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Complex-Valued Symmetric Radial Basis Function Classifier for Quadrature Phase Shift Keying Beamforming Systems

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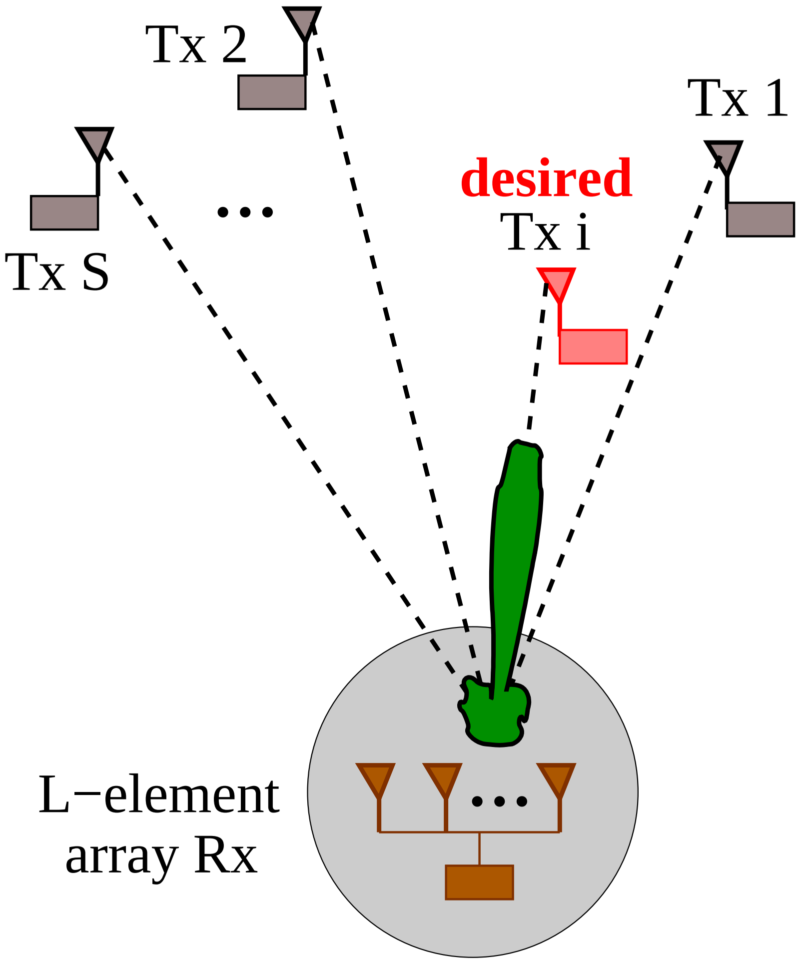
Outline

- Existing **linear** beamforming techniques, and motivations for **nonlinear** beamforming
- Signal model and optimal Bayesian detection with an inherent **symmetry** property for QPSK beamforming
- **Complex-valued** symmetric radial basis function classifier by incorporating *a priori* knowledge
- **Multi-class** Fisher ratio of **class separability** measure based **orthogonal forward selection**
- Simulation investigation, and performance comparison

Motivations



- Classical beamforming is **linear** with a **beampattern** interpretation of beamformer's weight vector
  - maximise response at desired user **direction** and place nulls at interferers' directions, **must** $L \geq S$
  - similar to **zero-forcing** equalisation, and suffers from **noise enhancement**
- Best linear beamforming is **minimum bit error rate** (L-MBER)
  - significantly enhance achievable system BER and user capacity

Motivations (continue)

- Beamforming can be viewed as **classification**, which classifies received channel-impaired signal into most-likely transmitted symbol point
- In comparison with linear beamforming, **nonlinear** detection offers
  - significantly better BER performance and much larger user capacity, at cost of higher complexity
- With **posterior** or **conditional probabilities** as **generalised beam-pattern** interpretation
  - This nonlinear detection can be viewed as **nonlinear beamforming**
- A practical case for **complex-valued** radial basis function network
  - A strong motivation for **grey-box** RBF classifier: the art of incorporating *a priori* knowledge

Signal Model

- S single-transmit-antenna users transmit on same carrier, receiver is equipped with L-element **antenna array**, channels are non-dispersive
- Received signal vector $\mathbf{x}(k) = [x_1(k)\ x_2(k) \cdots x_L(k)]^T$ is

$$\mathbf{x}(k) = \mathbf{P}\,\mathbf{b}(k) + \mathbf{n}(k) = \bar{\mathbf{x}}(k) + \mathbf{n}(k)$$

- $\mathbf{n}(k) = [n_1(k)\ n_2(k) \cdots n_L(k)]^T$ is noise vector, and **system matrix**

$$\mathbf{P} = [A_1\mathbf{s}_1\ A_2\mathbf{s}_2 \cdots A_M\mathbf{s}_S]$$

- $\mathbf{s}_m$: **steering vector** of source m, A_m: m-th non-dispersive channel tap
- User i is **desired** user, and transmitted symbol vector $\mathbf{b}(k) = [b_1(k)\ b_2(k) \cdots b_S(k)]^T$ with QPSK symbol set

$$b_m(k) \in \{b^{[1]} = +1+j,\ b^{[2]} = -1+j, b^{[3]} = -1-j,\ b^{[4]} = +1-j\}, 1 \le m \le S$$

Signal Space

- Denote $N_b = 4^S$ **legitimate sequences** of $\mathbf{b}(k)$ as $\mathbf{b}_q$, $1 \leq q \leq N_b$
- Noiseless **channel state** $\bar{\mathbf{x}}(k)$ takes values from set

$$\bar{\mathbf{x}}(k) \in \mathcal{X} = \{\bar{\mathbf{x}}_q = \mathbf{P}\,\mathbf{b}_q, 1 \leq q \leq N_b\}$$

which can be divided into **four subsets** conditioned on $b_i(k) = b^{[m]}$

$$\mathcal{X}^{[m,i]} \triangleq \{\bar{\mathbf{x}}_q^{[m,i]} \in \mathcal{X}, 1 \leq q \leq N_{sb} : b_i(k) = b^{[m]}\}, \ 1 \leq m \leq 4$$

- **Conditional probabilities** of receiving $\mathbf{x}(k)$ given $b_i(k) = b^{[m]}$ are

$$p^{[m,i]}(\mathbf{x}(k)) = \sum_{q=1}^{N_{sb}} \beta_q e^{-\frac{\|\mathbf{x}(k) - \bar{\mathbf{x}}_q^{[m,i]}\|^2}{2\sigma_n^2}}, \ 1 \leq m \leq 4$$

$N_{sb} = N_b/4 = 4^{M-1}$, noise power is $2\sigma_n^2$ and all priors β_q are equal
- $p^{[m,i]}(\mathbf{x}(k))$ can be interpreted as **generalised beampatterns**

Optimal Bayesian Detector

- **Optimal detection** strategy is

$$\hat{b}_i(k) = b^{[m^*]} \quad \text{with} \quad m^* = \arg \max_{1 \le m \le 4} p^{[m,i]}(\mathbf{x}(k))$$

- Define complex-valued Bayesian **decision variable**

$$y_{\mathrm{Bay},i}(k) \triangleq b^{[1]} \cdot p^{[1,i]}(\mathbf{x}(k)) + b^{[2]} \cdot p^{[2,i]}(\mathbf{x}(k)) + b^{[3]} \cdot p^{[3,i]}(\mathbf{x}(k)) + b^{[4]} \cdot p^{[4,i]}(\mathbf{x}(k))$$

- Optimal **Bayesian** detection is: $\hat{b}_i(k) = \mathrm{sgn}(y_{\mathrm{Bay},i}(k))$, where

$$\mathrm{sgn}(y) = \begin{cases} b^{[1]} = +1 + j, & y_R \ge 0 \text{ and } y_I \ge 0, \\ b^{[2]} = -1 + j, & y_R < 0 \text{ and } y_I \ge 0, \\ b^{[3]} = -1 - j, & y_R < 0 \text{ and } y_I < 0, \\ b^{[4]} = +1 - j, & y_R \ge 0 \text{ and } y_I < 0, \end{cases}$$

Symmetry of Bayesian Solution

- Four state subsets satisfy following **symmetric** properties

$$\mathcal{X}^{[2,i]} = +j \cdot \mathcal{X}^{[1,i]}, \ \mathcal{X}^{[3,i]} = -1 \cdot \mathcal{X}^{[1,i]}, \ \mathcal{X}^{[4,i]} = -j \cdot \mathcal{X}^{[1,i]}$$

- Thus **Bayesian solution** becomes, for $\bar{\mathbf{x}}_q^{[1,i]} \in \mathcal{X}^{[1,i]}$,

$$y_{\mathrm{Bay},i}(k) = \sum_{q=1}^{N_{sb}} \left\{ b^{[1]}\beta \cdot e^{-\frac{\|\mathbf{x}(k)-\bar{\mathbf{x}}_q^{[1,i]}\|^2}{2\sigma_n^2}} + b^{[2]}\beta \cdot e^{-\frac{\|\mathbf{x}(k)-j\cdot\bar{\mathbf{x}}_q^{[1,i]}\|^2}{2\sigma_n^2}} \right.$$

$$\left. +b^{[3]}\beta \cdot e^{-\frac{\|\mathbf{x}(k)+\bar{\mathbf{x}}_q^{[1,i]}\|^2}{2\sigma_n^2}} + b^{[4]}\beta \cdot e^{-\frac{\|\mathbf{x}(k)+j\cdot\bar{\mathbf{x}}_q^{[1,i]}\|^2}{2\sigma_n^2}} \right\}$$

- If system **channel matrix** $\mathbf{P}$ can be estimated, as in **uplink**, subset $\mathcal{X}^{[1,i]}$ can be calculated and Bayesian solution is specified
- In **downlink**, receiver only has access to desired user's training data, estimating $\mathbf{P}$ is difficult, and other adaptive means has to be adopted

Symmetric RBF Network

- Consider **complex-valued radial basis function** network

$$y(k) = \sum_{q=1}^{M} \theta_q \phi_q(\mathbf{x}(k))$$

θ_q: complex-valued **weight**, $\phi_q(\mathbf{x}(k))$: complex-valued **RBF node**

- In view of known symmetric underlying signal space,

$$\begin{aligned} \phi_q(\mathbf{x}) \quad = \quad & b^{[1]} \cdot \varphi(\|\mathbf{x} - \mathbf{c}_q\|/\rho) + b^{[2]} \cdot \varphi(\|\mathbf{x} - j \cdot \mathbf{c}_q\|/\rho) \\ & + b^{[3]} \cdot \varphi(\|\mathbf{x} + \mathbf{c}_q\|/\rho) + b^{[4]} \cdot \varphi(\|\mathbf{x} + j \cdot \mathbf{c}_q\|/\rho) \end{aligned}$$

$\varphi(\bullet)$: real-valued **basis function**, $\mathbf{c}_q$: RBF **centre**, ρ^2: **RBF variance**

- **Task**: construct a **sparse** CV-SRBF classifier when given a block of training data $D_K = \{\mathbf{x}(k), d(k) = b_i(k)\}_{k=1}^{K}$

Training Model

- Given ρ^2, use $\mathbf{c}_q = \mathbf{x}(q)$, $1 \le q \le M = K$, define **modelling residual** $\varepsilon(q) = d(q) - y(q) \Rightarrow$ over **training set** D_K

$$\mathbf{d} = \mathbf{\Phi}\boldsymbol{\theta} + \boldsymbol{\varepsilon}$$

$$\mathbf{d} = [d(1)\ d(2) \cdots d(K)]^T,\ \boldsymbol{\varepsilon} = [\varepsilon(1)\ \varepsilon(2) \cdots \varepsilon(K)]^T,\ \boldsymbol{\theta} = [\theta_1\ \theta_2 \cdots \theta_M]^T$$

- **Complex-valued regression matrix**

$$\mathbf{\Phi} = [\boldsymbol{\phi}_1\ \boldsymbol{\phi}_2 \cdots \boldsymbol{\phi}_M] \in \mathcal{C}^{K\times M}$$

with **column** vectors $\boldsymbol{\phi}_q = [\phi_q(\mathbf{x}(1))\ \phi_q(\mathbf{x}(2)) \cdots \phi_q(\mathbf{x}(K))]^T$, $1 \le q \le M$

- Goal: select subset model containing $M_{\rm spa}$ ($\ll M$) significant RBF nodes
  - RBF variance ρ^2: determined via **cross validation**
  - **Model size**: terminate selection when $M_{\rm spa} = N_{sb}$

Orthogonal Decomposition

- **Orthogonal decomposition** of $\mathbf{\Phi}$: $\mathbf{\Phi} = \mathbf{\Omega}\mathbf{A}$

$$\mathbf{A} = \begin{bmatrix} 1 & \alpha_{1,2} & \cdots & \alpha_{1,M} \\ 0 & 1 & \ddots & \vdots \\ \vdots & \ddots & \ddots & \alpha_{M-1,M} \\ 0 & \cdots & 0 & 1 \end{bmatrix}$$

with **complex-valued** $\alpha_{q,l}$, $1 \leq q < l \leq M$, and **orthogonal matrix**

$$\mathbf{\Omega} = [\boldsymbol{\omega}_1 \ \boldsymbol{\omega}_2 \cdots \boldsymbol{\omega}_M] = \begin{bmatrix} \omega_{1,1} & \omega_{1,2} & \cdots & \omega_{1,M} \\ \omega_{2,1} & \omega_{2,2} & \cdots & \omega_{2,M} \\ \vdots & \vdots & \vdots & \vdots \\ \omega_{K,1} & \omega_{K,2} & \cdots & \omega_{K,M} \end{bmatrix}$$

- Equivalent model

$$\mathbf{d} = \mathbf{\Omega}\boldsymbol{\gamma} + \boldsymbol{\varepsilon}$$

with complex-valued weight vector $\boldsymbol{\gamma} = [\gamma_1 \ \gamma_2 \cdots \gamma_M]^T = \mathbf{A}\boldsymbol{\theta}$

Multi-Class Fisher Ratio

- Divide training data $\mathbf{X} = \{\mathbf{x}(k)\}_{k=1}^{K}$ into $M_C = 4$ **classes**

$$\mathbf{X}^{[q]} \triangleq \{\mathbf{x}(k) \in \mathbf{X} : d(k) = b^{[q]}\}, \; 1 \le q \le M_C$$

Number of samples in $\mathbf{X}^{[q]}$ is $K^{[q]}$ with $\sum_{q=1}^{M_C} K^{[q]} = K$

- **Mean** and **variance** of samples belonging to **class** $\mathbf{X}^{[q]}$ in **direction** $\boldsymbol{\omega}_l$

$$m_{q,l} = \frac{1}{K^{[q]}} \sum_{k=1}^{K} \delta\left(d(k) - b^{[q]}\right) \omega_{k,l}, \;\; \sigma_{q,l}^2 = \frac{1}{K^{[q]}} \sum_{k=1}^{K} \delta\left(d(k) - b^{[q]}\right) \left(\omega_{k,l} - m_{q,l}\right)^2$$

where $\delta(x) = 1$ for $x = 0 + j0$ and $\delta(x) = 0$ for $x \neq 0 + j0$

- **Fisher ratio** of **class separation** between $\mathbf{X}^{[p]}$ and $\mathbf{X}^{[q]}$ in direction $\boldsymbol{\omega}_l$

$$F_{p,q,l} = \left(m_{p,l} - m_{q,l}\right)^2 / \left(\sigma_{p,l}^2 + \sigma_{q,l}^2\right)$$

Ratio of **interclass difference** to **intraclass spread**

OFS Based on FRCSM

- **Average** Fisher ratio of class separation in direction $\boldsymbol{\omega}_l$

$$F_l = \frac{2}{(M_C - 1)M_C} \sum_{p=1}^{M_C-1} \sum_{q=p+1}^{M_C} F_{p,q,l}$$

  Fisher ratio provides a good **class separability** measure

- **Orthogonal decomposition** makes computation of Fisher ratio of class separation measure very efficient

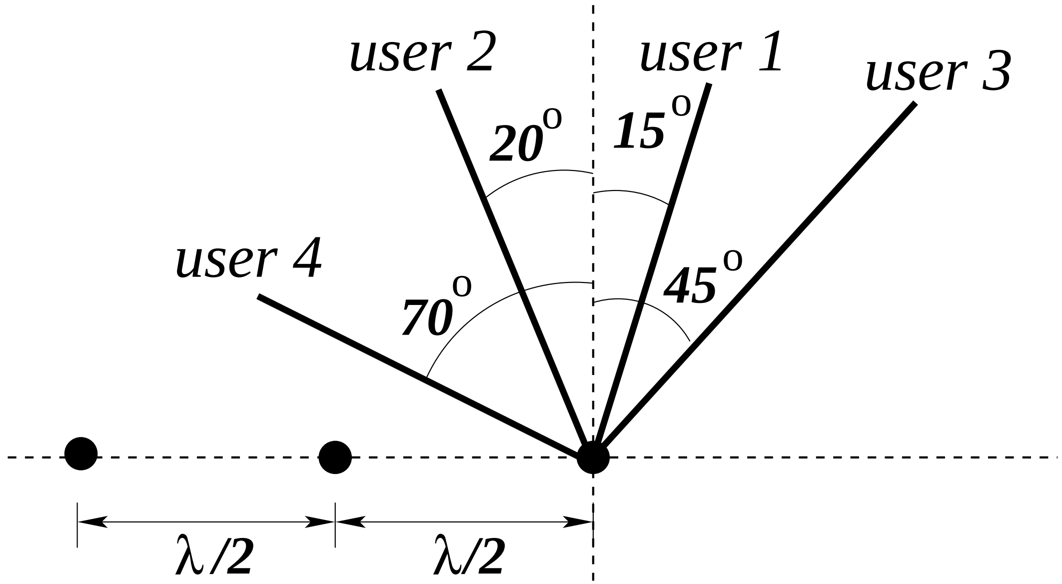
- Based on FRCSM, significant RBF nodes is selected in an OFS procedure

- At l-th stage of **orthogonal forward selection** procedure
  - A node is chosen as l-th term in selected CV-SRBF classifier if it produces **largest** F_l among candidates $\boldsymbol{\omega}_p$, $l \le p \le M$

- Procedure is terminated with a **sparse** classifier of $M_{\text{spa}} = N_{sb}$ terms

Simulation Set Up

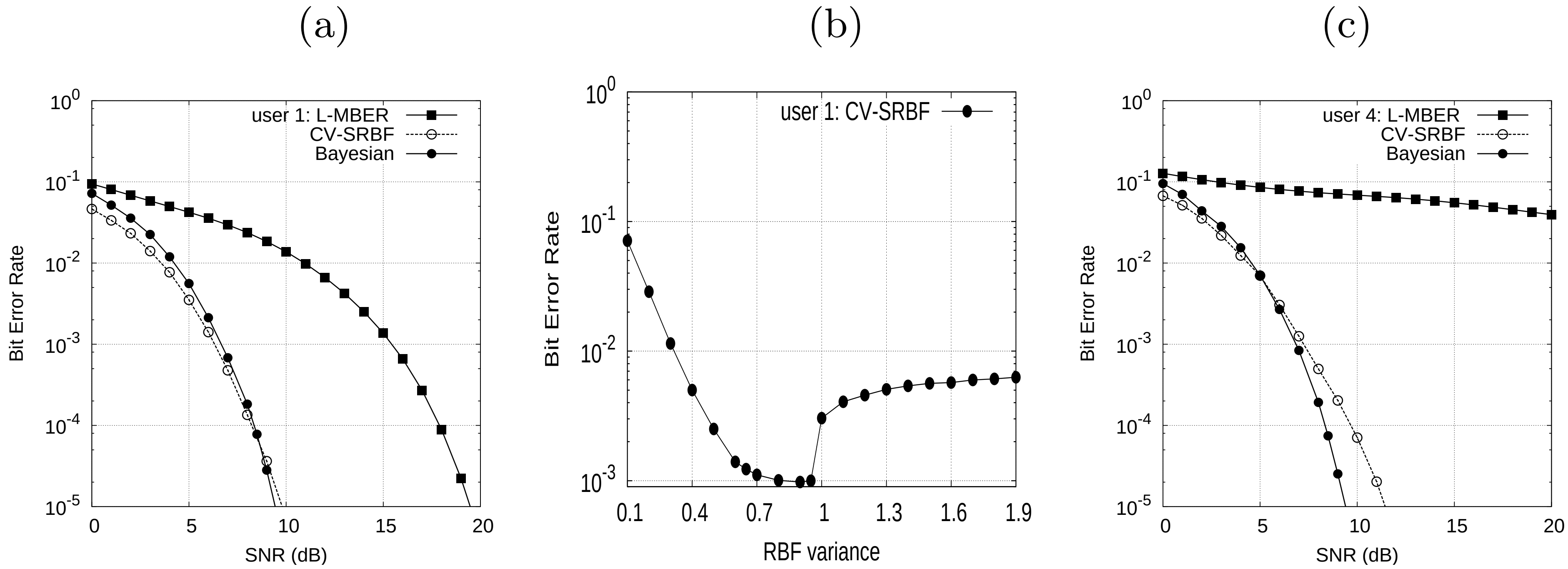
- Three-element antenna array having half wavelength spacing to support four QPSK users
- Angular locations of four users as illustrated
- Simulated channel conditions were $A_i = 1 + j0$, $1 \leq i \leq 4$
- All four users had an equal signal power



- Given each SNR, $K = 600$ training data were generated to train CV-SRBF classifier
- Since number of signal states $N_{sb} = 64$, $M_{\text{spa}} = 64$ terms were selected using OFS based on FRCSM

Simulation Results

(a) User-one bit error rate performance comparison, (b) Influence of RBF variance ρ^2 on bit error rate performance of user-one CV-SRBF classifier given SNR$=6$ dB, and (c) User-four bit error rate performance comparison



Conclusions

- We propose complex-valued symmetric radial basis function classifier for QPSK nonlinear beamforming
- Grey-box model by incorporating *a priori* knowledge
- Orthogonal forward selection based on multi-class Fisher ratio of class separability measure
- Select sparse CV-SRBF classifier from training data efficiently with excellet test bit error rate performance