On Separation Principle for a Class of Networked Control Systems

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Outline

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Some Basics

An NCS is a control system in which a control loop is closed via a shared communication network.

The advantages:

- Low installation cost.
- Reducing system wiring.
- Easy maintenance.

The basic problems:

- packet dropout.
- packet delay.
- Bandwidth constraint.

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Related Works

Separation Principle was shown to hold for:

- Network is only located in sensor-to-controller (S/C) channel [7].
- Channels are modeled as Bernoulli process [5] .
- Connections of S/C and controller-to-actuator (C/A) are on/off simultaneously [17].

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Our Contributions

Necessary & Sufficient conditions

- Separation principle for NCS
  - S/C: Markov chain
  - C/A: Markov chain
- Stabilisation control for NCS
  - S/C: Bernoulli process
  - C/A: Markov chain

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Networked Control System

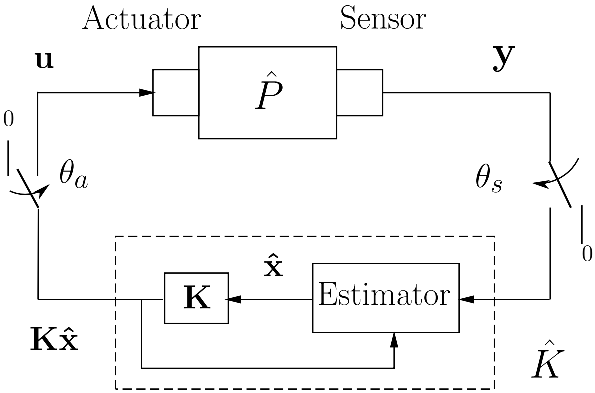

Figure: Networked control system $\hat{P}_K$.

Network Assumptions

Packet dropouts indicators:

$$\begin{cases} \theta_s(k) \in \{0, 1\}, & \text{S/C} \\ \theta_a(k) \in \{0, 1\}, & \text{C/A} \end{cases}$$

Assumption 1:

- $\theta_s(k)$ and $\theta_a(k)$ are driven by independent Markov chains, with transition probability matrices $[\pi_{t,r}]$ and $[\lambda_{i,j}]$.

Assumption 2:

- $\theta_a(k)$ is driven by a Markov chain.
- $\theta_s(k)$ is driven by a Bernoulli process, with $\text{Prob}(\theta_s(k) = 1) = \alpha$ and $\text{Prob}(\theta_s(k) = 0) = 1 - \alpha$.

Plant & Controller

The plant $\hat{P}$:

$$\begin{cases} \mathbf{x}(k+1) = \mathbf{A}\mathbf{x}(k) + \mathbf{B}\mathbf{u}(k) \\ \mathbf{y}(k) = \mathbf{C}\mathbf{x}(k) \end{cases}$$

- $\mathbf{x}(k)$: state, $\mathbf{u}(k)$: input, and $\mathbf{y}(k)$: output.

The controller $\hat{K}$:

$$\begin{cases} \hat{\mathbf{x}}(k+1) = \mathbf{A}\hat{\mathbf{x}}(k) + \mathbf{B}\mathbf{u}(k) + \theta_s(k)\mathbf{L}(\mathbf{C}\hat{\mathbf{x}}(k) - \mathbf{y}(k)) \\ \mathbf{u}(k) = \theta_a(k)\mathbf{K}\hat{\mathbf{x}}(k), \end{cases}$$

- $\mathbf{K}$: state feedback gain, $\mathbf{L}$: observer gain.
- $\theta_s(k) = 1$: standard observer law, $\theta_s(k) = 0$: imitation law.

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NCS Dynamics

The NCS $\hat{P}_K$ is in the form of Markovian jump linear system:

$$\bar{\mathbf{x}}(k+1) = \bar{\mathbf{A}}_{\theta_a(k)\theta_s(k)}\bar{\mathbf{x}}(k), \ \forall k \in \mathbb{N},$$

- State of $\hat{P}_K$: $\bar{\mathbf{x}}(k) \triangleq [\mathbf{x}^{\mathrm{T}}(k)\ \mathbf{e}^{\mathrm{T}}(k)]^{\mathrm{T}}$ with $\mathbf{e}(k) = \mathbf{x}(k) - \hat{\mathbf{x}}(k)$.
- $\theta_a(k)\theta_s(k) \in \{00, 01, 10, 11\}$ with

$$\bar{\mathbf{A}}_{00} = \begin{bmatrix} \mathbf{A} & \mathbf{0} \\ \mathbf{0} & \mathbf{A} \end{bmatrix}, \ \bar{\mathbf{A}}_{01} = \begin{bmatrix} \mathbf{A} & \mathbf{0} \\ \mathbf{0} & \mathbf{A}+\mathbf{LC} \end{bmatrix},$$

$$\bar{\mathbf{A}}_{10} = \begin{bmatrix} \mathbf{A}+\mathbf{BK} & -\mathbf{BK} \\ \mathbf{0} & \mathbf{A} \end{bmatrix}, \bar{\mathbf{A}}_{11} = \begin{bmatrix} \mathbf{A}+\mathbf{BK} & -\mathbf{BK} \\ \mathbf{0} & \mathbf{A}+\mathbf{LC} \end{bmatrix}.$$

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Theorem One

The NCS $\hat{P}_K$ under **Assumption 1** is stochastically stable if and only if the following two conditions hold.

- (i) there exist matrices $\mathbf{Q}_0 > 0$ and $\mathbf{Q}_1 > 0$ such that

$$\begin{cases} \hat{\mathbf{A}}_0^T\big(\pi_{0,0}\mathbf{Q}_0 + \pi_{0,1}\mathbf{Q}_1\big)\hat{\mathbf{A}}_0 - \mathbf{Q}_0 < 0 \\ \hat{\mathbf{A}}_1^T\big(\pi_{1,0}\mathbf{Q}_0 + \pi_{1,1}\mathbf{Q}_1\big)\hat{\mathbf{A}}_1 - \mathbf{Q}_1 < 0 \end{cases}$$

  where $\hat{\mathbf{A}}_0 = \mathbf{A},\ \hat{\mathbf{A}}_1 = \mathbf{A} + \mathbf{LC}$;
- (ii) there exist matrices $\mathbf{P}_0 > 0$ and $\mathbf{P}_1 > 0$ such that

$$\begin{cases} \mathbf{\Phi}_0^T\big(\lambda_{0,0}\mathbf{P}_0 + \lambda_{0,1}\mathbf{P}_1\big)\mathbf{\Phi}_0 - \mathbf{P}_0 < 0 \\ \mathbf{\Phi}_1^T\big(\lambda_{1,0}\mathbf{P}_0 + \lambda_{1,1}\mathbf{P}_1\big)\mathbf{\Phi}_1 - \mathbf{P}_1 < 0 \end{cases}$$

  where $\mathbf{\Phi}_0 = \mathbf{A},\ \mathbf{\Phi}_1 = \mathbf{A} + \mathbf{BK}$.

Remarks on Theorem One

Theorem One provides *necessary and sufficient* conditions for ensuring stochastic stability of $\hat{P}_K$, where

- S/C and C/A channels are modelled by two independent Markov chains.

Condition (i) only involves observer gain matrix **L**, while condition (ii) only involves state feedback gain matrix **K** $\Rightarrow$

- State feedback control and observer can be designed separately and independently.

Theorem Two

The NCS $\hat{P}_K$ under **Assumption 2** is stochastically stable if and only if the following two conditions hold.

- (i) there exists a matrix $\mathbf{Q} > 0$ such that

$$\alpha(\mathbf{A} + \mathbf{LC})^T \mathbf{Q}(\mathbf{A} + \mathbf{LC}) + (1 - \alpha)\mathbf{A}^T \mathbf{QA} - \mathbf{Q} < 0$$

- (ii) there exist matrices $\mathbf{P}_0 > 0$ and $\mathbf{P}_1 > 0$ such that

$$\begin{cases} \mathbf{\Phi}_0^T \left(\lambda_{0,0}\mathbf{P}_0 + \lambda_{0,1}\mathbf{P}_1\right)\mathbf{\Phi}_0 - \mathbf{P}_0 < 0 \\ \mathbf{\Phi}_1^T \left(\lambda_{1,0}\mathbf{P}_0 + \lambda_{1,1}\mathbf{P}_1\right)\mathbf{\Phi}_1 - \mathbf{P}_1 < 0 \end{cases}$$

Remark: When the S/C channel is modelled by a Bernoulli process, condition (i) is simplified accordingly.

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Theorem Three

$\hat{P}_K$ under **Assumption 2** is stochastically stable if and only if

- there exist matrices $\mathbf{Q} > 0$ and $\mathbf{Y}$; $\overline{\mathbf{P}}_0 > 0$, $\overline{\mathbf{P}}_1 > 0$ and $\mathbf{X}$ such that the following three LMIs hold

$$\begin{bmatrix} -\mathbf{Q} & * & * \\ \sqrt{\alpha}(\mathbf{QA}+\mathbf{YC}) & -\mathbf{Q} & * \\ \sqrt{1-\alpha}\mathbf{QA} & \mathbf{0} & -\mathbf{Q} \end{bmatrix} < 0; \quad \begin{bmatrix} -\overline{\mathbf{P}}_0 & * & * \\ \sqrt{\lambda_{0,0}}\mathbf{A}\overline{\mathbf{P}}_0 & -\overline{\mathbf{P}}_0 & * \\ \sqrt{\lambda_{0,1}}\mathbf{A}\overline{\mathbf{P}}_0 & \mathbf{0} & -\overline{\mathbf{P}}_1 \end{bmatrix} < 0,$$

$$\begin{bmatrix} -\overline{\mathbf{P}}_1 & * & * \\ \sqrt{\lambda_{1,0}}(\mathbf{A}\overline{\mathbf{P}}_1+\mathbf{BX}) & -\overline{\mathbf{P}}_0 & * \\ \sqrt{\lambda_{1,1}}(\mathbf{A}\overline{\mathbf{P}}_1+\mathbf{BX}) & \mathbf{0} & -\overline{\mathbf{P}}_1 \end{bmatrix} < 0.$$

- State feedback and observer gains are given by $\mathbf{K} = \mathbf{X}\overline{\mathbf{P}}_1^{-1}$ and $\mathbf{L} = \mathbf{Q}^{-1}\mathbf{Y}$.

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Plant and Network

Unstable 4^{th}-order NCS with plant model

$$\mathbf{A} = \begin{bmatrix} 1.4 & 1 & 1 & -1.1 \\ -1.3 & -0.9 & 0.5 & 0.5 \\ 0.3 & -0.2 & -1 & 0 \\ -0.5 & -0.3 & -0.5 & -1 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} -0.7 & -1 \\ 0 & -0.9 \\ 0.8 & 0.6 \\ 0.1 & 0 \end{bmatrix},$$

$$\mathbf{C} = \begin{bmatrix} -1 & 0.6 & -0.3 & 0 \\ 0.5 & 0.5 & 0.2 & -1 \end{bmatrix}.$$

The network specified by (**Assumption 2**)

- C/A: Markov chain with transition probability matrix

$$[\lambda_{i,j}] = \begin{bmatrix} 0.6 & 0.4 \\ 0.2 & 0.8 \end{bmatrix}$$

- S/C: Bernoulli process with $\alpha = 0.7$.

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Stabilisation Control

Solve synthesis of stochastic stabilisation control with **Theorem 3**:

- Applying Matlab LMI Control Toolbox to solve the three LMIs, obtaining

$$\mathbf{Q} > 0,\ \mathbf{Y};\ \overline{\mathbf{P}}_0 > 0,\ \overline{\mathbf{P}}_1 > 0,\ \mathbf{X}$$

- State feedback gain **K** and observer gain **L** are obtained as

$$\mathbf{K} = \mathbf{X}\overline{\mathbf{P}}_1^{-1} = \begin{bmatrix} 0.6786 & 0.9181 & 1.5972 & 1.8288 \\ 0.2455 & -0.0407 & 0.1753 & -1.2452 \end{bmatrix},$$

$$\mathbf{L} = \mathbf{Q}^{-1}\mathbf{Y} = \begin{bmatrix} 0.3124 & -1.2852 \\ 0.1239 & 1.2140 \\ -0.0061 & -0.3237 \\ -0.1387 & -0.6579 \end{bmatrix}.$$

Simulation

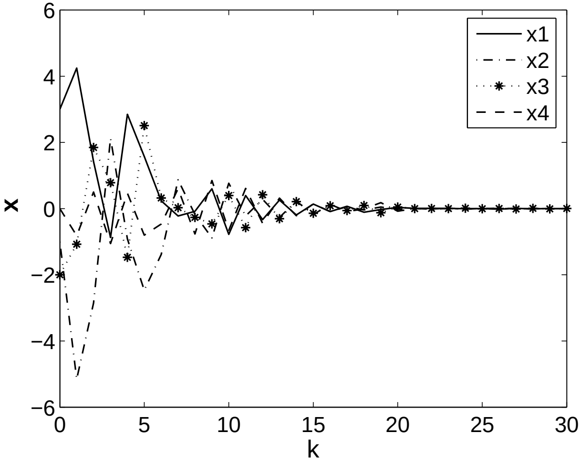

Figure: State trajectories of the plant $\hat{P}$.

Simulation

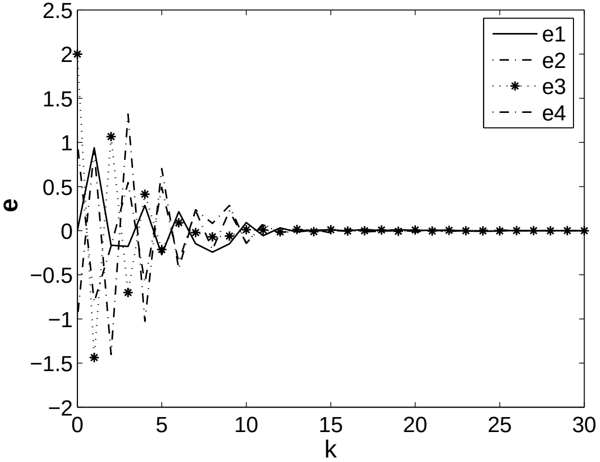

Figure: Error trajectories between state and estimator of $\hat{P}_K$.

Summary

We consider observer-based NCSs with random packet dropouts occurring independent in both S/C and C/A channels.

- Establish separation principle for NCSs where S/C and C/A channels are modelled as two independent Markov chains;
- Derive LMI stabilisation control solution for NCSs where C/A and S/C channels are governed by Markov chain and Bernoulli process, respectively.

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