

- In-phase C1 bit error probability

$$P_{E_R,C1}(\mathbf{w}) = \frac{1}{2N_{\text{sub}}} \sum_{\bar{y}_R^{(q)} \in \mathbb{Y}_R^{(+1,+1)}} \left(Q(g_R^{(q)}(\mathbf{w})) + Q(g_R^{(q,a)}(\mathbf{w})) \right)$$

$$N_{\text{sub}} = N_b/16,\ Q(u) = \frac{1}{\sqrt{2\pi}} \int_u^\infty e^{-\frac{v^2}{2}} dv,\ b_1^{(q)} = b_{R_1}^{(q)} + \mathrm{j} b_{I_1}^{(q)} \text{ 1st element of } \mathbf{b}^{(q)},$$

$$g_R^{(q)}(\mathbf{w}) = \frac{\operatorname{sgn}(b_{R_1}^{(q)})\bar{y}_R^{(q)}}{\sigma_n \sqrt{\mathbf{w}^{\mathrm{H}}\mathbf{w}}},\ g_R^{(q,a)}(\mathbf{w}) = \frac{2c_1 + \operatorname{sgn}(b_{R_1}^{(q)})\bar{y}_R^{(q)}}{\sigma_n \sqrt{\mathbf{w}^{\mathrm{H}}\mathbf{w}}},$$

- Quadrature C1 bit error probability

$$P_{E_I,C1}(\mathbf{w}) = \frac{1}{2N_{\text{sub}}} \sum_{\bar{y}_I^{(q)} \in \mathbb{Y}_I^{(+1,+1)}} \left(Q(g_I^{(q)}(\mathbf{w})) + Q(g_I^{(q,a)}(\mathbf{w})) \right)$$

with

$$g_I^{(q)}(\mathbf{w}) = \frac{\operatorname{sgn}(b_{I_1}^{(q)})\bar{y}_I^{(q)}}{\sigma_n \sqrt{\mathbf{w}^{\mathrm{H}}\mathbf{w}}},\ g_I^{(q,a)}(\mathbf{w}) = \frac{2c_1 + \operatorname{sgn}(b_{I_1}^{(q)})\bar{y}_I^{(q)}}{\sigma_n \sqrt{\mathbf{w}^{\mathrm{H}}\mathbf{w}}}$$

C2 Bit Error Rate

- With some accurate approximation, in-phase C2 bit error probability

$$P_{E_R,C2}(\mathbf{w}) \approx \frac{1}{2N_{\text{sub}}} \sum_{\bar{y}_R^{(q)} \in \mathbb{Y}_R^{(+1,+1)}} \left(2Q\left(g_R^{(q)}(\mathbf{w}) \right) + Q\left(g_R^{(q,a)}(\mathbf{w}) \right) \right)$$

- Quadrature C2 bit error probability

$$P_{E_I,C2}(\mathbf{w}) \approx \frac{1}{2N_{\text{sub}}} \sum_{\bar{y}_I^{(q)} \in \mathbb{Y}_I^{(+1,+1)}} \left(2Q\left(g_I^{(q)}(\mathbf{w}) \right) + Q\left(g_I^{(q,a)}(\mathbf{w}) \right) \right)$$

1. **Class 2 error probability** approximately **twice** of **class 1 error probability**
2. For 16QAM, **complexity** of calculating bit error rate is **similar** to that of calculating symbol error rate

MBER Solution

- MBER beamformer solution is defined as

$$\mathbf{w}_{\text{MBER}} = \arg\min_{\mathbf{w}} P_E(\mathbf{w})$$

- MBER beamformer design may be obtained based on a gradient-descent numerical optimisation
  1. Gradient of $P_E(\mathbf{w})$ requires extensive computation
  2. Slow convergence and local minima problem
- Alternatively, evolutionary algorithms, such as **differential evolution** (DE) algorithm can be used
  - DE is characterised by **a) initialisation**, **b) mutation**, **c) re-combination** and **d) selection** operations invoked for exploring the search space in an iterative procedure, until some termination criteria are met

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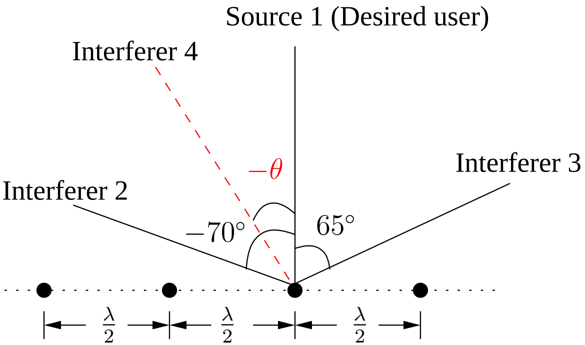
Simulation Systems



1. **Full-rank**: four-element antenna array supporting four users
   - Minimum **angular separation** with desired user $\theta < 65°$
   - $\mathrm{E_b/N_o}$: average bit energy over channel noise power
   - All channel taps A_i are identical
2. **Rank-deficient**: three-element array supporting four users

Benchmarks for Comparison

- Two beamforming receiver designs are used as benchmarks
  1. Conventional **minimum mean square error** (MMSE) solution that minimises MSE metric $E[|b_1(k) - y(k)|^2]$

     $$\mathbf{w}_{\mathrm{MMSE}} = \left(\mathbf{H}\,\mathbf{H}^{\mathrm{H}} + \frac{2\sigma_n^2}{\sigma_b^2}\mathbf{I}_L \right)^{-1} \mathbf{h}_1$$

     $2\sigma_n^2$: channel noise power, σ_b^2: average symbol power
  2. Our previous **minimum symbol error rate** (MSER) solution that minimises symbol error rate

     $$\mathrm{SER}(\mathbf{w}) = \mathrm{Prob}\{\hat{b}_1(k) \neq b_1(k)\}$$

     $\hat{b}_1(k)$: detected symbol for $b_1(k)$
- Same DE algorithm used to obtain MBER and MSER solutions

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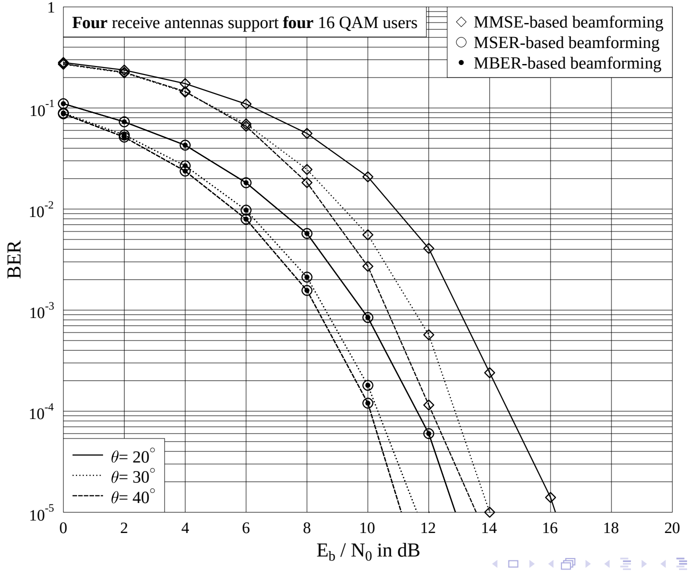
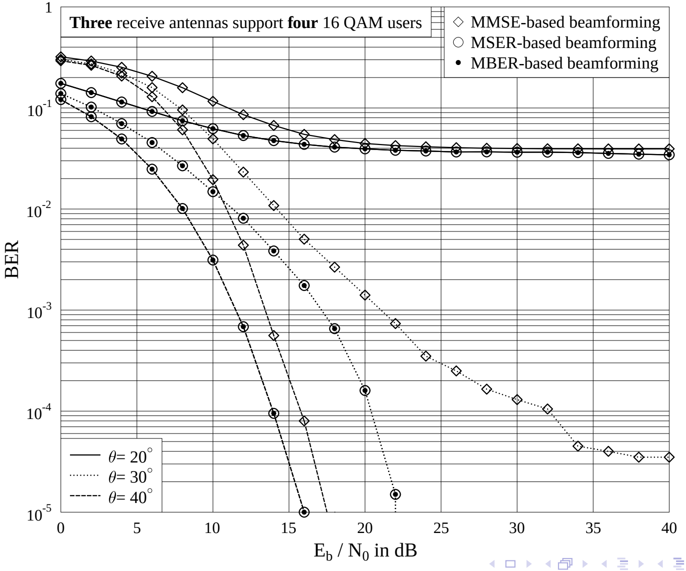
Bit Error Rate (full-rank)

- $P_s = 100$, $\gamma = 0.4$, $C_r = 0.4$, $G_{\max} = 200$



Bit Error Rate (rank-deficient)

- $P_s = 100$, $\gamma = 0.4$, $C_r = 0.4$, $G_{\max} = 200$



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Summary

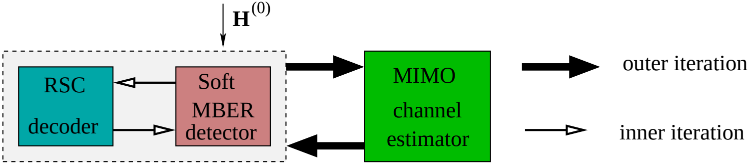
- We have proposed a minimum bit error rate beamforming receiver for multi-user SDMA based QAM systems
- More specifically, for 16QAM MIMO systems
  1. Derive explicitly bit error rate expression
  2. Show MBER design has a similar complexity to that of MSER design
  3. Confirm both MBER and MSER designs achieve same performance, in terms of BER
- Future work will incorporate minimum bit error rate design in applications to unknown MIMO channel

Adaptive Applications

- Previously we have developed a stochastic-gradient based adaptive MSER algorithm: least symbol error rate
  - Same approach can be adopted for adaptive MBER design
- More powerfully, previously we have applied MSER design in joint channel estimation and turbo detection

  J. Zhang, S. Chen, X. Mu, and L. Hanzo, "Turbo multi-user detection for OFDM/SDMA systems relying on differential evolution aided iterative channel estimation," *IEEE Trans. Communications*, to appear

  http://eprints.ecs.soton.ac.uk/23148/
  - Same approach can be adopted by using MBER design



State-of-the-Art

- Existing schemes require an **extra iterative loop** between channel estimator and turbo detection/decoder
- We have recently developed a new scheme where **channel estimator** is **embedded** in the **original** turbo iterative procedure