Coded MIMO With Asymmetric Constellation Sizes

Chen Qian, Zhaocheng Wang, *Senior Member, IEEE*, Zhaohua Lu, Linglong Dai, *Member, IEEE*, and Sheng Chen, *Fellow, IEEE*

***Abstract*—An asymmetric constellation scheme for coded multiple-input–multiple-output (MIMO) transmission is proposed, which applies different constellation mappings to different transmit streams and carefully selects the coding rates for different transmit streams. An improved power allocation is derived to naturally incorporate with the coding rate selection and to further enhance the achievable performance. The proposed scheme provides more flexible choices of data rate selection, and by employing fixed-complexity sphere decoding (FSD) detection, it achieves better performance with reduced detection complexity in comparison with the conventional MIMO using the FSD-based detection with the same constellation set for all streams.**

***Index Terms*—Coding rate selection, constellation set, extrinsic information transfer (EXIT) chart, multiple-input–multiple-output (MIMO), power allocation.**

I. Introduction

Multiple-input–multiple-output (MIMO) has been considered as a promising technology for wireless communications due to its potential to improve the spectral efficiency and the reliability over fading channels. For example, the 3rd Generation Partnership Project (3GPP) Long-Term Evolution Advanced (LTE-A) [1] supports up to 8×8 MIMO transmission. Recently, the next-generation mobile broadcasting system, known as digital video broadcasting next-generation handheld (DVB-NGH), has also considered to exploit the spatial multiplexing to improve the spectral efficiency by using 2×2 MIMO [2].

Most existing studies on MIMO technology in the literature focus on the assumption that all transmit streams use the same constellation set, and the investigation on the different constellation sets applied to different streams is seldom carried out. However, by transmitting different constellation-mapped data streams through different antennas, more options of data rate selection will be provided and the flexibility of the multiple service support can be improved. Although the analysis in this area is limited, this kind of systems has been adopted by some commercial standards, such as LTE-A and DVB-NGH.

Many existing MIMO schemes are not suitable for transmitting different constellation-mapped data over different antennas. Some existing unequal error protection (UEP) schemes use different constellation sets to provide different protections for the information sequences with different importance or priorities [3], [4]. However, it is difficult to apply these schemes when the different data streams all have the same importance. Moreover, the data rates or system throughputs provided by UEP schemes vary according to the detection conditions, and they cannot provide a fixed data rate to meet a required system throughput all the time. The vertical Bell Laboratories layered space–time (V-BLAST) architecture [5] can transmit independent data streams over different transmit antennas to exploit spatial multiplexing. It is well known that owing to MIMO interchannel interference, the optimal maximum-likelihood detection suffers from exponentially growing complexity. Low-complexity suboptimal detection schemes such as serial interference cancelation are typically applied. When different constellation sets are used for different streams, however, if the same code and equal power allocation are applied to all streams, then the error propagation of the transmit stream with the worst performance will be serious and this will degrade the overall performance severely. Thus, the coding rate and the power allocation have to be investigated carefully.

Power allocation is important to realize spatial multiplexing under MIMO channels and has been studied in detail [6]–[9]. However, most power-allocation algorithms, including the famous “water-filling” algorithm, “mercury–water-filling” algorithm [8], and the optimum precoder, aim at maximizing the mutual information on the complex MIMO vector channel [9], and they only consider the uncoded systems without turbo detection while imposing the assumption that the transmitter has perfect knowledge of channel conditions, which may be unrealistic.

In this paper, we have proposed a coded MIMO transmission scheme with different constellation sets for different transmit streams, and we carefully select the coding rates and allocate power values for different constellation-mapped data streams to ensure good performance. The basic idea is to “match” different streams with different constellation sets by properly selecting coding rates and power allocation so that all the streams have approximately the same performance. For illustration purposes and concise analysis, we will mainly consider the case of two different constellation sizes, but the concept and approach can be applied generically. Unlike the conventional power-allocation-based approach, the proposed method is designed to utilize the turbo detection at the receiver and does not rely on the feedback of channel state information (CSI) to the transmitter. In contrast to the method based on experimental results given in [10], a simple closed-form approach to power allocation is presented and the fixed-complexity sphere decoding (FSD) detection algorithm for the proposed scheme is also discussed. The proposed scheme offers more options of data rate selection, and furthermore, it is simple and efficient. The convergence analysis with the aid of extrinsic information transfer (EXIT) chart also demonstrates the effectiveness of our asymmetric constellation scheme. Simulation results show that, given the same data rate, our scheme achieves better performance in comparison with the conventional MIMO counterpart using the FSD-based detection algorithm while imposing lower implementation complexity.

II. System Model

Consider the MIMO system with N transmit antennas and M receive antennas, which is shown in Fig. 1. N independent bit streams $\{\mathbf{b}_n\}_{n=1}^N$ are encoded by N channel encoders to obtain the encoded bit streams $\{\mathbf{c}_n\}_{n=1}^N$. After interleaving, every B_n bit from the nth stream is mapped to a symbol selected from the constellation set $\mathcal{S}_n$ with $Q_n = 2^{B_n}$ as its cardinality. The constellation set used by each stream can be different. The symbol streams $\{\mathbf{s}_n\}_{n=1}^N$ are then passed to the

Manuscript received February 7, 2014; revised May 20, 2014 and July 23, 2014; accepted October 2, 2014. Date of publication October 7, 2014; date of current version September 15, 2015. This work was supported in part by the National High Technology Research and Development Program of China under Grant 2012AA011704, by the National Natural Science Foundation of China under Grant 61271266, by Beijing Natural Science Foundation under Grant 4142027, by ZTE under Fund Project CON1307250001, and by the Foundation of Shenzhen Government. The review of this paper was coordinated by Prof. Y. Fang.

C. Qian, Z. Wang, and L. Dai are with Tsinghua National Laboratory for Information Science and Technology (TNList), Department of Electronic Engineering, Tsinghua University, Beijing 100084, China (e-mail: qc8802@gmail.com; cwang@mail.tsinghua.edu.cn; daill@tsinghua.edu.cn).

Z. Lu is with ZTE Corporation, Shenzhen 518000, China (e-mail: lu.zhaohua@zte.com.cn).

S. Chen is with the School of Electronics and Computer Science, University of Southampton, Southampton SO17 1BJ, U.K., and also with King Abdulaziz University, Jeddah 22254, Saudi Arabia (e-mail: sqc@ecs.soton.ac.uk).

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Digital Object Identifier 10.1109/TVT.2014.2361646

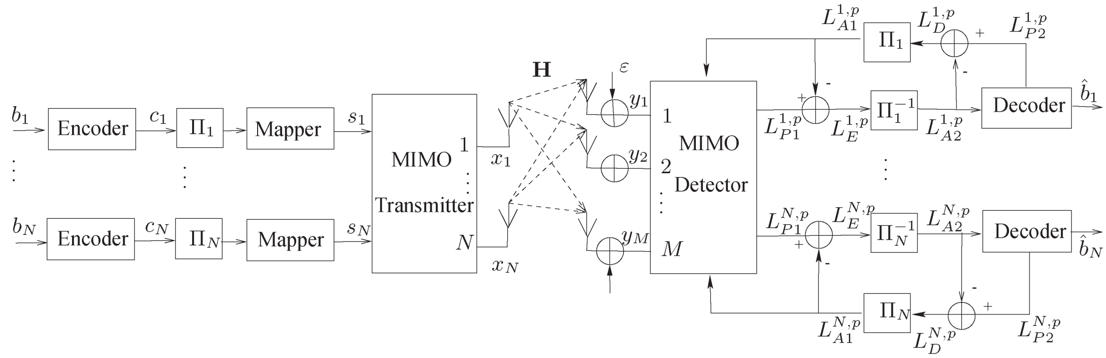

Fig. 1. $N \times M$ MIMO system with turbo detection and decoding, where Π_i and Π_i^{-1} denote the interleaver and de-interleaver for the ith data stream, respectively.

MIMO transmitter to perform power allocation and normalization and to generate the transmit vector $\mathbf{x} = [x_1\ x_2, \ldots, x_N]^T$, where $(\ )^T$ denotes the transpose operator. The N symbols in $\mathbf{x}$ are transmitted over the N transmit antennas, respectively.

Assuming flat fading, the channel is described by the well-known narrow-band MIMO model given by

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \boldsymbol{\varepsilon} \tag{1}$$

where $\mathbf{y} = [y_1\ y_2, \ldots, y_M]^T \in \mathbb{C}^{M\times 1}$ and $\boldsymbol{\varepsilon} = [\varepsilon_1\ \varepsilon_2, \ldots, \varepsilon_M]^T \in \mathbb{C}^{M\times 1}$ denote the received signal vector and the noise vector, respectively, whereas $\mathbf{H} = [h_{i,j}] \in \mathbb{C}^{M\times N}$ is the MIMO channel matrix with $h_{i,j}$ denoting the channel coefficient between the jth transmit antenna and the ith receive antenna. Under the assumption of Rayleigh fading, coefficient $h_{i,j}$ follows the complex Gaussian distribution with zero mean and $\mathbb{E}[|h_{i,j}|^2] = 1$, where $\mathbb{E}[\ ]$ denotes the expectation operation. In this paper, we assume that the receiver has ideal knowledge about the channel. We also point out that, at the cost of a minimum training overhead, the semiblind joint iterative channel estimation and turbo detection–decoding scheme in [11] and [12] is capable of attaining the optimal turbo detection–decoding solution associated with the perfect CSI while only imposing similar complexity of the latter.

The receiver performs turbo detection to improve the overall performance, where the extrinsic information is exchanged between the MIMO detector and the channel decoders. As shown in Fig. 1, the MIMO detector outputs the *a posteriori* log-likelihood ratio (LLR) $L_{P1}^{n,p}$, where superscripts n and p denote the pth bit of the nth stream. Then, the extrinsic LLR of the MIMO detector $L_E^{n,p}$ is obtained as $L_E^{n,p} = L_{P1}^{n,p} - L_{A1}^{n,p}$, in which $L_{A1}^{n,p}$ is the *a priori* LLR of the MIMO detector. The extrinsic LLR $L_E^{n,p}$ is de-interleaved to obtain the *a priori* LLR $L_{A2}^{n,p}$ for the channel decoder of the nth stream. The decoder of the nth stream takes $L_{A2}^{n,p}$ as the input and computes the *a posteriori* LLR $L_{P2}^{n,p}$ and the extrinsic LLR $L_D^{n,p}$. Then, $L_D^{n,p}$ is interleaved and delivered to the MIMO detector as the *a priori* LLR $L_{A1}^{n,p}$ for the next iteration. For the first iteration, $L_{A1}^{n,p} = 0$ since there is no *a priori* information. After several iterations, the hard decision is performed on $L_{P2}^{n,p}$ and the estimation of the transmitted bits $\widehat{\mathbf{b}} = [\widehat{b}_1\ \widehat{b}_2, \cdots, \widehat{b}_N]^T$ is obtained.

It should be emphasized that, although the narrow-band MIMO channel model described in (1) is assumed in this paper, our proposed scheme can be extended to the generic frequency-selective MIMO case with the aid of orthogonal frequency-division multiplexing (OFDM) and symbol-level interleaving. Specifically, with OFDM, each subcarrier can be regarded as experiencing a narrow-band channel, whereas with symbol-level interleaving, the equivalent channels of adjacent subcarriers can be regarded as independent.

III. Proposed Scheme

Most existing studies on MIMO detection in the literature always assume that the same modulation and coding scheme is applied to all streams since, in this way, the mutual information is maximized. However, adopting the same constellation set for all the data streams limits the flexibility of choosing various data rates with different qualities of service for multimedia services. Hereby, we propose an asymmetric constellation scheme for MIMO transmission, in which different modulation and coding schemes are applied to different transmit streams. The selection of the appropriate constellation sizes and coding rates for different streams and the power allocation of different streams are also investigated.

We concentrate on the case where two different constellation sets are applied. Denote $\mathcal{S}_1$ and $\mathcal{S}_2$ as the two constellation sets with cardinality $Q_1 = 2^{B_1}$ and $Q_2 = 2^{B_2}$, where B_1 and B_2 are the numbers of bits per symbol for these two constellation sets, respectively. Without loss of generality, assume that $Q_1 < Q_2$. The numbers of transmit streams applied $\mathcal{S}_1$ and $\mathcal{S}_2$ are N_1 and N_2, respectively. For a MIMO system with N transmit antennas, $N_1 + N_2 = N$. The values of N_1 and N_2 can be adjusted according to various application scenarios and antenna configurations. Hereby, we let $N_1 = N_2$ to simplify the analysis.

A. Coding Rate Selection

For the proposed scheme, the coding rates should be carefully selected to improve the performance. Denote R_1 and R_2 as the coding rates of the transmit streams with the constellation sets $\mathcal{S}_1$ and $\mathcal{S}_2$, respectively. Then, the overall data rate is given by $R_{\text{overall}} = R_1 B_1 N_1 + R_2 B_2 N_2$. Since, at the receiver, turbo detection is applied to improve the overall performance, the tools for convergence analysis, such as EXIT charts, can be used to select appropriate channel code, as proposed in [13]. However, an EXIT chart-aided algorithm is complicated, particularly for the case of independently coded data streams transmitted over different antennas, where a multidimensional EXIT chart is required.

It is well known that at the same signal-to-noise ratio (SNR), the detection performance of data streams with a small constellation set is much better than that of data streams with a larger constellation set. If the same coding rate is applied, implying the same error correction capability, the turbo detection performance of a larger modulation set-based stream remains much worse, and this will affect the turbo detection performance of other streams. Consequently, the overall performance is degraded, and less gain can be obtained from iterations. Therefore, applying the same coding scheme to all the streams is not a good idea given the fact that they have different constellation sets. Since $B_1 < B_2$, an appropriate rule to select coding rate is to choose $R_1 > R_2$ and to ensure that the data rates of various transmit streams with different modulation and coding schemes are close, which means that R_1B_1 is close to R_2B_2. By selecting the coding rates and constellations to make the data rates of different streams close, the turbo detection performance of different streams will be similar, and this avoids the aforementioned performance degradation caused by the adverse effects from a poorer performance stream. Later, we will show that this simple rule is also beneficial to the convergence of turbo detection.

B. Power Allocation

If the coding rates can be chosen exactly as $R_1B_1 = R_2B_2$, the power allocation is not required, as will be shown in (5). However, for a practical system with finite available choices of constellation sets and coding schemes, it is difficult to satisfy this condition. Thus, the investigation of power allocation is necessary. For a system with turbo detection, the power allocation, in theory, can be optimized by taking into account the convergence property of the turbo detection process. However, it is difficult to obtain a closed-form solution for such a complicated optimization problem, and moreover, it is very hard to visualize the solution since a multidimensional EXIT chart is required for MIMO systems. Here, we propose an alternative way to obtain a suitable power allocation.

For turbo detection-based MIMO, the extrinsic information on the MIMO detector depends on the *a priori* information from all the channel decoders. The overall performance is mainly limited by the most unreliable *a priori* information. For the asymmetric constellation scheme considered in this paper, the streams with the largest effective data rate $C_i = B_iR_i$ [in bits/symbol] provide the most unreliable information, which will degrade the achievable overall system performance significantly. Thus, the main objective of power allocation is to make the performance of the streams with different constellation sets nearly the same at the given data rates. To get the exact solution, the mutual information curves for the constellations $\mathcal{S}_1$ and $\mathcal{S}_2$ can be used. The minimum SNRs, i.e., ρ_1 and ρ_2, which achieve the coding rate R_1 for constellation set $\mathcal{S}_1$ and the coding rate R_2 for $\mathcal{S}_2$, are obtained through the mutual information curves. Since power allocation corresponds to the shift of the mutual information curves along the SNR-axis, different power allocations can be tested to find the appropriate power allocation, which makes ρ_1 and ρ_2 almost the same.

However, for the commonly used modulation schemes such as quadrature amplitude modulation (QAM) and phase-shift keying (PSK), the mutual information curves are difficult to obtain and they have no closed-form solutions. Hereby, we propose a simplified approach to obtain the approximate solution by applying the channel capacity with Gaussian input. For the single-input–single-output Gaussian channel, the capacity measured in [bits/symbol] is computed as

$$C = \log_2(1+\rho) \tag{2}$$

where $\rho = E_s/N_0$ denotes the SNR with E_s being the average symbol energy and $N_0/2$ being the two-sided power spectral density of the channel noise. For constellation set $\mathcal{S}_i$, $i = 1, 2$, the maximum data rate is calculated as $C_i = B_iR_i$ [bits/symbol], and the corresponding minimum SNR ρ_i to achieve C_i is given by

$$\rho_i = 2^{B_iR_i} - 1. \tag{3}$$

Assume that the average power values for $\mathcal{S}_1$ and $\mathcal{S}_2$ before the power allocation are $E_{s_1} = E_{s_2} = 1$, and after the power allocation, the average symbol power values for $\mathcal{S}_1$ and $\mathcal{S}_2$ become $1-x$ and $1+x$, respectively. Then, the SNRs in decibels for attaining the maximum data rates of $\mathcal{S}_1$ and $\mathcal{S}_2$ are $\rho_1|_{\mathrm{dB}} - 10\log_{10}(1-x)$ and $\rho_2|_{\mathrm{dB}} - 10\log_{10}(1+x)$, respectively. For the proposed scheme, the two SNRs are preferred to be equal, leading to

$$\rho_1|_{\mathrm{dB}} - 10\log_{10}(1-x) = \rho_2|_{\mathrm{dB}} - 10\log_{10}(1+x). \tag{4}$$

By solving (4), parameter x is calculated as

$$x = \frac{\rho_2 - \rho_1}{\rho_2 + \rho_1} = \frac{2^{B_2R_2} - 2^{B_1R_2}}{2^{B_2R_2} + 2^{B_1R_1} - 2}. \tag{5}$$

The ratio of the two average power values for the streams with the two different constellation sets is given by

$$\frac{P_1}{P_2} = \frac{1-x}{1+x}. \tag{6}$$

Although (6) is derived under the assumption of Gaussian input and thus it is an approximation for practical transmitted signals, it is simple and effective, and moreover, it does not require any knowledge of the CSI. As this power allocation realizes the principle of matching independent streams with different modulation and coding schemes, it is capable of significantly enhancing the overall system performance. Furthermore, as will be shown in Section IV-C, this power allocation also guarantees the convergence of the turbo detection, and therefore, it enables the proposed asymmetric constellation and coding scheme to benefit from turbo detection iterations.

C. Receiver Implementation

Some conventional MIMO detection algorithms are still applicable for the proposed scheme. For example, the FSD algorithm proposed in [14] and [15] is most suitable since the detection process is controlled by the node distribution vector $\mathbf{n}_{\mathcal{S}} = [n_1\ n_2, \ldots, n_N]^T$, where n_i is the number of possible symbols reserved for the ith transmit antenna. However, it has to be slightly modified to incorporate with the asymmetric constellation scheme.

First, unlike the case of the conventional FSD algorithm, the channel ordering is independent from the channel conditions. Since the smallest constellation set always has the best performance, detection should be started from these streams. In other words, the channel ordering is simply to start the detection from the streams associated with the smallest constellation set. As a result, the complex matrix inversion to find the most reliable layer, as required by the conventional FSD algorithm, is avoided, and this reduces the computational complexity.

Second, unlike the conventional FSD algorithm, the node distribution vector should be chosen as $\mathbf{n}_{\mathcal{S}} = [K \cdots K \quad Q_1 \cdots Q_1]^T$, where the number of Q_1's is equal to N_1, and $1 \leq K < Q_2$. N_1 streams with constellation set $\mathcal{S}_1$ are detected first, and an exhaustive search is performed over the first N_1 streams. K is a parameter to balance the tradeoff between the performance and the complexity. By appropriately choosing K, a candidate set containing $Q_1^{N_1}K^{N_2}$ possible vectors is obtained for LLR calculation. Considering that Q_1 is less than the equivalent constellation size $Q = 2^B$, where $B = (B_1 + B_2)/2$, K can often be chosen to be much smaller than Q_2, and the computational complexity of our proposed scheme is reduced.

It should be noted that the proposed scheme with power allocation significantly benefits from turbo detection. Rather than updating the candidate set for LLR calculation in each iteration, the candidate set obtained at the first iteration is used for LLR calculation for the subsequent iterations. This way, the receiver complexity is further significantly reduced with negligible performance loss. Our scheme is most suitable for small N and M with large Q, which is aligned with the wireless mobile broadcasting scenarios.

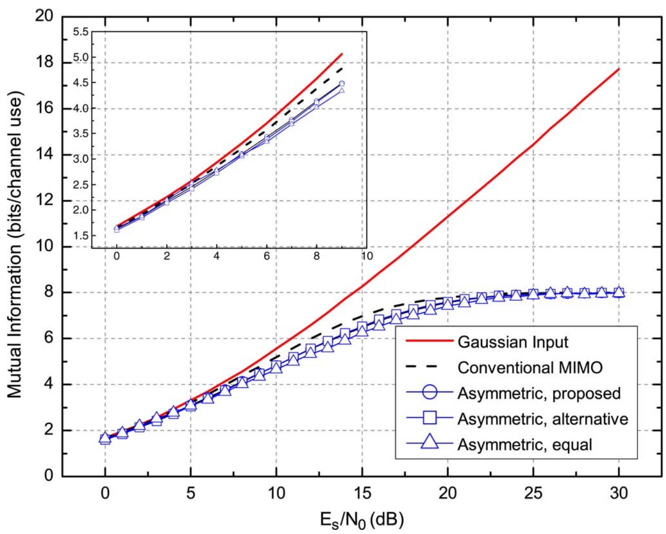

Fig. 2. Capacity comparison for the conventional 2 × 2 MIMO with the 16-QAM modulation and the 2 × 2 MIMO with the asymmetric modulations of QPSK and 64-QAM. For the MIMO with the asymmetric constellation sets, equal power allocation and asymmetric power allocation are applied.

IV. Numerical Results

A. Capacity Analysis

Here, the mutual information on the proposed asymmetric modulation and coding system and the conventional same modulation and coding system is evaluated. Since we assume an uncorrelated fast-fading channel environment, different transmitted symbols experience different fading channel coefficients, and therefore, the ergodic capacity is used. For Gaussian input, the capacity of the MIMO channel is calculated as

$$C = \mathbb{E}\left[\log_2\left(\mathbf{I}_M + \frac{\rho}{M}\mathbf{H}^H\mathbf{H}\right)\right] \quad (7)$$

where $\mathbf{I}_M$ is the $M \times M$ identity matrix, and $(\ )^H$ denotes Hermitian transpose operator, whereas $\rho = E_s/N_0$ is the MIMO channel SNR. For the commonly used modulation constellations, such as QAM and PSK, the mutual information is computed according to [16]

$$I(\mathbf{x},\mathbf{y}) = -\mathbb{E}\left[\log_2\left(\frac{1}{Q^N(2\pi\sigma^2)^M}\sum_{\mathbf{x}\in\mathcal{S}^{N\times 1}} e^{-\frac{1}{2\sigma^2}\|\mathbf{y}-\mathbf{H}\mathbf{x}\|^2}\right)\right] + M\log_2(2\pi e\sigma^2) \quad (8)$$

where $\mathcal{S}^{N\times 1}$ denotes the set containing all the possible transmit vectors, and $2\sigma^2 = N_0$ is the channel noise power. Monte Carlo simulation is used to obtain the first item in (8).

Fig. 2 shows the mutual information for the 2 × 2 MIMO system with the asymmetric constellation sets of quaternary PSK (QPSK) and 64-QAM applied to the two streams, respectively. Let P_1 and P_2 denote the average symbol power values for the QPSK and 64-QAM modulations, respectively. Three power allocations were applied: a) the *proposed* power allocation in which $P_1/P_2 = 0.4748$ is based on (5); b) the *alternative* power allocation of $P_1/P_2 = B_1/B_2 = 1:3$, which is a close approximation to the proposed allocation; and c) the *equal* power allocation. Other power allocations such as mercury–water-filling algorithm were not applied since they are not suitable for coded systems with turbo detection. The conventional 2 × 2 MIMO with 16-QAM modulation is also included in Fig. 2 for comparison. For all the MIMO schemes shown in Fig. 2, the same average symbol power of $\mathbb{E}[\mathbf{x}^H\mathbf{x}] = 1$ was applied to ensure a fair comparison. As expected, unlike the Gaussian MIMO case, all the practical MIMO systems with finite constellation sizes saturate at the same data rate, which is 8 bits per channel use, at high SNR.

In addition, as expected, the conventional MIMO with same constellation set attains higher capacity than the MIMO with asymmetric constellation sets as the former maximizes the mutual information. However, the capacity gap between the two MIMO systems is small. For example, at 4 bits per channel use, the gap between the conventional scheme and the proposed scheme with unequal power allocation is less than 0.6 dB, whereas at 6 bits per channel use, the gap is around 1 dB. It is shown in Fig. 2 that the proposed power allocation achieves slightly better mutual information than the equal power allocation, whereas the alternative power allocation of $P_1/P_2 = 1:3$ has almost the same mutual information compared with the proposed allocation.

B. Turbo Detection–Decoding Performance Analysis

As predicted by the previous capacity analysis, when the optimal detection algorithm is applied, the conventional MIMO scheme outperforms the asymmetric MIMO scheme. However, we now demonstrate that, with well-defined and widely adopted error correction codes and applying practical suboptimal turbo detection techniques such as our low-complexity modified FSD algorithm, the MIMO of asymmetric constellation sizes with the proposed power allocation outperforms the conventional MIMO in terms of achievable bit error rate (BER).

The same 2 × 2 MIMO system was considered. The proposed asymmetric MIMO scheme applied QPSK and 64-QAM constellations to the two transmit streams. For the QPSK modulation, the low-density parity-check (LDPC) code used in digital video broadcasting second generation (DVB-S2) with code length of 64 800 and code rate of 4/5 [17] was applied, whereas for the 64-QAM modulation, the LDPC code used in DVB-S2 with code length of 64 800 and code rate of 2/5 [17] was applied. Thus, 1.6 bits were transmitted per channel use with the QPSK modulated stream, whereas the stream of 64-QAM modulation achieved 2.4 bits per channel use. These two code rates were chosen to be as close as possible considering the practical constraint of the finite codes available. The overall data rate was 4 bits per channel use. Two different power allocations were applied, namely, the proposed allocation with $P_1/P_2 = 0.4748$ and the alternative power allocation with $P_1/P_2 = 1/3$. To ensure a fair comparison, for the conventional 2 × 2 MIMO with 16-QAM constellation, the LDPC code in DVB-S2 with code length of 64 800 and code rate of 1/2 [17] was applied, which also had the overall data rate of 4 bits per channel use. Turbo detection–decoding was applied, and the number of turbo detection iterations was set to 2. For each turbo detection iteration, the maximum iteration number of LDPC decoding was set to 25. As a result, at most, 50 iterations for LDPC decoding were performed for each stream. The modified FSD algorithm, outlined in Section III-C, was applied at the receiver of the proposed asymmetric MIMO system with the node distribution vector $\mathbf{n}_\mathcal{S} = [4\ \ 4]^T$. For the conventional MIMO, the conventional FSD algorithm [14], [15] with the node distribution vector $\mathbf{n}_\mathcal{S} = [1\ \ 16]^T$ was used at the receiver. Thus, the sizes of the two search candidate sets for the two receiver detectors were kept the same, namely, 16. This further ensured a fair comparison. As explained in Section III-C, the modified FSD detector for the proposed asymmetric MIMO has inherently lower complexity than the FSD detector for the conventional MIMO.

Fig. 3 shows the BERs of the proposed asymmetric MIMO scheme with two different power allocations in comparison with those of the conventional MIMO scheme. Observe that the proposed MIMO with asymmetric constellation sets outperforms the conventional MIMO scheme with same constellation set, and moreover, it benefits more from turbo gain. Specifically, at the BER level of 10^{-6}, the proposed

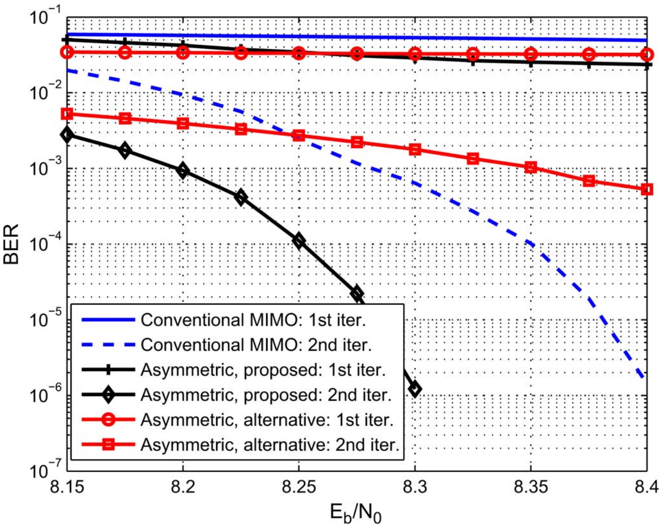

Fig. 3. Bit error rate comparison for the conventional 2 × 2 MIMO with the 16-QAM modulation and the 2 × 2 MIMO with the asymmetric modulations of QPSK and 64-QAM. For the MIMO with the asymmetric constellation sets, the proposed power allocation and the alternative power allocation of $P_1/P_2 = 1 : 3$ are applied.

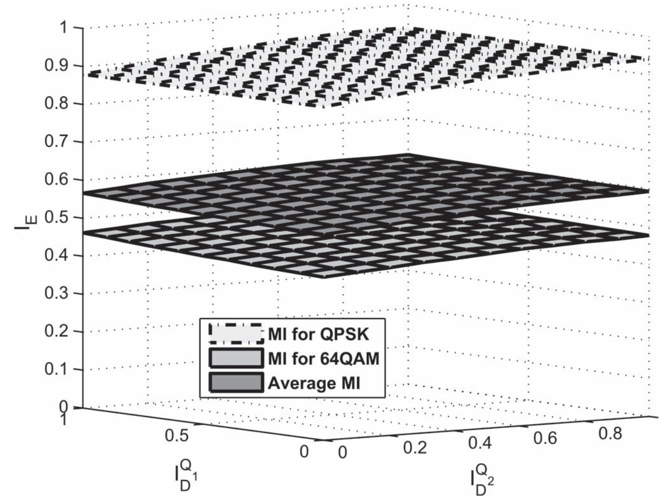

Fig. 4. Mutual information for the MIMO with the asymmetric constellation sets of QPSK and 64-QAM at $E_b/N_0 = 8.3$ dB. The modified FSD algorithm outlined in Section III-C is applied.

asymmetric MIMO scheme outperforms the conventional MIMO by approximately 0.1 dB. More significantly, this gain is attained with reduced complexity. In Fig. 3, further observe that the asymmetric MIMO with the proposed power allocation significantly outperforms the same asymmetric MIMO with the alternative power allocation of $P_1/P_2 = 1/3$. More specifically, at the first outer detection iteration, the two power-allocation schemes have almost the identical BER performance. However, at the second iteration, the BER of the proposed power allocation is significantly smaller than that of the alternative power allocation. This demonstrates the effectiveness of this simple closed-form power allocation, which does not require feedback of any CSI to the transmitter. It should be also noted that, although the results in Fig. 3 were obtained with the overall data rate of 1/2, the same conclusion could be drawn for other scenarios, for example, the system with the overall data rate of 3/5.

However, it is worth emphasizing that, if the optimal maximum *a posteriori* probability (MAP) detector with exponentially growing complexity is applied, instead of the low-complexity suboptimal FSD-based detector, the conventional MIMO scheme will outperform the proposed asymmetric MIMO scheme, as predicted by the capacity analysis in Fig. 2. Space limitation precludes including the results obtained based on the optimal MAP detection.

C. Turbo Detection–Decoding Convergence Analysis

The EXIT chart [18] is a powerful tool to analyze the performance of turbo detection–decoding since it can visualize the trajectory of detection and decoding process. We utilize the 2-D EXIT chart to analyze the convergence properties of the asymmetric MIMO scheme with various power allocations. According to [18], under the assumption that the coded bits are equiprobably selected from $\{0, 1\}$, the mutual information I_E between the coded bit $c \in \mathcal{C} = \{0, 1\}$ and the corresponding extrinsic LLR L_E is given by

$$I_E = \frac{1}{2}\sum_{c\in\mathcal{C}} \int_{-\infty}^{+\infty} p_{L_E|c}(x|\mathcal{C}) \times \log_2 \frac{2p_{L_E|c}(x|\mathcal{C})}{p_{L_E|c}(x|c=0)+p_{L_E|c}(x|c=1)}\, dx \quad (9)$$

where $p_{L_E|c}(x|\mathcal{C})$ denotes the probability density function of L_E conditioned on $c \in \mathcal{C}$ and can be obtained by using histogram of the LLRs L_E through Monte Carlo simulations.

For the generic MIMO system with independent streams transmitted on each antenna, the mutual information on MIMO detector is hard to visualize since it depends on the *a priori* information on all the streams, which leads to a high-dimensional function. For the proposed asymmetric constellation scheme, the modulation and coding scheme is the same for all the streams in the same group. Thus, for the asymmetric MIMO with two different groups of streams, the mutual information on the nth group is expressed as

$$I_E^n = T\left(I_D^{Q_1}, I_D^{Q_2}; \frac{E_b}{N_0}\right) \quad (10)$$

where $I_D^{Q_n}$ and I_E^n, $n = 1, 2$, are the average mutual information on the decoder and the MIMO detector for the streams with the constellation order Q_n, respectively, whereas $T(\cdot)$ denotes the EXIT function of the MIMO detector, and it is related to the detection algorithm used, which is the modified FSD algorithm outlined in Section III-C. An example of $Q_1 = 4$ and $Q_2 = 64$ with $E_b/N_0 = 8.3$ dB is shown in Fig. 4. The mutual information on the MIMO detector is divided into two planes. The plane with higher extrinsic information belongs to the constellation with lower order, which means that these streams can obtain more reliable information. The average mutual information is also shown in Fig. 4, and it lies between the two planes associated with the two different constellation orders. The separation of the mutual information for the different streams of the MIMO detector simplifies the convergence analysis.

To simplify the analysis, the EXIT chart can be projected onto 2-D planes [13], [19]. In [19], the average projected EXIT curve is applied to analyze the convergence property. However, since I_E^n, $n = 1, 2$, depends on both $I_D^{Q_1}$ and $I_D^{Q_2}$, the average projected EXIT chart cannot represent the actual convergence property. Here, a more accurate method is applied for the convergence analysis. After the projection, the relationship between I_E^n and $I_D^{Q_n}$ can be written as

$$\begin{cases} I_E^1 = T_{I_D^{Q_2}}\left(I_D^{Q_1}; \frac{E_b}{N_0}\right), & 0 \le I_D^{Q_2} \le 1 \\ I_E^2 = T_{I_D^{Q_1}}\left(I_D^{Q_2}; \frac{E_b}{N_0}\right), & 0 \le I_D^{Q_1} \le 1 \end{cases} \quad (11)$$

where $T_{I_D^{Q_n}}(\cdot)$ denotes the projected EXIT function. Thus, the projected range of the MIMO detector's mutual information can be viewed

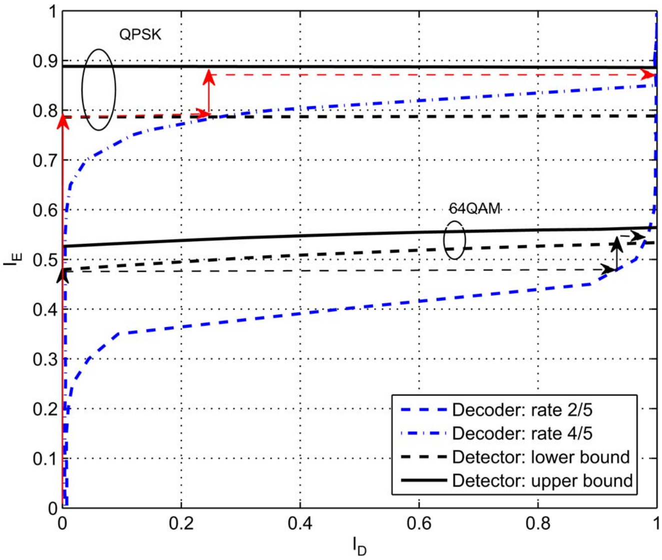

Fig. 5. Projected EXIT chart for the 2 × 2 MIMO system with asymmetric constellation orders of $Q_1 = 4$ and $Q_2 = 64$. The SNR is $E_b/N_0 = 8.3$ dB, and the proposed power allocation is applied.

to consist of a set of disjoint lines, each of which is determined by a given value of the other variable. Considering that $I_D^{Q_1}$ and $I_D^{Q_2}$ are obtained from the previous decoding iteration, (11) can be used for the convergence analysis. Moreover, we need to just display the upper and lower bounds of I_E^n for $n = 1, 2$ to carry out the EXIT chart-based analysis.

Fig. 5 shows the convergence property of the 2 × 2 MIMO with QPSK and 64-QAM modulation schemes and the proposed power allocation. In Fig. 5, the two staircase trajectories denote the decoding paths for the streams with QPSK and 64-QAM, respectively. In each trajectory, the solid-line part represents the detection whereas the dashed-line part corresponds to the decoding. The convergence path starts from the original point with the detection. Since there is no *a priori* information, the mutual information on detection is determined by the lower bound of the detection range with $I_D^{Q_n} = 0$, which leads to $I_E^1 = 0.74$ for QPSK and $I_E^2 = 0.5$ for 64-QAM. After decoding, we find that $I_D^{Q_1} = 0.11$ for QPSK and $I_D^{Q_2} = 0.93$ for 64-QAM. For the second iteration, transfer functions $T_{I_D^{Q_2}=0.93}(I_D^{Q_1}; E_b/N_0)$ and $T_{I_D^{Q_1}=0.11}(I_D^{Q_2}; E_b/N_0)$ are used. At the end of the second iteration, we observe that $I_D^{Q_1} = 1$ and $I_D^{Q_2} = 0.99$. With more iterations, $I_D^{Q_2}$ can approach 1. For any practical purpose, the performance after two iterations is satisfied, and the turbo detection–decoding process is regarded as convergence after two iterations.

For the same asymmetric MIMO system with the equal power allocation, we can similarly analyze its convergence property with the aid of the projected EXIT chart shown in Fig. 6. It is shown in Fig. 6 that, although $I_D^{Q_1}$ reaches 1 after one iteration, $I_D^{Q_2} = 0.92$ after the second iteration. Moreover, since the upper bound for the stream with 64-QAM inserts with the curve of the 4/5 code at $I_D^{Q_2} = 0.92$, $I_D^{Q_2}$ cannot be improved with more iterations. This result leads to the much worse performance for the equal power allocation in comparison with the proposed power allocation.

Fig. 7 shows the projected EXIT chart for the same 2 × 2 asymmetric MIMO system with the power allocation of $P_1/P_2 = 1/3$. Observe in Fig. 7 that now the $I_D^{Q_1} = 1$ point cannot be reached, making the performance worse compared with that of the proposed power allocation as also confirmed in the BER performance comparison in Fig. 3.

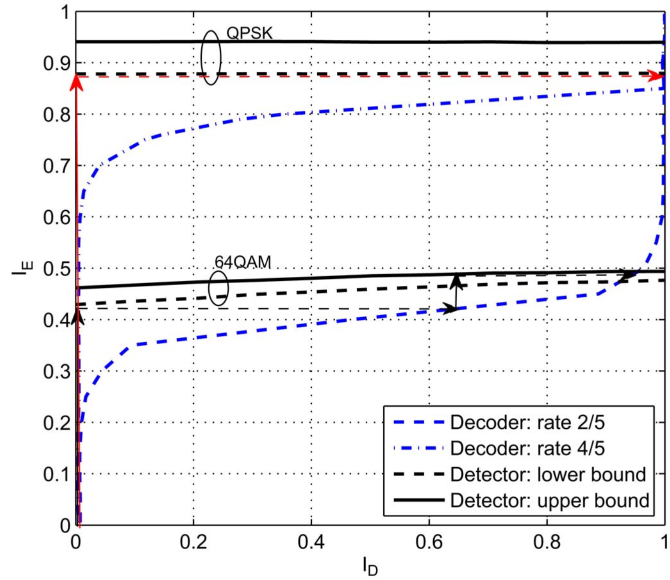

Fig. 6. Projected EXIT chart for the 2 × 2 MIMO system with asymmetric constellation orders of $Q_1 = 4$ and $Q_2 = 64$. The SNR is $E_b/N_0 = 8.3$ dB, and the equal power allocation is applied.

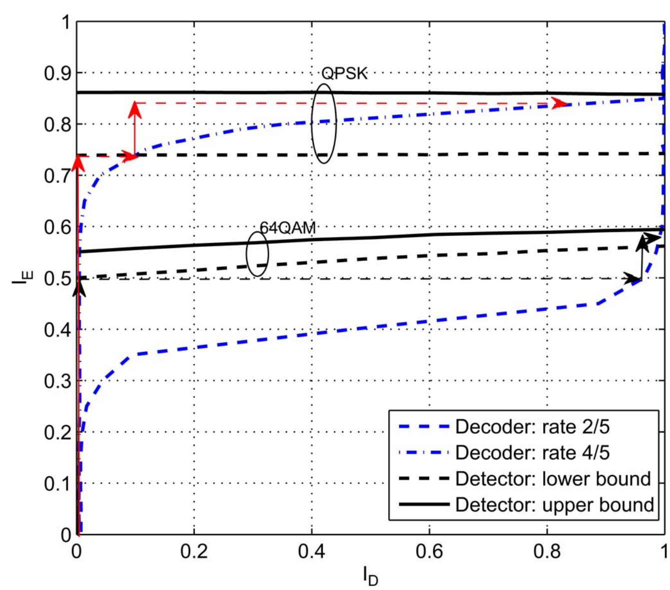

Fig. 7. Projected EXIT chart for the 2 × 2 MIMO system with asymmetric constellation orders of $Q_1 = 4$ and $Q_2 = 64$. The SNR is $E_b/N_0 = 8.3$ dB, and the power allocation $P_1/P_2 = 1 : 3$ is applied.

The previous convergence analysis clearly demonstrates that the asymmetric constellation scheme with the proposed power allocation benefits much more from the turbo detection compared with the other power allocation schemes.

V. Conclusion

We have proposed a novel asymmetric constellation scheme for coded MIMO transmission where different constellation mappings are applied to different transmit streams. A simple yet effective rule has been suggested to select the coding rates of different constellation-mapped streams to improve the overall performance. A highly effective power-allocation method, derived in closed form, further enhances the performance when turbo detection is applied. This power allocation does not require the feedback of any CSI to the transmitter and is optimal for Gaussian-distributed inputs. The modified FSD algorithm has been applied for the detection at the receiver. With well-defined practical error correction codes and the turbo FSD-based detection and decoding, the results obtained have demonstrated that, given the same overall data rate, our proposed scheme outperforms the conventional MIMO system that applies the same constellation set to all transmit streams and uses the FSD-based detection and decoding while imposing lower receiver detection complexity than the latter. The convergence analysis with the aid of the EXIT chart has also demonstrated that the proposed power allocation is suitable for the asymmetric constellation scheme with turbo detection.

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Maximum SINR-Based Receive Combiner for Cognitive MU-MIMO Systems

Hyukmin Son, Seonghyun Kim, and
Sanghoon Lee, *Senior Member, IEEE*

***Abstract*—In this correspondence, we seek to maximize the signal-to-interference-plus-noise ratio (SINR) of cognitive users (CUs) by designing an optimal receive combiner (ORC), taking into account interference from the primary base station (PBS) to CUs [inter-PBS interference (IPI)], multiuser interference (MUI) among CUs, and desired channel gains. By employing the Rayleigh–Ritz quotient result, we formulate an optimal receiver design and then determine the ORC by solving tfshe derived standard eigenvalue problem. The ORCs are derived as closed forms according to the presence or absence of interference beam information at each CU. When the interference beam information is available, the maximum SINR is given as the largest eigenvalue of the received SINR. In the absence of interference beam information, the ORC based on prediction is proposed based on the prediction of the worst IPI and MUI directions and asymptotically analyzed in terms of the number of transmit antennas. Simulation results demonstrate that the ORC maximizes the sum rate of the cognitive multiuser multiple-input/multiple-output system compared with the reference receive combiners.**

***Index Terms*—Cognitive radio (CR), multiple-input multiple-output (MIMO), null space, receive combiner, spectrum sharing.**

I. Introduction

In a cognitive multiple-input/multiple-output (MIMO) system, an increase in spatial dimensionality can be utilized to share the licensed spectrum of primary users (PUs) with cognitive users (CUs). The main motivation of conventional systems stems from ensuring the quality of service (QoS) of PUs by appropriately controlling interference from the cognitive base station (CBS) to the PUs [1], [2]. However, to maximize spectral efficiency for the cognitive radio (CR) system, it is also important to improve the signal-to-interference-plus-noise ratio (SINR) of CUs. Toward this end, it is necessary to take into account multiuser interference (MUI) among CUs, the desired channel gain, and the interference from the primary base station (PBS) to CUs, which is termed as the inter-PBS interference (IPI). As an effort to reduce MUI, prior work has mainly focused on finding linear precoding solutions at the CBS rather than including these three factors altogether [3]–[5]. Moreover, the transceiver schemes presented in [3]–[5] are valid only when the CBS has the perfect channel state information (CSI) of CUs, under the assumption that the CBS has knowledge of the IPI, MUI, and the desired channel gain. These assumptions are impractical due to the excessive feedback overhead from CUs. Even if Chen *et al.* in [6] proposed a practical IPI estimation method using a covariance matrix, it is still restricted to having perfect CSI at the

Manuscript received January 9, 2014; revised September 17, 2014 and October 19, 2014; accepted October 29, 2014. Date of publication November 4, 2014; date of current version September 15, 2015. This work was supported in part by the Basic Science Research Program through the National Research Foundation of Korea funded by the Ministry of Education (NRF-2013R1A1A2A10011764) and in part by the Ministry of Science, ICT and Future Planning, Korea, under the Convergence Information Technology Research Center support program (NIPA-2014-H0401-14-1006) supervised by the National IT Industry Promotion Agency. The review of this paper was coordinated by Dr. E. K. S. Au. *(Corresponding author: Sanghoon Lee.)*

The authors are with the Department of Electrical and Electronic Engineering, Yonsei University, Seoul 120-749, Korea (e-mail: sbpgood@yonsei.ac.kr; sh-kim@yonsei.ac.kr; slee@yonsei.ac.kr).

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Digital Object Identifier 10.1109/TVT.2014.2366951