Pareto Repeated Weighted Boosting Search for Multiple-Objective Optimisation

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Outline

Global Optimisation

- Attaining **global** or near global **optimal solutions** at **affordable** computational **costs** are critical in engineering applications
- We have successful record in applications of **computational intelligence** methods, such as

  adaptive simulated annealing, genetic algorithms, ant colony / particle swarm optimisation, differential evolution algorithm
- Key metrics in assessing a method
  - **Capability**: high successful rate to attain global solutions in challenging problems
  - **Complexity**: fast convergence speed and reasonably low computational costs
  - **Simplicity**: few algorithmic parameters need tuning and easy of programming

RWBS Algorithm

- **Repeated weighted boosting search** is a **guided stochastic** search or meta-heuristic algorithm
  - Ease of implementation/programming
  - very few number of tuning parameters, and
  - capable of achieving levels of performance comparable with standard benchmark techniques, such as GA and ASA
- Successfully apply to various image and signal processing problems as well as wireless communication designs, e.g.
  - Tunable radial basis function data modelling
  - Blind joint channel estimation and data detection
  - Joint timing and channel estimation
- Original RWBS algorithm is for **single-objective** optimisation

S. Chen, X. X. Wang and C. J. Harris, "Experiments with repeating weighted boosting search for optimization in signal processing applications," *IEEE Trans. Systems, Man and Cybernetics, Part B*, vol. 35, no. 4, 682–693, 2005.

Outline

Contributions of This Work

- Extend RWBS algorithm to **multiple-objective** optimisation
    - More specifically, arm the RWBS with a **Pareto-ranking** scheme combined with a **sharing** process
    - Similar to state-of-the-art multiple-objective GA, known as non-dominated sorting genetic algorithm (NSGA-II)
    - Resulting algorithm is therefore referred to as **Pareto** repeated weighted boosting search
- Performance of Pareto RWBS algorithm was assessed using some well-known benchmark problems
    - It offers promising level of performance in solving these multiple-objective optimisation problems,
    - while retaining the attractive properties of the original RWBS

Outline

Original RWBS

- Consider optimisation problem

$$\min_{\mathbf{u}\in\mathbb{U}^n} J(\mathbf{u})$$

$\mathbf{u} = [u_1 \ u_2 \cdots u_n]^{\mathrm{T}}$: decision variable vector, $\mathbb{U}^n$: feasible set of $\mathbf{u}$, and $J(\mathbf{u})$: cost function

- RWBS: population based guided stochastic search
  1. Stochastic search component, outer loop – "**generations**"
     - Random population initialisation with elitism
  2. Local search component, inner loop – "**weighted boosting search**"
     - Convex combination and reflection, with adaptive weighting that **boosts** weak local optimiser
- Algorithmic parameters: population size P_{s}, generations N_{g}, WBS iterations N_{B}

Algorithm

- **Outer loop**: generations for $g = 1 : N_g$
  - Random generation initialisation $\mathbf{u}_i^{(g)}$, $2 \le i \le P_s$, with elitism $\mathbf{u}_1^{(g)} = \mathbf{u}_{\text{best}}^{(g-1)}$
  - Equal initial weightings $\delta_i(0)$ and cost evaluations $J_i = J(\mathbf{u}_i^{(g)})$, $1 \le i \le P_s$
  - **Inner loop**: weighted boosting search $t = 1 : N_B$
    1. Boosting

       1) Best and worst members: $\mathbf{u}_{\text{best}}^{(g)}$ and $\mathbf{u}_{\text{worst}}^{(g)}$, according to costs $\{J_i\}$

       2) Adapt weightings $\delta_i(t)$, $1 \le i \le P_s$, according to costs $\{J_i\}$
    2. Updating

       1) Convex combination $\mathbf{u}_{P_s+1} = \sum_{i=1}^{P_s} \delta_i(t)\mathbf{u}_i^{(g)}$

       2) Reflection $\mathbf{u}_{P_s+2} = \mathbf{u}_{\text{best}}^{(g)} + (\mathbf{u}_{\text{best}}^{(g)} - \mathbf{u}_{P_s+1})$

       3) Best of $\mathbf{u}_{P_s+1}, \mathbf{u}_{P_s+2}$ replaces $\mathbf{u}_{\text{worst}}^{(g)}$ in population
  - **End of Inner loop**: gth generation solution $\mathbf{u}_{\text{best}}^{(g)}$
- **End of Outer loop**: solution $\mathbf{u}_{\text{best}}^{(N_g)}$

Outline

Multiple-Objective Optimisation

- Consider optimisation problem

$$\min_{\mathbf{u}\in\mathbb{U}^n} f\big(J_1(\mathbf{u}), J_2(\mathbf{u}), \cdots, J_N(\mathbf{u})\big)$$

  $J_i(\mathbf{u})$: ith objective function, N: number of objective functions, and f: objective preference function

- True **multiple-objective** optimisation: **no objective preference** structure is available
- Set of **optimal** solutions is characterised by **Pareto-frontier**, and two key aspects of designing efficient Pareto optimisation
  1. Mechanism drives solutions toward Pareto frontier, Pareto **ranking**: promote **non-dominated** solutions
  2. Mechanism ensures distribution of solutions across Pareto frontier, **sharing**: encourage **spread** of solutions

Pareto RWBS

- **Elitism count**: to aid identifying suitable set of Pareto-optimal solutions, P_{e} population members are kept between generations
- **Pareto-ranking**: fast-non-dominated-sort procedure of (Deb *et al.*, 2002) is used to calculated Pareto-ranking

  $$\{R_i\}_{i=1}^{P_{\mathrm{s}}} = \textbf{FastNonDominatedSort}\{J_{i,o}, 1 \le i \le P_{\mathrm{s}}, 1 \le o \le N\}$$

  where $J_{i,o} = J_o\big(\mathbf{u}_i^{(g)}\big)$, $1 \le o \le N$
- **Cost mapping**: given scaling parameter P_{r} and mean distance of $\mathbf{u}_i^{(g)}$ to other points

  $$D_i = \frac{1}{P_{\mathrm{s}}} \sum_{j \neq i} \|\mathbf{u}_i^{(g)} - \mathbf{u}_j^{(g)}\|, 1 \le i \le P_{\mathrm{s}},$$

  distance and ranking **adjusted** costs

  $$\hat{J}_i = \frac{P_{\mathrm{r}} R_i}{D_i}, \ 1 \le i \le P_{\mathrm{s}}$$

Algorithm

- **Outer loop**: generations for $g = 1 : N_{\text{g}}$

  – Pareto generation initialisation: $\mathbf{u}_i^{(g)} = \mathbf{u}_{\text{best},i}^{(g-1)}$, $1 \le i \le P_{\text{e}}$, and randomly generate rest of population $\mathbf{u}_i^{(g)}$, $P_{\text{e}} + 1 \le i \le P_{\text{s}}$

  – Equal initial weightings $\delta_i(0)$ and cost evaluations

$$J_{i,o} = J_o\big(\mathbf{u}_i^{(g)}\big), 1 \le i \le P_{\text{s}}, 1 \le o \le N$$

  – **Inner loop**: weighted boosting search $t = 1 : N_{\text{B}}$

  1. Pareto Boosting
  2. Pareto Updating

  – **End of Inner loop**: choose P_{e} best solutions $\{\mathbf{u}_{\text{best},i}^{(g)}\}_{i=1}^{P_{\text{e}}}$

  For $i = 1 : P_{\text{e}}$

  i) Perform **Pareto Ranking, Distance Measure and Cost Mapping** $\{\mathbf{u}_j^{(g)}, J_{j,o}, 1 \le o \le N\}_{j=1}^{P_{\text{s}}-(i-1)} \rightarrow \{\hat{J}_j\}_{j=1}^{P_{\text{s}}-(i-1)}$

  ii) Find $j_{\text{best}} = \arg \min\limits_{1 \le j \le P_{\text{s}}-(i-1)} \hat{J}_j$, set $\mathbf{u}_{\text{best},i}^{(g)} = \mathbf{u}_{j_{\text{best}}}^{(g)}$, and remove $\mathbf{u}_{j_{\text{best}}}^{(g)}$

- **End of Outer loop**: solution set $\{\mathbf{u}_i^{(N_{\text{g}})}\}_{i=1}^{P_{\text{s}}}$

Inner Loop

1. Pareto Boosting

   1) Perform **Pareto Ranking, Distance Measure and Cost Mapping**

$$\{\mathbf{u}_i^{(g)}, J_{i,o}, 1 \le o \le N\}_{i=1}^{P_\mathrm{s}} \to \{\hat{J}_i\}_{i=1}^{P_\mathrm{s}}$$

   Find $i_\mathrm{best} = \arg \min_{1 \le i \le P_\mathrm{s}} \hat{J}_i$ and denote $\mathbf{u}_\mathrm{best}^{(g)} = \mathbf{u}_{i_\mathrm{best}}^{(g)}$

   2) Adapt weightings $\delta_i(t)$, $1 \le i \le P_\mathrm{s}$, according to costs $\{\hat{J}_i\}$

2. Pareto Updating

   1) Convex combination $\mathbf{u}_{P_\mathrm{s}+1} = \sum_{i=1}^{P_\mathrm{s}} \delta_i(t)\mathbf{u}_i^{(g)}$

   2) Reflection $\mathbf{u}_{P_\mathrm{s}+2} = \mathbf{u}_\mathrm{best}^{(g)} + (\mathbf{u}_\mathrm{best}^{(g)} - \mathbf{u}_{P_\mathrm{s}+1})$

   3) Compute $J_{i,o}(\mathbf{u}_i)$, $1 \le o \le N$ and $i = P_\mathrm{s}+1, P_\mathrm{s}+2$

   4) Removes two worst points to keep population size P_s: For $i = 1 : 2$

      i) Perform **Pareto Ranking, Distance Measure and Cost Mapping** $\{\mathbf{u}_j^{(g)}, J_{j,o}, 1 \le o \le N\}_{j=1}^{P_\mathrm{s}+2-(i-1)} \to \{\hat{J}_j\}_{j=1}^{P_\mathrm{s}+2-(i-1)}$

      ii) Find $j_\mathrm{worst} = \arg \max_{1 \le j \le P_\mathrm{s}+2-(i-1)} \hat{J}_j$, and remove $\mathbf{u}_{j_\mathrm{worst}}^{(g)}$

Outline

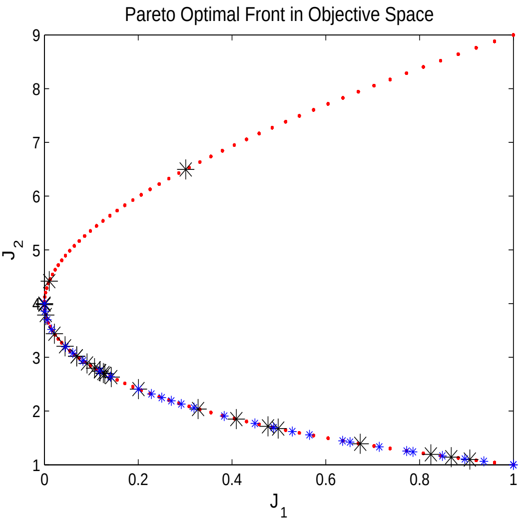
SCH Function

- One-dimensional, exhibits convex Pareto-frontier

$$J_1(u) = u^2$$
$$J_2(u) = (u-2)^2$$

$$u \in [-1,\ 1]$$

- **Red** dot: feasible solutions visualising Pareto-frontier
- **Blue** smaller asterisk: NSGA-II
- **Black** larger asterisk: Pareto-RWBS



Outline

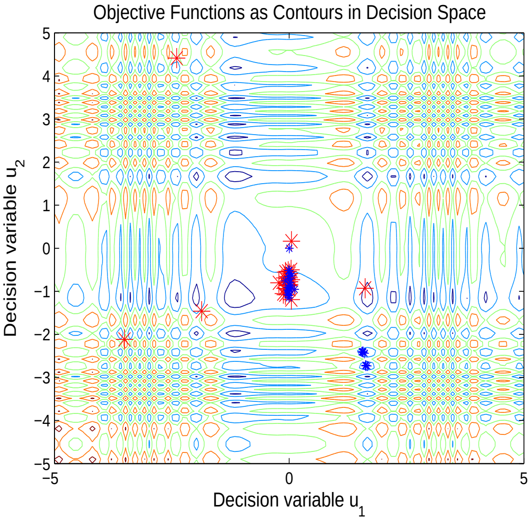
KUR Function

- Two-dimensional, exhibits non-convex Pareto-frontier

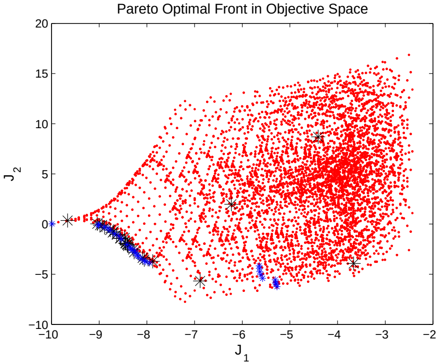
$$J_1 = -10e^{-0.2\sqrt{u_1^2+u_2^2}}$$
$$J_2 = \sum_{i=1}^{2} \left(|u_i|^{0.8} + 5\sin(u_i^3)\right)$$

$$u_i \in [-5,\ 5],\ i = 1, 2$$

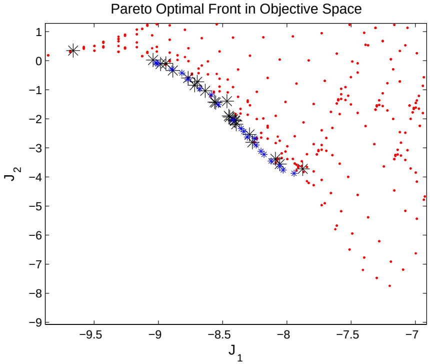
- Overlaid contours: objective functions
- **Blue** smaller asterisk: NSGA-II
- **Red** larger asterisk: Pareto-RWBS



KUR Function (continue)



(a)



(b)

(a) Full objective space, and (b) close-up objective space

- **Red** dot: feasible solutions visualising Pareto-frontier
- **Blue** smaller asterisk: NSGA-II
- **Black** larger asterisk: Pareto-RWBS

Outline

Multi-Modal Function

- Two-dimensional, difficult multi-modal Pareto-frontier

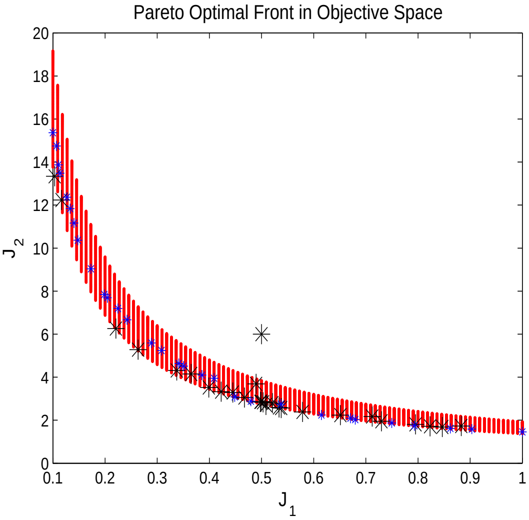
$$J_1(\mathbf{u}) = u_1$$

$$g(u_2) = 2.0 - e^{-\left(\frac{u_2 - 0.2}{0.004}\right)^2} - 0.8 e^{-\left(\frac{u_2 - 0.6}{0.4}\right)^2}$$

$$J_2(\mathbf{u}) = \frac{g(u_2)}{u_1}$$

$$u_1 \in [0.1,\ 1],\ u_2 \in [0,\ 1]$$

- **Red** dot: feasible solutions visualising Pareto-frontier
- Asterisk: **blue** smaller for NSGA-II; **black** larger for Pareto-RWBS



Outline

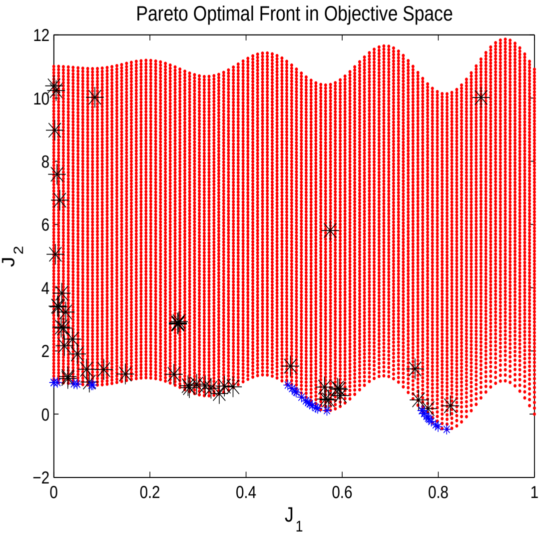
Discontinuous Function

- Two-dimensional, challenging discontinuous Pareto-frontier

$$J_1(\mathbf{u}) = u_1$$
$$g(u_2) = 1 + 10u_2$$
$$J_2(\mathbf{u}) = g(u_2)\left(1 - \left(\frac{J_1(\mathbf{u})}{g(u_2)}\right)^{\alpha} - \frac{J_1(\mathbf{u})}{g(u_2)}\sin\left(2\pi q J_1(\mathbf{u})\right)\right)$$

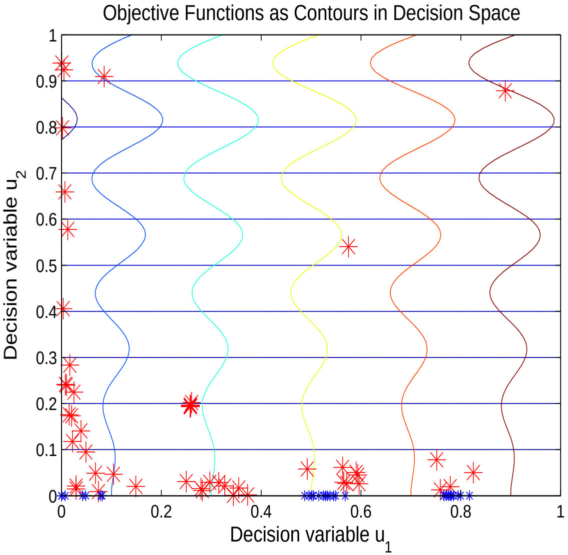
$\alpha = 2,\ q = 4,\ u_1, u_2 \in [0,\ 1]$

- **Red** dot: feasible solutions visualising Pareto-frontier
- **Blue** smaller asterisk: NSGA-II;
- **Black** larger asterisk: Pareto-RWBS



Discontinuous Function (continue)

- Overlaid contours: objective functions
- **Blue** smaller asterisk: NSGA-II
- **Red** larger asterisk: Pareto-RWBS



Outline

Summary

- Pareto RWBS algorithm for multiple-objective optimisation
  - Provide Pareto-**ranking** scheme and **sharing** process to RWBS originally for single-objective optimisation
- Pareto RWBS performs **on par** with NSGA-II algorithm
  - while retaining attractive properties: simplicity, ease of implementation and small number of tuning parameters
- Scopes to further improve Pareto RWBS:
  - improve distribution of its solutions along Pareto-frontier
  - improve accuracy of solutions in terms of their distances to Pareto-frontier

Further Work

- This Pareto RWBS generates **single** convex combination of all candidates
- Future work will investigate **selective combining**
  - develop a selection operator to select which members are used in a **set** of convex combinations
  - thus create a number of new individuals at each inner iteration
  - This is similar to the way a GA proceeds
- We hypothesise this approach will improve performance
  - in terms of solutions' distribution along Pareto-frontier and solutions' accuracy to Pareto-frontier