Particle Swarm Optimisation Aided Multiuser Transmission Schemes for MIMO Communication

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Outline

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Bio-Inspired Algorithms

- Bio-inspired computational intelligence has found wide-ranging applications in all walks of engineering
- Examples of bio-inspired computational intelligence algorithms
  - Evolutionary methods, such as **genetic algorithms**
  - Bio-inspired **ant colony optimisation**
  - Swarm intelligence, such as **particle swarm optimisation**
  - Many many more

Outline

Communication Applications

- What critical to a communication signal processing application are: performance and complexity
- A bio-inspired algorithm must offer near optimal solution with affordable cost
- Communication Research Group at Southampton has a long and successful record in applying
  - genetic algorithms and ant colony optimisation

  to **multiuser receiver** designs
- Our new contribution: Particle swarm optimisation aided **multiuser transmission** for MIMO communication

Outline

PSO Flowchart

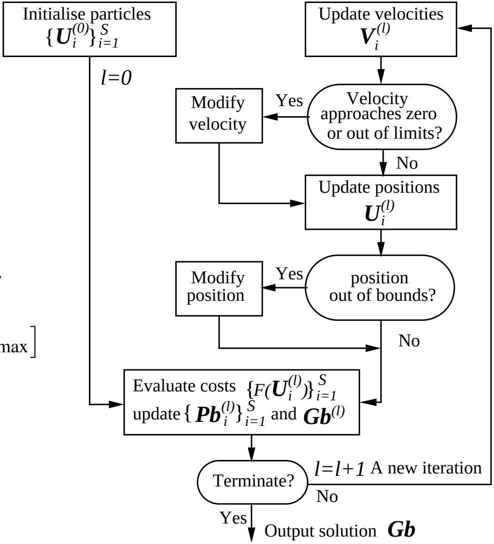
- Solving generic optimisation

$$\mathbf{U}_{\text{opt}} = \arg \min_{\mathbf{U} \in \mathsf{U}^{N\times M}} F(\mathbf{U})$$

U is an $N \times M$ complex-valued parameter matrix to be optimised, $F(\bullet)$ is cost, and search space $\mathsf{U}^{N\times M}$ defined by

$$\mathsf{U} = \left[-U_{\max}, U_{\max}\right] + j\left[-U_{\max}, U_{\max}\right]$$

- A swarm of particles, $\{\mathbf{U}_i^{(l)}\}_{i=1}^{S}$, are "flying" in search space, where S is swarm size and l denotes iteration index



PSO Algorithm Adopted

PSO: A population based stochastic optimisation method inspired by social behaviour of bird flocks or fish schools

- Each particle remembers its best position visited $\Longrightarrow$ *cognitive information*, $\mathbf{Pb}_i^{(l)}$, $1 \le i \le S$
- Every particle knows best position visited among entire swarm $\Longrightarrow$ *social information*, $\mathbf{Gb}^{(l)}$
- Each particle has a **velocity** $\mathbf{V}_i^{(l)} \in \mathrm{V}^{N \times M}$ to direct its "flying", and velocity space is defined by

$$\mathrm{V} = \big[-V_{\max}, V_{\max} \big] + j \big[-V_{\max}, V_{\max} \big]$$

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PSO Procedure

- *a) Initialisation*: Set iteration index $l = 0$ and randomly generate $\{\mathbf{U}_i^{(l)}\}_{i=1}^{S}$ in search space $\mathrm{U}^{N\times M}$
- *b) Evaluation*: Particle $\mathbf{U}_i^{(l)}$ has cost $F(\mathbf{U}_i^{(l)})$, based on which $\mathbf{Pb}_i^{(l)}$, $1 \le i \le S$, and $\mathbf{Gb}^{(l)}$ are updated
- *c) Update*: Velocities and positions are updated

$$\mathbf{V}_i^{(l+1)} = \xi * \mathbf{V}_i^{(l)} + c_1 * \varphi_1 * (\mathbf{Pb}_i^{(l)} - \mathbf{U}_i^{(l)}) + c_2 * \varphi_2 * (\mathbf{Gb}^{(l)} - \mathbf{U}_i^{(l)})$$

$$\mathbf{U}_i^{(l+1)} = \mathbf{U}_i^{(l)} + \mathbf{V}_i^{(l+1)}$$

where $\varphi_1 = rand()$ and $\varphi_2 = rand()$

- *d) Termination*: If maximum number of iterations $I_{\max}$ is reached, terminate with solution $\mathbf{Gb}^{(I_{\max})}$; otherwise, $l = l + 1$ and goto *b)*

PSO Algorithmic Parameters

- Inertial weight: $\xi = rand()$, $\xi = 0$ or a small positive constant
- Time varying acceleration coefficients

$$c_1 = (0.5 - 2.5) * l/l_{\max} + 2.5, \;\; c_2 = (2.5 - 0.5) * l/l_{\max} + 0.5$$

  - Initially, large cognitive component and small social component help particles to exploit better search space
  - Later, small cognitive component and large social component help particles to converge quickly to a minimum
- For our application, typically, S in range of 20 to 40, and $l_{\max}$ in range of 25 to 40
- Search space, $U_{\max}$, is specified by problem, and related velocity space, $V_{\max}$, can be determined empirically

Computational Complexity

- Let complexity of evaluating cost function once be C_{single}
- Since number of cost function evaluations is

$$N_{\text{total}} = S \times I_{\max},$$

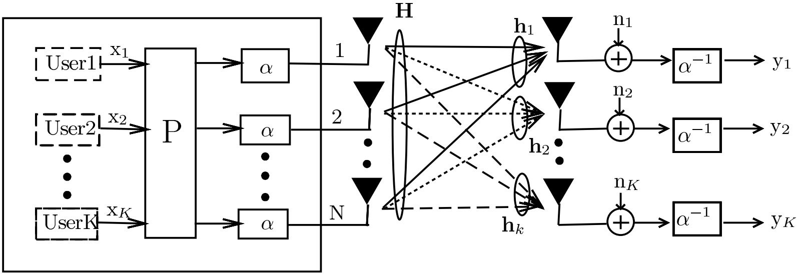
complexity of the algorithm is

$$C_{\text{PSO}} = N_{\text{total}} \times C_{\text{single}} = I_{\max} \times S \times C_{\text{single}}$$

- Choice of S and $I_{\max}$ should ensure achieving optimal solution with minimum complexity
- Attraction of PSO is that the algorithm can be easily tuned to attain optimum with small N_{total}

Outline

Linear MUT



- Base station employs *N* transmit antennas to communicate with *K* single-receive-antenna mobile stations, i.e. downlink
- MSs unable to perform multiuser detection $\Longrightarrow$ Do multiuser transmission at BS instead to combat multiuser interference
- The scheme is “linear” as BS uses *linear* precoding to preprocess transmitted signals

System Model

Linear MUT system model

$$\mathbf{y} = \mathbf{H}^T \mathbf{P}\,\mathbf{x} + \alpha^{-1}\mathbf{n}$$

- $\mathbf{x} = [x_1\ x_2 \cdots x_K]^T$, x_k: **transmitted** 4-QAM **symbol** to kth MS
- $N \times K$ complex-valued **precoder matrix** $\mathbf{P} = [\mathbf{p}_1\ \mathbf{p}_2 \cdots \mathbf{p}_K]$
- Complex-valued MIMO **channel matrix** $\mathbf{H} = [\mathbf{h}_1\ \mathbf{h}_2 \cdots \mathbf{h}_K]$
- Complex-valued Gaussian white noise vector $\mathbf{n} = [n_1\ n_2 \cdots n_K]^T$
- $\alpha = \sqrt{\mathrm{E_T}/\|\mathbf{Px}\|^2}$ for fullfilling power constraint
- $\mathbf{y} = [y_1\ y_2 \cdots y_K]^T$, y_k: kth MS's **received signal**

Given $\mathbf{H}$, $\mathbf{x}$ and statistics of $\mathbf{n}$, design $\mathbf{P} \Longrightarrow$ kth MS can use received y_k directly as sufficient statistics to detect transmitted symbol x_k

Linear MBER MUT Design

- **Minimum mean square error** design $\mathbf{P}_{\mathrm{MMSE}}$ has appealing simplicity but is not optimal
- Optimal **minimum bit error rate** design

$$\begin{aligned} \mathbf{P}_{\mathrm{MBER},\mathbf{x}} &= \arg\min_{\mathbf{P}} P_{e,\mathbf{x}}(\mathbf{P}) \\ \text{s.t.} &\quad \|\mathbf{P}\mathbf{x}\|^2 = \mathrm{E_T} \end{aligned}$$

- **Bit error rate** for 4-QAM symbol vector $\mathbf{x}$

$$P_{e,\mathbf{x}}(\mathbf{P}) = \left(P_{e_I,\mathbf{x}}(\mathbf{P}) + P_{e_Q,\mathbf{x}}(\mathbf{P})\right)/2$$

$$P_{e_I,\mathbf{x}} = \frac{1}{K}\sum_{k=1}^{K} Q\left(\frac{\operatorname{sgn}(\Re[x_k])\Re[\mathbf{h}_k^T\mathbf{P}\mathbf{x}]}{\sigma_n}\right) \quad P_{e_Q,\mathbf{x}} = \frac{1}{K}\sum_{k=1}^{K} Q\left(\frac{\operatorname{sgn}(\Im[x_k])\Im[\mathbf{h}_k^T\mathbf{P}\mathbf{x}]}{\sigma_n}\right)$$

$Q(\bullet)$ is Gaussian error function, and σ_n^2 noise variance

Outline

Low-Complexity PSO Aided Solution

- MBER design is typically solved by sequential quadratic programming (SQP) algorithm $\Longrightarrow$ high complexity
- Low-complexity alternative: using PSO to solve

$$\mathbf{P}_{\mathrm{MBER},\mathbf{x}} = \arg \min_{\mathbf{P} \in \mathrm{U}^{N \times K}} F(\mathbf{P})$$

by defining cost $F(\mathbf{P}) = P_{e,\mathbf{x}}(\mathbf{P}) + G_{\mathbf{x}}(\mathbf{P})$ with penalty function

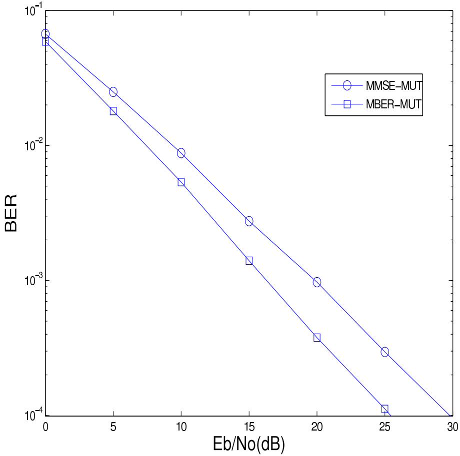
$$G_{\mathbf{x}}(\mathbf{P}) = \begin{cases} 0, & \|\mathbf{Px}\|^2 - \mathrm{E_T} \leq 0 \\ \lambda(\|\mathbf{Px}\|^2 - \mathrm{E_T}), & \|\mathbf{Px}\|^2 - \mathrm{E_T} > 0 \end{cases}$$

- Empirical PSO algorithmic parameter tuning:
  - One of the initial particles set to $\mathbf{P}_{\mathrm{MMSE}}$
  - Search limit $U_{\max} = 1$ while velocity limit $V_{\max} = 1$
  - Remove previous velocity's influence with $\xi = 0$

Experimental Results

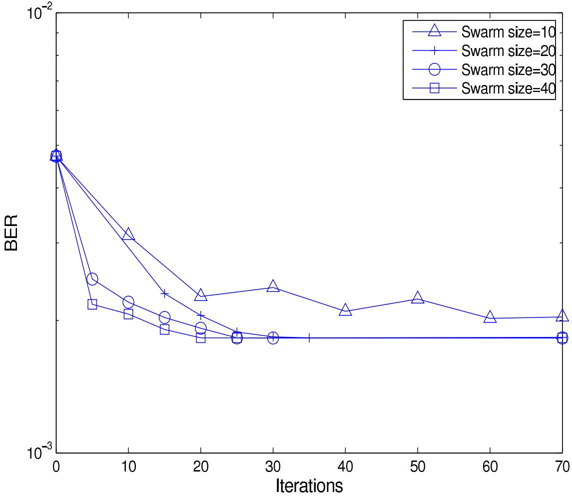
BER **performance** of PSO-aided linear **MBER-MUT** design for the 4×4 MIMO system, in comparison with **MMSE-MUT** benchmark

- MIMO system employed $N = 4$ transmit antennas at BS to communicate with $K = 4$ MSs
- All the simulation results were obtained by averaging over 100 channel realisations
- Appropriate swarm size $S = 20$ was found empirically
- Maximum number of iterations, $I_{\max}$, was in range of 20 to 30, depending on channel SNR



Convergence versus Swarm Size

Convergence of PSO aided linear MBER-MUT design with different swarm sizes for the 4×4 MIMO system given SNR$=$ 15 dB



Swarm Size versus Complexity

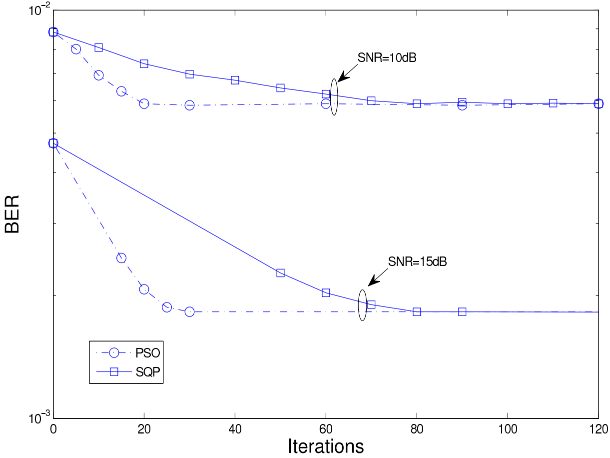
Complexity (Flops) of PSO aided design with different swarm sizes for the 4×4 MIMO system given SNR$=$ 15 dB

Swarm size S	**20**	30	40
Iterations $I_{\max}$	**30**	25	20
C_{PSO} (Flops)	**402,840**	$503,450$	$536,960$

- $S = 10$ insufficient for PSO to attain optimal solution
- PSO with $S = 20$, 30 and 40 converged to optimal solution after $I_{\max} = 30$, 25 and 20, respectively
- $S = 20$ **optimal** in terms of complexity

Convergence Comparison

Convergence performance of PSO and SQP based MBER-MUT schemes for the 4×4 MIMO system given two SNR values



Complexity Comparison

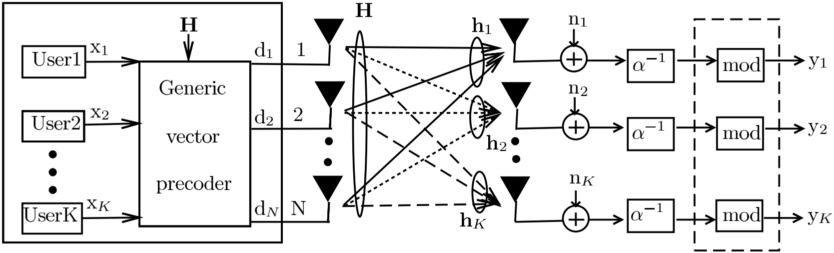
Complexity (Flops) and run time (s) of PSO and SQP aided designs for the 4×4 MIMO system given two SNR values

(SNR= 10 dB)	SQP	PSO
Iterations	70	20
Complexity (Flops)	$3,180,170$	$268,560$
Run time (s)	7412.1	664.9
(SNR= 15 dB)	SQP	PSO
Iterations	80	30
Complexity (Flops)	$3,634,480$	$402,840$
Run time (s)	8457.3	957.4

- PSO-aided design imposed approximately **twelve times lower** complexity than SQP counterpart at SNR= 10 dB
- PSO-aided design imposed approximately **nine times lower** complexity than SQP counterpart at SNR= 15 dB

Outline

Nonlinear MUT



- Base station employs N transmit antennas to communicate with K single-receive-antenna mobile stations, i.e. downlink
- The scheme is “nonlinear” as BS uses **vector precoding** to preprocess Tx signals, and each MS has a modulo device
- Capable of outperforming linear MUT, particularly in **rank-deficient** case of $N < K$, at cost of higher complexity

Conventional Design

- Given $N \times K$ channel matrix $\mathbf{H}$ and K-element symbol vector $\mathbf{x}$, MMSE VP generates N-element effective symbol vector

$$\mathbf{d} = \mathbf{P}(\mathbf{x} + \omega)$$

  according to MMSE criterion, to mitigate multiuser interference
  - $\mathbf{P}$ is $N \times K$ precoding matrix
  - ω is K-element discrete-valued perturbation vector
- Received signal vector $\hat{\mathbf{y}} = [\hat{y}_1 \cdots \hat{y}_K]^T$ before modulo device is

$$\hat{\mathbf{y}} = \mathbf{H}^T \mathbf{d} + \alpha^{-1} \mathbf{n}$$

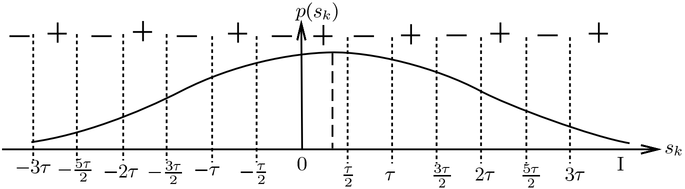
- **Modulo operation** is invoked for each $\hat{y}_k$

$$y_k = mod_\tau(\hat{y}_k) = \hat{y}_k - \lfloor \frac{\Re[\hat{y}_k] + \frac{\tau}{2}}{\tau} \rfloor \tau - j \lfloor \frac{\Im[\hat{y}_k] + \frac{\tau}{2}}{\tau} \rfloor \tau$$

  - $\lfloor \bullet \rfloor$ denotes integer floor operator
  - τ is a positive number determined by modulation scheme
- kth MS uses y_k to detect transmitted information symbol x_k

Generalised VP Design

- We would like to directly design effective symbol vector **d** according to MBER criterion
- Signed decision variable, $s_k = \text{sgn}(\Re[x_k])\Re[\hat{y}_k]$, has probability density function



  - A decision error occurs when s_k falls into intervals $[\frac{2m+1}{2}\tau, \ (m+1)\tau)$ for $-\infty < m < \infty$ (marked by $-$)

- Accurate approximation for BER of the in-phase component associated kth MS, $P_{e_I,k}(\mathbf{d})$, can readily be derived

MBER VP Design

- For 4-QAM case, average BER of in-phase component of $\mathbf{y}$ is

$$P_{e_I,\mathbf{x}}(\mathbf{d}) = \frac{1}{K}\sum_{k=1}^{K} P_{e_I,k}(\mathbf{d})$$

- Similarly, average BER for quadrature-phase component of $\mathbf{y}$ is

$$P_{e_Q,\mathbf{x}}(\mathbf{d}) = \frac{1}{K}\sum_{k=1}^{K} P_{e_Q,k}(\mathbf{d})$$

- Average BER of $\mathbf{y}$ is then

$$P_{e,\mathbf{x}}(\mathbf{d}) = \frac{1}{2}\left(P_{e_I,\mathbf{x}}(\mathbf{d}) + P_{e_Q,\mathbf{x}}(\mathbf{d})\right)$$

- Optimal effective symbol vector $\mathbf{d}_{\text{opt}}$ can be found by solving

$$\mathbf{d}_{\text{opt}} = \arg \min_{\mathbf{d}\in \mathrm{U}^N} P_{e,\mathbf{x}}(\mathbf{d})$$

Outline

Why PSO

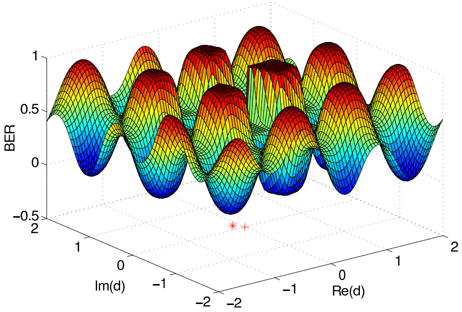
- MBER optimisation

$$\mathbf{d}_{\text{opt}} = \arg \min_{\mathbf{d} \in \mathrm{U}^N} P_{e,\mathbf{x}}(\mathbf{d})$$

  is a non-convex optimisation with many local minima

- PSO algorithm offers an effective means to solve this challenging problem
- $U_{\max} = 1.2$ and $V_{\max} = 0.2$
- Inertia weight $\xi = rand()$
- One of initial particles set to improved MMSE-VP solution

BER surface as a function of effective symbol vector $\mathbf{d}$ for 4-QAM system with $N = 1$ and $K = 1$, given SNR$= 16$ dB

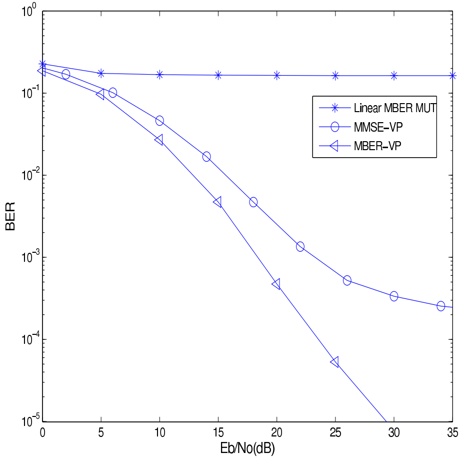


Mark $*$ is MBER generalied VP solution while mark $+$ is MMSE VP solution

Experimental Results

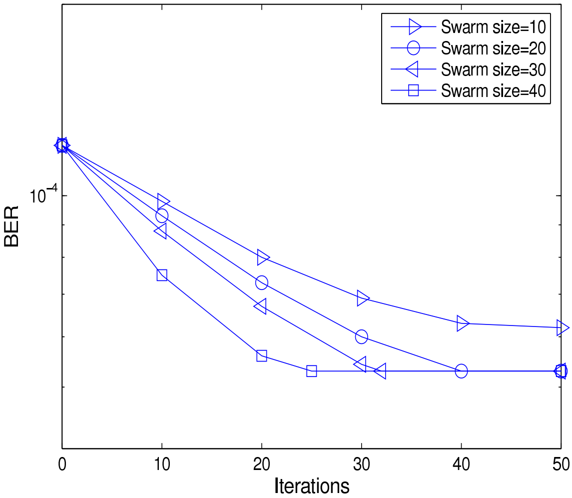
Performance comparison of linear MBER-MUT, nonlinear MMSE-VP and PSO-aided MBER generalised VP for 2×4 MIMO system

- MIMO system employed $N = 2$ transmit antennas at BS to communicate with $K = 4$ MSs
- All the simulation results were obtained by averaging over 100 channel realisations
- Appropriate swarm size $S = 20$ was found empirically
- Maximum number of iterations, $I_{\max}$, was in range of 20 to 45, depending on channel SNR



Convergence versus Swarm Size

Convergence of PSO aided MBER generalised VP design with different swarm sizes for the 2×4 MIMO system given SNR$=$ 25 dB



Swarm Size versus Complexity

Complexity of PSO aided MBER-VP design with different swarm sizes for the 2×4 MIMO system given SNR$= 25$ dB

Swarm size S	**20**	30	40
Iterations $I_{\max}$	**40**	32	25
Complexity (Flops)	**4,064,937**	$4,149,627$	$4,174,077$

- $S = 10$ insufficient for PSO to attain optimal MBER generalised VP solution
- PSO with $S = 20$, 30 and 40 converged to optimal solution after $I_{\max} = 40$, 32 and 25, respectively
- $S = 20$ **optimal** in terms of complexity

Complexity Comparison

Complexity (Flops) and run time (s) required by MMSE-VP design and PSO-aided MBER-VP design for the 2×4 MIMO system given two SNR values

(SNR= 25 dB)	MMSE-VP	MBER-VP
Complexity (Flops)	$2,508,638$	$4,064,937$
Run time (s)	4787.3	8878.9
(SNR= 30 dB)	MMSE-VP	MBER-VP
Complexity (Flops)	$2,609,600$	$4,471,060$
Run time (s)	4981.9	9565.8

- Complexity of PSO aided MBER-VP design is no more than twice of MMSE-VP design
- PSO aided MBER-VP design achieves significantly better performance than MMSE-VP design

Summary

- PSO has been invoked for designing MUT schemes for MIMO systems
- PSO aided designs are capable of attaining global or near global optimal solutions at affordable computational costs
- PSO aided linear MBER MUT design imposes significantly lower computational complexity than state-of-the-art SQP-based linear MBER MUT design
- PSO aided nonlinear MBER generalised VP design outperforms powerful nonlinear MMSE VP solution considerably, at cost of slightly increased complexity