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THE OPTIMAL CONTROLLER REALIZATION USING THE δ OPERATOR FOR SAMPLED DATA SYSTEMS WITH FINITE WORD LENGTH CONSIDERATION

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Abstract

This paper present a new effective algorithm for the optimal realization of digital controller structures using the delta operator for sampled data systems subject to Finite-Word-Length (FWL) constraints. The problem is formulated as a non-linear programming to provide an easy and efficient optimization tool to solve such complex problem. Simulation results of the optimum realizations of digital controller structures are presented to illustrate the effectiveness of the proposed strategy.

1 Introduction

The recent advances in fixed-point implementation of digital controllers such as the design of dedicated fixed-point Digital Signal Processors (DSP) and new Digital Control Processors (DCP) architectures made Finite Word Length (FWL) implementation an important issue in modern digital control engineering design applications. Improved control performance and increased levels of integration are especially important in many areas such as consumer electronic products, automotive and electro-mechanical control systems. This is because hardware controller implementation with fixed-point arithmetic offer the advantages of speed, memory space, cost and simplicity over floating-point arithmetic.

The FWL effects have been well studied in digital signal processing, especially in digital filter implementation for a long time [robe]. The results have recently been extended to the study of FWL effects of digital controller on control systems. [moro], [qiu], [fial], [madi], [li2], [iste2], and [whid] dealt with FWL implementation issues using different approaches. In all the above study of FWL effects of digital controller on control systems, the digital controllers were described and realized with the usual shift operator z. It is known that a discrete time system can also be described and

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realized with a different operator called delta operator [midd]. The delta operator parametrization provides a unified formulation between the continuous time and discrete time systems. [geve], [li1] and [iste1] have promoted the use of the delta operator as opposed to the shift operator in control applications. This paper intends to present a FWL stability measure based on the delta operator and discuss the algorithm of finding the optimal controller realization whose measure is biggest in all realizations.

2 The delta operator

From a continuous time transfer function *G(s)*, as the results of a discretization procedure with the shift operator *z* and a sampling period *h*, a discrete time transfer function $G_z(z)$ can be obtained. Define

$$\delta = \frac{z-1}{h} \quad (1)$$

then the transfer function $G_z(z)$ can be re-expressed in δ form:

$$G_z(z) = G_\delta(\delta) \quad (2)$$

which means $G_z(z)$ and $G_\delta(\delta)$ are two different but equivalent parametrizations representing the same object. These two input-output relationships can be represented by a shift operator (resp. δ-operator) state space model as follows [geve]:

$$\begin{cases} zx_z(k) = A_z x_z(k) + B_z u(k) \\ y(k) = C_z x_z(k) + D_z u(k) \end{cases}$$
$$\begin{cases} \delta x_\delta(k) = A_\delta x_\delta(k) + B_\delta u(k) \\ y(k) = C_\delta x_\delta(k) + D_\delta u(k) \end{cases} \quad (3)$$

The following relationships relate the internal and external representations:

$$\begin{aligned} G_z(z) &= C_z(zI - A_z)^{-1}B_z + D_z \\ &= G_\delta(\delta) = C_\delta(\delta I - A_\delta)^{-1}B_\delta + D_\delta \\ A_z &= hA_\delta + I \\ B_z &= hB_\delta \\ C_z &= C_\delta \\ D_z &= D_\delta \end{aligned} \quad (4)$$

Define

$$S_\rho = \{(A_\rho, B_\rho, C_\rho, D_\rho): G_\rho(\rho) = C_\rho(\rho I - A_\rho)^{-1}B_\rho + D_\rho\}$$

where $\rho = z$ or δ is called generalized operator. Hence if $(A_\rho, B_\rho, C_\rho, D_\rho) \in S_\rho$, for any nonsingular *T*, there is $(T^{-1}A_\rho T, T^{-1}B_\rho, C_\rho T, D_\rho) \in S_\rho$. With $A_z = hA_\delta + I$, it is easily seen that

Lemma 1: $\lambda_i(A_z) = 1 + h\lambda_i(A_\delta)$, where $\lambda_i(\cdot)$ is the eigenvalue.

It is well known that the discrete time system (A_z, B_z, C_z, D_z) is stable if and only if $|\lambda_i(A_z)| < 1$. From lemma 1, we can get the condition of the stability of the discrete time system described with the delta operator:

Lemma 2: The discrete time system $(A_\delta, B_\delta, C_\delta, D_\delta)$ is stable if and only if $\left|\lambda_i(A_\delta) + \frac{1}{h}\right| < \frac{1}{h}$.

3 FWL Stability Measure

Considering the sampled data system Σ_1 shown as Figure 1.

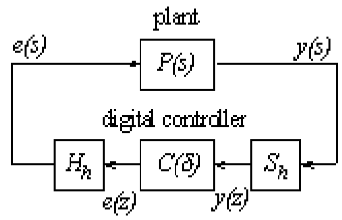

Figure 1. Sampled data system with digital controller

where *P(s)* is the continuous time plant, $C(\delta)$ is the discrete time controller, S_h is the sampler with sampling period *h*,

$$y(z) = S_h y(s) : y(k) = y(kh)$$

H_h is the hold device with sampling period *h*.

$$e(s) = H_h e(z) : e(t) = e(k), (kh < t \le (k+1)h)$$

Suppose *P(s)* is strictly proper. Let $(A_P, B_P, C_P, 0)$ be a state description of *P(s)* i.e. $P(s) = C_P(sI - A_P)^{-1}B_P$ where $A_P \in R^{m\times m}, B_P \in R^{m\times l}, C_P \in R^{q\times m}$. Let (A_C, B_C, C_C, D_C) be a state description of $C(\delta)$, i.e. $C(\delta) = C_C(\delta I - A_C)^{-1}B_C + D_C$ where $A_C \in R^{n\times n}, B_C \in R^{n\times q}, C_C \in R^{l\times n}, D_C \in R^{l\times q}$. In this paper, (A_C, B_C, C_C, D_C) is also called a realization of $C(\delta)$. The realizations of $C(\delta)$ are not unique. In fact, if

$\left(A_C^0, B_C^0, C_C^0, D_C^0\right)$ is a realization of $C(\delta)$. Then so is $\left(T^{-1}A_C^0 T, T^{-1}B_C^0, C_C^0 T, D_C^0\right)$ for any similarity transformation $T \in R^{n\times n}$. Considering the behaviour of the sampled data system Σ_1 only at its sampling instants, we arrive at a discrete time feedback system Σ_2:

$$\begin{cases} y(z) = S_h P(s) H_h e(z) \\ e(z) = C(\delta) y(z) \end{cases} \tag{5}$$

The plant $P(\delta) = S_h P(s) H_h = C_\delta (\delta I - A_\delta)^{-1} B_\delta$ is the discretized *P(s)* whose state description is $(A_\delta, B_\delta, C_\delta, 0)$, where

$$A_\delta = \frac{1}{h}\left(e^{A_P h} - I\right) \in R^{m\times m}, \qquad B_\delta = \frac{1}{h}\int_0^h e^{A_P \tau} B_P d\tau \in R^{m\times l}, C_\delta = C_P \in R^{q\times m} \tag{6}$$

It can be easily seen that the corresponding state description $(\overline{A}, \overline{B}, \overline{C}, \overline{D})$ of the discrete time closed loop system Σ_2 without FWL effect is given by:

$$\begin{aligned}\overline{A} &= \begin{bmatrix} A_\delta + B_\delta D_C C_\delta & B_\delta C_C \\ B_C C_\delta & A_C \end{bmatrix} \\ &= \begin{bmatrix} A_\delta & 0 \\ 0 & 0 \end{bmatrix} + \begin{bmatrix} B_\delta & 0 \\ 0 & I_n \end{bmatrix}\begin{bmatrix} D_C & C_C \\ B_C & A_C \end{bmatrix}\begin{bmatrix} C_\delta & 0 \\ 0 & I_n \end{bmatrix} \\ &= M_0 + M_1 X M_2 = \overline{A}(X)\end{aligned} \tag{7}$$

$$\overline{B} = \begin{bmatrix} B_\delta \\ 0 \end{bmatrix}, \overline{C} = [C_\delta \quad 0], \overline{D} = 0 \tag{8}$$

where $M_0 \in R^{(m+n)\times(m+n)}, M_1 \in R^{(m+n)\times(l+n)}, \quad M_2 \in R^{(q+n)\times(m+n)}$ are fixed matrix that depend on *P(s)* and *h*; $X \in R^{(l+n)\times(q+n)}$ is called the controller matrix; I_n denotes $n\times n$ identity matrix. Suppose $C(\delta)$ has been given to make the sampled data system Σ_1 stable, and the realization of $C(\delta)$ is $X_0 = \begin{bmatrix} D_C^0 & C_C^0 \\ B_C^0 & A_C^0 \end{bmatrix}$. Since the sampled data system Σ_1 is stable if and only if system Σ_2 is stable [chen], it follows from lemma 2 that $\forall i \in \{1, \cdots, m+n\}$, $\left|\lambda_i(\overline{A}(X_0)) + \frac{1}{h}\right| < \frac{1}{h}$.

When realization $(A_C^0, B_C^0, C_C^0, D_C^0)$ of $C(\delta)$ is implemented with a DCP, the controller matrix

$$X = \begin{bmatrix} p_1 & p_2 & \cdots & p_{q+n} \\ p_{q+n+1} & p_{q+n+2} & \cdots & p_{2(q+n)} \\ \vdots & \vdots & \cdots & \vdots \\ p_{(l+n-1)(q+n)+1} & p_{(l+n-1)(q+n)+2} & \cdots & p_{(l+n)(q+n)} \end{bmatrix} \tag{9}$$

is perturbed into:

$$X_0 + \Delta X = \begin{bmatrix} D_C^0 & C_C^0 \\ B_C^0 & A_C^0 \end{bmatrix} + \begin{bmatrix} \Delta p_1 & \Delta p_2 & \cdots & \Delta p_{q+n} \\ \Delta p_{q+n+1} & \Delta p_{q+n+2} & \cdots & \Delta p_{2(q+n)} \\ \vdots & \vdots & \cdots & \vdots \\ \Delta p_{(l+n-1)(q+n)+1} & \Delta p_{(l+n-1)(q+n)+2} & \cdots & \Delta p_{(l+n)(q+n)} \end{bmatrix} \tag{10}$$

due to the FWL effect, where each element of ΔX is bounded by $\frac{\varepsilon}{2}$, i.e.

$$\mu(\Delta X) \stackrel{\Delta}{=} \max_{i\in\{1,\cdots,(l+n)(q+n)\}} |\Delta p_i| \le \frac{\varepsilon}{2} \tag{11}$$

For a fixed point processor of B_s bits,

$$\varepsilon = 2^{-(B_s - B_X)} \tag{12}$$

where 2^{B_X} is the smallest normalization factor such that each parameter of $2^{-B_X} X_0$ is absolutely not bigger than 1. With the perturbation ΔX, $\lambda_i\left(\overline{A}(X_0)\right)$ may be moved to $\lambda_i\left(\overline{A}(X_0 + \Delta X)\right)$. The sampled data system Σ_1 is unstable if and only if there is $i \in \{1, \cdots, m+n\}$ such that $\left|\lambda_i\left(\overline{A}(X_0 + \Delta X)\right) + \frac{1}{h}\right| \ge \frac{1}{h}$.

When ΔX is small, $\forall i \in \{1, \cdots, m+n\}$, we have [li2], [iste2]

$$\begin{aligned}\Delta\lambda_i &= \lambda_i\left(\overline{A}(X_0 + \Delta X)\right) - \lambda_i\left(\overline{A}(X_0)\right) \\ &= \sum_{j=1}^{N} \left.\frac{\partial \lambda_i}{\partial p_j}\right|_{X=X_0} \Delta p_j\end{aligned} \tag{13}$$

where $N = (l+n)\times(q+n)$ is the number of elements of X. It follows that:

$$\begin{aligned}|\Delta\lambda_i| &\le \sum_{j=1}^{N} \left|\left.\frac{\partial \lambda_i}{\partial p_j}\right|_{X=X_0}\right| |\Delta p_j| \\ &\le \mu(\Delta X) \sum_{j=1}^{N} \left|\left.\frac{\partial \lambda_i}{\partial p_j}\right|_{X=X_0}\right|\end{aligned} \tag{14}$$

Thus $\forall i \in \{1, \cdots, m+n\}$, if

$$\mu(\Delta X) < \frac{\frac{1}{h} - \left|\lambda_i\left(\overline{A}(X_0)\right) + \frac{1}{h}\right|}{\sum_{j=1}^{N} \left|\left.\frac{\partial \lambda_i}{\partial p_j}\right|_{X=X_0}\right|} \tag{15}$$

we have

$$\left|\lambda_i\left(\overline{A}(X_0 + \Delta X)\right) + \frac{1}{h}\right| \le \frac{1}{h} \tag{16}$$

which means the sampled data system Σ_1 is stable. Let

$$\mu_1(X_0) \overset{\Delta}{=} \min_{i\in\{1,\cdots,m+n\}} \frac{\frac{1}{h} - \left|\lambda_i\left(\overline{A}(X_0)\right) + \frac{1}{h}\right|}{\sum_{j=1}^{N}\left|\frac{\partial \lambda_i}{\partial p_j}\right|_{X=X_0}} \tag{17}$$

From (14)--(16), we can reach to the following:

Theorem 1: System Σ_1 with FWL effect is stable when $\mu(\Delta X) < \mu_1(X_0)$.

We can compute $\mu_1(X_0)$ using the following lemma, which was proved by [li2] and [iste2] and presented here for completeness.

Lemma 3: Assume $\overline{A}(X)$ described in (7) is diagonalizable and has $\{\lambda_i\} = \{\lambda_i(\overline{A}(X))\}$ as its eigenvalues, let x_i be a right eigenvector of $\overline{A}(X)$ corresponding to the eigenvalue λ_i. Denote $M_x = [x_1 \quad x_2 \quad \cdots \quad x_{m+n}]$ and $M_y = [y_1 \quad y_2 \quad \cdots \quad y_{m+n}] = M_x^{-H}$, where y_i is called the reciprocal left eigenvector corresponding to λ_i. Then $\forall i \in \{1, \cdots, m+n\}$

$$\frac{\partial \lambda_i}{\partial X} = M_1^T y_i' x_i^T M_2^T \tag{18}$$

where y_i' is conjugate to y_i, "T" denotes the transpose operation, "H" denotes the transpose and conjugate operation.

Based on $\mu_1(X_0)$, we can compute

$$\hat{B}_{s1}^{\min} = Int(-\log_2 \mu_1(X_0)) - 1 + B_X$$

where $Int(x)$ rounds x to the nearest integer towards positive infinity. From (11), (12) and theorem 1, we know that the sampled data system Σ_1 is stable when X_0 is implemented with a DCP of $\hat{B}_{s1}^{\min}$ bits.

4 Optimal Realization

From the last section, we know that there are different realizations for a given $C(\delta)$, and the stability robustness measure $\mu_1(X)$ is a function of the realization X. Hence there is an interesting problem of finding out the realization such that $\mu_1(X)$ is maximized. This realization is called optimal realization in such a sense. The digital controller implemented with an optimal realization means the minimum hardware requirements in terms of less word length (i.e optimized controller data path hardware design) and such that the closed loop sampled data system remains stable.

It is known that any realization of $C(\delta)$ can be represented as

$$X_T = \begin{bmatrix} I_l & 0 \\ 0 & T^{-1} \end{bmatrix} X_0 \begin{bmatrix} I_q & 0 \\ 0 & T \end{bmatrix} \tag{19}$$

where I_l is $l \times l$ identity matrix, I_q is $q \times q$ identity matrix, X_0 is the initial realization of $C(\delta)$, $T \in R^{n\times n}$ and $\det(T) \neq 0$. Then from (7),

$$\overline{A}(X_T) = \begin{bmatrix} I_m & 0 \\ 0 & T^{-1} \end{bmatrix} \overline{A}(X_0) \begin{bmatrix} I_m & 0 \\ 0 & T \end{bmatrix} \tag{20}$$

Let x_i^0 be a right eigenvector of $\overline{A}(X_0)$ corresponding to the eigenvalue $\lambda_i^0 = \lambda_i(\overline{A}(X_0))$, y_i^0 be the reciprocal left eigenvector corresponding to x_i^0. Applying Lemma 3, we have

$$\begin{aligned} &\left.\frac{\partial \lambda_i}{\partial X}\right|_{X=X_T} \\ &= \begin{bmatrix} B_\delta^T & 0 \\ 0 & I_n \end{bmatrix} \begin{bmatrix} I_m & 0 \\ 0 & T^T \end{bmatrix} (y_i^0)'(x_i^0)^T \begin{bmatrix} I_m & 0 \\ 0 & T^{-T} \end{bmatrix} \begin{bmatrix} C_\delta^T & 0 \\ 0 & I_n \end{bmatrix} \\ &= \begin{bmatrix} I_l & 0 \\ 0 & T^T \end{bmatrix} \left.\frac{\partial \lambda_i}{\partial X}\right|_{X=X_0} \begin{bmatrix} I_q & 0 \\ 0 & T^{-T} \end{bmatrix}. \end{aligned} \tag{21}$$

For complex matrix

$$M = \begin{bmatrix} M_{11} & \cdots & M_{1,n+q} \\ \vdots & \cdots & \vdots \\ M_{n+l,1} & \cdots & M_{n+l,n+q} \end{bmatrix} \in C^{(n+l)\times(n+q)} \tag{22}$$

denote

$$\|M\|_s = \sum_{i=1}^{n+l}\sum_{j=1}^{n+q} |M_{ij}| \tag{23}$$

We can describe the optimal FWL realization problem of digital controller as the optimization problem:

$$\begin{aligned} \upsilon &= \frac{1}{\max_{X_T} \mu_1(X_T)} \\ &= \min_{\substack{T\in R^{n\times n} \\ \det(T)\neq 0}} \max_{i\in\{1,\cdots,m+n\}} \left\| \begin{bmatrix} I_l & 0 \\ 0 & T^T \end{bmatrix} \frac{\left.\frac{\partial \lambda_i}{\partial X}\right|_{X=X_0}}{\frac{1}{h} - \left|\lambda_i^0 + \frac{1}{h}\right|} \begin{bmatrix} I_q & 0 \\ 0 & T^{-T} \end{bmatrix} \right\|_s \\ &\overset{\Delta}{=} \min_{\substack{T\in R^{n\times n} \\ \det(T)\neq 0}} \max_{i\in\{1,\cdots,m+n\}} \left\| \begin{bmatrix} I_l & 0 \\ 0 & T^T \end{bmatrix} \Phi_i \begin{bmatrix} I_q & 0 \\ 0 & T^{-T} \end{bmatrix} \right\|_s \end{aligned} \tag{24}$$

where Φ_i is a fixed complex matrix. Define

$$f(T)=\max_{i\in\{1,\cdots,m+n\}}\left\|\begin{bmatrix} I_l & 0 \\ 0 & T^T \end{bmatrix}\Phi_i\begin{bmatrix} I_q & 0 \\ 0 & T^{-T} \end{bmatrix}\right\|_s \qquad (25)$$

then the optimal controller realization problem can be posed as

$$\upsilon=\min_{\substack{T\in R^{n\times n}\\ \det T\neq 0}} f(T) \qquad (26)$$

The above problem is a nonconvex nonlinear programming problem. We intend to search for the minimum of problem (26) with iterative optimization methods, i.e. a sequence $\{T_0, T_1, T_2, \cdots\}$ which converges to the minimum T_{opt} is generated. In the iterative procedure we can neglect the constraint $\det T \neq 0$, i.e. we solve the problem

$$\upsilon=\min_{T\in R^{n\times n}} f(T) \qquad (27)$$

with iterative methods. There are two reasons for us to do so:

- $\Omega=\{T \mid \det T = 0, T\in R^{n\times n}\}$ is only a manifold in space $R^{n\times n}$. Hence the case is rare that the iterate T_i moves into Ω when we search the space $R^{n\times n}$ for $T_{opt}\notin\Omega$ by an iterative sequence from the start point $T_0\notin\Omega$.
- Even if it happens that T_i moves into Ω in the iterative procedure, we can add a small perturbation τI_n to T_i such that $T_i+\tau I_n\notin\Omega$. This small perturbation would not affect the convergence of the iterative sequence to T_{opt}.

In this paper, the simplex search method is applied to solve problem (27) which is an unconstrained nonlinear programming problem. There are many existing optimization software which uses the simplex search method, for example, the *fmins* function in MATLAB Ver5.1 optimization toolbox. Clearly, the above algorithm is locally optimal. In order to "globalize" the algorithm, we repeatedly run the algorithm starting from various initial points to obtain "randomized" solutions for υ; then pick the smallest solution obtained.

5 Illustrative Example

To show how the optimization approach presented earlier can be used efficiently for the parameterization issues of optimal FWL controller realization with improved stability bounds and minimum word-length requirements. We consider an example to confirm our theoretical results.

$P(\delta)$ is given by:

$$A_\delta=\begin{bmatrix} 0&0&0&0&0\\ 1&0&0&0&-0.0139\\ 0&1&0&0&-20.8663\\ 0&0&1&0&-28.9275\\ 0&0&0&1&-6.9450 \end{bmatrix}, B_\delta=\begin{bmatrix}1\\0\\0\\0\\0\end{bmatrix}$$

$$C_\delta=[0.0130 \quad 0.0759 \quad -2.3950 \quad -2.5700 \quad 52.7147]$$

The initial controller realization X_0 of $C(\delta)$ is given by:

$$A_C^0=\begin{bmatrix} 0&0&0&0&0&-0.0018\\ 1&0&0&0&0&-89.7102\\ 0&1&0&0&0&-154.4319\\ 0&0&1&0&0&-120.0748\\ 0&0&0&1&0&-50.2874\\ 0&0&0&0&1&-9.5696 \end{bmatrix}, B_C^0=\begin{bmatrix}1\\0\\0\\0\\0\\0\end{bmatrix}, D_C^0=0.0460$$

$$C_C^0= [0.9804 \quad -2.7180 \quad 3.9832 \quad -3.3420 \quad 2.5162 \quad -2.5142]$$

The corresponding transition matrix $\overline{A}(X_0)$ can then be formed using (7), from which the corresponding eigenvalue sensitivity matrices Φ_i can be computed using Lemma 3 and hence problem (27) can be constructed. For problem (27), we get the solution by using the method developed in this paper:

$$T_{opt}=\begin{bmatrix} 6.006 & -102.7 & -135.3 & -89.49 & -18.01 & -16.65\\ -15.37 & 19.65 & -6.005 & -197.5 & -73.42 & -74.63\\ -5.620 & -10.58 & -14.88 & -134.0 & -3.812 & -100.4\\ -4.038 & -7.664 & -2.726 & -44.56 & -11.65 & -108.8\\ -58.62 & -37.89 & -3.250 & -20.75 & -31.56 & -88.97\\ -71.65 & -55.63 & -11.16 & -29.65 & 1.348 & -25.12 \end{bmatrix}$$

and $\upsilon=f(T_{opt})=1.3338\times10^5$. The results for the different finite realizations are summarized in Table 1.

Realization	$\mu_1(X)$	$\hat{B}_{s1}^{\min}$
X_0	4.6347×10^{-9}	34
X_{opt}	7.4972×10^{-6}	24

Table 1. Stability measures and stabilized word lengths

For the different realizations, Table 1 summarizes the results and show their relevant stability robustness measure $\mu_1(X)$, and the word length $\hat{B}_{s1}^{\min}$. The comparative results clearly show that optimal realization X_{opt} needs only 24 bits and provides larger stability measure while the non-optimal realization X_0 requires 34 bits with much lower stability bound.

6 Conclusions

In this paper we have presented an efficient approach for the new measure based on the delta operator of sampled data control systems with FWL consideration. It has also been shown that the optimal realization problem for digital controller with FWL consideration can be interpreted as nonlinear programming problems in convex set. The computation of the relevant FWL optimization problem was solved using the simplex search algorithm to illustrate that such problem can be efficiently and easily computed using existing mathematical programming techniques. The theoretical results were verified using a numerical controller example which illustrate that the optimum realizations based on the optimization method presented here greatly improves the stability robustness of the relevant controller realizations with minimum word-length characteristics compared to non-optimal realizations.

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