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Particle Swarm Optimisation Aided Minimum Bit Error Rate Multiuser Transmission

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Motivations

- **Multiuser transmission** preprocessing technique is attractive for system with noncooperative mobile receivers

  $\Rightarrow$ Low-complexity high power-efficiency mobile terminals

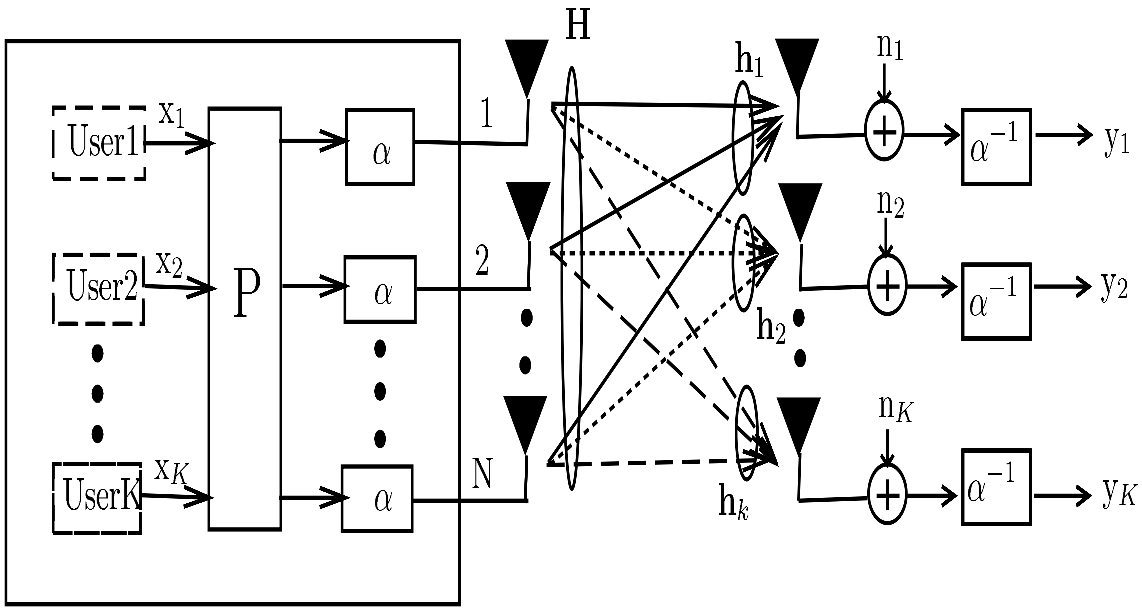
- **Minimum mean square error** MUT design has appealing simplicities but is limited in achievable bit error rate

- **Minimum bit error rate** MUT design enhances achievable BER performance

  $\Rightarrow$ Standard MBER design based on **sequential quadratic programming** imposes high complexity

- **Particle swarm optimisation** aided MBER MUT design to significantly reduce complexity

System Model



- N **transmit antennas** and K **mobile users**
- **Transmit symbol vector** $\mathbf{x} = [x_1 \; x_2 \cdots x_K]^T$
- **Precoder** $\mathbf{P} = [\mathbf{p}_1 \cdots \mathbf{p}_K]$
- Flat fading MIMO **channel** $\mathbf{H} = [\mathbf{h}_1 \; \mathbf{h}_2 \cdots \mathbf{h}_K]$
- AWGN $\mathbf{n}$: $E[\mathbf{n}\mathbf{n}^H] = 2\sigma_n^2 \mathbf{I}_K$

- Transmit **power constraint** $\mathrm{E_T} \Rightarrow$ scaling factor $\alpha = \sqrt{\mathrm{E_T}/E[\|\mathbf{Px}\|^2]}$
- **Receive signal vector** $\mathbf{y} = [y_1 \; y_2 \cdots y_K]^T$ is given by

$$\mathbf{y} = \mathbf{H}^T \mathbf{P}\mathbf{x} + \alpha^{-1}\mathbf{n}$$

y_k, $1 \le k \le K$, are sufficient statistics for detecting x_k, $1 \le k \le K$

Bit Error Rate

- Average **bit error rate** for QPSK signalling is given by

$$P_e(\mathbf{P}) = \big(P_{e_I}(\mathbf{P}) + P_{e_Q}(\mathbf{P})\big)/2$$

- Average BER of **in-phase** component of $\mathbf{y}$

$$P_{e_I}(\mathbf{P}) = \frac{1}{KM^K} \sum_{q=1}^{M^K} \sum_{k=1}^{K} Q\left(\frac{\mathrm{sgn}(\Re[x_k^{(q)}])\Re[\mathbf{h}_k^T \mathbf{P} \mathbf{x}^{(q)}]}{\sigma_n} \right)$$

where $M^K = 4^K$ is the number of equiprobable legitimate transmit symbol vectors $\mathbf{x}^{(q)}$ for QPSK signalling, $x_k^{(q)}$ the kth element of $\mathbf{x}^{(q)}$, $1 \leq q \leq M^K$, and $Q(\bullet)$ the standard Gaussian error function

- Average BER of **quadrature-phase** component of $\mathbf{y}$

$$P_{e_Q}(\mathbf{P}) = \frac{1}{KM^K} \sum_{q=1}^{M^K} \sum_{k=1}^{K} Q\left(\frac{\mathrm{sgn}(\Im[x_k^{(q)}])\Im[\mathbf{h}_k^T \mathbf{P} \mathbf{x}^{(q)}]}{\sigma_n} \right)$$

MBER MUT Design

- MBER MUT **solution** is defined as

$$\mathbf{P}_{\text{TxMBER}} = \arg\min_{\mathbf{P}} P_e(\mathbf{P}) \quad \text{s.t.} \quad E[\|\mathbf{P}\mathbf{x}\|^2] = \text{E}_T$$

- Solving this optimisation by SQP algorithm has total **complexity**

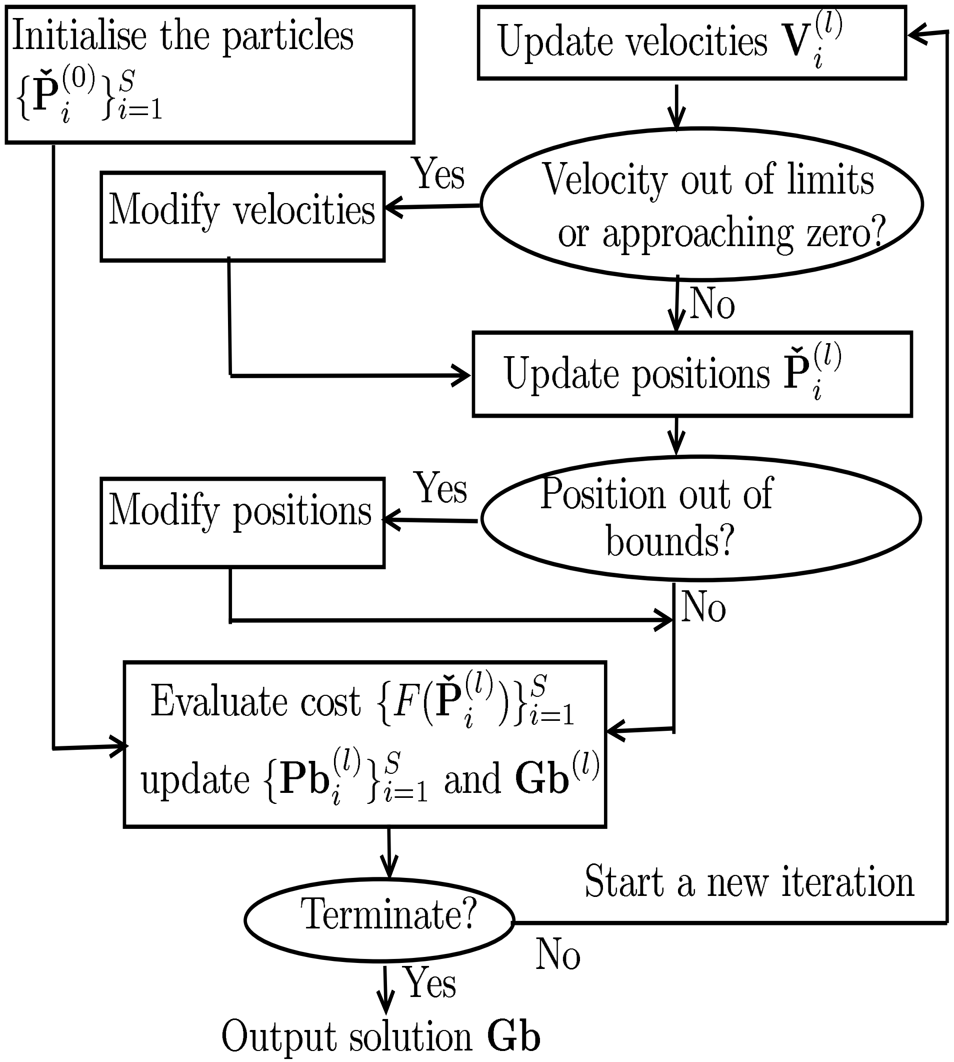
$$\text{CT}_{\text{SQP}} = I_{\text{SQP}} \times \text{C1}_{\text{SQP}}$$

where I_{SQP} is number of iterations and C1_{SQP} complexity per iteration

Complexity per iteration C1_{SQP} of SQP-MBER-MUT for QPSK signalling, where N is the number of transmit antennas, K the number of mobile users, $M = 4$ is the size of symbol constellation

Flops
$K \times (8 \times N^2 \times K^2 + 6 \times N \times K + 6 \times N + 8 \times K + 4) \times M^K + \mathcal{O}(8 \times N^3 \times K^3) + 8 \times N^2 \times K^2$ $+16 \times N \times K^2 + 8 \times N^2 \times K + 12 \times N \times K + 6 \times K^2 - 2 \times N^2 + N - 2 \times K + 11$

Particle Swarm Optimisation



- **Optimisation** for MBER-MUT

$$\mathbf{P}_{\text{TxMBER}} = \arg \min_{\mathbf{P} \in \mathsf{S}^{N \times K}} F(\mathbf{P})$$

- **Cost** function $F(\mathbf{P}) = P_e(\mathbf{P}) + G(\mathbf{P})$ with penalty function

$$G(\mathbf{P}) = 0$$

$$\text{if } E[\|\mathbf{Px}\|^2] - \mathrm{E_T} \le 0;$$

$$G(\mathbf{P}) = \lambda(E[\|\mathbf{Px}\|^2] - \mathrm{E_T})$$

$$\text{if } E[\|\mathbf{Px}\|^2] - \mathrm{E_T} > 0$$

- **Search space** $\mathsf{S}^{N \times K}$ with search range for each precoder coefficient

$$\mathsf{S} = [-P_{max},\ P_{max}] + j[-P_{max},\ P_{max}]$$

PSO algorithm

PSO is a population based stochastic optimisation method inspired by **social behaviour** of bird flocking or fish schooling: a swarm of particles $\{\check{\mathbf{P}}_i^{(l)}\}_{i=1}^{S}$ evolves in search space $\mathsf{S}^{N\times K}$, where S is swarm size and index l denotes iteration

- *a) Swarm initialisation* With iteration index $l=0$, set $\check{\mathbf{P}}_1^{(l)}$ to MMSE solution and randomly generate rest of initial particles, $\{\check{\mathbf{P}}_i^{(l)}\}_{i=2}^{S}$, in search space $\mathsf{S}^{N\times K}$
- *b) Swarm evaluation* Particle $\check{\mathbf{P}}_i^{(l)}$ has cost $F(\check{\mathbf{P}}_i^{(l)})$, based on which cognitive information $\mathbf{Pb}_i^{(l)}$, $1 \le i \le S$, and social information $\mathbf{Gb}^{(l)}$ are updated
- *c) Swarm update* Each particle has a velocity, $\mathbf{V}_i^{(l)} \in \mathsf{V}^{N\times K}$, to direct its flying

$$\mathbf{V}_i^{(l+1)} = w * \mathbf{V}_i^{(l)} + rand() * c_1 * (\mathbf{Pb}_i^{(l)} - \check{\mathbf{P}}_i^{(l)}) + rand() * c_2 * (\mathbf{Gb}^{(l)} - \check{\mathbf{P}}_i^{(l)})$$

$$\check{\mathbf{P}}_i^{(l+1)} = \check{\mathbf{P}}_i^{(l)} + \mathbf{V}_i^{(l+1)}$$

- *d) Termination condition check.* If maximum number of iterations, I_{PSO}, is reached, terminate with solution $\mathbf{Gb}^{(I_{\text{PSO}})}$; otherwise, $l = l+1$ and go to *b)*

Complexity of PSO Based Design

- There are well-defined rules for choosing PSO algorithmic parameters
- PSO based MBER-MUT design has total **complexity**

$$\mathrm{CT}_{\mathrm{PSO}} = I_{\mathrm{PSO}} \times \mathrm{C1}_{\mathrm{PSO}}$$

where I_{PSO} is number of iterations and $\mathrm{C1}_{\mathrm{PSO}}$ complexity per iteration

Complexity per iteration $\mathrm{C1}_{\mathrm{PSO}}$ of PSO-MBER-MUT for QPSK signalling, where N is the number of transmit antennas, K the number of mobile users, $M = 4$ is the size of symbol constellation, S is the swarm size

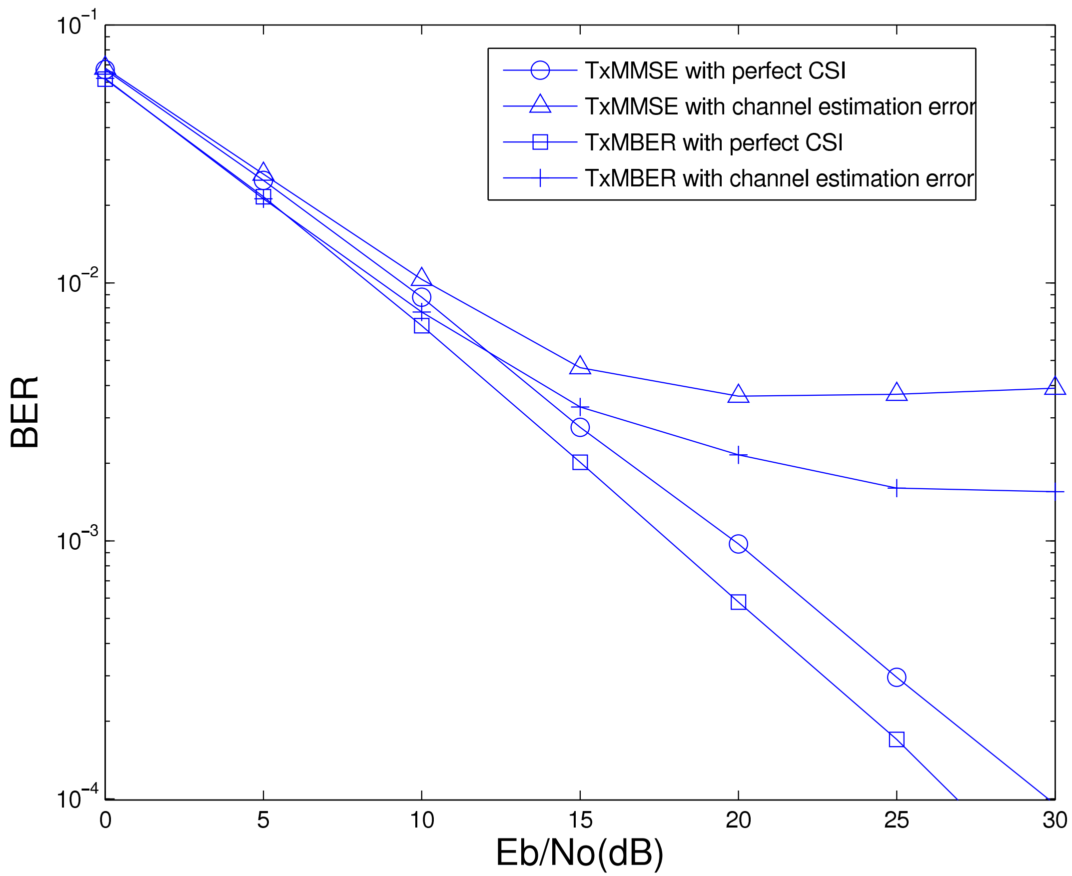
Flops
$((16 \times N \times K + 7 \times K + 6 \times N + 1) \times M^K + 20 \times N \times K + 2) \times S + 8$

- Since $\mathrm{C1}_{\mathrm{PSO}} < \mathrm{C1}_{\mathrm{SQP}}$ and typically $I_{\mathrm{PSO}} \ll I_{\mathrm{SQP}}$, PSO-MBER-MUT design imposes lower complexity than SQP-MBER-MUT design, i.e.

$$\mathrm{CT}_{\mathrm{PSO}} \ll \mathrm{CT}_{\mathrm{SQP}}$$

Simulation Results

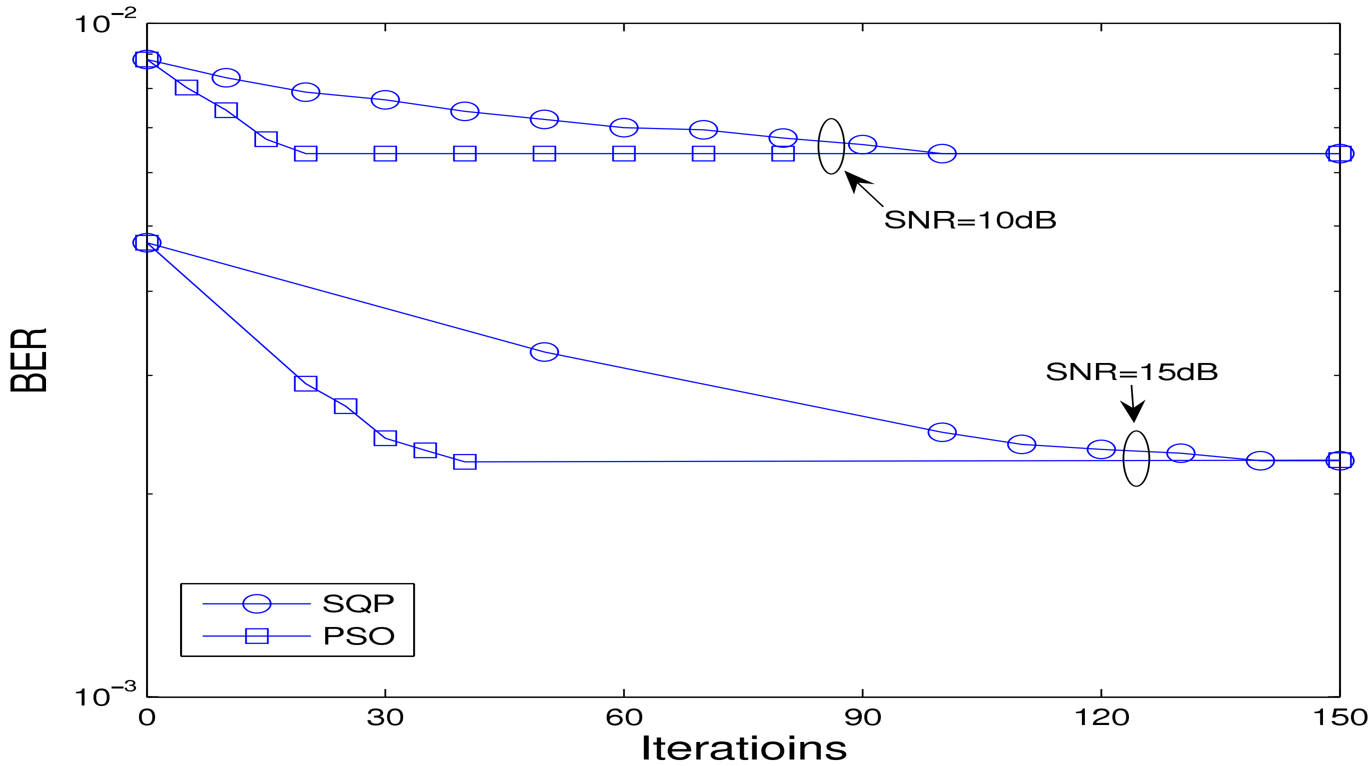
- BS with $N = 4$ transmit antennas communicated over 4×4 flat Rayleigh fading channels with $K = 4$ single-receive-antenna QPSK mobile users
- $S = 20$ and $I_{\mathrm{PSO}} = 20$ to 40, depending on SNR value, which were adequate as PSO algorithm arrived at MBER solution with lowest $\mathrm{CT}_{\mathrm{PSO}}$
- Results obtained by averaging over 100 channel realisations



- Channel taps $h_{i,k}$: uncorrelated complex-value Gaussian with $E[|h_{i,k}|^2] = 1$
- In the case of channel estimation error, AWGN with variance 0.01 was added to each tap

Convergence Comparison

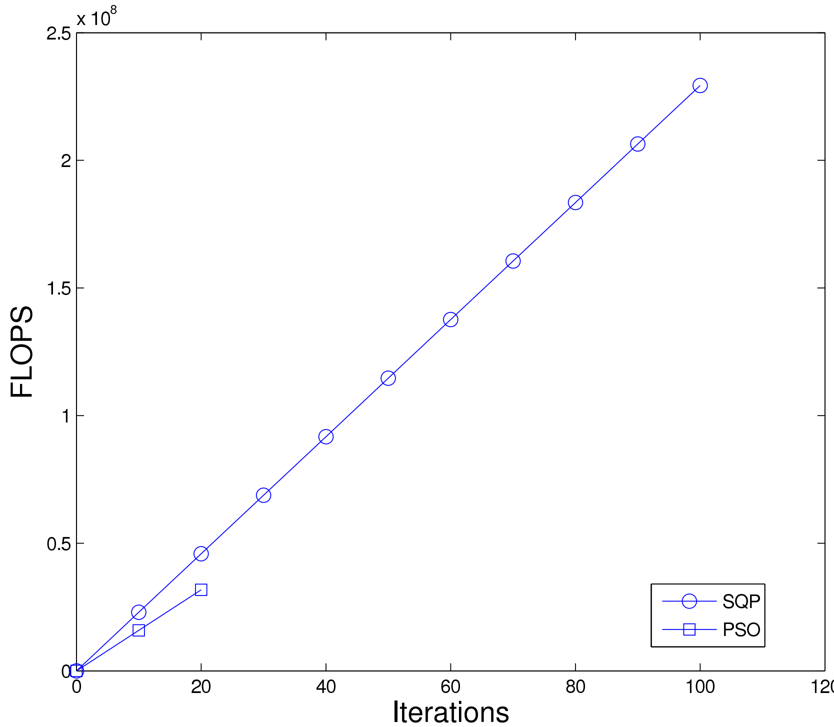
- Given SNR=Eb/No=10 dB, PSO-MBER-MUT converged after $I_{\mathrm{PSO}} = 20$ iterations while SQP-MBER-MUT converged after $I_{\mathrm{SQP}} = 100$ iterations
- Given SNR=Eb/No=15 dB, PSO-MBER-MUT converged after $I_{\mathrm{PSO}} = 40$ iterations while SQP-MBER-MUT converged after $I_{\mathrm{SQP}} = 140$ iterations



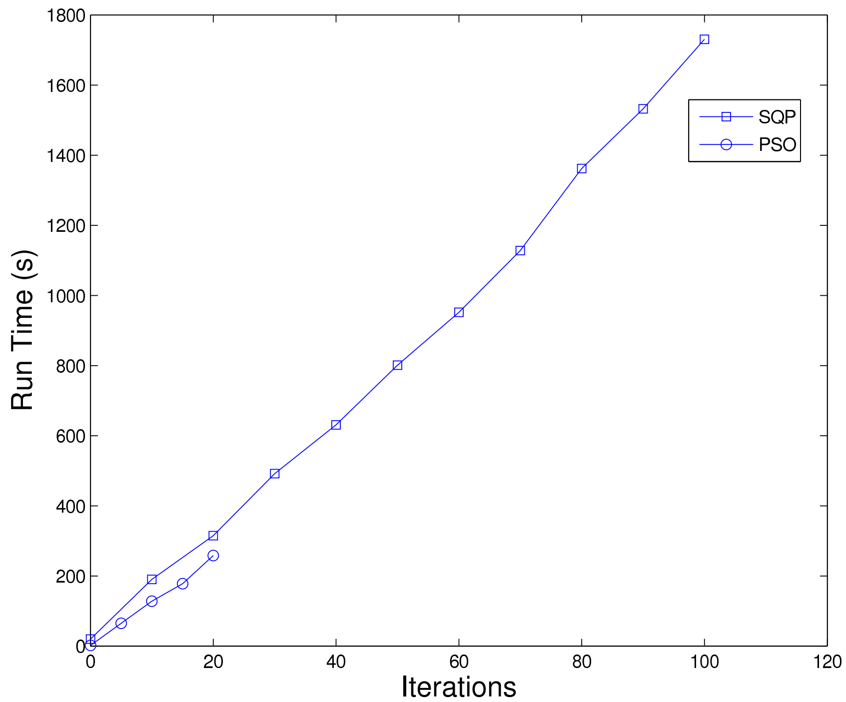
Complexity Comparison

◯ SNR=10 dB: Complexity in terms of (a) total Flops required, and (b) run times recorded, both yielding a complexity ratio $CT_{SQP} : CT_{PSO} \approx 7 : 1$

(a)



(b)



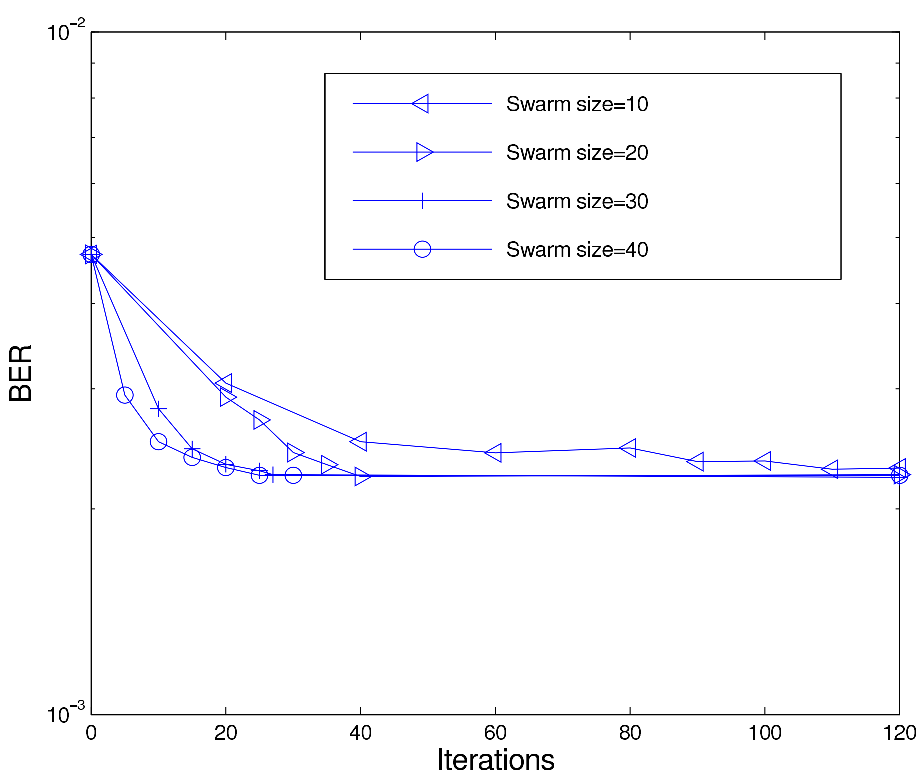
◯ SNR=15 dB: yielding a complexity ratio $CT_{SQP} : CT_{PSO} \approx 5 : 1$

Choice of Swarm Size

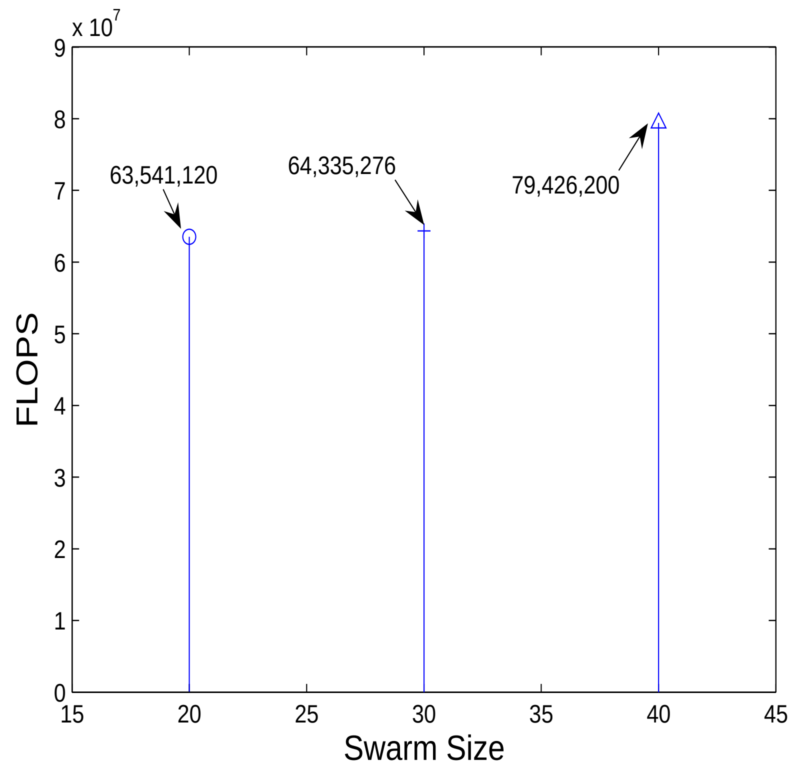
◯ Swarm size $S = 20$ was found to be appropriate

SNR=15 dB: (a) convergence performance, and (b) total complexity required

(a)



(b)



Conclusions

We have proposed PSO assisted MBER-MUT algorithm

- Which offers a much **lower computational complexity** than existing SQP based MBER-MUT algorithm

Simulating system of four transmit antennas and four QPSK mobile users over flat Rayleigh fading channels has confirmed

- PSO-based MBER-MUT imposes approximately **five to seven times lower** complexity than SQP-based MBER-MUT