Blind Joint Maximum Likelihood Channel Estimation and Data Detection for Single-Input Multiple-Output Systems

S. Chen, X.C. Yang and L. Hanzo

School of Electronics and Computer Science
University of Southampton, Southampton SO17 1BJ, U.K.

Abstract— **A blind adaptive scheme is proposed for joint maximum likelihood (ML) channel estimation and data detection of single-input multiple-output (SIMO) systems. The joint ML optimization over channel and data is decomposed into an iterative optimization loop. An efficient global optimization algorithm called the repeated weighted boosting search is employed at the upper level to identify optimally the unknown SIMO channel model, and the Viterbi algorithm is used at the lower level to produce the maximum likelihood sequence estimation of the unknown data sequence. A simulation example is used to demonstrate the effectiveness of this joint ML optimization scheme for blind adaptive SIMO systems. Our simulation study shows that this scheme requires very few received data samples to achieve a near optimal solution of the joint ML SIMO channel estimation and data detection.**

I. Introduction

The single-input multiple-output (SIMO) system, consisting of a single-antenna transmitter and a receiver equipped with multiple antennas, has enjoyed popularity owing to its simplicity. A space-time equalizer (STE) based on this SIMO structure is capable of mitigating the channel impairment arising from hostile multipath propagation. For the sake of improving the achievable system throughput, blind adaptation of the STE is attractive, since this avoids the reduction of the effective throughput by invoking training. Blind space-time equalization of the SIMO system can be performed by directly adjusting the STE's parameters using the constant modulus algorithm (CMA) type adaptive scheme [1]-[3]. Blind space-time equalization performance can further be improved by aiding the CMA with a soft decision-directed scheme [4]. Research for blind adaptive SIMO systems has also been focused on blind channel identification [5]-[7]. Once the SIMO channel impulse responses (CIRs) have been identified, various designs, such as the minimum mean square error or minimum bit error rate [8], can be invoked for the STE. Alternatively, the decoupled weighted iterative least squares with projection (DW-ILSP) algorithm [9],[10] can be adopted. The DW-ILSP algorithm may be viewed as a batch expectation-maximization type algorithm, which iteratively performs channel estimation and symbol detection. In general, however, the DW-ILSP algorithm cannot achieve the optimal joint maximum likelihood (ML) channel estimation and data detection for the SIMO system.

In the content of single-input single-output (SISO) blind channel equalization, the optimal joint ML channel estimation and data detection can be realized or approximated closely using for example the blind trellis search or per-survivor processing techniques [11],[12], the quantized channel approach [13], and the combined genetic algorithm (GA) for channel estimation and Viterbi algorithm (VA) for data detection [14]. This paper develops a blind adaptive scheme of joint ML channel estimation and data detection for the SIMO system. The proposed algorithm decomposes the joint optimization over channel and data into an iterative optimization loop by combining a global optimization search method, referred to as the repeated weighted boosting search (RWBS) [15], for an optimal estimation of the SIMO channel and the VA for the maximum likelihood sequence estimation of the transmitted data sequence. Specifically, at the upper level, the RWBS algorithm [15] searches the channel parameter space to optimize the ML criterion, while at the lower level, the VA decodes data based on the given channel model and feeds back the corresponding likelihood metric to the RWBS algorithm. The effectiveness of this joint ML estimation scheme for blind equalization of the SIMO system is demonstrated by a simulation example. We also point out that a GA can be used in the place of the RWBS algorithm to optimize the SIMO channel estimate. In this case the proposed scheme becomes an extension of the joint ML channel and data estimation scheme using the GA originally developed for the SISO system [14]. Our simulation results suggests that both the RWBS-based and GA-based schemes attain a similarly convergence speed but the RWBS-based methods achieves a more accurate blind SIMO channel estimation.

II. The proposed blind joint ML estimation algorithm

Consider the SIMO system employing a single transmitter antenna and L (> 1) receiver antennas. The symbol-rate sampled antennas' outputs $x_l(k)$, $1 \le l \le L$, are given by

$$x_l(k) = \sum_{i=0}^{n_c-1} c_{i,l} s(k-i) + n_l(k) \qquad (1)$$

where $n_l(k)$ is the complex-valued Gaussian white noise associated with the lth channel and $E[|n_l(k)|^2] = 2\sigma_n^2$, $\{s(k)\}$ is the transmitted symbol sequence and is assumed to take values from the quadrature phase shift keying (QPSK) symbol set $\{\pm 1 \pm j\}$, and $c_{i,l}$ are the CIR taps associated with the lth receive antenna. For notational simplicity, we have assumed that each of the L channels has the same length of n_c. Let

$$\begin{aligned}\mathbf{x} &= [x_1(1)\, x_1(2) \cdots x_1(N)\, x_2(1) \cdots \\ &\quad x_L(1)\, x_L(2) \cdots x_L(N)]^T \qquad (2)\\ \mathbf{s} &= [s(-n_c+2) \cdots s(0)\, s(1) \cdots s(N)]^T \qquad (3)\\ \mathbf{c} &= [c_{0,1}\, c_{1,1} \cdots c_{n_c-1,1}\, c_{0,2} \cdots \\ &\quad c_{0,L}\, c_{1,L} \cdots c_{n_c-1,L}]^T \qquad (4)\end{aligned}$$

be the vector of $N \times L$ received signal samples, the corresponding transmitted data sequence and the vector of the SIMO CIRs, respectively. The probability density function of the received data vector $\mathbf{x}$ conditioned on the SIMO CIR $\mathbf{c}$ and the symbol squence $\mathbf{s}$ is

$$p(\mathbf{x}|\mathbf{c},\mathbf{s}) = \frac{1}{(2\pi\sigma_n^2)^{NL}} \times e^{-\frac{1}{2\sigma_n^2}\sum_{k=1}^{N}\sum_{l=1}^{L}\left|x_l(k)-\sum_{i=0}^{n_c-1}c_{i,l}s(k-i)\right|^2} \qquad (5)$$

The joint ML estimate of $\mathbf{c}$ and $\mathbf{s}$ is obtained by maximizing $p(\mathbf{x}|\mathbf{c},\mathbf{s})$ over $\mathbf{c}$ and $\mathbf{s}$ jointly. Equivalently, the joint ML estimate is the minimum of the cost function

$$J_{\mathrm{ML}}(\hat{\mathbf{c}},\hat{\mathbf{s}}) = \frac{1}{N}\sum_{k=1}^{N}\sum_{l=1}^{L}\left|x_l(k) - \sum_{i=0}^{n_c-1}\hat{c}_{i,l}\hat{s}(k-i)\right|^2 \qquad (6)$$

namely

$$(\hat{\mathbf{c}}^*,\hat{\mathbf{s}}^*) = \arg\left[\min_{\hat{\mathbf{c}},\hat{\mathbf{s}}} J_{\mathrm{ML}}(\hat{\mathbf{c}},\hat{\mathbf{s}})\right] \qquad (7)$$

The joint minimization process (7) can also be solved using an iterative loop first over the data sequences $\hat{\mathbf{s}}$ and then over all the possible channels $\hat{\mathbf{c}}$:

$$(\hat{\mathbf{c}}^*,\hat{\mathbf{s}}^*) = \arg\left[\min_{\hat{\mathbf{c}}}\left(\min_{\hat{\mathbf{s}}} J_{\mathrm{ML}}(\hat{\mathbf{c}},\hat{\mathbf{s}})\right)\right] \qquad (8)$$

The inner or lower-level optimization can readily be carried out using the standard VA. In order to guarantee a joint ML estimate, the search algorithm used in the outer or upper-level optimization should be capable of finding a global optimal or near optimal channel estimate efficiently. We employ the RWBS guided random search algorithm [15] to perform the outer optimization task. The detailed RWBS algorithm as a generic global optimizer is given in Appendix. The proposed blind joint ML optimization scheme can now be summarized.

Outer level Optimization. The RWBS algorithm searches the SIMO channel parameter space to find a global optimal estimate $\hat{\mathbf{c}}^*$ by minimizing the mean square error (MSE)

$$J_{\mathrm{MSE}}(\hat{\mathbf{c}}) = J_{\mathrm{ML}}(\hat{\mathbf{c}},\tilde{\mathbf{s}}^*) \qquad (9)$$

TABLE I

THE SIMULATED SIMO SYSTEM.

l	Channel impulse response		
1	0.365-0.274j	0.730+0.183j	-0.440+0.176j
2	0.278+0.238j	-0.636+0.104j	0.667-0.074j
3	-0.639+0.249j	-0.517-0.308j	0.365+0.183j
4	-0.154+0.693j	-0.539-0.077j	0.268-0.358j

Inner level optimization. Given the channel estimate $\hat{\mathbf{c}}$, the VA provides the ML decoded data sequence $\tilde{\mathbf{s}}^*$, and feeds back the corresponding value of the likelihood metric $J_{\mathrm{ML}}(\hat{\mathbf{c}},\tilde{\mathbf{s}}^*)$ to the upper level.

Let C_{VA} be the complexity of the VA required to decode a data sequence of $N \times L$ samples, and denote N_{VA} the total number of VA calls required for the RWBS algorithm to converge. The complexity of the proposed scheme is obviously $N_{\mathrm{VA}} \times C_{\mathrm{VA}}$. Note that the proposed blind SIMO equalization scheme is capable of achieving a (near) optimal joint ML solution for SIMO channel estimation and data detection with a very small N. The RWBS algorithm is a simple yet efficient global search algorithm. In several global optimization applications investigated in [15], including the blind joint ML channel estimation and data detection for the SISO system, the RWBS algorithm achieved a similar global convergence speed as the GA and was seen to be more accurate than the GA-based scheme. The RWBS algorithm has additional advantages of requiring minimum programming effort and having very few algorithmic parameters that require to set.

III. SIMULATION EXAMPLE

In the simulation, the number of receive antennas was $L = 4$, the transmitted data symbols were QPSK, and the SIMO CIRs, listed in Table I, were simulated. The length of data samples was $N = 50$. In practice, the value of the likelihood metric $J_{\mathrm{MSE}}(\hat{\mathbf{c}})$ is all that the upper level optimizer can see, and the convergence of the algorithm can only be observed through the MSE (9). In simulation, the performance of the algorithm can also be assessed by the mean tap error which is defined as

$$\mathrm{MTE} = \|\mathbf{c} - a \cdot \hat{\mathbf{c}}\|^2 \qquad (10)$$

where

$$a = \begin{cases} +1, & \text{if } \hat{\mathbf{c}} \rightarrow +\mathbf{c} \\ -1, & \text{if } \hat{\mathbf{c}} \rightarrow -\mathbf{c} \\ -j, & \text{if } \hat{\mathbf{c}} \rightarrow +j\mathbf{c} \\ +j, & \text{if } \hat{\mathbf{c}} \rightarrow -j\mathbf{c} \end{cases} \qquad (11)$$

Note that since $(\hat{\mathbf{c}}^*,\hat{\mathbf{s}}^*)$, $(-\hat{\mathbf{c}}^*,-\hat{\mathbf{s}}^*)$, $(-j\hat{\mathbf{c}}^*,+j\hat{\mathbf{s}}^*)$ and $(+j\hat{\mathbf{c}}^*,-j\hat{\mathbf{s}}^*)$ are all the solutions of the joint ML estimation problem (7), the channel estimate $\hat{\mathbf{c}}$ can converges to $\mathbf{c}$, $-\mathbf{c}$, $j\mathbf{c}$ or $-j\mathbf{c}$. Since the CIRs can always be normalized,

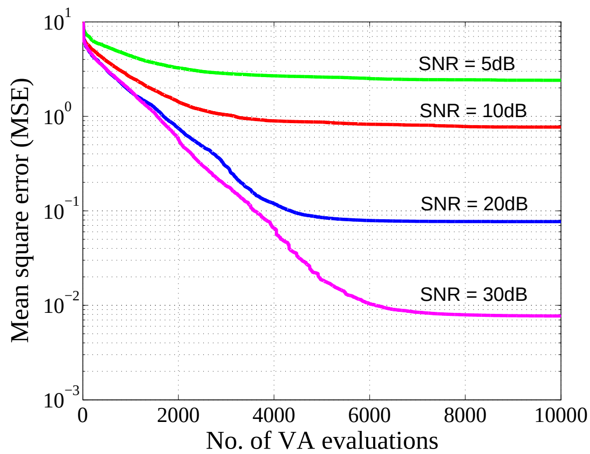

Fig. 1. Mean square error against number of VA evaluations averaged over 50 runs using the RWBS for the SIMO channel listed in Table I. The length of data samples $N = 50$.

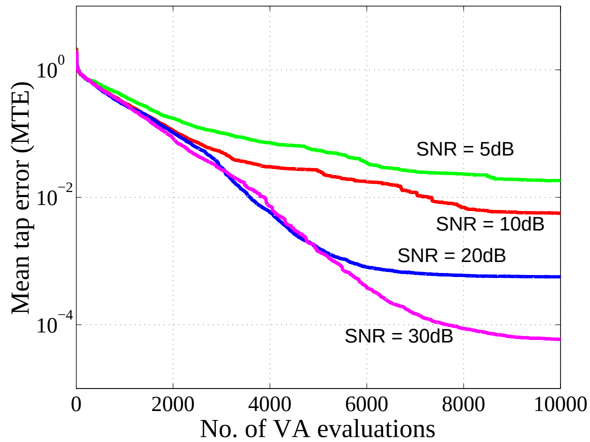

Fig. 2. Mean tap error against number of VA evaluations averaged over 50 runs using the RWBS for the SIMO channel listed in Table I. The length of data samples $N = 50$.

uniformly random sampling initialization with $c_{i,j} \in [-1, 1]$ was adopted by the RWBS algorithm.

Figs. 1 and 2 show the evolutions of the MSE and MTE averaged over 50 runs and for different values of signal to noise ratio (SNR), respectively, obtained by the proposed blind joint ML optimization scheme using the RWBS. From Fig. 1, it can be seen that the MSE converged to the noise floor. Phase ambiguity of $90°$, $180°$ or $270°$ associated with the blind ML estimate for $\mathbf{s}$ cannot be resolved by the blind adaptive scheme itself. In practice, this ambiguity is resolved either by adopting differential encoding or by employing a few pilot training symbols. We adopt the complete blind adaptive scheme of using differential encoding. Fig. 3 depicts the bit error rate (BER) of the blind joint ML optimization scheme with differential encoding, in comparison with the BERs of the optimal maximum likelihood sequence estimation in the known channel case with and without differential encoding. It is seen that the proposed blind scheme only induced half dB degradation in SNR compared with the optimal solution with differential encoding. We also investigated using the GA to perform the upper-level optimization, and the results obtained by this GA-based blind joint ML estimation scheme are presented in Figs. 4 and 5. Comparing Fig. 1 with Fig. 4, it can be seen that both the RWBS and GA based schemes have similar convergence speed, in terms of the total number of required VA evaluations. It can also be seen that the true estimation accuracy of the RWBS-based scheme is more accurate than the GA-based one, as confirmed by comparing Fig. 2 with Fig. 5.

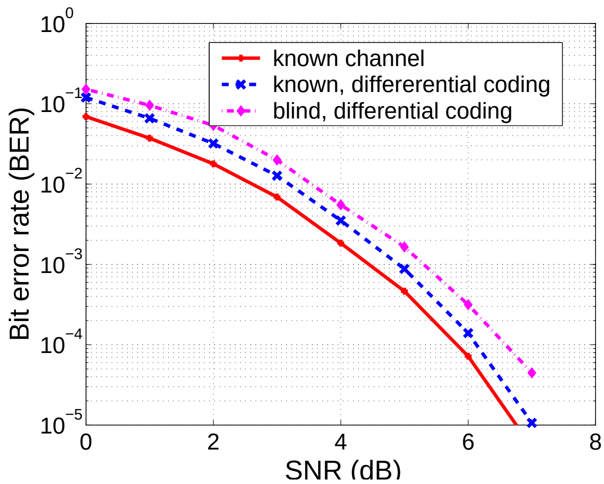

Fig. 3. Comparison of bit error rate performance using the maximum likelihood sequence detection for the SIMO channel listed in Table I. The length of data samples for the blind scheme is $N = 50$.

IV. Conclusions

A batch scheme using the global optimization method, called the RWBS, has been developed for blind space-time equalization of the SIMO system based on the joint ML channel estimation and data detection. The proposed algorithm provides the best performance over other types of blind adaptive schemes for SIMO systems, at the expense of computational complexity. Our simulation study has shown that this blind joint ML optimization scheme requires very few received data samples to achieve a near optimal solution of the joint maximum likelihood SIMO channel estimation and data detection.

Appendix. Repeated Weighted Boosting Search

Consider solving the generic optimization problem

$$\min_{\mathbf{u} \in \mathcal{U}} J(\mathbf{u}) \tag{12}$$

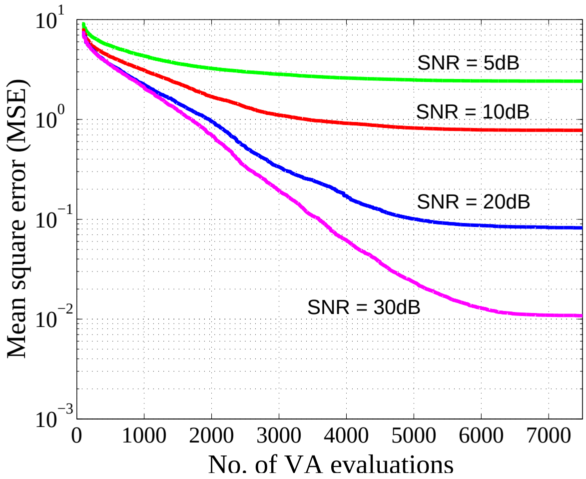

Fig. 4. Mean square error against number of VA evaluations averaged over 50 runs using the GA for the SIMO channel listed in Table I. The length of data samples $N = 50$.

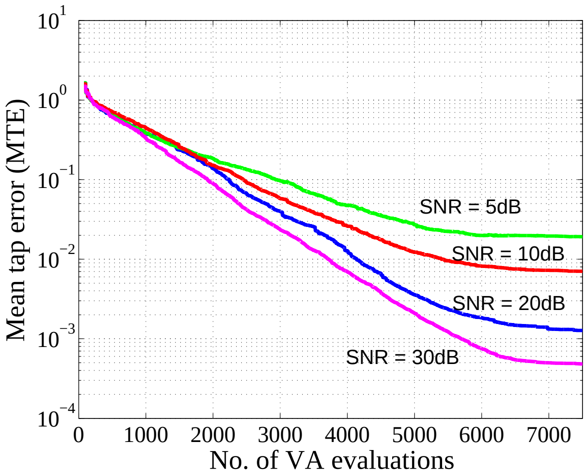

Fig. 5. Mean tap error against number of VA evaluations averaged over 50 runs using the GA for the SIMO channel listed in Table I. The length of data samples $N = 50$.

where $\mathcal{U}$ defines the feasible set of $\mathbf{u}$, with the RWBS algorithm [15]. The algorithm is detailed in the following.

Specify the following RWBS algorithmic parameters: P_S – population size, N_G – number of generations in the repeated search, and ξ_B – accuracy for terminating the inner weighted boosting search.

Outer loop: generations For $g = 1 : N_G$

Generation initialization: Initialize the population by setting $\mathbf{u}_1^{(g)} = \mathbf{u}_{\text{best}}^{(g-1)}$ and randomly generating rest of the population members $\mathbf{u}_i^{(g)} \in \mathcal{U}$, $2 \le i \le P_S$, where $\mathbf{u}_{\text{best}}^{(g-1)}$ denotes the solution found in the previous generation. If $g = 1$, $\mathbf{u}_1^{(g)}$ is also randomly chosen

Weighted boosting search initialization: Assign the initial distribution weightings $\delta_i(0) = \frac{1}{P_S}$, $1 \le i \le P_S$, for the population, and calculate the cost function value of each point

$$J_i = J(\mathbf{u}_i^{(g)}),\ 1 \le i \le P_S$$

Inner loop: weighted boosting search Set $t = 0$; $t = t+1$

Step 1: Boosting

1. Find

$$i_{\text{best}} = \arg \min_{1 \le i \le P_S} J_i \quad \text{and} \quad i_{\text{worst}} = \arg \max_{1 \le i \le P_S} J_i$$

Denote $\mathbf{u}_{\text{best}}^{(g)} = \mathbf{u}_{i_{\text{best}}}^{(g)}$ and $\mathbf{u}_{\text{worst}}^{(g)} = \mathbf{u}_{i_{\text{worst}}}^{(g)}$

2. Normalize the cost function values

$$\bar{J}_i = \frac{J_i}{\sum_{m=1}^{P_S} J_m},\ 1 \le i \le P_S$$

3. Compute a weighting factor β_t according to

$$\eta_t = \sum_{i=1}^{P_S} \delta_i(t-1)\bar{J}_i,\ \beta_t = \frac{\eta_t}{1-\eta_t}$$

4. Update the distribution weightings for $1 \le i \le P_S$

$$\delta_i(t) = \begin{cases} \delta_i(t-1)\beta_t^{\bar{J}_i}, & \text{for } \beta_t \le 1 \\ \delta_i(t-1)\beta_t^{1-\bar{J}_i}, & \text{for } \beta_t > 1 \end{cases}$$

and normalize them

$$\delta_i(t) = \frac{\delta_i(t)}{\sum_{m=1}^{P_S} \delta_m(t)},\ 1 \le i \le P_S$$

Step 2: Parameter updating

1. Construct the (P_S+1)th point using the formula

$$\mathbf{u}_{P_S+1} = \sum_{i=1}^{P_S} \delta_i(t)\mathbf{u}_i^{(g)}$$

2. Construct the (P_S+2)th point using the formula

$$\mathbf{u}_{P_S+2} = \mathbf{u}_{\text{best}}^{(g)} + \left(\mathbf{u}_{\text{best}}^{(g)} - \mathbf{u}_{P_S+1}\right)$$

If $\mathbf{u}_{P_S+1}$ or $\mathbf{u}_{P_S+2}$ is outside the feasible set $\mathcal{U}$, it can always be projected back to $\mathcal{U}$.

3. Compute the cost function values $J(\mathbf{u}_{P_S+1})$ and $J(\mathbf{u}_{P_S+2})$ for these two points and find

$$i_* = \arg \min_{i=P_S+1, P_S+2} J(\mathbf{u}_i)$$

4. The pair $(\mathbf{u}_{i_*}, J(\mathbf{u}_{i_*}))$ then replaces $(\mathbf{u}_{\text{worst}}^{(g)}, J_{i_{\text{worst}}})$ in the population

If $\|\mathbf{u}_{P_S+1} - \mathbf{u}_{P_S+2}\| < \xi_B$, exit **inner loop**

End of inner loop

The solution found in the gth generation is $\mathbf{u} = \mathbf{u}_{\text{best}}^{(g)}$

End of outer loop

This yields the solution $\mathbf{u} = \mathbf{u}_{\mathrm{best}}^{(N_G)}$

The inner loop, the weighted boosting search, can be viewed as a local optimizer that searches for a local minimum within the convex region defined by the initial population. The outer loop resembles a multistar [16], which is a tested strategy for converting a local optimizer into a global one, with a randomly sampling initialization. To guarantee a global optimal solution as well as to achieve a fast convergence, the algorithmic parameters, P_S, N_G and ξ_B, need to be set carefully. The appropriate values for these algorithmic parameters depends on the dimension of $\mathbf{u}$ and how hard the objective function to be optimized. Generally, these algorithmic parameters have to be found empirically, just as in any global optimization algorithm. The elitist initialization is very useful, as it keeps the information obtained by the previous search generation, which otherwise would be lost due to the randomly sampling initialization. In the inner loop optimization, there is no need for every members of the population to converge to a (local) minimum, and it is sufficient to locate where the minimum lies. Thus, the accuracy for stopping the weighted boosting search, ξ_B, can be set to a relatively large value. This makes the search efficient, achieving convergence with a small number of the cost function evaluations. The population size P_S and the number of generations N_G should be set to be sufficiently large so that the parameter search space will be sampled sufficiently to guarantee a global optimal solution.

REFERENCES

[1] J.J. Shynk and R.P. Gooch, "The constant modulus array for cochannel signal copy and direction finding," *IEEE Trans. Signal Processing*, Vol.44, No.3, pp.652–660, 1996.

[2] J.J. Shynk, A.V. Keerthi and A. Mathur, "Steady-state analysis of the multistage constant modulus array," *IEEE Trans. Signal Processing*, Vol.44, No.4, pp.948–962, 1996.

[3] C.-Y. Chi, C.-Y. Chen, C.-H. Chen and C.-C. Feng, "Batch processing algorithms for blind equalization using higher-order statistics," *IEE Signal Processing Letters*, Vol.20, No.1, pp.25–49, 2003.

[4] S. Chen, A. Wolfgang and L. Hanzo, "Constant modulus algorithm aided soft decision-directed blind space-time equalization for SIMO channels," in *Proc. VTC2004 Fall* (Los Angeles, USA), Sept.26-29, 2004, pp.1718–1722.

[5] H. Gazzah, P.A. Regalia and J.-P. Delmas, "Asymptotic eigenvalue distribution of block Toeplitz matrices and application to blind SIMO channel identification," *IEEE Trans. Information Theory*, Vol.47, No.3, pp.1243–1251, 2001.

[6] D. Luengo, I. Santamaria, J. Ibanez, L. Vielva and C. Pantaleon, "A fast blind SIMO channel identification algorithm for sparse sources," *IEE Signal Processing Letters*, Vol.10, No.5, pp.148–151, 2003.

[7] I. Santamaria, J. Via and C.C. Gaudes, "Robust blind identification of SIMO channels: a support vector regression approach," in *Proc. ICASSP'2004*, 17-21 May, 2004, Vol.5, pp.673–676.

[8] S. Chen, X.C. Yang and L. Hanzo, "Space-time equalisation assisted multiuser detection for SDMA systems," to be presented at *VTC2005 Spring* (Stockholm, Sweden), May 30 - June 1, 2005.

[9] A. Ranheim, "A decoupled approach to adaptive signal separation using an antenna array," *IEEE Trans. Vehicular Technology*, Vol.48, No.3, pp.676–682, 1999.

[10] A. Dogandzic and A. Nehorai, "Generalized multivariate analysis of variance - A unified framework for signal processing in correlated noise," *IEEE Signal Processing Magzine*, Vol.20, No.5, pp.39–54, 2003.

[11] N. Seshadri, "Joint data and channel estimation using blind trellis search techniques," *IEEE Trans. Communications*, Vol.42, No.2/3/4, pp.1000–1011, 1994.

[12] R. Raheli, A. Polydoros and C.-K. Tzou, "Per-survivor processing: a general approach to MLSE in uncertain environments," *IEEE Trans. Communications*, Vol.43, No.2/3/4, pp.354–364, 1995.

[13] E. Zervas, J. Proakis and V. Eyuboglu, "A quantized channel approach to blind equalization," in *Proc. ICC'92* (Chicago, USA), 1992, Vol.3, pp.351.8.1–351.8.5.

[14] S. Chen and Y. Wu, "Maximum likelihood joint channel and data estimation using genetic algorithms," *IEEE Trans. Signal Processing*, Vol.46, No.5, pp.1469–1473, 1998.

[15] S. Chen, X.X. Wang and C.J. Harris, "Experiments with repeating weighted boosting search for optimization in signal processing applications," *IEEE Trans. Systems, Man and Cybernetics, Part B*, to appear, June 2005.

[16] F. Schoen, "Stochastic techniques for global optimization: a survey of recent advances," *J. Global Optimization*, Vol.1, pp.207–228, 1991.