PHYS2006 Classical Mechanics – Problem Sheet 2

Solutions to be handed in to your problem class leader by **Tuesday 17 February 2015 before the Problem Class**

A. Two circular loops are made from lengths of the same thin uniform wire. One has twice the radius of the other. What is the ratio of the moments of inertia of the two loops about axes through their centres perpendicular to the plane of the loops?

B.

1. Show that the moment of inertia of a uniform right circular cylinder of mass m and radius a about its central axis is
$$\frac{1}{2}ma^2$$
2. Show that the moment of inertia of a uniform thin spherical shell of mass m and radius a about a diameter is
$$\frac{2}{3}ma^2$$

C. A block is free to slide on a smooth horizontal table. A hopper on the block contains N pellets, each of mass m, and a firing mechanism which can eject these pellets horizontally with speed u relative to the block. If the mass of the block, hopper and firing mechanism is M, and m/M is so small that you can treat the problem as approximately continuous, show that the block attains a speed of
$$u\ln\left(1+\frac{Nm}{M}\right)$$
when all the pellets have been fired, if it starts from rest. Assume all the pellets are fired in the same direction. If you want to achieve the greatest final speed for the block, is it better to eject the pellets one-by-one as above, or to eject them all at once with speed u relative to the block? Give the reasoning behind your answer.

D. A raindrop is falling through a cloud, collecting water as it falls. If the raindrop has mass m and velocity v, show that
$$mg = v\dot{m} + m\dot{v}.$$
Assume that the drop remains spherical and that the rate of accretion is proportional to the cross-sectional area of the drop multiplied by the speed of fall. Show that this implies that
$$\dot{m} = kvm^{2/3},$$
for some constant k. Using this equation and the assumption that the drop starts from rest when it is infinitesimally small, find m as a function of x. Hence show that the acceleration of the raindrop is constant and equal to $g/7$.