Selected topics in

Phenomenology

Stefano Moretti

School of Physics and Astronomy
University of Southampton
Southampton SO17 1BJ, UK

and

Particle Physics Department
Rutherford Appleton Laboratory
Chilton, Didcot, Oxon OX11 0QX

Foreword

The tradition is that this course is given using transparencies, unlike the other courses in the school and that the transparencies are simply reproduced in the proceedings. This year I have used a mixture of slides and whiteboard. These notes attempt to combine all the material I used throughout the course and also contain some which I could not treat extensively in the lecture theater. In preparing my course I used material from Nigel Glover, Mike Seymour and Michael Krämer, who preceded me as lecturers of the Phenomenology course. I am greatly indebted to them for letting me using it. I have also ripped off some slides from Gavin Salam's talks and presentations for some of the QCD topics and from Laura Reina in the case of Higgs physics. A special thank goes to Dan Tovey for letting me use many slides from one of his talks for that very last lecture the day after the school dinner (and aftermath !). The references I have used to prepare the course are collected at the end and should ideally provide a good starting point for those who want to learn more about some of the topics. I would like to thank Tim Greenshaw for organising the school so well and for his support throughout. Lots of thanks also go to the other lecturers, to the tutors and primarily to the students. Finally, I am grateful to Margaret Evans for all the practical arrangements and for coping with my extreme lateness in preparing these notes: I was again the last one ...

Introduction

- Kant's Critique of Pure Reason (as interpreted by Wikipedia):

  1. Phenomenon: Phenomena constitute the world as we experience it, as opposed to the world as it exists independently of our experiences (thing-in-themselves, 'das ding an sich'). Humans cannot, according to Kant, know things-in-themselves, only things as we experience them.

  2. Noumenon: "Thing in itself (Ding an sich)" is an allegedly unknowable, undescribable reality that, in some way, lies "behind" observed phenomena. Noumena are sometimes spoken of, though the very notion of individuating items in "the noumenal world" is problematic, since the very notions of number and individuality are among the categories of the understanding, which are supposed to apply only to phenomena, not noumena.

- (The concept of 'Phenomena' led to a tradition of philosophy known as Phenomenology: Hegel, Heidegger, etc. – which we will ignore here !)

- Phenomenon in the general sense: stands for any observable event; phenomena make up the raw data of science.

- Famous quotes: "No phenomenon is a phenomenon until it is an observed phenomenon" (Niels Bohr).

- (I will nonetheless discuss Supersymmetry ...)

My definition of (high energy) phenomenology

- Branch of high-energy physics that seeks knowledge by:

1. Exploiting the hints and clues available in observable phenomena (aka experimental data), without any preconception on the theory governing the latter.

2. Parametrise theories into a set of observables (predictions) that can directly be tested by experiment, thus confirming or disproving the former.

- Phenomenology: bridge between theory and experiment !

Outline

- Introduction: The Standard Model & Beyond
- Tests of the Standard Model
  - QCD: running coupling; infrared safety; factorisation; parton distribution functions; jet production; searches for new physics
  - Electro-Weak (EW) Physics: weak interactions from unitarity; Z line-shape; precision tests; W boson production; indirect search for the Higgs boson
- Higgs Boson Hunting
  - The Higgs mechanism
  - The Higgs picture
  - The Higgs profile
  - Collider searches
- Supersymmetry (SUSY)
  - Why supersymmetry ?
  - The hierarchy problem and gauge coupling unification
  - The Minimal Supersymmetric Standard Model (MSSM)
  - Indirect searches: $g-2$
  - Collider searches
- Epilogue

Introduction

- Current theoretical framework of particle physics is

Standard Model (SM)

- SM is $SU(3) \times SU(2) \times U(1)$ gauge theory with

Matter fields:

$$\begin{pmatrix} u \\ d \end{pmatrix}_L \begin{pmatrix} s \\ c \end{pmatrix}_L \begin{pmatrix} b \\ t \end{pmatrix}_L \qquad d_R \;\; u_R \;\; s_R \;\; c_R \;\; b_R \;\; t_R \text{ (quarks)}$$

$$\begin{pmatrix} e \\ \nu_e \end{pmatrix}_L \begin{pmatrix} \mu \\ \nu_\mu \end{pmatrix}_L \begin{pmatrix} \tau \\ \nu_\tau \end{pmatrix}_L \qquad e_R \qquad \mu_R \qquad \tau_R \qquad \text{(leptons)}$$

Force fields:

$$\gamma, W^\pm, Z, g \text{ (Vector bosons)}$$

and a

$$H \text{ (Higgs scalar)}$$

Q: Why do we believe in the Standard Model ?

A: Because confirmed by experiment !

Q: Why look Beyond the Standard Model (BSM) ?

A: Because SM lacks explanation of fundamental quantities !

SM Flaws

- SM does not explain quantum numbers:

→ EM charge, weak isospin, hypercharge and colour

- Contains (at least) 19 arbitrary parameters:

3 gauge couplings

1 CP-violating vacuum angle

6 quark masses

3 charged lepton masses

3 weak mixing angles

1 CP-violating CKM phase

1 W mass

1 Higgs mass

and (possibly) 9 more parameters in the neutrino sector:

3 neutrino masses

3 neutrino mixing angles

3 CP-violating phases

- More crucially: it does not incorporate gravity !

Beyond the Standard Model

- Three kind of problems:

  1. Mass:
  - What is the origin of particle masses ?
  - Are the masses due to a Higgs boson ?
  - What sets the scale of fermion masses ?

  2. Unification:
  - Is there a theory unifying all particle interactions ?

  3. Flavour:
  - Why are there so many types of quarks and leptons ?
  - What is the origin of CP-violation ?

- Solutions should incorporate gravity (space-time origin/structure)
- String theory best (only ?) candidate, but not yet predictive !
- Supersymmetry (SUSY) to play a role in solving problems:

1. (Gauge) coupling unification best with light sparticles;
2. Mass hierarchy needs light sparticles for stabilisation;
3. SUSY seems essential for the consistency of string theory.

Jargon: a sparticle is a SUSY particle !

What New Physics (NP) ?

- Which way to go about ?

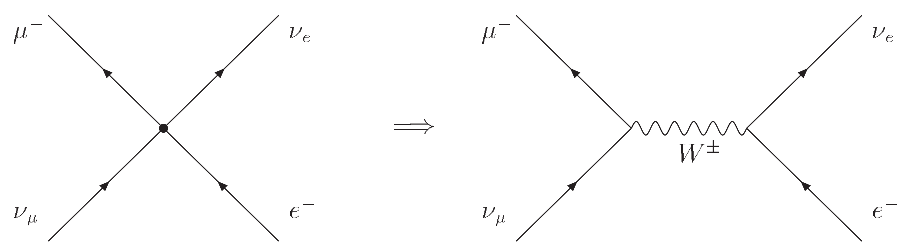
1. Come up with theory, devise model for it, get out predictions, compare with experiment !
2. Treat SM as effective theory below some high scale Λ: NP described by operators of dimension ≥ 6 suppressed by powers of E^2/Λ^2 ($E \to$ relevant energy).

Historic example: Fermi's theory of weak interactions,

- $\mu^- \to e^- \bar{\nu}_e \nu_\mu$ decay described by effective Lagrangian:

$$\mathcal{L} = \frac{G_F}{\sqrt{2}} [\bar{\nu}_\mu \gamma_\lambda (1-\gamma_5)\mu][\bar{e}\gamma^\lambda (1-\gamma_5)\nu_e].$$

- From experiment $G_F \approx 1.17 \times 10^{-5}$ GeV^{-2} (Fermi coupling).
- As $\Lambda \approx M_W$, W appears as deviations from effective theory.



- Hence, precision tests of the SM can reveal NP !

- Crucial question for phenomenology is:

What is the scale of new physics ? $\Lambda \lesssim 1$ TeV ? Higher ?

- We do not know for sure, so we push up collider energies !

Test of the SM: QCD

Outline

- Importance of QCD
- The QCD coupling
- $e^+e^- \to$ hadrons
- Infrared safe quantities
- Jets
- Parton shower
- Hadronisation
- Deeply inelastic scattering
- Hadron-Hadron collisions
- New physics searches

Importance of QCD

QCD is the correct* theory of strong interactions

(*in the described sense of a low-energy effective theory)

→ Why QCD studies ?

1) A Quantum Field Theory (QFT) with unique features:

   – asymptotic freedom
   – infrared slavery (confinement)

2) We need to understand QCD also to search for NP:

   – for new particles hadro-production (Tevatron and LHC)
   – to predict the SM backgrounds to NP signals

- QCD degrees of freedom: quarks & gluons (aka partons).
- Will study their interactions in e^+e^-, $e^\pm p$, pp and $p\bar{p}$.
- (See Nick's course for Lagrangian & Feynman rules)

- The QCD Lagrangian is given by:

$$\mathcal{L} = -\frac{1}{4}F^A_{\mu\nu}F^{A\,\mu\nu} + \sum_{\text{flavours}} \bar{q}_a(i\not{D} - m)_{ab}q_b$$
$$+\mathcal{L}_{\text{gauge-fixing}} + \mathcal{L}_{ghost}$$

where $F^A_{\alpha\beta}$ is the field strength tensor derived from the gluon field $\mathcal{A}^a_\alpha$,

$$F^A_{\mu\nu} = \partial_\mu \mathcal{A}^A_\nu - \partial_\nu \mathcal{A}^A_\mu - g f^{ABC} \mathcal{A}^B_\mu \mathcal{A}^\mu_\nu$$

and the indices A, B, C run over the eight colour degrees of freedom of the gluon field. The quark fields q_a are in the triplet representation of the SU(3) colour group and D is the covariant derivative:

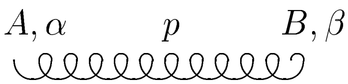
$$(D_\mu)_{ab} = \partial_\mu \delta_{ab} + ig(t^c \mathcal{A}^c_\mu)_{ab}$$

The t are matrices in the fundamental representation of SU(3) and satisfy:

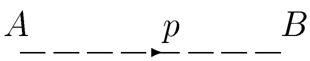
$$[t^A, t^B] = if^{ABC}t^C$$

For a discussion of the gauge-fixing and ghost terms of the QCD Lagrangian see Nick's course.

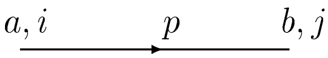
The **Feynman rules** can be derived from the QCD Lagrangian:



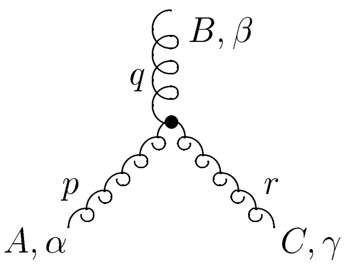
$$\delta^{AB}\left[-g^{\alpha\beta}+(1-\lambda)\frac{p^\alpha p^\beta}{p^2+i\varepsilon}\right]\frac{i}{p^2+i\varepsilon}$$



$$\delta^{AB}\frac{i}{p^2+i\varepsilon}$$

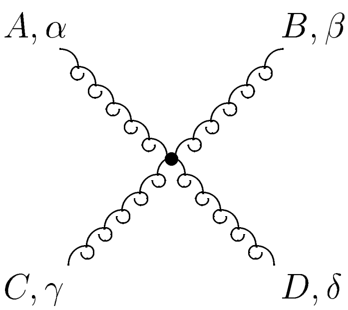


$$\delta^{ab}\frac{i}{\left(\not{p}-m+i\varepsilon\right)_{ji}}$$

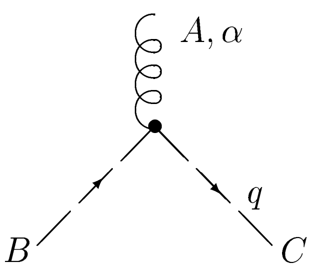


$$\begin{aligned}-gf^{ABC}\Big[&g^{\alpha\beta}\,(p-q)^\gamma\\&+g^{\beta\gamma}\,(q-r)^\alpha\\&+g^{\gamma\alpha}\,(r-p)^\beta\,\Big]\end{aligned}$$

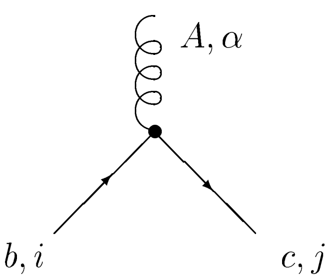
(all momenta incoming)



$$\begin{aligned}&-ig^2 f^{XAC}f^{XBD}\left(g_{\alpha\beta}g_{\gamma\delta}-g_{\alpha\delta}g_{\beta\gamma}\right)\\&-ig^2 f^{XAD}f^{XBC}\left(g_{\alpha\beta}g_{\gamma\delta}-g_{\alpha\gamma}g_{\beta\delta}\right)\\&-ig^2 f^{XAB}f^{XCD}\left(g_{\alpha\gamma}g_{\beta\delta}-g_{\alpha\delta}g_{\beta\gamma}\right)\end{aligned}$$

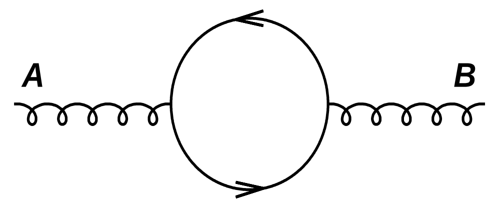


$$gf^{ABC}q^\alpha$$

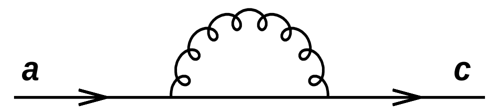


$$-ig\left(t^A\right)_{cb}\left(\gamma^\alpha\right)_{ji}$$

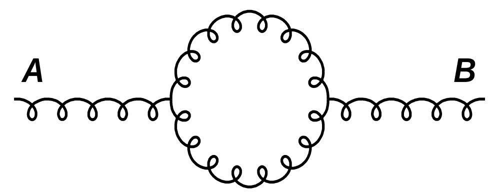
$$\mathrm{Tr}(t^A t^B) = T_R \delta^{AB}\,, \quad T_R = \tfrac{1}{2}$$



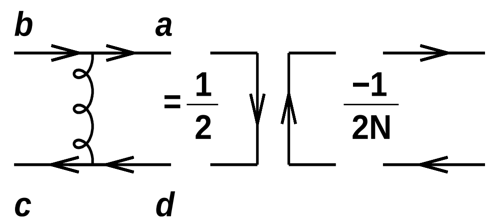
$$\sum_A t^A_{ab} t^A_{bc} = C_F \delta_{ac}\,, \quad C_F = \frac{N_c^2 - 1}{2N_c} = \frac{4}{3}$$



$$\sum_{C,D} f^{ACD} f^{BCD} = C_A \delta^{AB}\,, \quad C_A = N_c = 3$$



$$t^A_{ab} t^A_{cd} = \frac{1}{2}\delta_{bc}\delta_{ad} - \frac{1}{2N_c}\delta_{ab}\delta_{cd}$$ (Fierz)



The QCD coupling

... is running !

- Quantum corrections alter particle masses and couplings.
- Ultraviolet divergences removed by renormalisation.

Renormalisation introduces a mass scale μ – the subtraction point of UV divergences – and the <u>renormalised</u> coupling α_s depends on μ:

$$\alpha_s \to \alpha_s(\mu) = \frac{1}{\beta_0 \ln(\mu^2/\Lambda^2)},$$

$\beta_0 = (11N_C - 2n_f)/12\pi$, $N_C = 3$, $n_f = \#$ of active flavours.

- $\Lambda \equiv \Lambda_{\rm QCD}$ (≈ 200 MeV) is an integration constant:

$$\mu^2 \frac{d\alpha_s}{d\mu^2} \equiv \beta(\alpha_s) = -\beta_0 \alpha_s^2 + \ldots$$

- **<u>Asymptotic freedom</u>: $\alpha_s \to 0$ as $\mu \to \infty$**

  $\to$ we can use perturbation theory for processes involving large momentum scales (small distances).

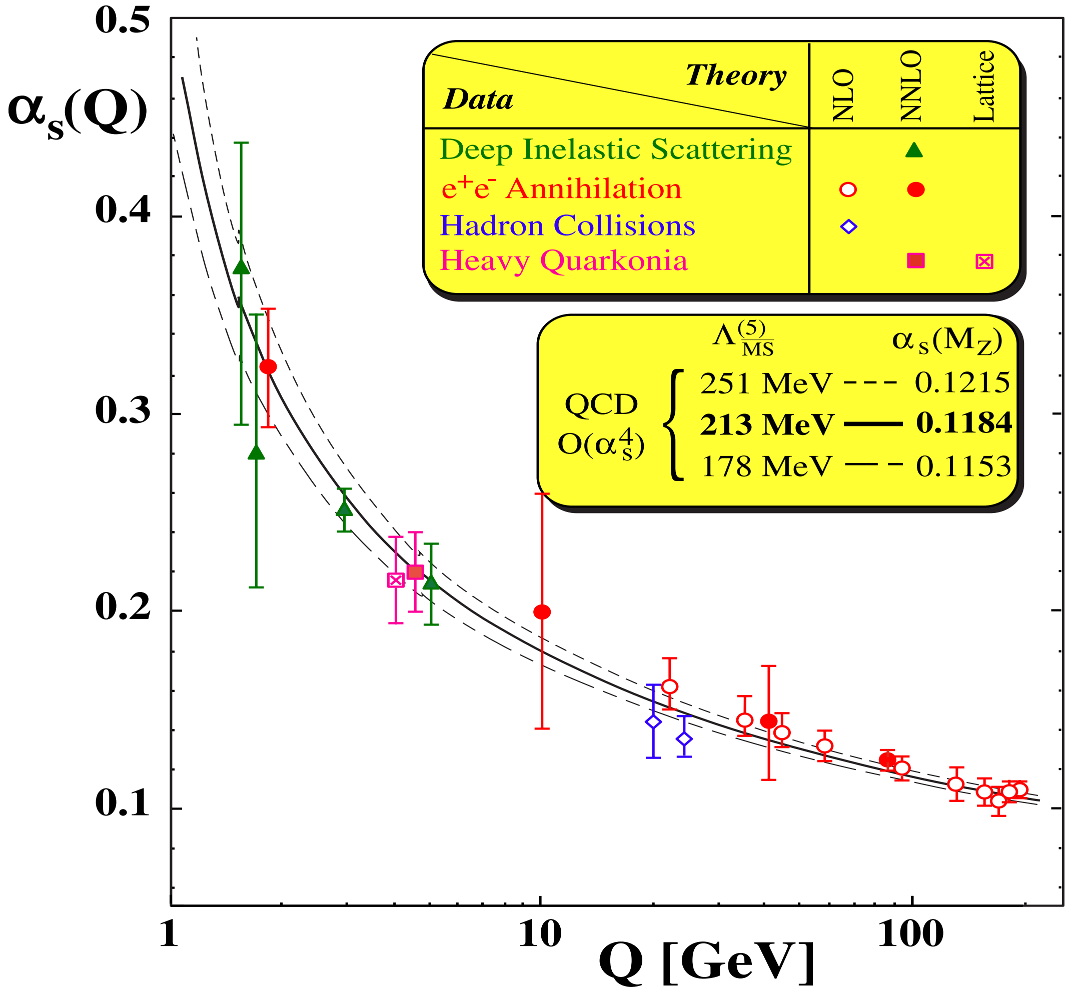
[Sign of β is crucial: in QED, $\beta < 0$ and α increases as $\mu \to \infty$.]

- **<u>Infrared slavery</u>: $\alpha_s \to \infty$ as $\mu \to \Lambda$**

  $\to$ confinement: quarks & gluons are only found in colour-singlet bound states.

  $\to$ we have to use non-perturbative methods (e.g. lattice) at low momentum scales (large distances).

• Running of α_s has been established experimentally !



(Compilation of data by Siggi Bethke)

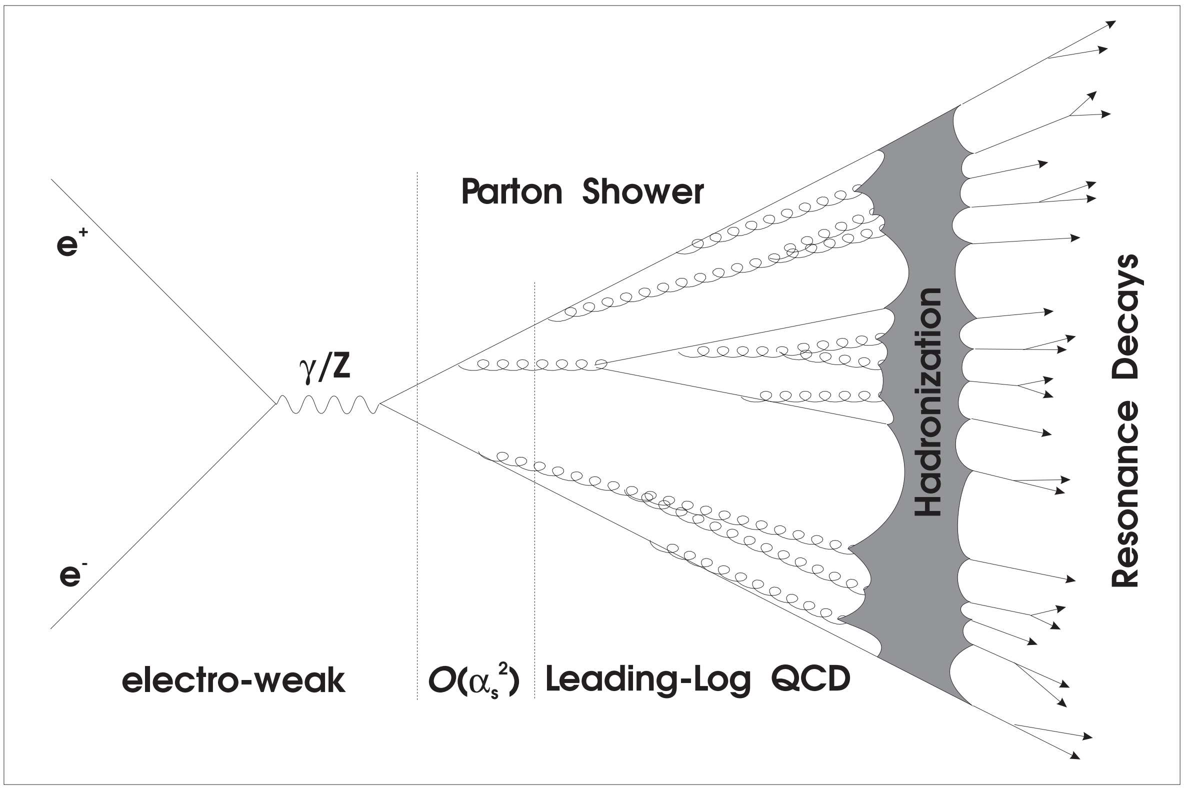
• But how do we measure α_s ?

$e^+e^- \to$ hadrons

- But QCD Feynman rules tell us only about partons !
- Hadron formation (long distance) is not perturbative !

  $\to$ how to calculate $e^+e^- \to$ hadrons ?

Plenty of physics between partons and hadrons !



- Each event has different hadronic final state !

  $\to$ how to sum over all of these ?

• Symmetries can help us !

→ Matrix Element (ME) to produce n hadrons $h_1 ... h_n$:

$$\mathcal{M} \sim \{\bar{v}(p_{e^+}) e \gamma_\mu u(p_{e^-})\} \; \frac{-g^{\mu\nu}}{q^2} \; T_\nu(n, q, \{p_{h_1} \ldots p_{h_n}\}),$$

with T_ν parametrisation of the unknown part.

→ Gives total cross section:

$$\sigma = \frac{1}{2s} \; \frac{1}{4} \; \frac{e^2}{s^2} \mathrm{Tr}(\not{p}_{e^+} \gamma^\mu \; \not{p}_{e^-} \gamma^\nu)$$

$$\times \sum_n \int d\mathrm{PS}_n \; T_\mu(n, q, \{p_{h_1} \ldots p_{h_n}\}) \; T^*_\nu(n, q, \{p_{h_1} \ldots p_{h_n}\}).$$

→ Define: $H_{\mu\nu}(q) \equiv \sum_n \int dPS_n \; T_\mu \; T^*_\nu$.

→ Impose Lorentz covariance:

$$H_{\mu\nu} = A g_{\mu\nu} + B q_\mu q_\nu, \qquad (A, B \text{ functions only of } q^2).$$

→ Impose gauge invariance:

$$q^\mu H_{\mu\nu} = q^\nu H_{\mu\nu} = 0 \;\; \Rightarrow \;\; A = -q^2 B.$$

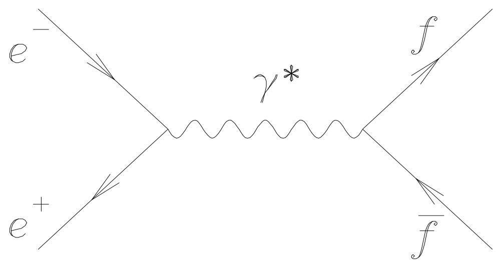
→ Hence, $\sigma = \frac{e^2}{2s} B(s)$ and $B(s)$ dimensionless.

→ Gives fundamental prediction:

$$R \equiv R(e^+ e^-) = \frac{\sigma(e^+ e^- \to \text{hadrons})}{\sigma(e^+ e^- \to \mu^+ \mu^-)} = \text{constant},$$

without knowing anything about hadron interactions !

$e^+e^- \to$ hadrons at leading order

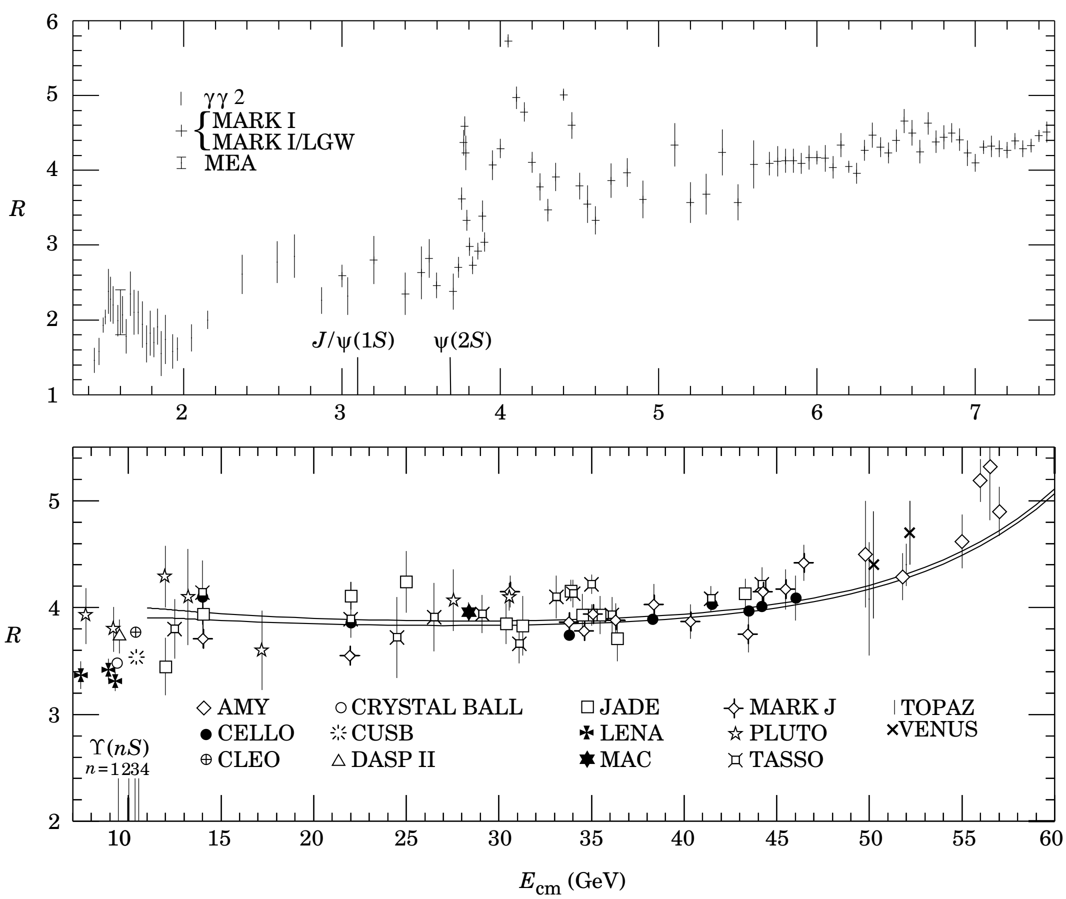


$$\sigma_0 = \frac{4\pi\alpha^2}{3s} N_C \sum_q e_q^2 \;(\text{if}\, f = q)$$

$$\Rightarrow R_0 \equiv \frac{\sigma_0(e^+e^- \to \text{hadrons})}{\sigma_0(e^+e^- \to \mu^+\mu^-)} = N_C \sum_q e_Q^2$$

$\to$ first evidence for colour ($N_C = 3$) !

- Kinematically allowed if $\sqrt{s} > 2m_q$, steps at $\sqrt{s} = 2m_q$.



(e.g. $\sqrt{s} = 34$ GeV, $R = \frac{11}{3}$, cf PETRA data: 3.88 ± 0.03)

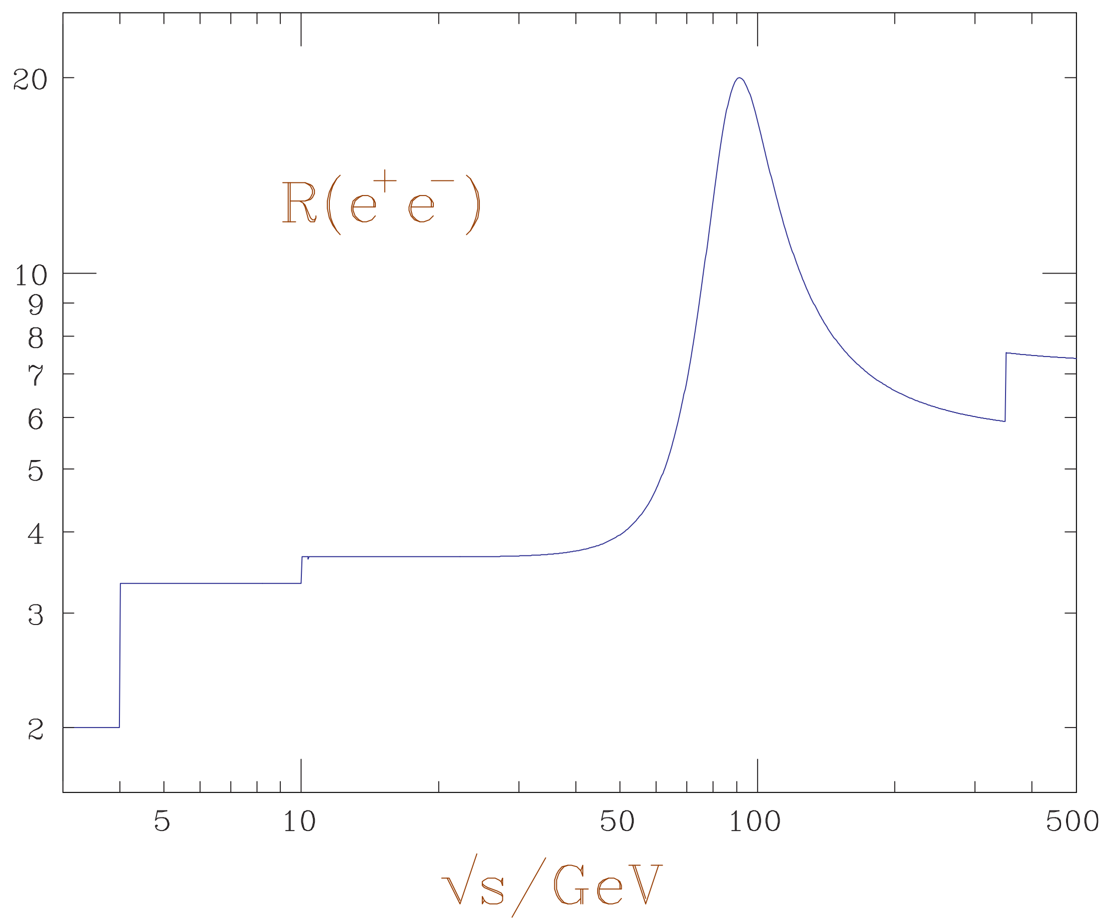
R at the Z peak

- Can also tell us about EW couplings:

$$R = N_C \frac{\sum_q \mathcal{A}_q}{\mathcal{A}_\mu} = 20.095, \qquad \mathcal{A}_f = v_f^2 + a_f^2.$$

(cf LEP average: 20.775 ± 0.027)

- In general, sensitive to $\gamma - Z$ interference:



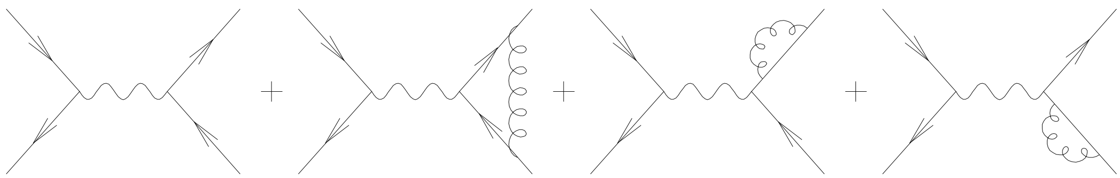
(in fact, $R = 19.984$ on Z peak)

$e^+e^- \to$ hadrons beyond leading order

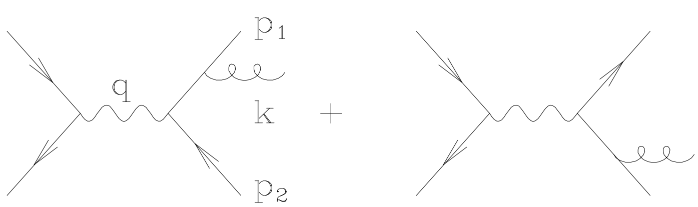
- α_s largest coupling: expect QCD corrections largest !

$\to$ start with them !

- At $\mathcal{O}(\alpha_s)$:



(a)



(b)

- Virtual corrections: <u>interfere</u> tree-level diagrams with one-loop ones in (a)

$\to$ can be negative

- Real corrections: <u>square</u> tree-level diagrams in (b)

$\to$ positive definite

(b) real gluon emission

- 3-body phase space: $d\Phi_3 = [\ldots]\, d\alpha\, d\beta\, d\gamma\, dx_1\, dx_2$

where α, β, γ are Euler angles and $x_1 = 2E_q/\sqrt{s}$ and $x_2 = 2E_{\bar{q}}/\sqrt{s}$ are energy fractions of final-state quark and antiquark.

- Applying Feynman rules and integrating over Euler angles:

$$\sigma^{q\bar{q}g} = 3\sigma_0 C_F \frac{\alpha_s}{2\pi} \int dx_1 dx_2 \frac{x_1^2 + x_2^2}{(1-x_1)(1-x_2)}$$

with integration region $0 \leq x_1, x_2 \leq 1$.

- Integral is divergent at $x_1, x_2 = 1$:

$$\begin{aligned} 1 - x_1 &= x_2 x_3\, (1 - \cos\theta_{qg})/2, \\ 1 - x_2 &= x_1 x_3\, (1 - \cos\theta_{\bar{q}g})/2, \end{aligned}$$

where $x_3 = 2E_g/\sqrt{s}$ (E_g gluon energy) and $\theta_{ig} (i = 1, 2)$ are angles between gluon and quarks.

$\rightarrow$ collinear divergence: $\theta_{qg} \rightarrow 0$ or $\theta_{\bar{q}g} \rightarrow 0$

$\rightarrow$ soft divergence: $E_g \rightarrow 0$

- Singularities indicate breakdown of perturbation theory when mass scales approach Λ.

- Fortunately, collinear/soft regions do not make important contributions to total cross section:

$\rightarrow$ they cancel !

• Make integral finite using e.g. dimensional regularisation:

$D = 4 - 2\epsilon \Rightarrow$

$$\sigma^{q\bar{q}g} = 3\sigma_0 C_F \frac{\alpha_s}{2\pi} H(\epsilon) \int dx_1 dx_2 \frac{(1-\epsilon)(x_1^2 + x_2^2) + 2\epsilon(1-x_3)}{(1-x_3)^\epsilon [(1-x_1)(1-x_2)]^{1+\epsilon}}$$

where $H(\epsilon) = \frac{3(1-\epsilon)(4\pi)^{2\epsilon}}{(3-2\epsilon)\Gamma(2-2\epsilon)} = 1 + \mathcal{O}(\epsilon)$.

• Hence

$$\sigma^{q\bar{q}g} = 3\sigma_0 C_F \frac{\alpha_s}{2\pi} H(\epsilon) \left[\frac{2}{\epsilon^2} + \frac{3}{\epsilon} + \frac{19}{2} - \pi^2 + \mathcal{O}(\epsilon) \right]$$

$\rightarrow$ soft/collinear divergences are regulated, appearing as poles at $D = 4$ ($\epsilon = 0$).

(b) virtual gluon exchange

$$\sigma^{q\bar{q}} = 3\sigma_0 \left\{ 1 + C_F \frac{\alpha_s}{2\pi} H(\epsilon) \left[-\frac{2}{\epsilon^2} - \frac{3}{\epsilon} - 8 + \pi^2 + \mathcal{O}(\epsilon) \right] \right\}$$

$\Rightarrow$ Adding real and virtual corrections, the infrared/collinear poles cancel and the result is finite as $\epsilon \to 0$:

$$R = R_0 \left\{ 1 + \frac{\alpha_s}{\pi} + \mathcal{O}(\alpha_s^2) \right\}$$

• Other regularisation schemes available: e.g. finite g mass $m_g \equiv \epsilon s$ (non-gauge invariant !).

$\Rightarrow$ R is an infrared safe quantity !

$\Rightarrow$ R is finite and regularisation scheme-independent !

$e^+e^- \to$ hadrons cross section at NLO

1. First α_s measurement:

$$\begin{aligned} R(\text{LEP}) &= 20.775 \pm 0.027 \\ R_0(M_Z) &= 19.984 \\ \to \alpha_s(M_Z) &= 0.124 \pm 0.004 \end{aligned}$$

2. Second α_s measurement:

$$\begin{aligned} R(\text{PETRA}) &= 3.88 \pm 0.03 \\ R_0(34\ \text{GeV}) &= 3.69 \\ \to \alpha_s &= 0.162 \pm 0.026 \\ \to \alpha_s(M_Z) &= 0.134 \pm 0.018 \text{ (upon running)} \end{aligned}$$

- PETRA agrees with LEP:

$\Rightarrow$ test of QCD in intervening energy range !

Note: τ-decays

- Related measurement: $R \equiv R(\tau) = \frac{\text{BR}\,(\tau \to \text{hadrons})}{\text{BR}\,(\tau \to \text{electrons, muons})}$

$\to$ one of best α_s measurements:

$$\begin{aligned} \alpha_s(m_\tau = 1.77\ \text{GeV}) &= 0.33 \pm 0.03 \\ \to \alpha_s(M_Z) &= 0.118 \pm 0.004 \text{ (upon running)} \end{aligned}$$

- Cross section now known through NNLO $\to$ fig

- And theoretical errors ? Where are they ?

Dependence of total cross section on only *hard* gluons is reflected in 'good behaviour' of perturbation series:

$$\sigma_{tot} = \sigma_{q\bar{q}} \left(1 + 1.045 \frac{\alpha_s(Q)}{\pi} + 0.94 \left(\frac{\alpha_s(Q)}{\pi} \right)^2 - 15 \left(\frac{\alpha_s(Q)}{\pi} \right)^3 + \cdots \right)$$

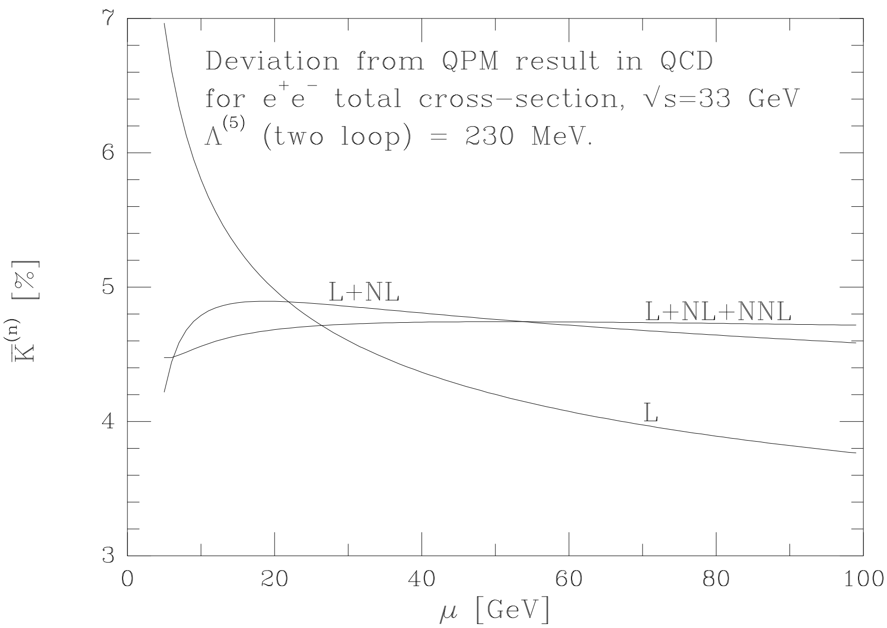
(Coefficients given for $Q = M_Z$)

Renormalisation scale dependence

- Recall $\alpha_s(\mu)$, μ arbitrary: would disappear to all orders ...

$\Rightarrow$ Use dependence as estimate of uncertainty due to truncating perturbative series $\rightarrow$ smaller at each order

- Vary μ (by some factor) to estimate theoretical uncertainty:



- What scale to use for <u>central</u> α_s value ?

1. Physical scale, $\mu = \sqrt{s}$
2. Principle of Minimal Sensitivity: where $d\sigma/d\mu = 0$
3. Fastest Apparent Convergence: where NLO=LO

$\rightarrow$ theoretical predictions rather subjective !

Infrared safe quantities

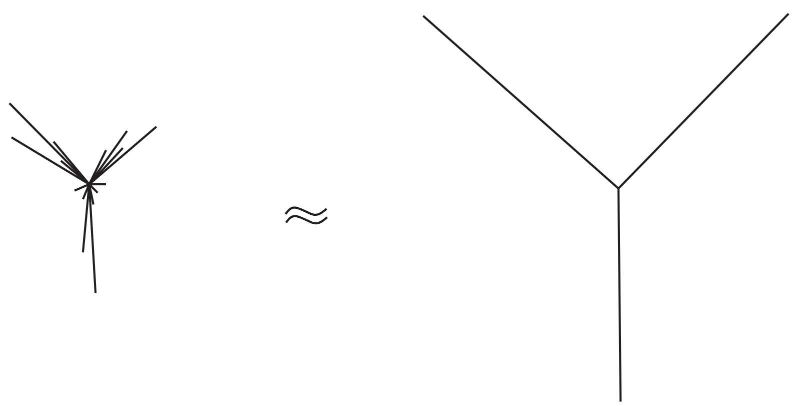
• $R(e^+e^-)$ and $R(\tau)$ are very inclusive quantities: total cross sections or decay rates !

• Infrared safety guaranteed by 'theorems', e.g. Bloch and Nordsieck (BN) plus Kinoshita, Lee and Nauenberg (KLN):

→ suitably defined quantities are free of singularities.

• Physical meaning: events with hadrons give approximately the same measurement as parton ones.

• Computational meaning: infinities cancel when adding real gluon emission and virtual gluon exchange.

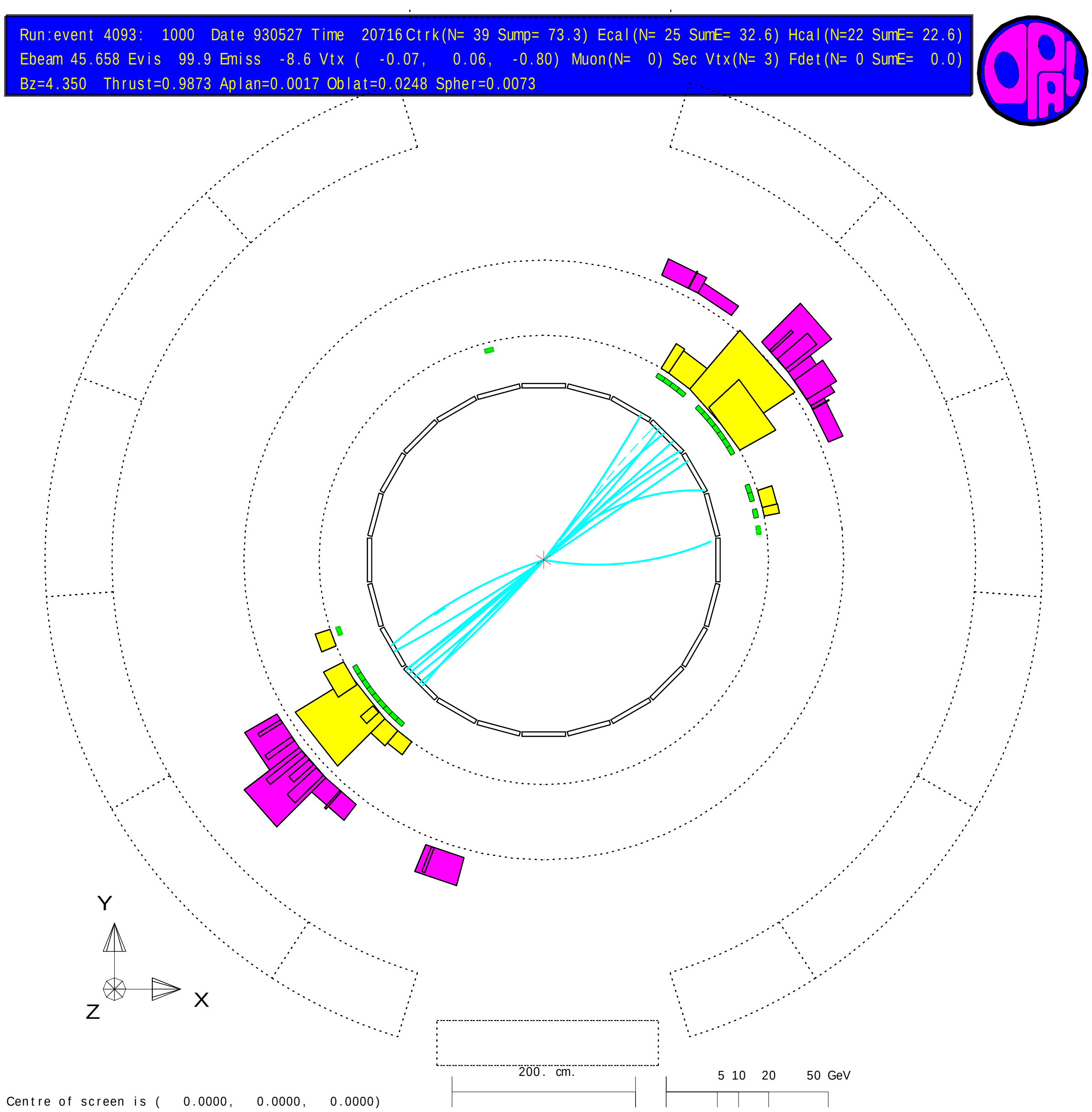


• BN & KLN apply also to more exclusive quantities: e.g.

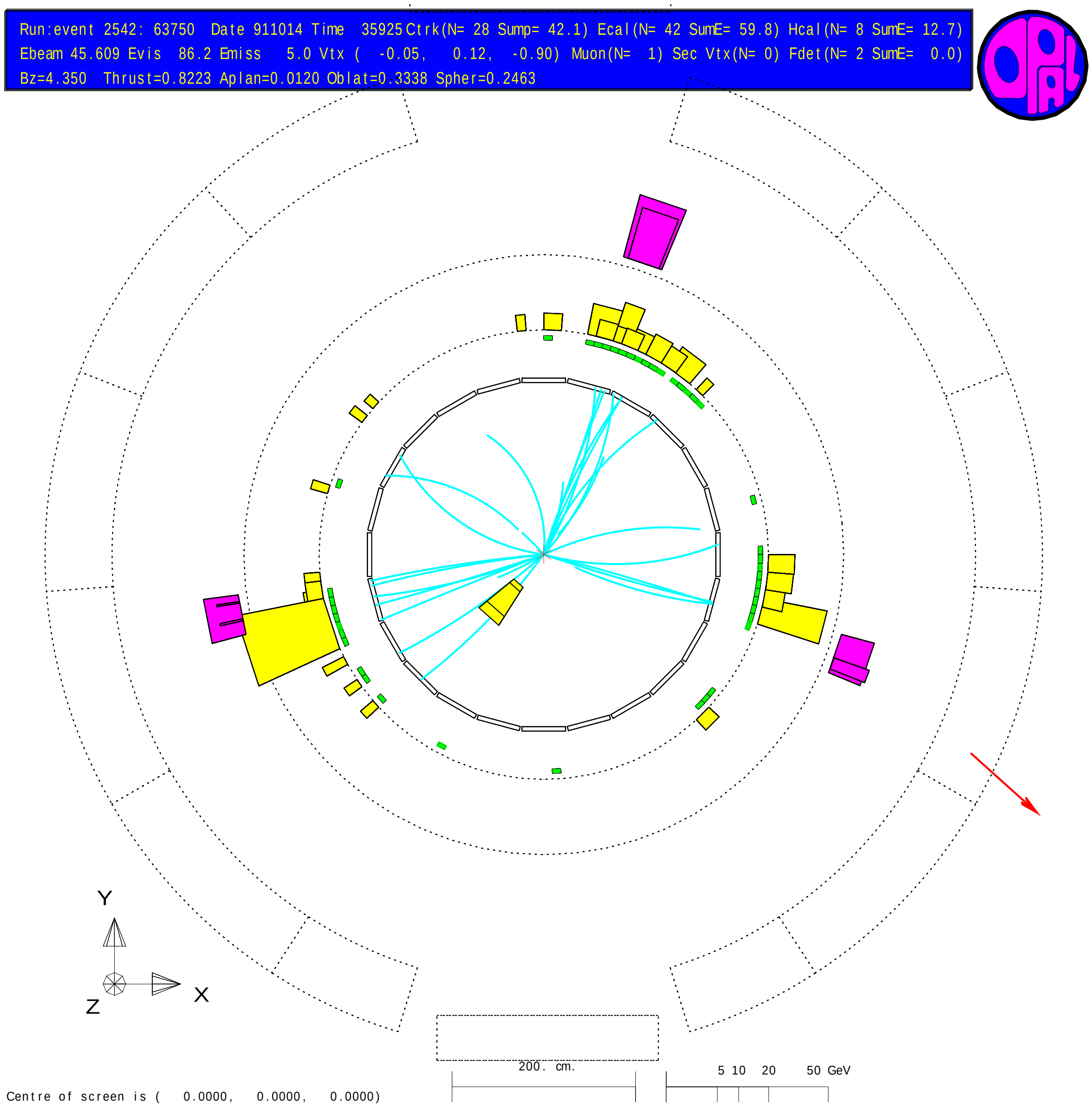
1. n-jets cross section, $n = 2, 3, ...$
2. Event-shape variables like thrust

Jets

Naively expect most events to look like:



with a fraction $\sim \alpha_s$ more like:



(and even a fraction $\sim \alpha_s^2$ four-jet like, etc)

Jet definition

- Intuitively, jet is a spray of collimated particles.
- Need a procedure: in e^+e^- use clustering algorithms.
- Start with a list of momenta $p_1^\mu, p_2^\mu, ..., p_n^\mu$.

(In perturbative calculations, they are parton momenta.)

- Three ingredients:

1. A measure of inter-jet distance: y_{ij}.

   $\to$ for each pair of final state momenta calculate, e.g.

$$y_{ij} = m_{ij}^2/s \text{ (Invariant Mass)}$$
$$y_{ij} = 2E_i\, E_j\,(1 - \cos\theta_{ij})/s \text{ (JADE)}$$
$$y_{ij} = 2\min\{E_i^2, E_j^2\}\,(1 - \cos\theta_{ij})/s \text{ (Durham)}$$

2. A resolution for the latter: y_{cut}.

   $\to \min\{y_{ij}, ...\} < y_{\text{cut}}$ combine i and j into k

3. A recombination procedure: e.g.

$$p_k^\mu = p_i^\mu + p_j^\mu \text{ (E-scheme)}$$
$$p_k^\mu = (|\mathbf{p}_i + \mathbf{p}_j|, \mathbf{p}_i + \mathbf{p}_j) \text{ (p-scheme)}$$
$$p_k^\mu = (|E_i + E_j|, \tfrac{E_i+E_j}{|\mathbf{p}_i+\mathbf{p}_j|}\mathbf{p}_i + \mathbf{p}_j) \text{ (E0-scheme)}$$

- Repeat till $\min\{y_{kl}, ...\} > y_{\text{cut}}$: remaining objects are jets.
- A n-parton final state can give any number of jets between n (all partons well-separated) and 2 (e.g. two energetic quarks accompanied by soft and collinear gluons).

Jet rates

- Define n-jet fraction $f_n(y)$ by ($y \equiv y_{\rm cut}$).

$$f_n(y) = \frac{\sigma_n(y)}{\sum_m \sigma_m(y)} = \frac{\sigma_n(y)}{\sigma_{\rm tot}},$$

- If $\sigma_{\rm tot} = \sigma_0(1 + \alpha_s/\pi + ...)$, then

$$\sum_n f_n(y) = 1.$$

- For $\mu = \sqrt{s}$ and $n = 2, 3$ and 4:

$$f_2(y) = 1 - \left(\frac{\alpha_s}{2\pi}\right) A(y) + \left(\frac{\alpha_s}{2\pi}\right)^2 (2A(y) - B(y) - C(y)) + ...,$$

$$f_3(y) = \left(\frac{\alpha_s}{2\pi}\right) A(y) + \left(\frac{\alpha_s}{2\pi}\right)^2 (B(y) - 2A(y)) + ...,$$

$$f_4(y) = \left(\frac{\alpha_s}{2\pi}\right)^2 C(y) + ...,$$

- Coupling constant α_s and functions $A(y), B(y)$ and $C(y)$ defined in some renormalisation scheme (e.g. $\overline{\rm MS}$ scheme).

- Terms of order $\mathcal{O}(\alpha_s^2)$ involving $A(y)$ take account of the normalisation to $\sigma_{\rm tot}$ rather than to σ_0.

- Example 1 of n-jet event rates (α_s^2 vs. OPAL)

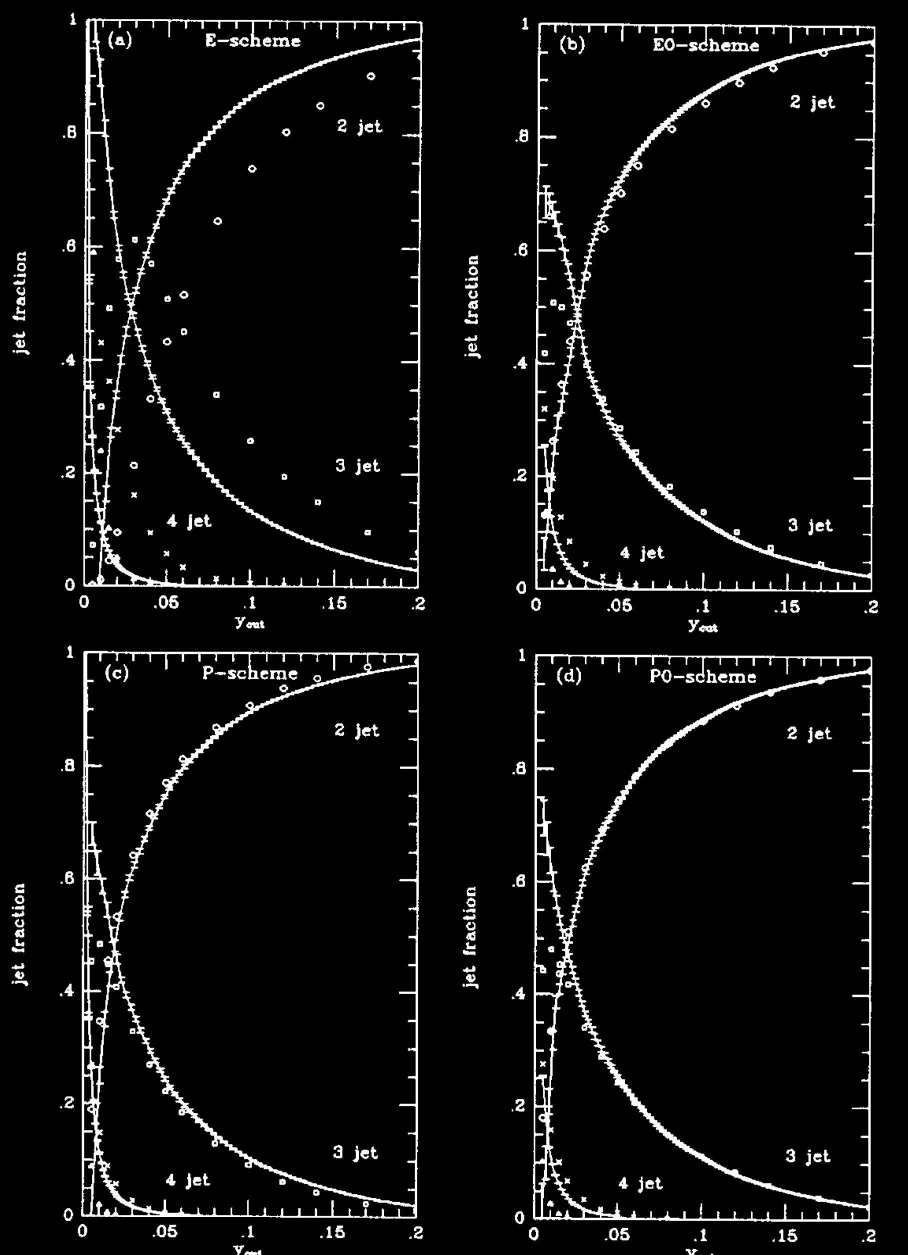
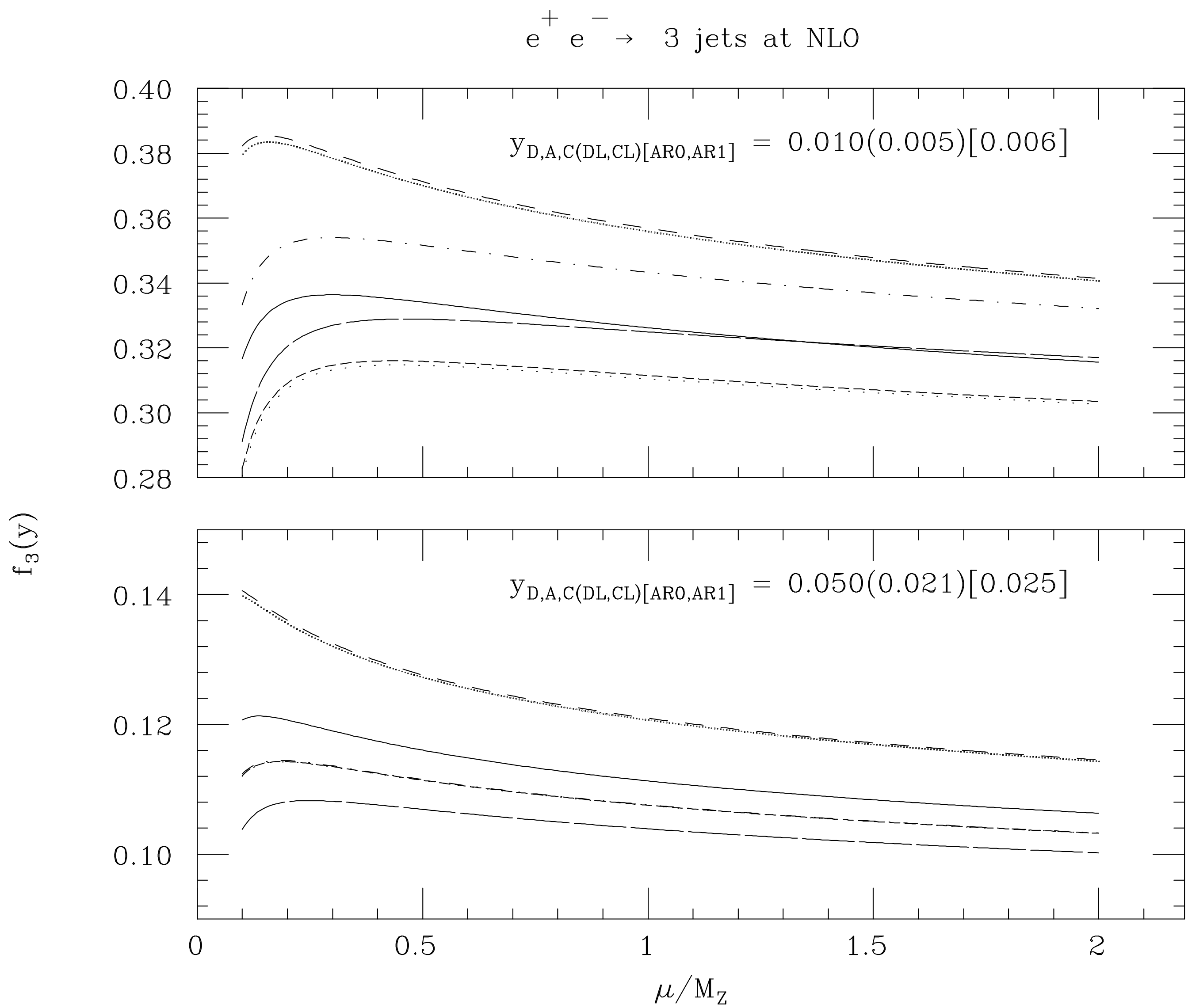


Fig. 5. Comparison of the $\mathcal{O}(\alpha_s^2)$ parton level two, three and four jet fractions (solid lines with statistical error bars) with the hadronic data (points) as given by the OPAL collaboration [3] in the (a) E-scheme, (b) E0-scheme, (c) P-scheme and the (d) P0-scheme.

- The μ-dependence of the three-jet rate is introduced by

$$\alpha_s \to \alpha_s(\mu), \qquad B(y) \to B(y) - A(y)\beta_0 \ln \frac{Q}{\mu}.$$



Event shape variables

- Attempt to find a more global measure of 2/3-jet separation
- E.g. Thrust (T):

$$T = \max_{\hat{\mathbf{n}}} \frac{\sum |\mathbf{p_i} \cdot \hat{\mathbf{n}}|}{\sum |\mathbf{p_i}|}$$

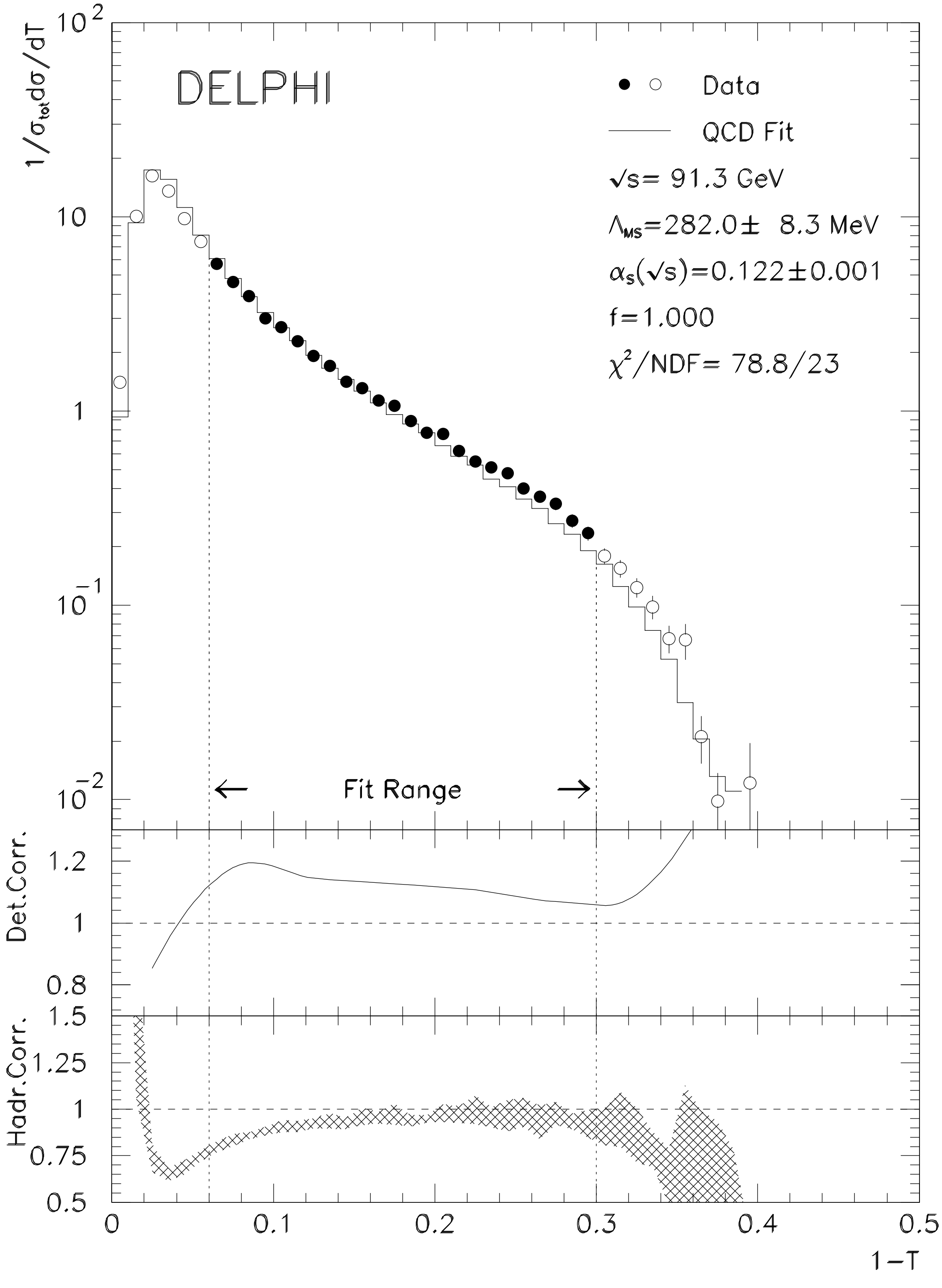
- Through order α_s^2:

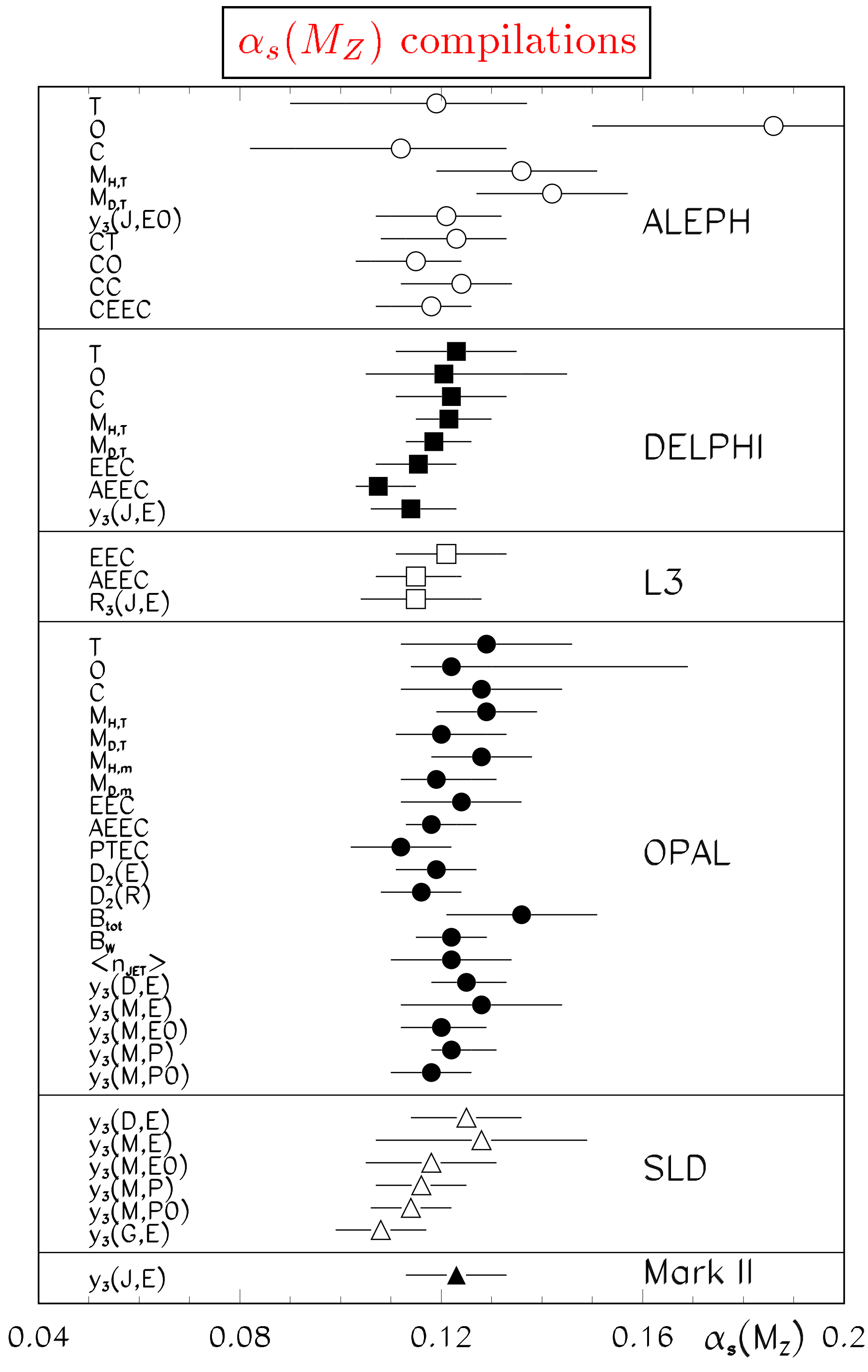
$$\frac{1}{\sigma_0}\frac{d\sigma}{dT} = \frac{\alpha_s(\mu)}{2\pi} A(T) + \left(\frac{\alpha_s(\mu)}{2\pi}\right)^2 \Bigg[\underbrace{2\pi A(T) \beta_0 \log \frac{\mu^2}{s}}_{\text{renormalisation scale dependence}} + B(T) \Bigg]$$

- LO term:

$$\begin{aligned} A(T) &= C_F \left[\frac{2(3T^2 - 3T + 2)}{T(1-T)} \log \frac{2T-1}{1-T} \right. \\ &\quad \left. - \frac{3(3T-2)(2-T)}{1-T} \right] \\ &\stackrel{T \to 1}{\sim} C_F \left[\frac{4}{1-T} \log \frac{1}{1-T} - \frac{3}{1-T} \right]. \end{aligned}$$

- NLO term $B(T)$ computed numerically !

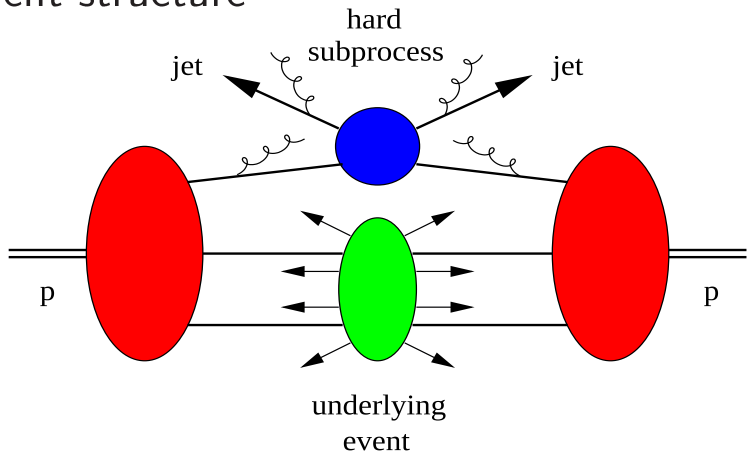

DELPHI
Data
QCD Fit
√s= 91.3 GeV
Λ_MS=282.0± 8.3 MeV
α_s(√s)=0.122±0.001
f=1.000
χ²/NDF= 78.8/23
1/σ_tot dσ/dT
Fit Range
Det.Corr.
Hadr.Corr.
1−T



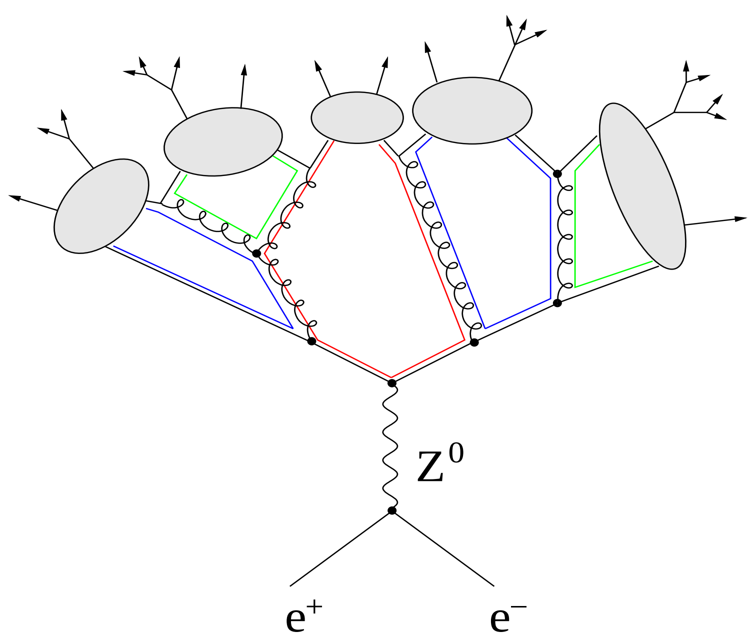
$\mathcal{O}(\alpha_s^2)$!

QCD Event Generators

- Basic event structure



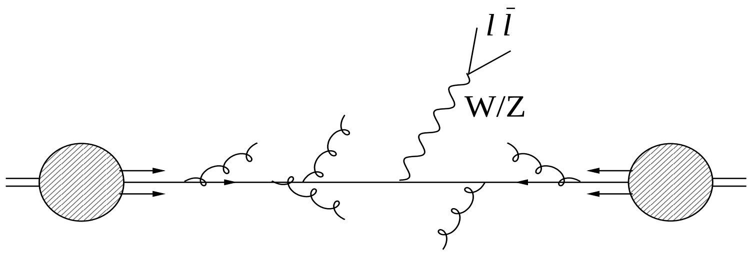
- Parton showers and hadronization



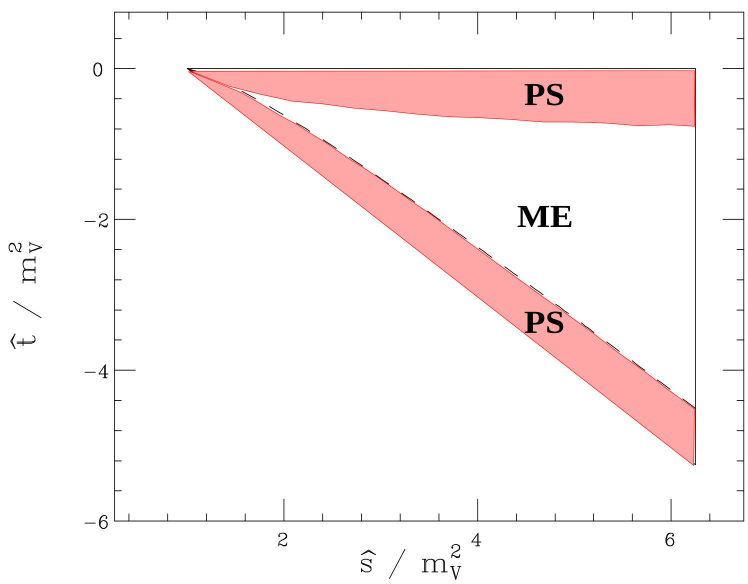
Matrix Element Corrections

- Parton showers inside cones do not populate whole phase space. We also have to include (less singular) matrix element corrections

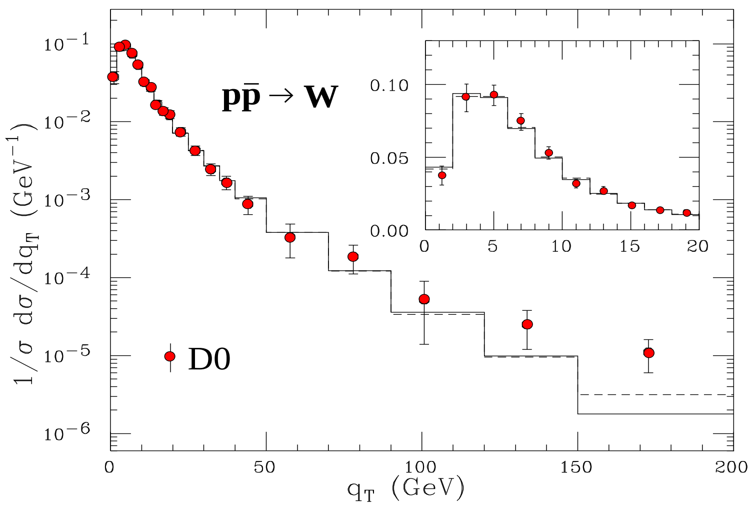
- For example, in W/Z hadroproduction

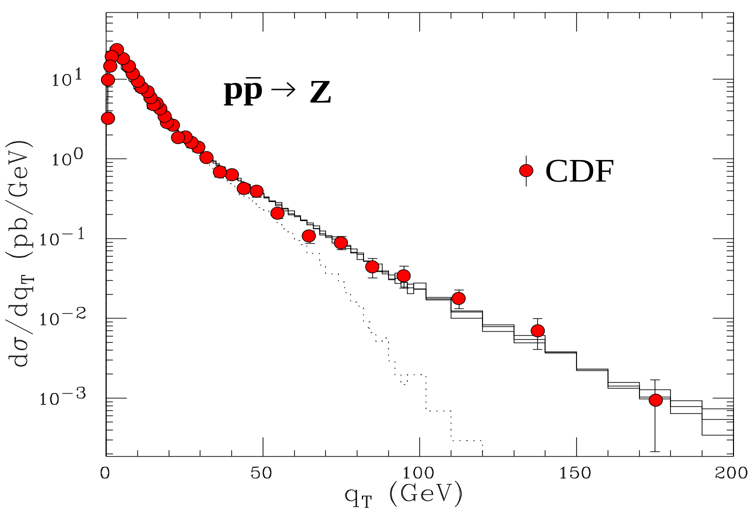


- Phase space for W + jet



- Comparisons with Tevatron data:





Hadronisation

- The formation of hadrons (long distance physics) is not described by perturbative QCD

- Space-time picture:

  - e^+ and e^- form γ (or Z) with virtual mass $Q = \sqrt{s}$, which fluctuates into q and $\bar{q}$.

    – By the uncertainty principle, fluctuation occurs at short distance/timescale $\sim 1/Q$.

    – At large Q, the rate $e^+e^- \to q\bar{q}(g)$ is given by perturbation theory.

  - At much later times $\sim 1/\Lambda$, quarks form hadrons.

    – Hadronisation modifies the outgoing state, but occurs too late to change the original probability for the event to happen.

$\Rightarrow \sigma(e^+e^- \to \text{hadrons}) =$

$$\sigma(e^+e^- \to \text{partons}) \times (1 + \mathcal{O}(\Lambda/Q)^n) \text{ (power corrections)}$$

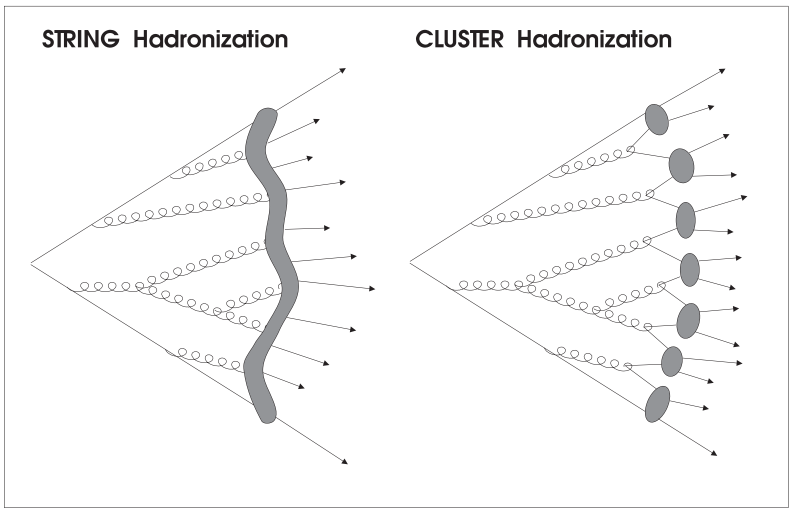
$\Rightarrow$ $\sigma(e^+e^- \to \text{hadrons})$ can be calculated in perturbative QCD for $Q \gg \Lambda$.

$\Rightarrow$ Need Monte Carlo (MC) approach for $Q \leq \Lambda$.

• Quarks and gluons produced in a short-distance process form themselves into hadrons: hadronisation.

• Hadronisation modelled to data in MC programs like

1. HERWIG (cluster hadronisation)
2. PYTHIA, ARIADNE (string hadronisation)



• General approach to hadronisation based on "parton-hadron duality": the flow of momentum and quantum numbers at hadron level follows that established at the partonic stage.

• E.g. flavour of quark initiating a jet found in hadron near the jet axis.

• Approach works because hadronisation is long-distance process which only involves small momentum transfers.

Deeply Inelastic Scattering (DIS)

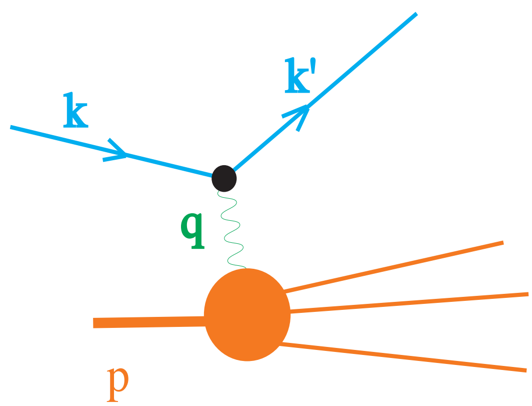
- First test of perturbative QCD was breaking of Bjorken scaling in deeply inelastic lepton hadron-scattering (DIS).

- DIS structure functions provide among most precise tests of QCD & determine Parton Distribution Functions (PDFs) of hadrons:

$\rightarrow$ can be used in predicting hadronic cross sections.

- Kinematics of DIS:

$\rightarrow$ Consider $l(k) + h(p) \rightarrow l'(k') + X$ (via γ, W or Z):



- Standard DIS variables are defined by

$$Q^2 = -q^2; \qquad x = \frac{-q^2}{2p \cdot q} \qquad \text{and} \qquad y = \frac{q \cdot p}{k \cdot p}.$$

- Scattering is called deeply inelastic if $Q^2 \gg \Lambda^2$.

• Structure functions parametrise target as 'seen' by γ, W, Z.

• Consider photon only:

$\rightarrow$ cross section can be written as $d\sigma \propto L^{\mu\nu}(k,q)\, W_{\mu\nu}(p,q)$.

• Structure of lepton tensor is determined by QED:

$$L^{\mu\nu} = \mathrm{Tr}\left(k\cdot\gamma\ \gamma^{\mu} k'\cdot\gamma\ \gamma^{\nu}\right)/2.$$

• Hadronic tensor $W^{\mu\nu}$ contains instead information about photon interaction with hadronic target and cannot be calculated in perturbation theory !

• Symmetry properties give restrictions on $W^{\mu\nu}$ form.

• Define two scalar structure functions, F_1 and F_2, dependent only on (invariants) x and Q^2:

$$\begin{aligned} W_{\mu\nu} &= -\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right) F_1(x,Q^2) \\ &\quad + \left(p_\mu - q_\mu \frac{p\cdot q}{q^2}\right)\left(p_\nu - q_\nu \frac{p\cdot q}{q^2}\right)\frac{1}{p\cdot q}\, F_2(x,Q^2). \end{aligned}$$

• Neglecting hadron mass w.r.t. Q^2, DIS cross section is

$$\frac{d\sigma}{dx\,dy} = \frac{4\pi\alpha^2}{2Q^2}\left[yF_1 + \frac{1-y}{xy}F_2\right].$$

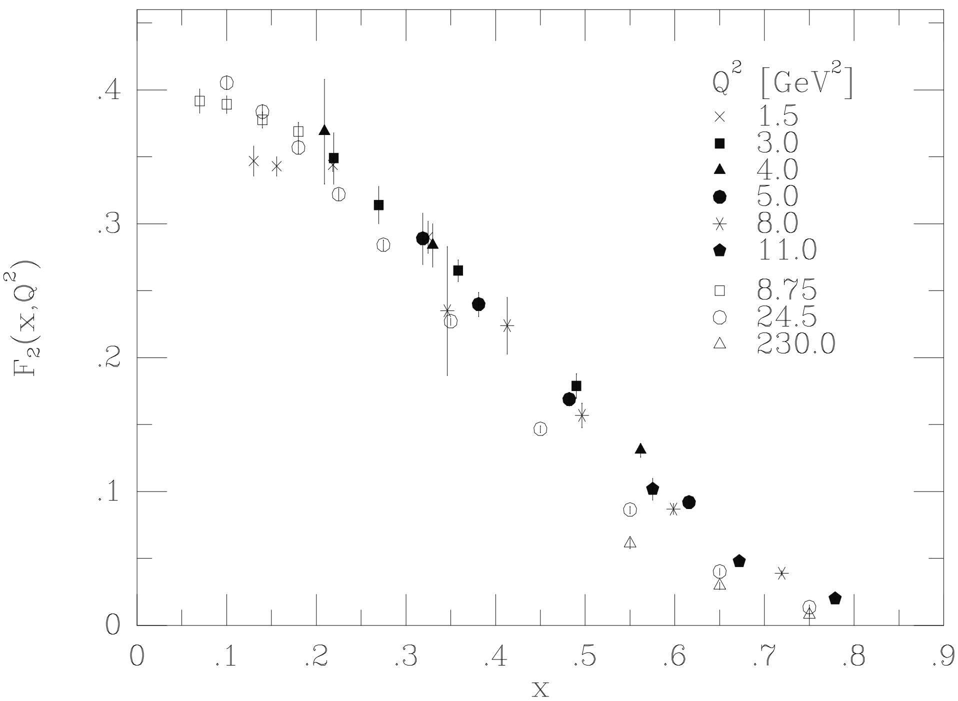
• In principle, can use y dependence to determine structure functions F_i in a DIS experiment.

• (Additional structure function, F_3, needed for W, Z.)

• Bjorken scaling limit defined as $Q^2 \to \infty$ with x fixed:

→ structure functions obey approximate scaling law, i.e.

$$F_i(x, Q^2) \to F_i(x).$$



• Even though the Q^2 values vary by three orders of magnitude, data approximately lie on universal curve.

• Scaling implies γ^*-scattering off point-like constituents.

[Otherwise (dimensionless) structure functions would depend on Q/Q_0, with $1/Q_0$ some length scale characterising size of constituents.]

• Observation of scaling was the motivation for parton model.

• Parton model: p made of point-like constituents $\rightarrow$ partons.

• Their interactions are over time scales of $\mathcal{O}(1/\Lambda)$: longer w.r.t. time it takes e^- to traverse Lorentz contracted proton.

• Can therefore consider partons as (approximately) free particles over the very short interaction time.

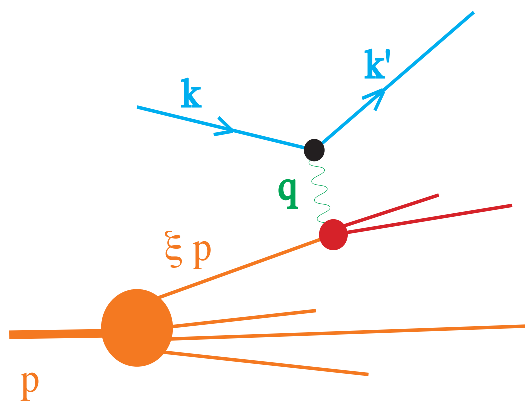
• Model leads to intuitive formula:

$$\frac{d\sigma^{(lh)}}{dx dQ^2} = \sum_a \int_0^1 d\xi \, f_{a/h}(\xi) \, \frac{d\sigma^{(la)}}{dx dQ^2},$$

$d\sigma^{(lh}$ $\rightarrow$ inclusive cross section for lepton-nucleon scattering;

$d\sigma^{(la)}$ *to* parton-electron one;

ξp, $0 < \xi < 1$, $\rightarrow$ parton momentum.

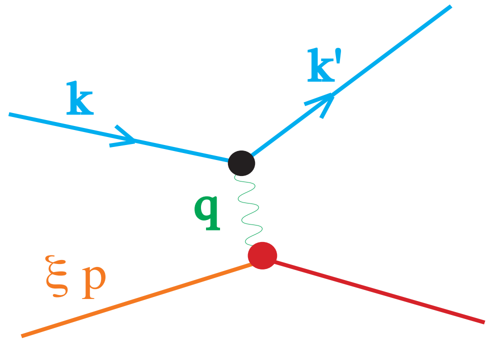


• Function f is called PDF:

$\rightarrow$ $f_{a/h}(\xi)\, d\xi$ gives the probability to find a parton with flavour $a = g, u, \bar{u}, d, \ldots$ in hadron h, carrying momentum fraction within $d\xi$ of ξ.

• PDFs are universal: i.e. independent of particular hard scattering process and can be determined from experiment.

- Hard scattering cross sections from perturbation theory:



- Using QED Feynman rules:

$$\frac{d\sigma}{dQ^2} = \frac{2\pi\alpha^2 e_q^2}{Q^4}\left[1 + (1-y)^2\right].$$

- Mass-shell constraint for outgoing quark

$$(\xi p + q)^2 = q^2 + 2\xi p \cdot q = -2p \cdot q(x - \xi) = 0$$

implies $x = \xi$.

- Write $\int_0^1 dx \delta(x - \xi) = 1$ and obtain

$$\frac{d\sigma}{dx dQ^2} = \frac{4\pi\alpha^2}{Q^4}\left[1 + (1-y)^2\right]\frac{1}{2}e_q^2\delta(x - \xi).$$

- At lowest order, structure functions are given by

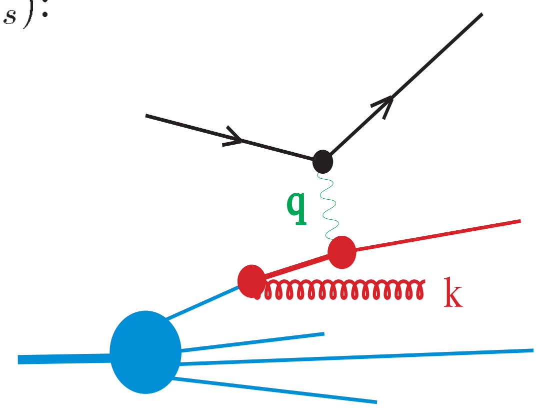
$$F_2(x, Q^2) = \sum_q e_q^2 x f_{/h}(x) = 2xF_1(x, Q^2).$$

→ Callan-Gross relation: from spin of partons !

- Do not confuse structure functions and PDFs !

• In higher order QCD, structure functions F_i are Q^2-dependent and scaling is broken by logarithms of Q^2.

• Through $\mathcal{O}(\alpha_s)$:



• Quark acquires large transverse momentum k_T with probability $\sim \alpha_s dk_T^2/k_T^2$ at large k_T.

• Integral extends up to the kinematic limit $k_T^2 \sim Q^2$ and gives rise to contributions $\propto \alpha_s \ln Q^2$ which break scaling.

• Also, k_T integral logarithmically divergent as $|k_T| \to 0$.

• Introducing k_T cut-off λ:

$$F_2(x,Q^2) = x \sum_q e_q^2 \int_x^1 \frac{d\xi}{\xi} f_{q/h}(\xi) \left[\delta\left(1 - \frac{x}{\xi}\right) \right.$$

$$\left. + \frac{\alpha_s}{\pi} \left\{ P_{qq}\left(\frac{x}{\xi}\right) \ln\left(\frac{Q^2}{\lambda^2}\right) + C\left(\frac{x}{\xi}\right) \right\} \right].$$

• $P_{qq}(\xi) = C_F(1+\xi^2)/(1-\xi)$ called splitting function and C is finite term due to virtual gluon exchange.

• Limit $k_T \to 0$ ($\lambda \to 0$) corresponds to long-range non-perturbative QCD: however,

$\to$ factorisation theorem: can separate from hard scattering.

QCD factorisation theorem

• Perturbative expansion can be rearranged such that contributions from long-range physics appear in PDFs while those short-distance appear in the hard-scattering cross section (Collins, Soper, Sterman).

• Separation requires introduction of factorisation scale μ_F.

• E.g. gluon emission with $k_T^2 \leq \mu_F^2$ is part of $f_{q/h}$ while with $k_T^2 \geq \mu_F^2$ is part of perturbative scattering.

• Through $\mathcal{O}(\alpha_s)$:

$$F_2(x, Q^2) = x \sum_q e_q^2 \int_x^1 \frac{d\xi}{\xi} f_{q/h}(\xi, \mu_F^2) \left[\delta\left(1 - \frac{x}{\xi}\right) + \frac{\alpha_s}{\pi} \left\{ P_{qq}\left(\frac{x}{\xi}\right) \ln\left(\frac{Q^2}{\mu_F^2}\right) + C_{FS}\left(\frac{x}{\xi}\right) \right\} \right]$$

(C_{FS} factorisation-scheme dependent finite correction).

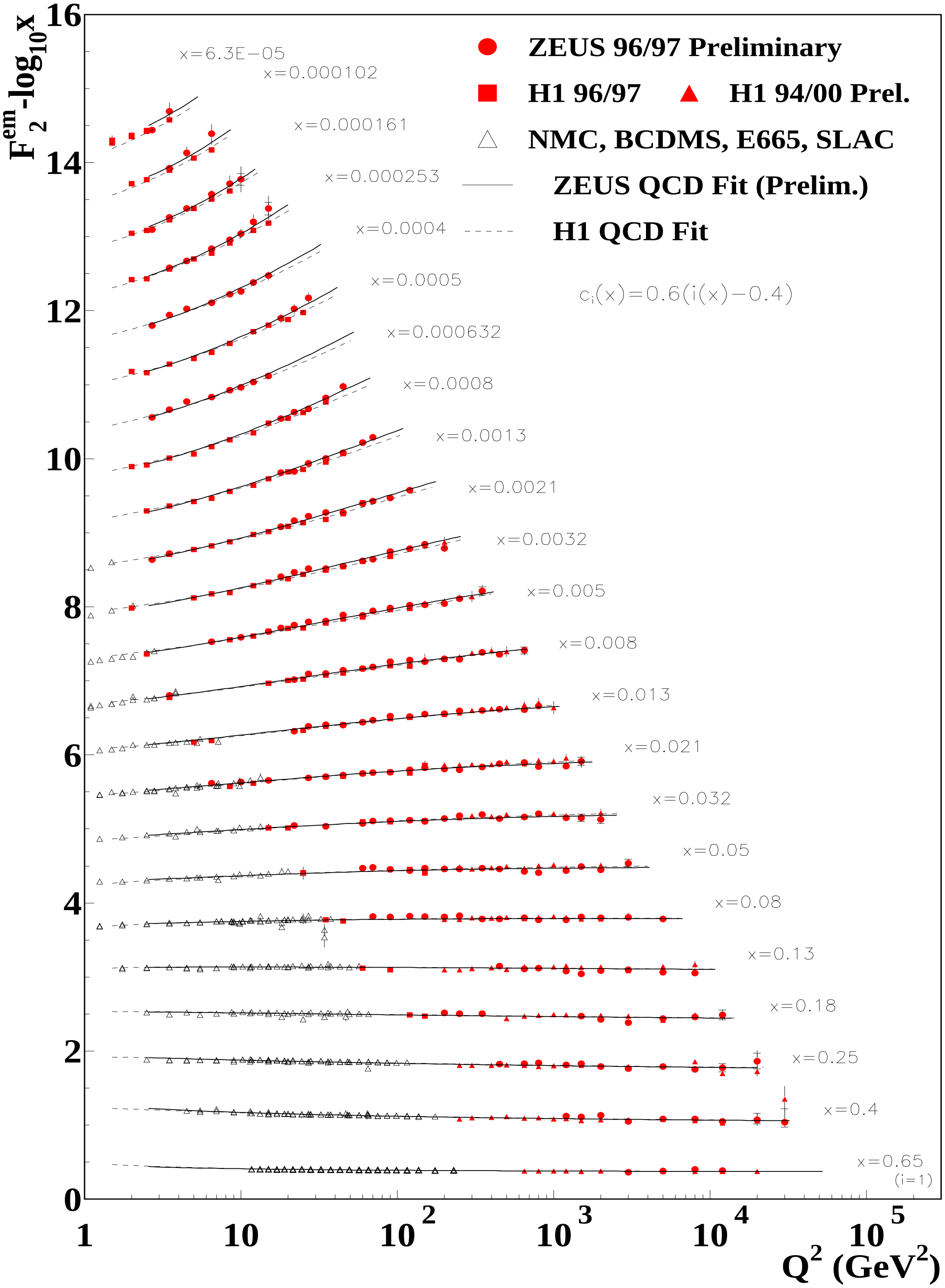
• Arbitrariness in how much of C_{FS} is factored into PDFs defines so-called 'factorisation scheme'.

• While PDFs and hard scattering cross section depend on μ_F, physical cross section does not.

• The more terms included in the perturbative expansion the weaker the dependence on μ_F.

• Factorisation turns QCD into a reliable calculational tool !

• QCD scaling violation observed experimentally:



• PDFs can be defined in terms of quark- and gluon-field operators.

• PDFs appear in QCD formulae for any process with $n \geq 1$ hadrons in initial state.

• PDFs could (in principle) be calculated in lattice QCD, yet determined from experiment.

• Dependence of PDFs on μ_F determined by Renormalisation Group Equation (RGE) [Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) equation]:

$$\frac{d}{d \ln \mu_F} f_{a/h}(x, \mu_F) = \sum_b \int_x^1 \frac{d\xi}{\xi} \, P_{ab}(x/\xi, \alpha_s(\mu_F)) \, f_{b/h}(\xi, \mu_F).$$

• Splitting function P_{ab} has perturbative expansion:

$$P_{ab}(x/\xi, \alpha_s(\mu_F)) = P_{ab}^{(1)}(x/\xi) \, \frac{\alpha_s(\mu_F)}{\pi} + P_{ab}^{(2)}(x/\xi) \left(\frac{\alpha_s(\mu_F)}{\pi}\right)^2 + \cdots$$

• First two terms known and used in numerical solutions.

• DGLAP-equation:

→ enables to relate PDFs measured at one scale to other scales and make corresponding predictions

PDFs

- Constrain PDFs by using many different beams/targets:

Nomenclature/isospin:
$f_{u/n}(x,Q^2) = f_{d/p}(x,Q^2) = f_{\bar{d}/\bar{p}}(x,Q^2) \equiv f_d(x,Q^2)$, etc.

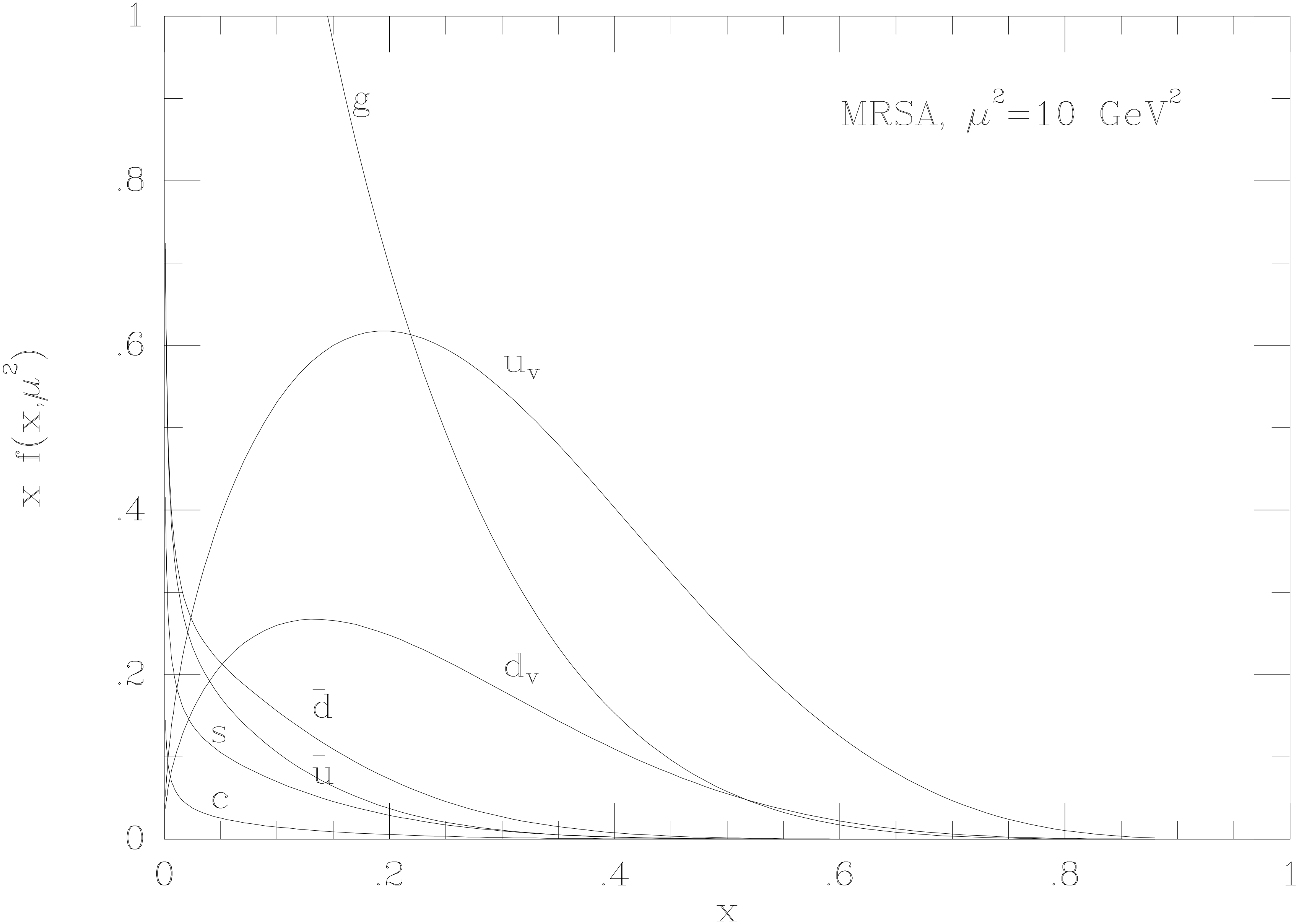
$$
\begin{aligned}
F_2^{ep}(x,Q^2) &= \frac{1}{9}xf_d + \frac{4}{9}xf_u + \frac{1}{9}xf_{\bar{d}} + \frac{4}{9}xf_{\bar{u}} + \frac{1}{9}xf_s + \ldots \\
F_2^{en}(x,Q^2) &= \frac{4}{9}xf_d + \frac{1}{9}xf_u + \frac{4}{9}xf_{\bar{d}} + \frac{1}{9}xf_{\bar{u}} + \frac{1}{9}xf_s + \ldots \\
F_2^{\nu p}(x,Q^2) &= 2xf_d + 2xf_{\bar{u}} + 2xf_s + 2xf_{\bar{c}} + \ldots \\
F_3^{\nu p}(x,Q^2) &= 2xf_d - 2xf_{\bar{u}} + 2xf_s - 2xf_{\bar{c}} + \ldots \\
F_2^{\bar{\nu} p}(x,Q^2) &= 2xf_u + 2xf_{\bar{d}} + 2xf_c + 2xf_{\bar{s}} + \ldots \\
F_3^{\bar{\nu} p}(x,Q^2) &= 2xf_u - 2xf_{\bar{d}} + 2xf_c - 2xf_{\bar{s}} + \ldots
\end{aligned}
$$

etc ...

→ global fits → fig

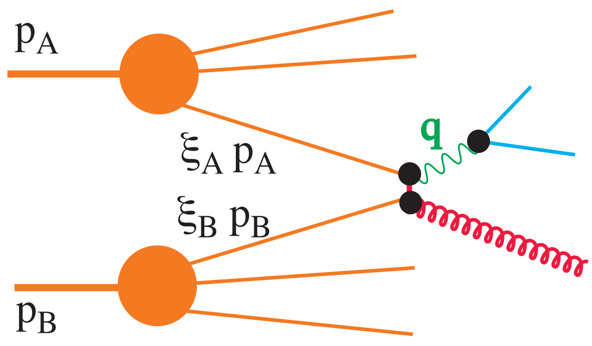
- E.g. MRS(T), CTEQ, GRV, etc.,

→ http://durpdg.dur.ac.uk/hepdata/pdf.html


MRSA, μ^2=10 GeV^2
g
u_v
d_v
$\bar{d}$
s
$\bar{u}$
c
x f(x,μ^2)
x
0
.2
.4
.6
.8
1

Hadron-hadron collisions

- Cross section for e.g. Z boson production



can be factored:

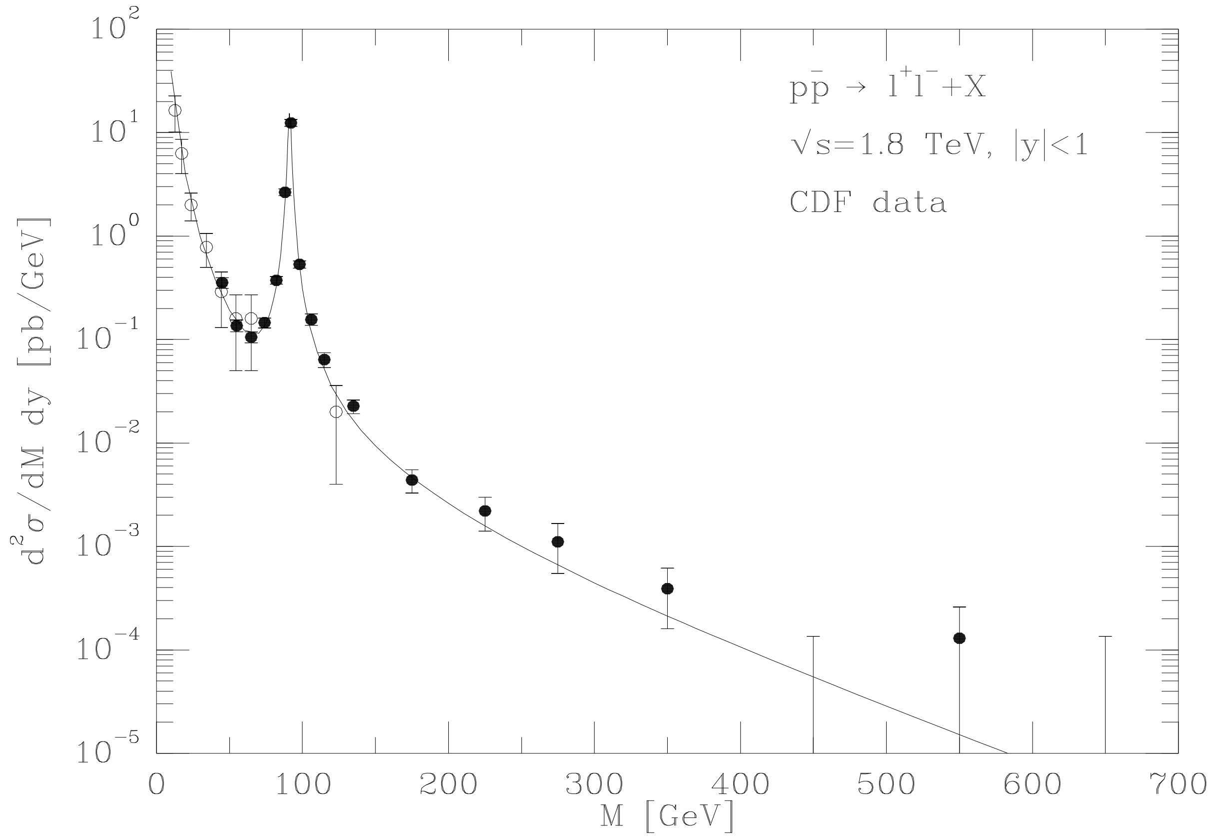
$$d\sigma(p_A, p_B) = \sum_{a,b} \int d\xi_A d\xi_B \; f_{a/A}(\xi_A, \mu_F) \; f_{b/B}(\xi_B, \mu_F)$$
$$\times d\hat{\sigma}_{ab}(\xi_A p_A, \xi_B p_B, \mu_F).$$

- Characteristic scale of hard scattering $Q^2 \gg \Lambda^2$ could be e.g. M_Z or p_Z^T.

- Factorisation formula holds up to Λ^2/Q^2 corrections.

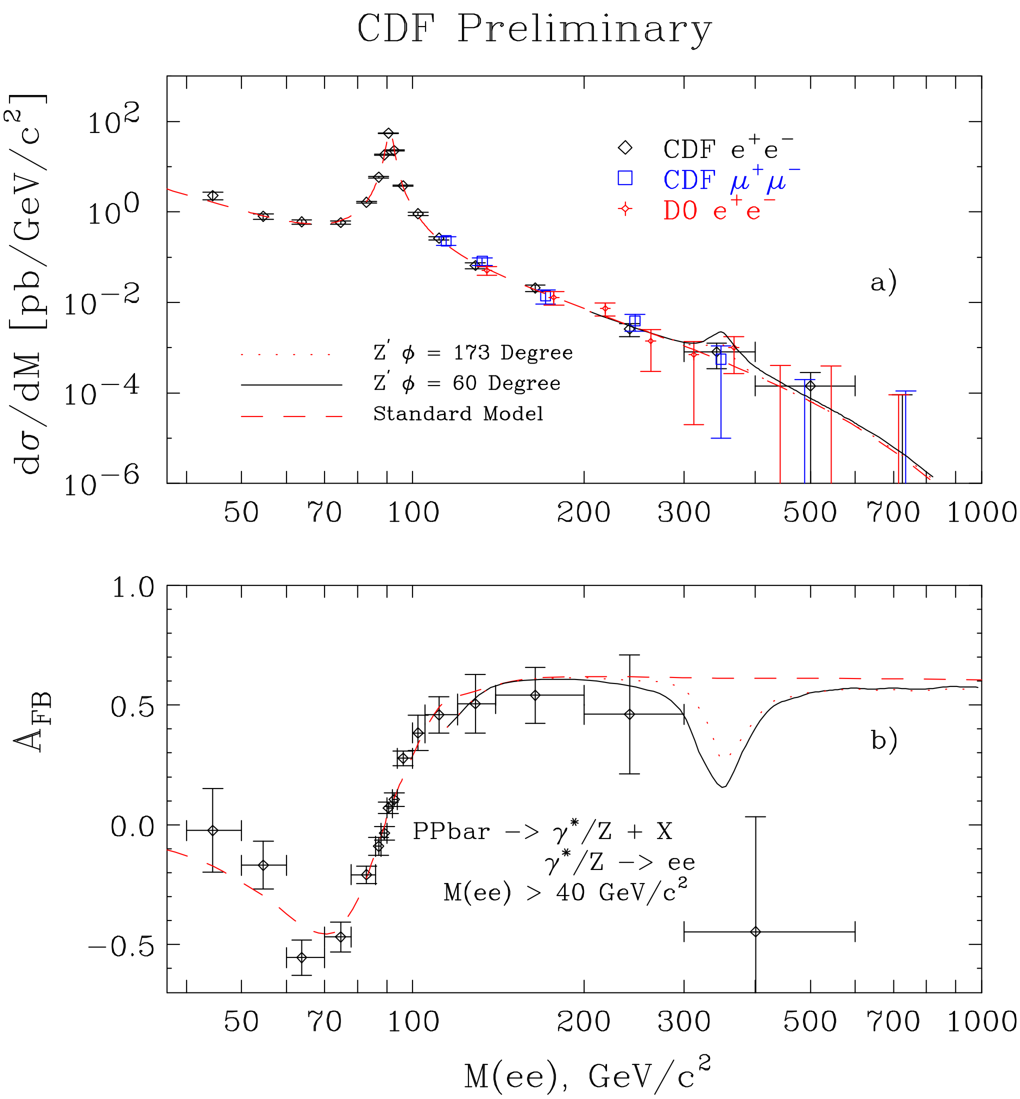
- To prove factorisation one needs to sum over graphs and use unitarity, causality and gauge invariance (Collins, Soper and Sterman).

• Historically, confirmed by lepton-pair hadro-production ($A + B \to l^+ l^- + X$, or 'Drell-Yan' process, DY), using the parton picture and the PDFs from DIS:



(Distribution is lepton pair invariant mass squared.)

NP in DY processes



(a) $d\sigma/dM$ distribution of e^+e^- (CDF and DØ) and $\mu^+\mu^-$ (CDF). SM (dashed) normalized ($\times 1.11$) to CDF data in Z mass region. (b) CDF A_{FB} versus mass compared to SM (dashed). Also shown are theoretical curves ($\times 1.11$) for $d\sigma/dM$ and A_{FB} for extra E_6 boson with $M_{Z'} = 350$ GeV and $?_{Z'} = 0.1\, M_{Z'}$, for $\phi = 60^0$ (solid) and 173^o (dotted).

Top mass

- Measured by Tevatron experiments (CDF & DØ).
- Hadro-production modes:

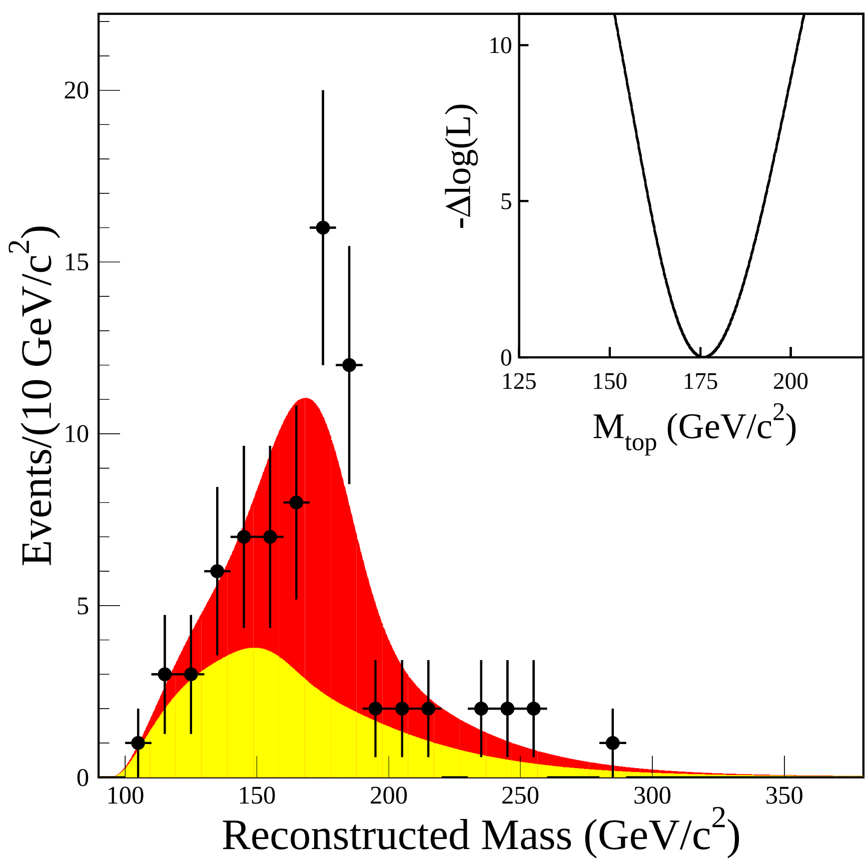
$$gg \to t\bar{t} \text{ (dominates at LHC)},$$

$$q\bar{q} \to t\bar{t} \text{ (dominates at Tevatron)}.$$

- Top decays (e.g. semi-leptonic, j = jet & $\ell = e, \mu$):

$$t\bar{t} \to (bW^+)(\bar{b}W^-) \to (bjj)(\bar{b}\ell\nu_\ell) \; + \; \text{C.C.}$$

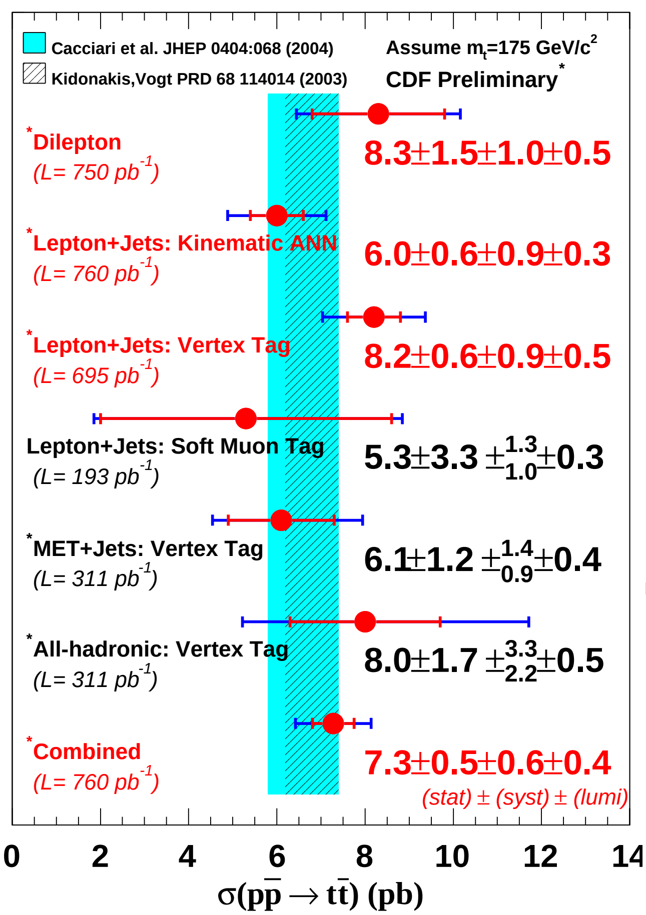
- 6-jet signatures has worse combinatorics !
- Reconstruct m_t from bjj invariant mass (e.g. DØ):

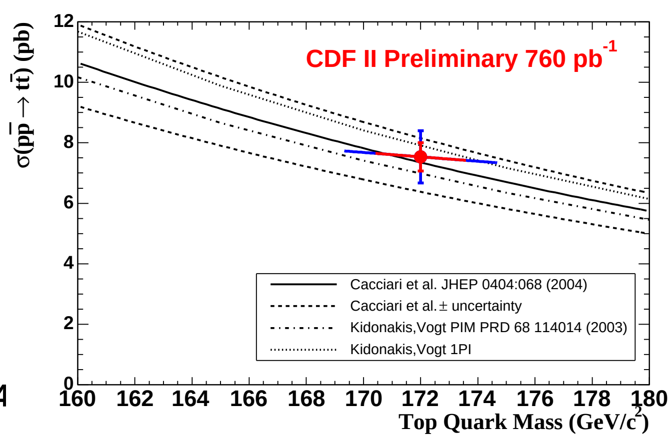


Top pair production cross section

- Measured by both Tevatron experiments (CDF & DØ).
- Achieved consistency with SM prediction (required NLO and resummation)

1. Many different channels
2. Compare to look for NP
3. Currently statistics limited (750 pb^{-1})
4. Assume $m_t = 175$ GeV



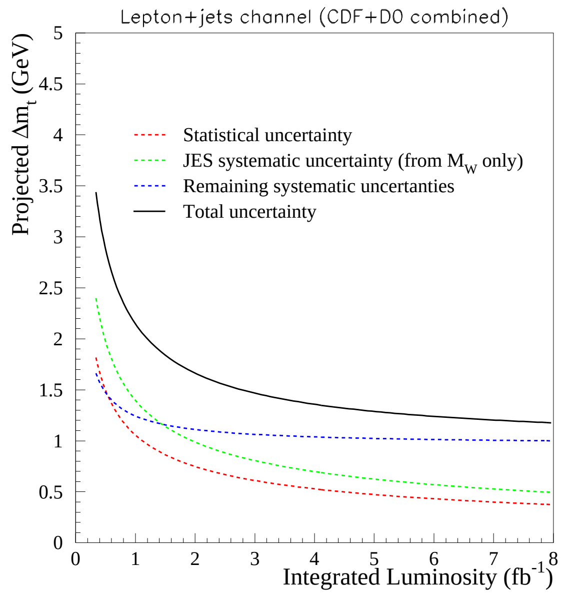


Top mass: Tevatron Summary

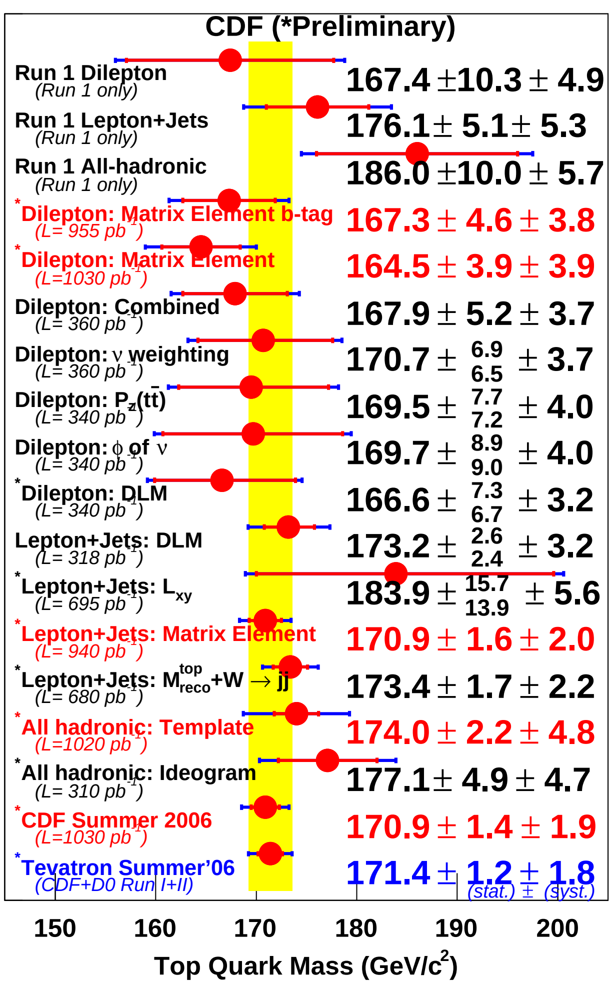
- Run 2 initial goal

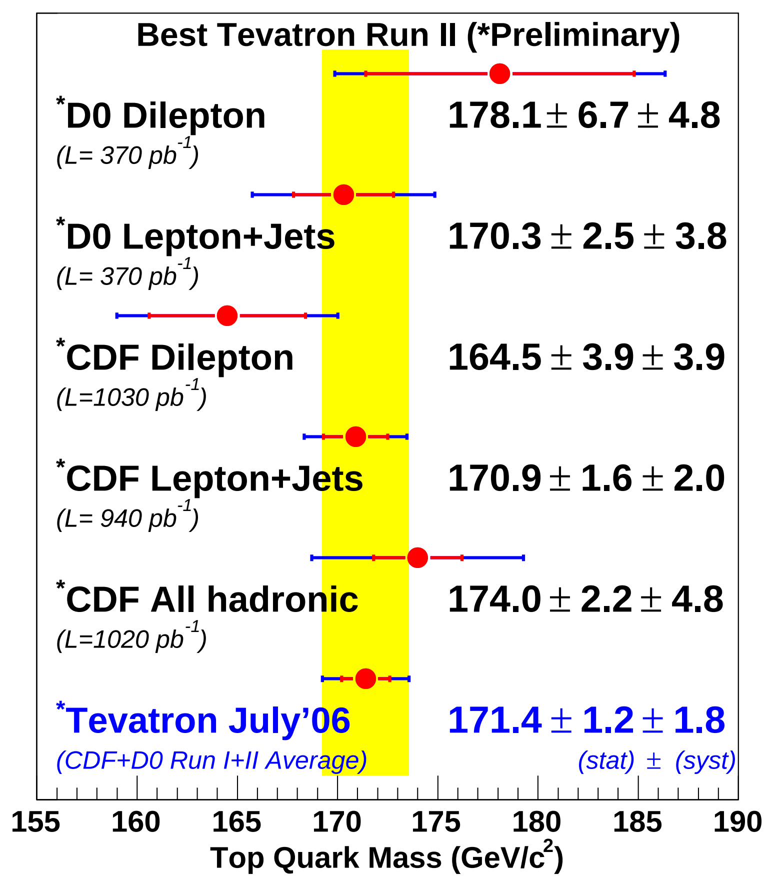
  was $\delta m_t = 2.0$–2.5 GeV

  (per experiment)



- Large systematics from jet energy scale



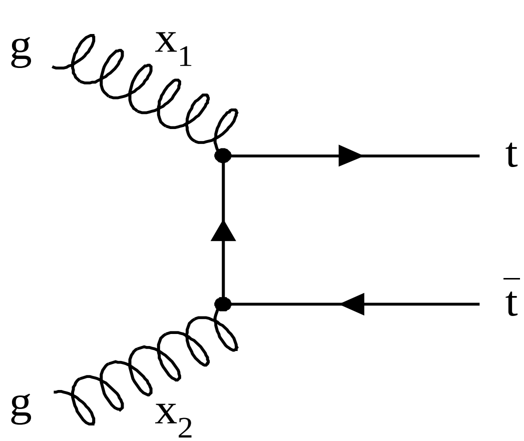


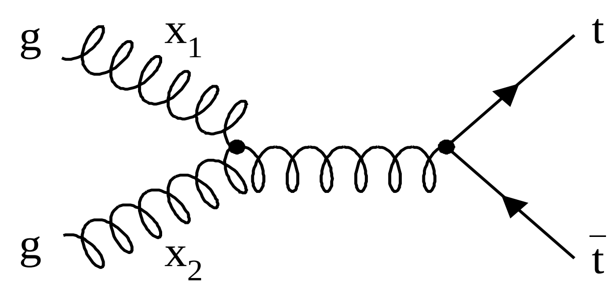
$t\bar{t}$ production at the LHC

$\sqrt{s} = 14\,\text{TeV} \quad \sigma(pp \to t\bar{t}) \simeq 800\,\text{pb} \quad \Rightarrow 8 \cdot 10^6$ events @ $\mathcal{L}_{\text{low}} = 10\,\text{fb}^{-1}$

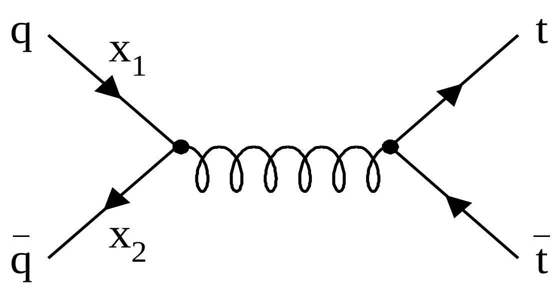
$\Rightarrow 8 \cdot 10^7$ events @ $\mathcal{L}_{\text{high}} = 10^5\,\text{pb}^{-1}$

90 %



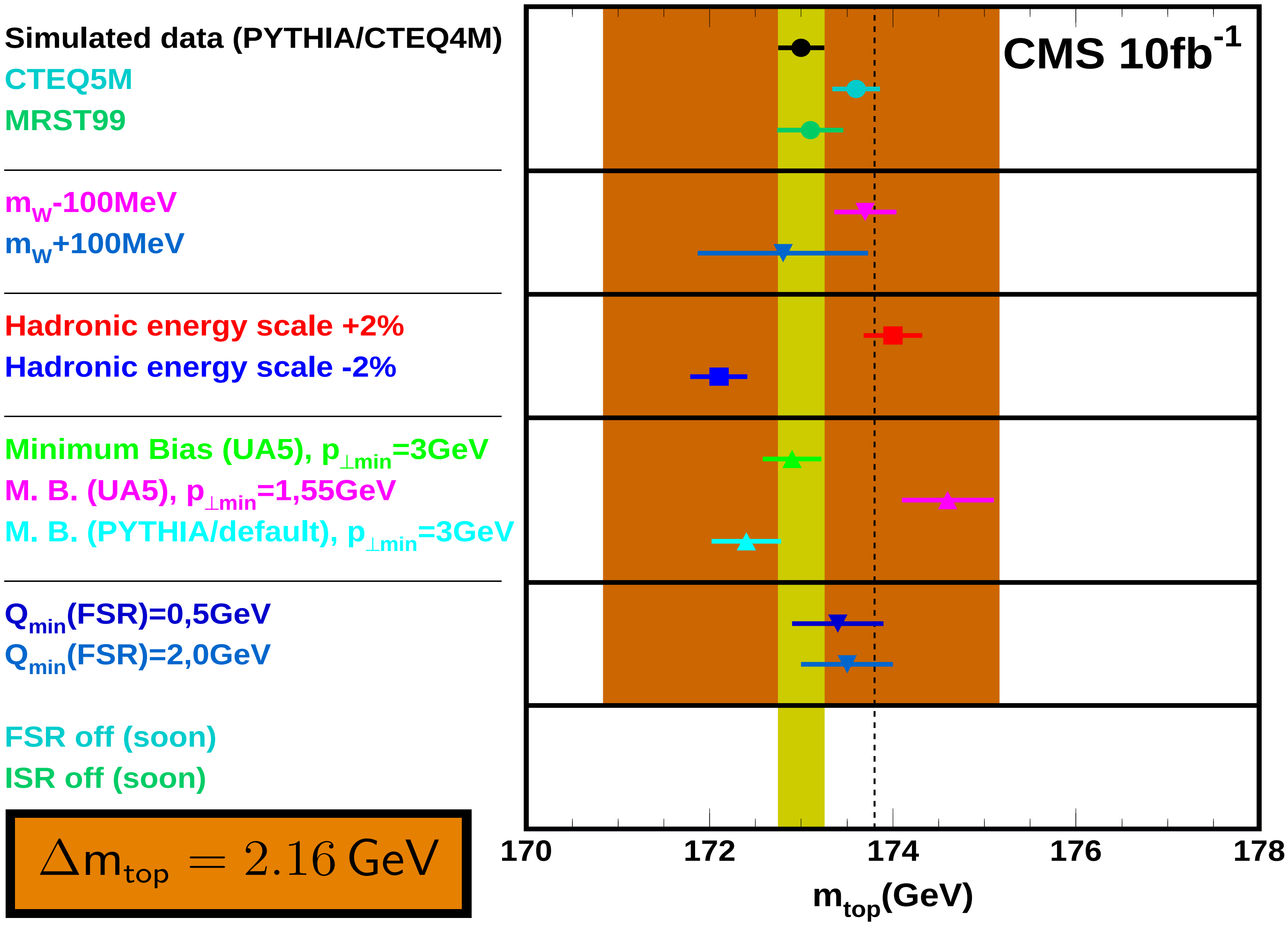


10 %



Systematical uncertainties of the top mass

(only hadronic decay in semileptonic channel)



$\Delta m_{top} = 2.16$ GeV

Crucial cuts:

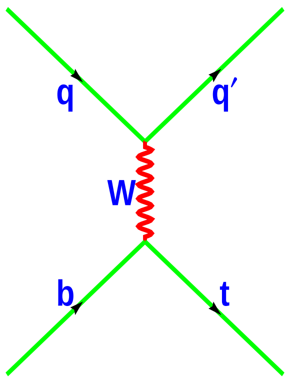
$p_\perp(\ell) > 20$ GeV

$\not{p}_\perp > 20$ GeV

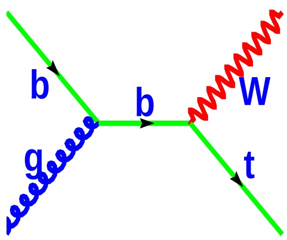
4 jets:
$p_\perp > 40$ GeV
$|\eta_{\text{jet axis}}| < 2.4$

demand 2 b jets

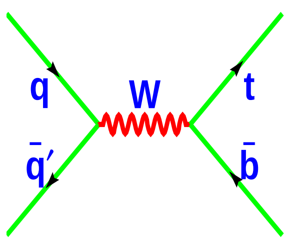
- At LHC it is possible to produce top quarks singly via the weak interaction



Wg
large LHC x-sec ≈ 245 pb
high rate, V_{tb}, polarized tops, etc.



Wt
LHC x-sec ≈ 50 pb
V_{tb}, new theoretical results recently.....



W*
LHC x-sec ≈ 10 pb
low th. errors, V_{tb}

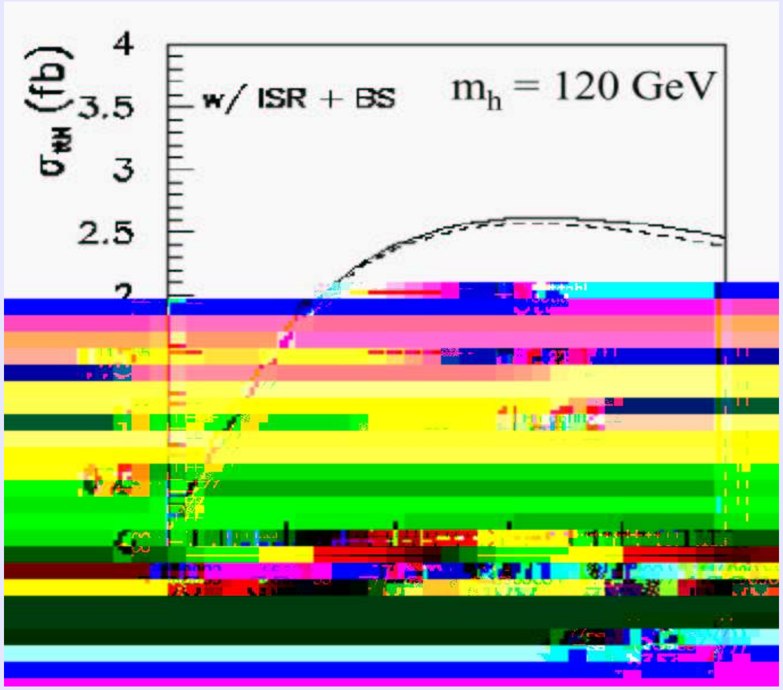
- "New Physics" can affect each rate differently
- Single top provides the best opportunity to study W-t-b vertex:
  - cross-section $\propto |V_{tb}|^2$
  - source of polarized tops (precise prediction)

Threshold Results

- Mass: Δm_t = 16 MeV, $\Delta\alpha_S$ = 0.0011
  - Using cross section only: Δm_t = 24 MeV, $\Delta\alpha_S$ = 0.0017.
  - Γ_t, g_{tth} fixed at SM values; assume m_h=120 GeV, $\alpha_s(M_Z)$=0.120.
  - Theory error: ~100 MeV.
- Width: allow to vary in a 3−parameter fit.
  - $\Delta\Gamma_t$ = 32 MeV, Δm_t = 18 MeV, $\Delta\alpha_S$ = 0.0015
  - 2% exp. uncertainty on width

$t\bar{t}H$ production and the Top Yukawa Coupling

- $e^+e^- \to$ ttH $\to$ WbWb bb
- Very complicted final state:
  - Up to 8 jets
  - 4 b's
  - Many kinematic constraints
- Tiny cross section (~2 fb), with backgrounds ~3 orders of magnitude higher.
- Interfering backgrounds from EWK (ttZ), QCD ($g \to bb$)
- Non−interfering backgrounds
  - Dominantly $e^+e^- \to$ tt
  - Formally smaller number of partons, but can enter the selection due to hard gluon radiation, detector effects, and their very large cross sections



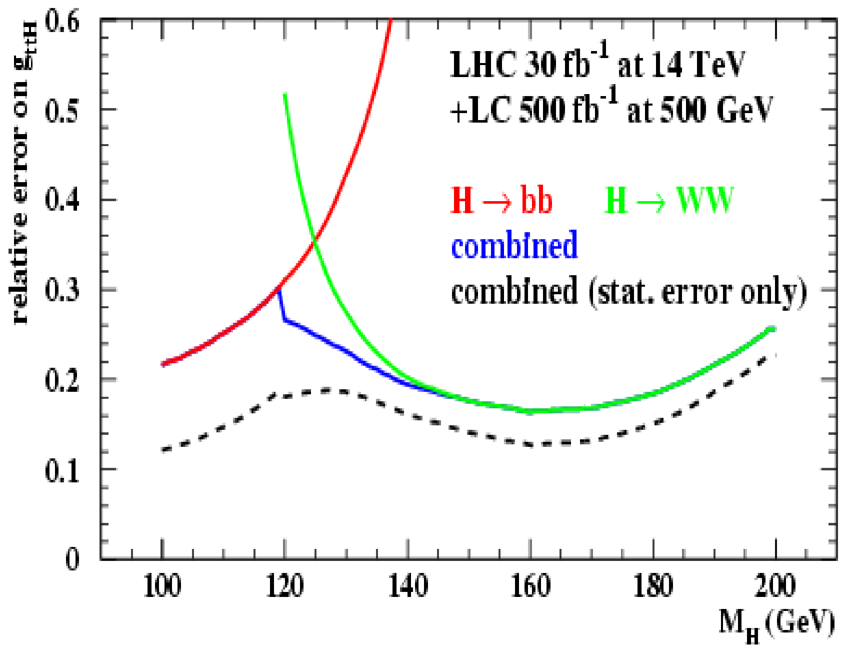
K. Desch
M. Schumacher
hep-ph/0407159

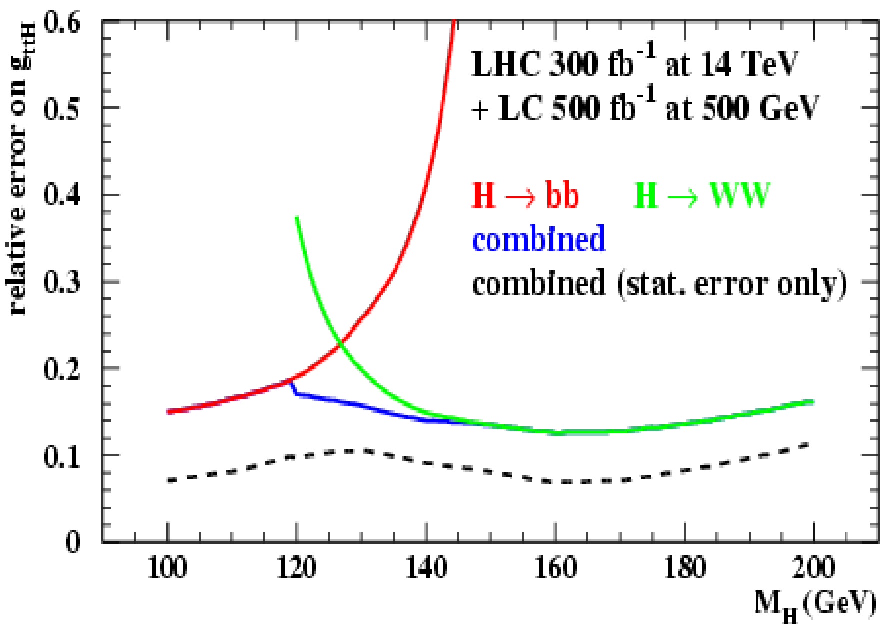
Top Yukawa Coupling

SM prediction is $$g_{ttH} = \frac{\sqrt{2} m_{top}}{246\ GeV} = 1.02 \pm 0.02$$

- **Important to test coupling between Higgs and top quark**
- **Combine LHC and LC for model independent measurement**
  - **LHC: pp →ttH+X – measure σ(ttH)xBR(H→WW) to 20-50%**
  - **ILC: e^+e^-→ZH - measure BR(H→WW) to 2%**
    - **Can do with 500 GeV Linear Collider**

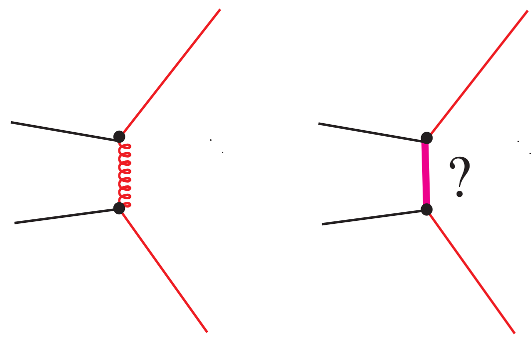
$$\sigma(ttH) \propto g_{ttH}^2$$





New physics searches

- NP can introduce new terms in SM (effective) Lagrangian.
- Imagine quarks scattering by gluon exchange to produce two jets supplemented by quarks exchanging new object with mass $M \sim \mathcal{O}(\text{TeV})$:



- At $\sqrt{s} \ll M$, NP details cannot be resolved.
- Effect can be emulated by new terms in QCD Lagrangian:

$$\Delta\mathcal{L} = \frac{\tilde{g}^2}{M^2}\, \bar{\psi}\gamma^\mu\psi\; \bar{\psi}\gamma_\mu\psi.$$

($\tilde{g}^2 \to$ strength of coupling between q and NP).

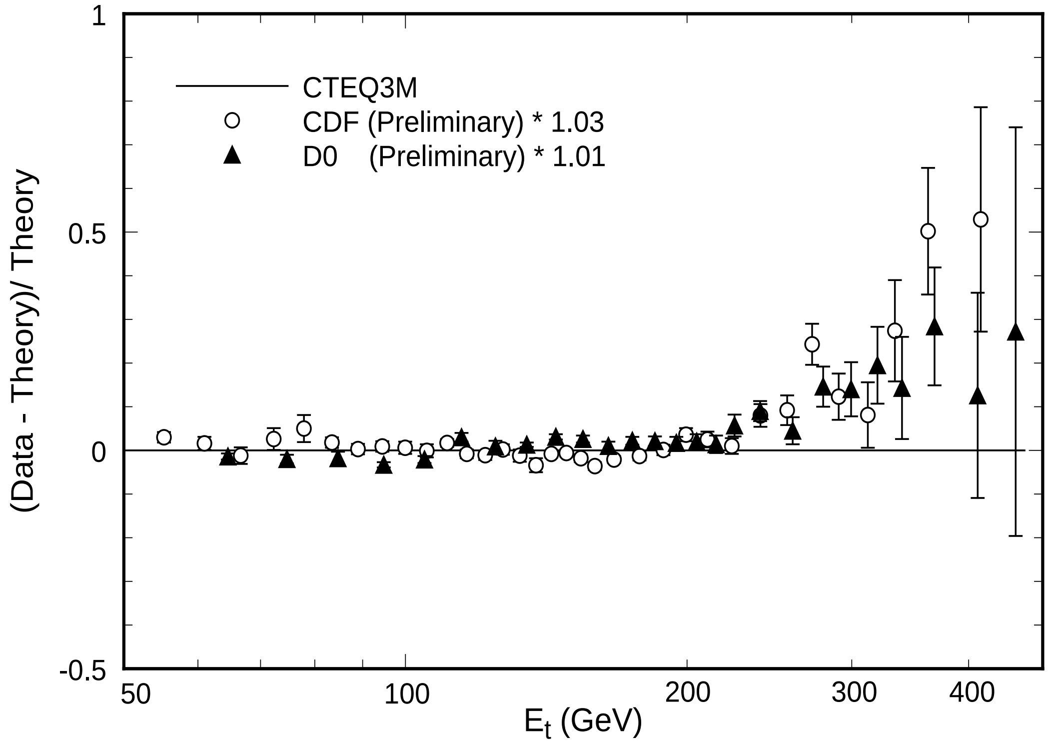
- Factor $1/M^2$ needed for dimensional reasons and implies that effect of NP is small.
- To observe deviation from SM need:

1. high-precision experiment, or
2. experiment looking for some effect forbidden in SM, or
3. an experiment at $\sqrt{s} \approx M$.

- E.g. consider $p\bar{p} \to \text{jet} + X$ as a function of $\approx E_T^{\text{jet}}$ of jet.
- For $E_T^{\text{jet}} \ll M$:

$$\frac{\text{Data} - \text{Theory}}{\text{Theory}} \propto \tilde{g}^2 \frac{E_T^2}{M^2}$$

- Compare experimental jet cross section to NLO QCD:



- Beware: observed effect can most likely be explained by theoretical uncertainty on gluon PDF at large x !

Test of SM: EW Physics

Outline

- Weak interactions from unitarity
- SM renormalisation
- e^+e^- annihilation near Z pole
- W production
- Indirect search for top and Higgs

Weak interactions from unitarity

• Weak interactions discovered in β-decay and described by effective Lagrangian (Fermi theory).

• For $\mu^- \to e^- \bar{\nu}_e \nu_\mu$, Lagrangian is:

$$\mathcal{L} = \frac{G_F}{\sqrt{2}} [\bar{\nu}_\mu \gamma_\lambda (1-\gamma_5)\mu][\bar{e}\gamma^\lambda (1-\gamma_5)\nu_e]$$

with $G_F \approx 1.17 \times 10^{-5}$ GeV^{-2} (Fermi coupling)

• Fermi theory as an effective low-energy theory and cannot be extended to arbitrarily high energies.

• Applying effective Lagrangian at high energies,

$$\mathcal{M}[\bar{\nu}_\mu e^- \to \mu^- \nu_e] \sim \frac{G_F s}{2\sqrt{2}\pi}$$

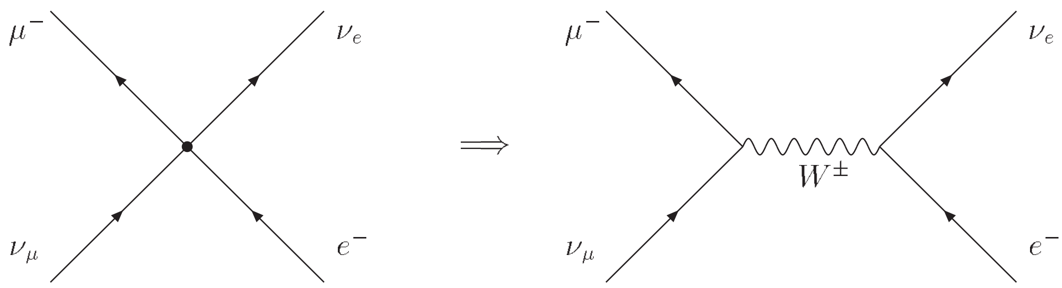
• Scattering amplitude must respect unitarity bound

$$|Re\mathcal{M}| \leq 1/2.$$

$\Rightarrow$ Theory cannot be applied at $s \gtrsim (600 \text{ GeV})^2$.

$\Rightarrow$ Can deduce structure of weak interactions from unitarity constraints (Llewellyn Smith and Cornwall, Levin and Tiktopoulos).

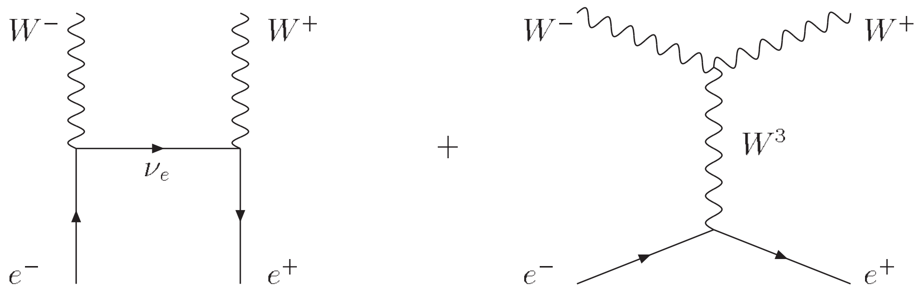
• Unitarity problem in $\bar{\nu}_\mu e^- \to \mu^- \nu_e$ solved by assuming weak interactions mediated by heavy charged vector bosons:



• W propagator dampens rise of scattering amplitudes as $\sqrt{s} \to \infty$ if $M_W \approx 100$ GeV:

$$\mathcal{M}[\bar{\nu}_\mu e^- \to \mu^- \nu_e] \to \frac{G_F s}{2\sqrt{2}\pi} \frac{M_W^2}{M_W^2 - s}$$

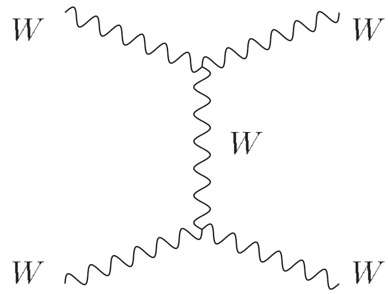
• Consider production of W^+W^- pairs in e^+e^- annihilation.



• Neutrino term grows quadratically and violates unitarity.

• Bad high-energy behaviour cured by exchange of a new neutral vector boson W^3 in s-channel !

• Amplitude for $W_L W_L \to W_L W_L$, as mediated by virtual W exchange and quadrilinear W boson coupling,



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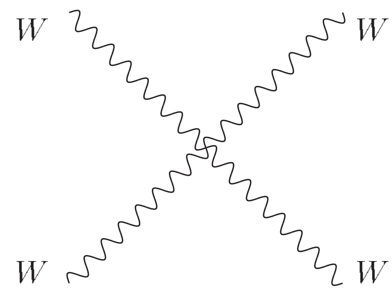
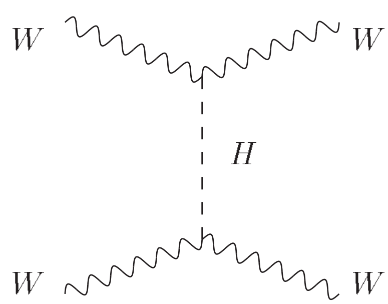


grows quadratically with energy !

• WW scattering amplitude can be damped by new interactions between W bosons at high-energy.

• If theory is to remain weakly interacting up to high energies, a new scalar particle, Higgs boson, must be introduced, which couples to a particle with a strength proportional to particle mass.

• Higgs boson exchange cancels bad high-energy behaviour so that amplitude fulfills unitarity requirement if $M_H \lesssim 1$ TeV.



$$\mathcal{M} \to \frac{G_F M_H^2}{4\sqrt{2}\pi}$$

• (Unitarity requirements can be exploited further to determine quartic W-Higgs interactions and Higgs self-interaction potential.)

SM renormalisation

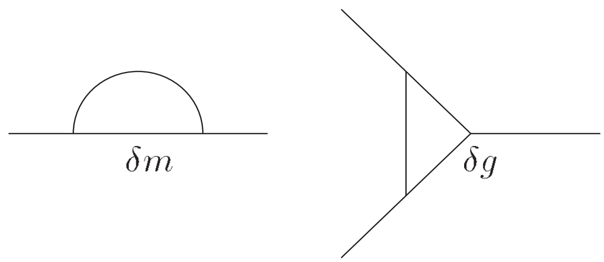
• Structure of EW interactions emerged from requirement of unitarity at high energies.

• Theoretically, SM is a non-Abelian gauge field theory.

• SM observables can be calculated to arbitrarily high precision in a systematic expansion after a few basic parameters are fixed experimentally.

• Quantum corrections in interacting field theories modify particle masses and couplings, i.e. interactions renormalise the fundamental parameters.

• Described by Feynman diagrams including loops



$$\sim \int \frac{d^4 k}{k^4} \sim \ln \Lambda_{\mathrm{cut}}^2$$

• Self-energy and vertex corrections are logarithmically divergent for large loop momenta and lead to contributions $\sim \ln \Lambda_{\mathrm{cut}}^2$ where Λ_{cut} is energy scale up to which SM is valid.

• Quantum corrections add to unobservable bare mass m_0 and bare coupling g_0 to generate the observable physical mass m and coupling g, i.e. $m_0 + \delta m = m$ and $g_0 + \delta g = g$.

• Renormalisation is sufficient to absorb all divergences and render all observables finite if $\Lambda_{\mathrm{cut}} \to \infty$.

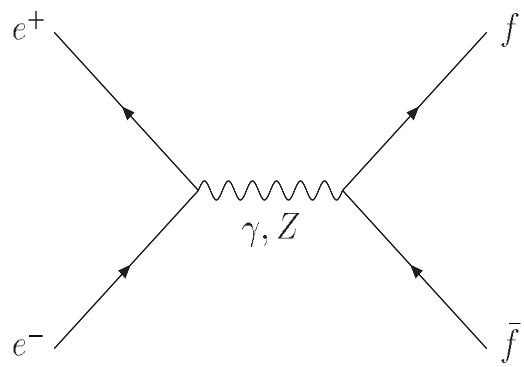
• SM is renormalisable ('t Hooft and Veltman).

• Once masses/couplings are fixed experimentally, all other observables are calculable to arbitrarily high precision.

e^+e^- annihilation near Z pole

• LEP1 and SLC experiments allowed tests of EW theory at quantum level.

• Consider $e^+e^- \to f\bar{f}$ $(f = q, \ell, \nu)$ in SM:



$$\sigma_\gamma(s) = \frac{4\pi\alpha^2}{3s} Q_f^2 N_f \qquad (N_q = N_C,\ N_{\ell,\nu} = 1)$$

$$\sigma_Z(s) = \frac{4\pi\alpha^2}{3s} \frac{s^2}{(s - M_Z^2)^2 + M_Z^2 ?_Z^2} \mathcal{A}_f \mathcal{A}_e N_f$$

with

$$\mathcal{A}_f = v_f^2 + a_f^2 = \frac{(t_{3f} - 2Q_f \sin^2\theta_W)^2 + t_{3f}^2}{4\sin^2\theta_W \cos^2\theta_W}$$

- Include leading logarithmic radiative corrections

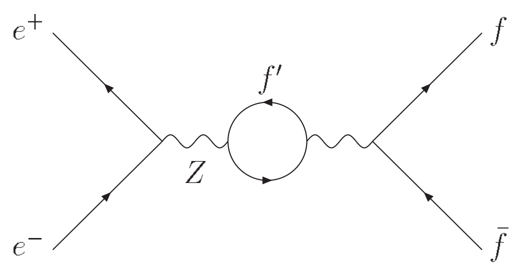
$$\alpha \to \alpha(s) \text{ (improved Born approximation)}$$

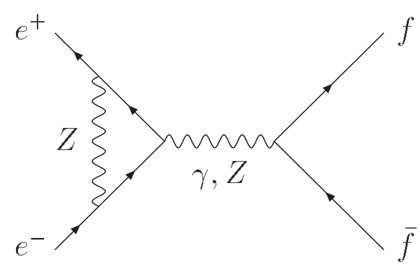
- SM cross section:

$$\sigma(s) = \frac{4\pi\alpha^2(s)}{3s} \frac{s^2}{(s-M_Z^2)^2 + (s^2/M_Z^2)?_Z^2} \left[1 + \underbrace{\Delta_Z}_{\gamma - Z \text{ interference}}\right] + \frac{4\pi\alpha^2(s)}{3s} Q_f^2 N_f$$

(also use running width).

- Most important EW corrections near Z resonance:

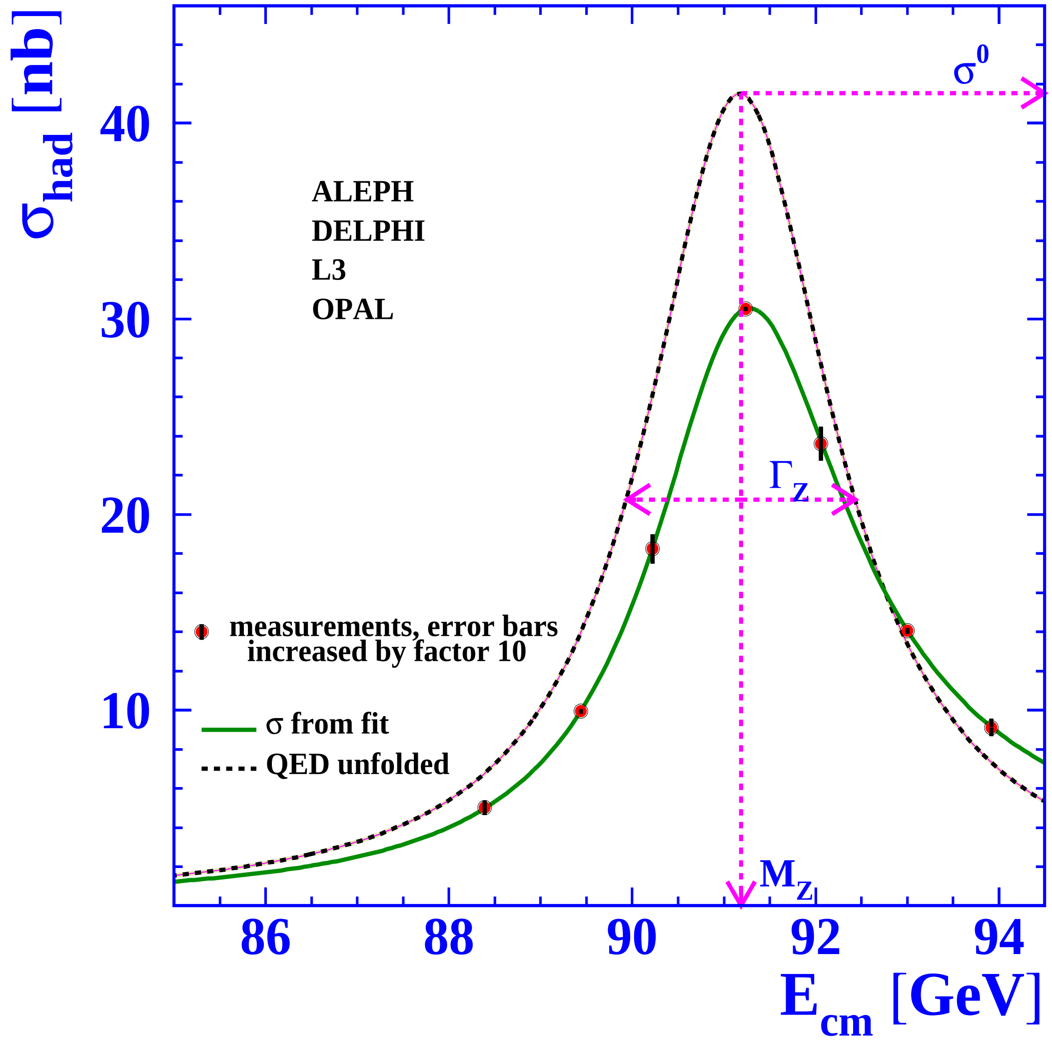




(Lead to ultraviolet divergences which have to be absorbed into renormalised masses and couplings.)

- In addition, QCD corrections have to be included in $q\bar{q}$.

• QED corrections due to photon Initial State Radiation (ISR) are crucial near resonance:



• First non-trivial SM test:

→ given measurements of α, M_Z, G_F and $?_Z$ predicts

$$\sigma_{\rm SM}(M_Z) = \frac{4\pi\alpha^2}{3?_Z^2}\mathcal{A}_f\mathcal{A}_e$$

Total Z Width

- Consider width to given final state fermion:

$$?_f = \frac{1}{3}\alpha \, M_Z \, \mathcal{A}_f$$

- Total width comes all possible final states:

$$?_Z = \sum_f ?_f = \sum_\ell ?_\ell + \sum_\nu ?_\nu + \sum_q ?_q$$

$$\rightarrow ?_Z = 2.4952 \pm 0.0023 \text{ GeV}$$

$\rightarrow$ Gives further non-trivial test of SM !

- Gives measurement of number of neutrino species:

$$N_\nu = 2.993 \pm 0.011$$

- Or limit on width to additional invisible particles:

$$?_{\text{inv}} = 499.0 \pm 1.5 \text{ MeV}$$

Forward-Backward asymmetry

- Line-shape and widths only sensitive to combinations of:

$$\mathcal{A}_f = v_f^2 + a_f^2$$

- $\cos\theta$-dependence also contains

$$\mathcal{B}_f = 2 v_f a_f$$

- Construct forward-backward asymmetry:

$$\begin{aligned} A_{FB} &\equiv \frac{\sigma_{\rm SM}(\theta < 90^\circ) - \sigma_{\rm SM}(\theta > 90^\circ)}{\sigma_{\rm SM}(\theta < 90^\circ) + \sigma_{\rm SM}(\theta > 90^\circ)} \\ &= \frac{3}{4}\,\frac{\mathcal{B}_e \mathcal{B}_f}{\mathcal{A}_e \mathcal{A}_f} \end{aligned}$$

$\rightarrow$ more and complementary tests !

Left-right asymmetry

- SLC had unique feature: highly polarized electrons

$$P_{e^-} \sim 69\%$$

- New asymmetry:

$$A_{LR} \equiv \frac{\sigma_{\rm SM}(e^+e^-_L) - \sigma_{\rm SM}(e^+e^-_R)}{\sigma_{\rm SM}(e^+e^-_L) + \sigma_{\rm SM}(e^+e^-_R)} = -\frac{\mathcal{B}_e}{\mathcal{A}_e}$$

- Note:

1. independent of final state
2. independent of angular range
3. much larger than A_{FB}

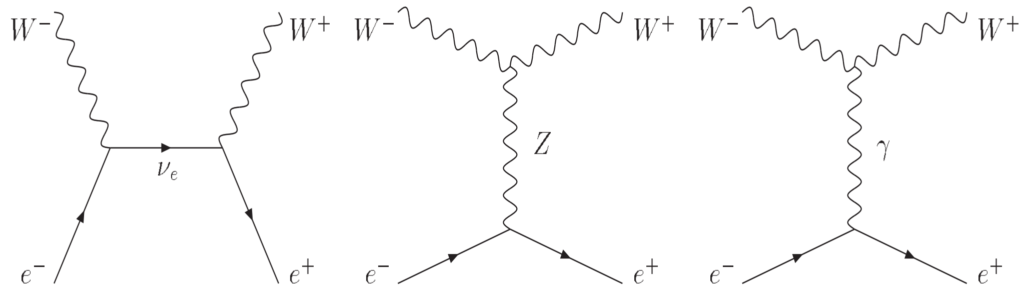
→ almost systematically error-free !

- Just need to measure polarisation well ...

- (SLD: world's best $\sin^2\theta_W$.)

W production

- Consider $e^+e^- \to W^+W^-$ (LEP2):



- Large cancellations at high energies (ditto):

each diagram $\sim \frac{G_F^2 s}{48\pi}$, $s \gg M_W^2$

but sum $\sim \frac{G_F^2 m_w^4}{s\pi} \log \frac{s}{M_W^2}$, $s \gg M_W^2$

- Very sensitive to Triple Gauge Couplings (TGCs)

$\to$ other very powerful SM test

W mass in e^+e^-

- Predicted by SM once α, G_F and M_Z measured

$\Rightarrow$ another strong SM test (symmetry breaking mechanism)

- Near threshold:

$$\sigma_{WW} \sim \frac{G_F^2 M_W^2}{2\pi} \underbrace{\sqrt{1 - \frac{4M_W^2}{s}}}_{\substack{\text{velocity of } W \\ \text{rapidly varying for } \sqrt{s} \sim 2M_W}}$$

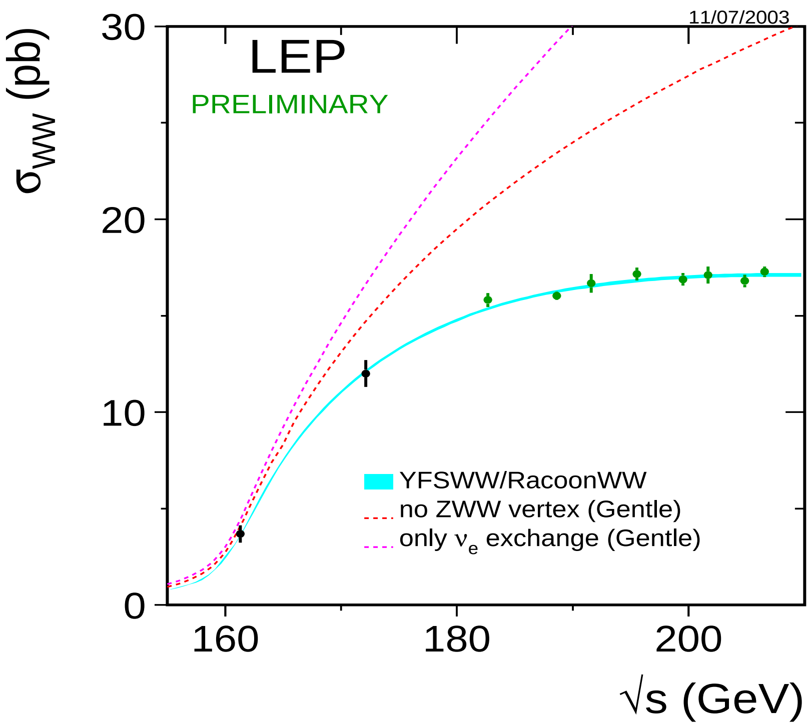
Very clean theoretically, but few events
$\Rightarrow$ large statistical errors $\rightarrow$ fig

- Above threshold:

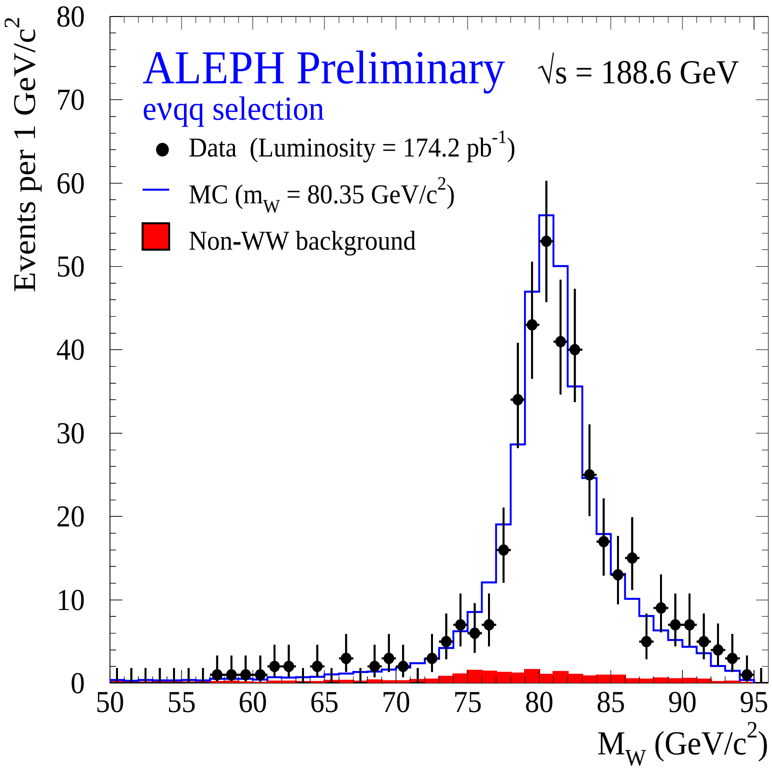
$\rightarrow$ Measure invariant mass of W decay products $\rightarrow$ fig

- LEP average:

$$M_W = 80.412 \pm 0.042 \text{ GeV}$$



11/07/2003
LEP
PRELIMINARY
σ_WW (pb)
√s (GeV)
YFSWW/RacoonWW
no ZWW vertex (Gentle)
only ν_e exchange (Gentle)
0
10
20
30
160
180
200



ALEPH Preliminary
√s = 188.6 GeV
eνqq selection
Data (Luminosity = 174.2 pb^-1)
MC (m_W = 80.35 GeV/c^2)
Non-WW background
Events per 1 GeV/c^2
M_W (GeV/c^2)
0
10
20
30
40
50
60
70
80
50
55
60
65
70
75
80
85
90
95

W mass in hadron-hadron

- W boson mass measured at hadron colliders (Tevatron).
- $W^{\pm} \to \ell\bar{\nu}_\ell/\bar{\ell}\nu_\ell$ decays provide small but clean sample.
- Neutrino lost $\Rightarrow p^\nu$ reconstructed from rest of event.
- Many hadrons lost in beam directions.

$\Rightarrow$ only transverse momentum conservation can be used

- Use:

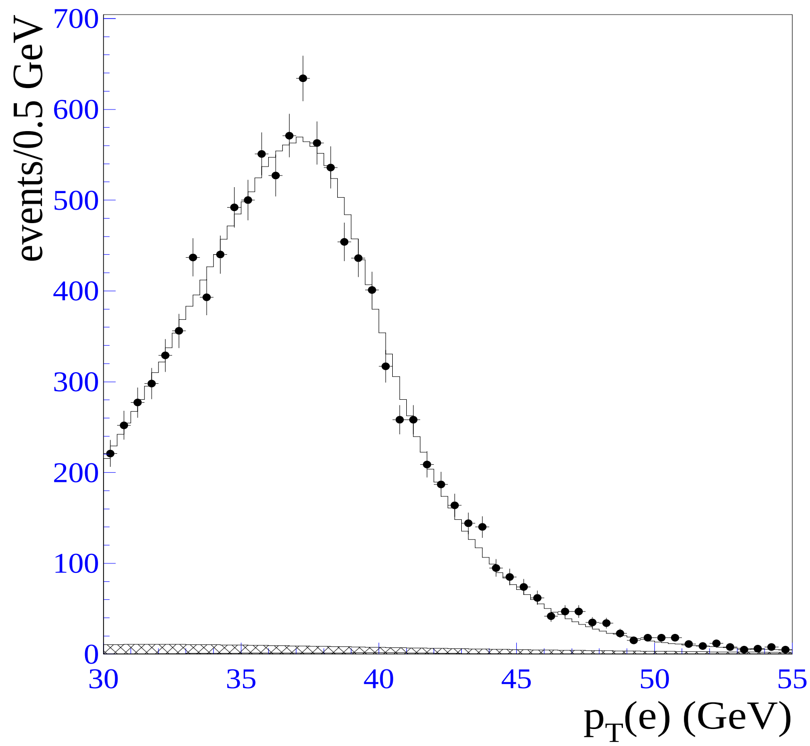
  1. lepton transverse momentum: $p_T(\ell)$ → fig
  2. transverse mass: $M_T^2 \equiv 2p_T^e p_T^\nu(1-\cos\phi)$ → fig

- (Insensitive to W transverse momentum !)
- Tevatron average:

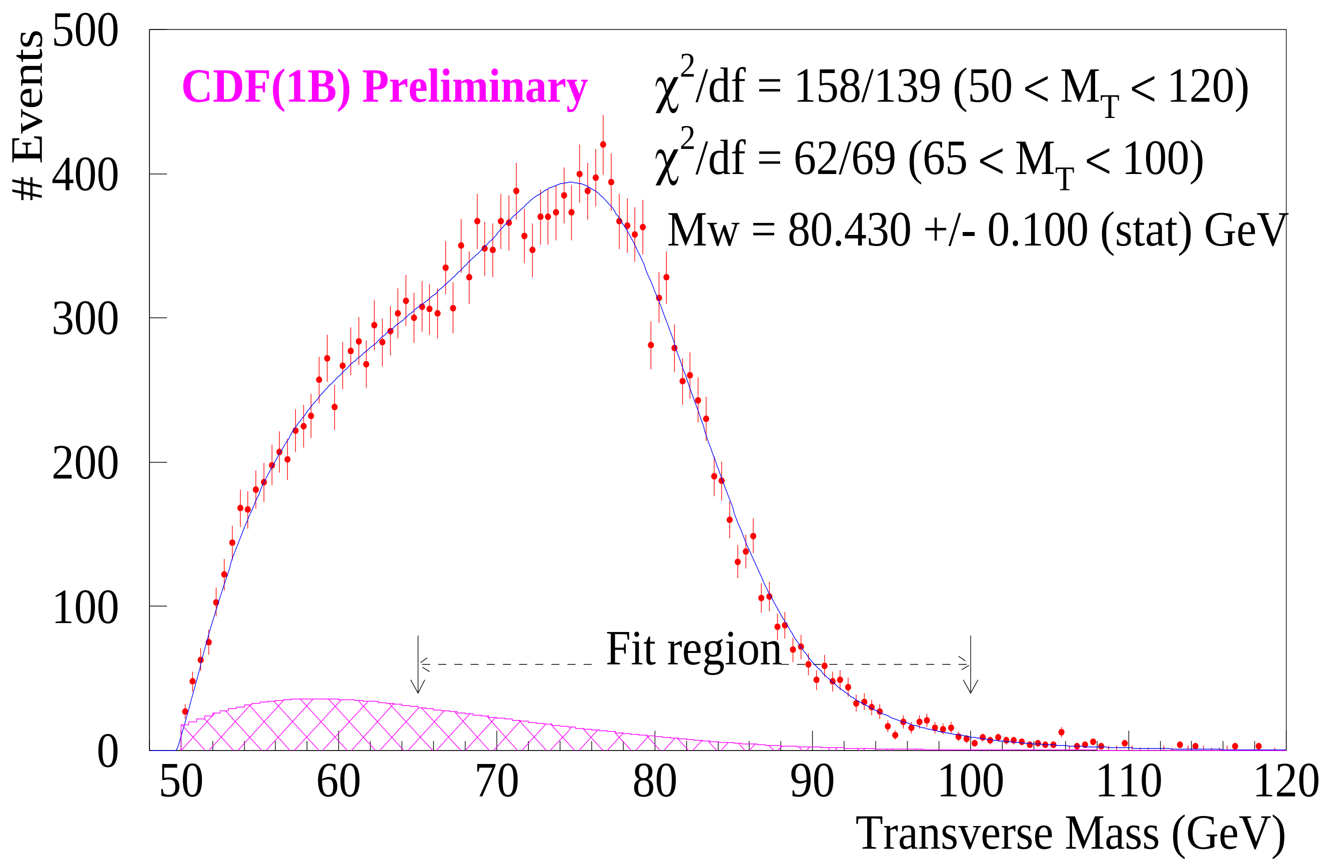
$$M_W = 80.452 \pm 0.059 \text{ GeV}$$

- World average:

$$M_W = 80.425 \pm 0.034 \text{ GeV}$$



events/0.5 GeV
700
600
500
400
300
200
100
0
30
35
40
45
50
55
p_T(e) (GeV)

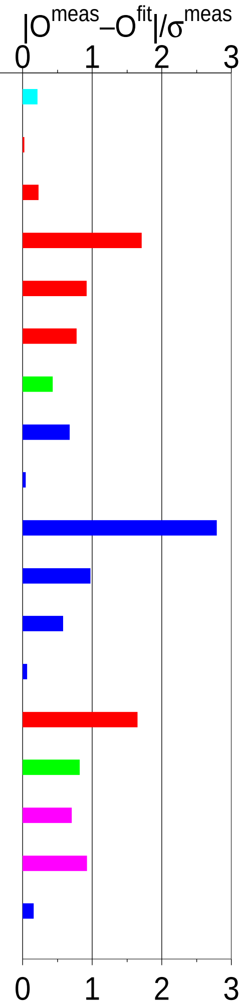


Events
CDF(1B) Preliminary
χ^2/df = 158/139 (50 < M_T < 120)
χ^2/df = 62/69 (65 < M_T < 100)
Mw = 80.430 +/- 0.100 (stat) GeV
Fit region
500
400
300
200
100
0
50
60
70
80
90
100
110
120
Transverse Mass (GeV)

Precision observables

Summer 2006

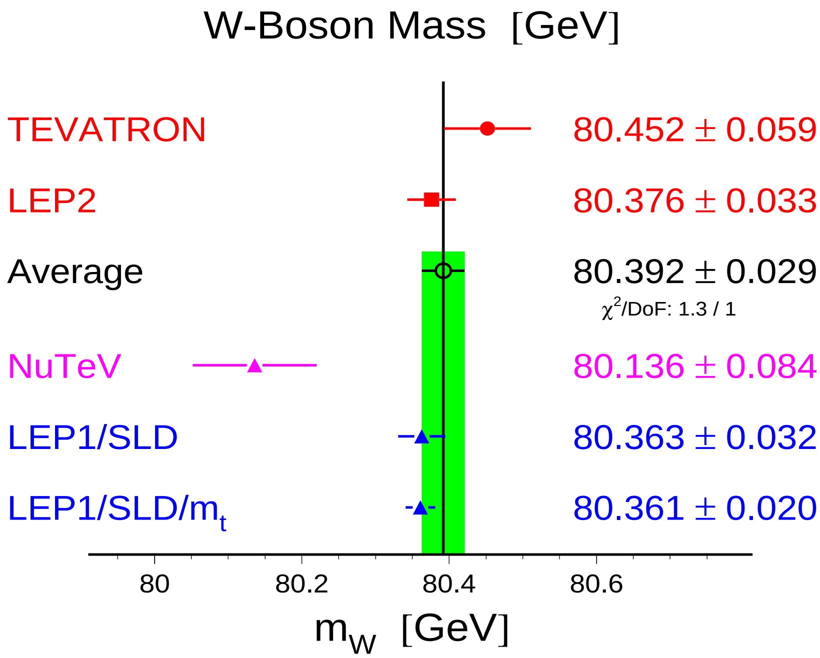
	Measurement	Fit
$\Delta\alpha_{had}^{(5)}(m_Z)$	0.02758 ± 0.00035	0.02766
m_Z [GeV]	91.1875 ± 0.0021	91.1874
Γ_Z [GeV]	2.4952 ± 0.0023	2.4957
σ_{had}^{0} [nb]	41.540 ± 0.037	41.477
R_l	20.767 ± 0.025	20.744
$A_{fb}^{0,l}$	0.01714 ± 0.00095	0.01640
$A_l(P_\tau)$	0.1465 ± 0.0032	0.1479
R_b	0.21629 ± 0.00066	0.21585
R_c	0.1721 ± 0.0030	0.1722
$A_{fb}^{0,b}$	0.0992 ± 0.0016	0.1037
$A_{fb}^{0,c}$	0.0707 ± 0.0035	0.0741
A_b	0.923 ± 0.020	0.935
A_c	0.670 ± 0.027	0.668
$A_l(SLD)$	0.1513 ± 0.0021	0.1479
$\sin^2\theta_{eff}^{lept}(Q_{fb})$	0.2324 ± 0.0012	0.2314
m_W [GeV]	80.392 ± 0.029	80.371
Γ_W [GeV]	2.147 ± 0.060	2.091
m_t [GeV]	171.4 ± 2.1	171.7



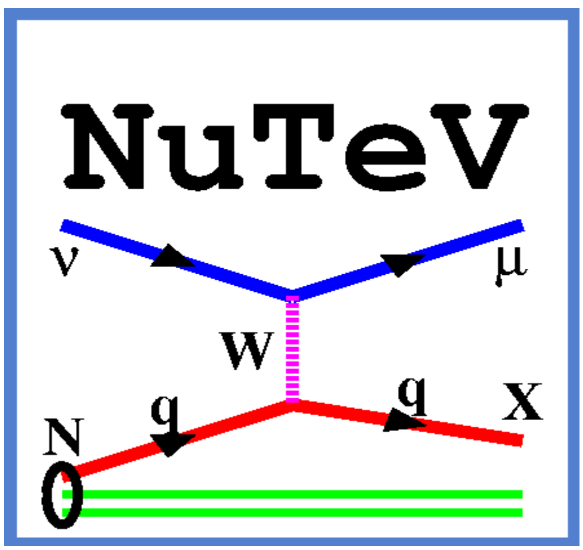
- (Pull is defined as deviation from theoretical prediction in units of corresponding one-standard deviation experimental uncertainty.)
- Includes latest top mass !

M_W (and NuTeV anomaly ?)

• Direct vs. indirect M_W determinations:



• NuTeV:



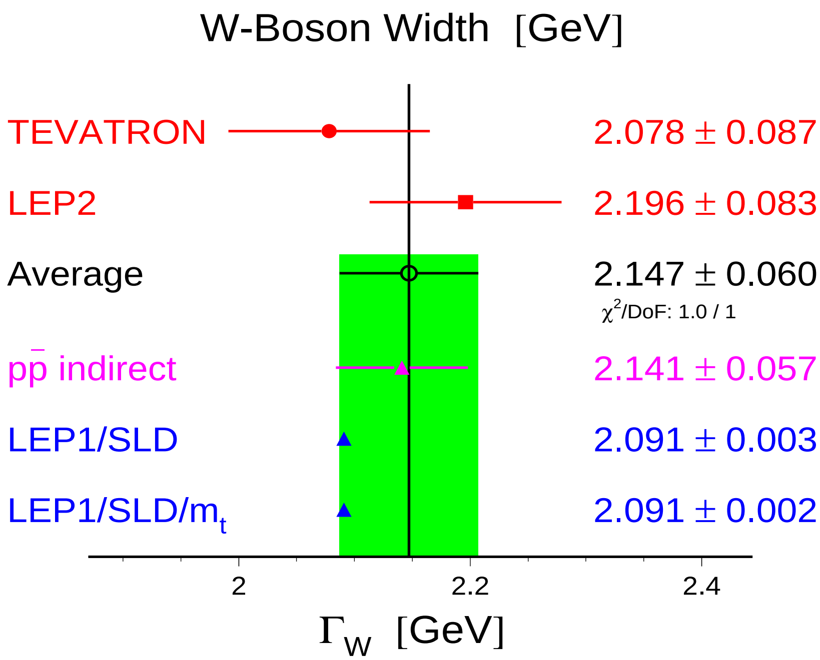
→ ratio of neutral to charged currents in neutrino-nucleon

• Measurement from NuTeV collaboration (when interpreted as a measurement of M_W) shows 2.6–2.8σ deviation.

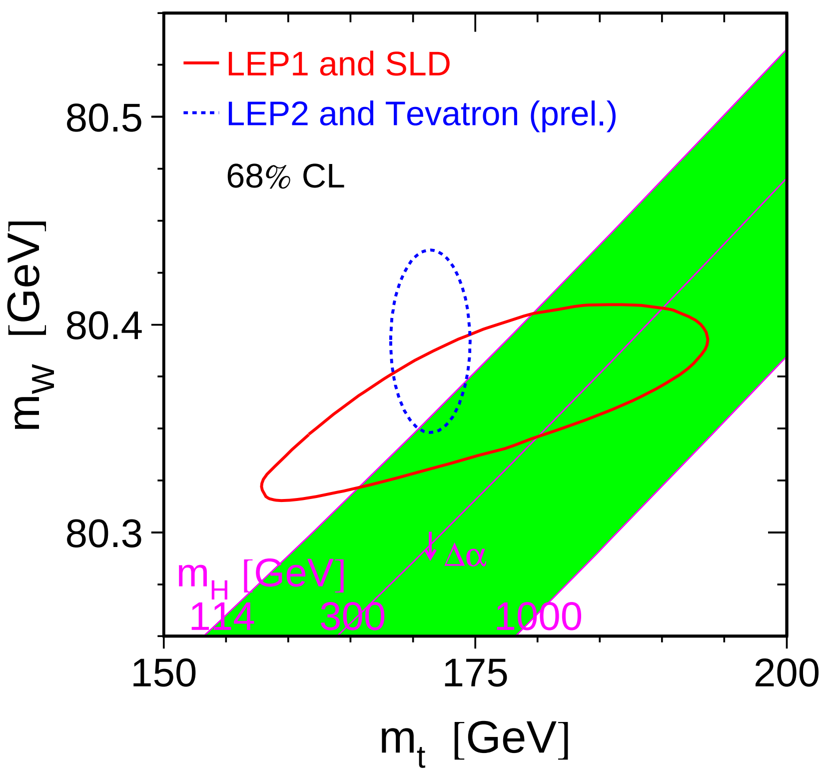
• (Some sort of) SM inconsistency ?

• (Can be viewed as PDF problem, etc.)

• Also W width:



• Can correlate m_t and M_W in global EW fit:



(Summer 2006 combination for $m_t = 171.4 \pm 2.1$ GeV)

Indirect search for top and Higgs

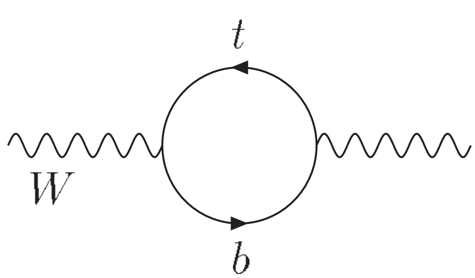
- Precision observables are affected by quantum fluctuations:
$\rightarrow$ give access to two high mass SM scales: m_t and M_H

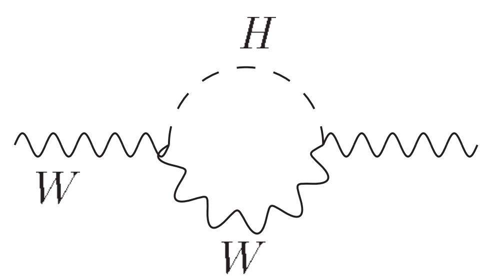
- t, H enter in loop corrections to EW observables.

- E.g. radiative corrections to M_W, M_Z vs. $\sin^2 \vartheta_W$ relation:

$$\sin^2 \theta_W = 1 - \frac{M_W^2}{M_Z^2}$$

- Quadratic dependence on m_t and logarithmic on M_H:





$\propto m_t^2$ $\qquad\qquad$ $\propto \log M_H$

- (Sensitivity also to BSM physics.)

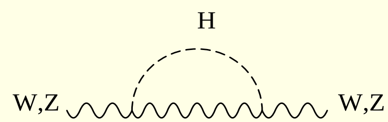
EW precision fits: perturbatively calculate observables in terms of few parameters:

$$M_Z, G_F, \alpha(M_Z), M_W, m_f, (\alpha_s(M_Z))$$

extracted from experiments with high accuracy.

- SM needs Higgs boson to cancel infinities, e.g.

$$M_W, M_Z \longrightarrow$$



- Finite logarithmic contributions survive, e.g. radiative corrections to $\rho = M_W^2/(M_Z^2 \cos^2\theta_W)$:

$$\rho = 1 - \frac{11g^2}{96\pi^2 \tan^2\theta_W} \ln\left(\frac{M_H}{M_W}\right)$$

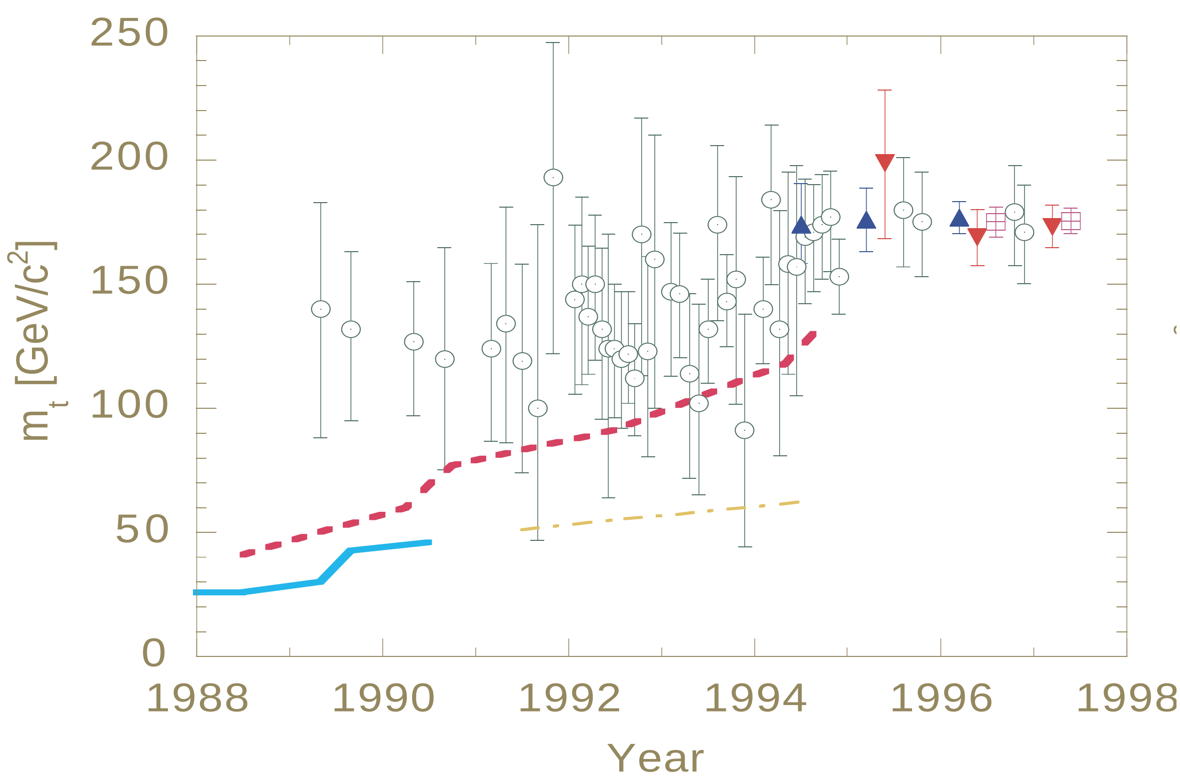
  Main effects in oblique radiative corrections (S,T-parameters)

- New physics at the scale Λ will appear as higher dimension effective operators.

- EW precision observables led to m_t <u>prediction</u> !

- Determinations of m_t from

  1. fits to EW observables (open circles)
  2. 95% confidence-level (CL) lower bounds on m_t from
     – direct searches in e^+e^- annihilations (solid line)
     – direct searches in $\bar{p}p$ collisions (broken line)
  3. from $?_W$ in $\bar{p}p \to (W \text{ or } Z) + X$ (dot-dashed line)
  4. direct measurements of m_t by CDF (triangles) and DØ (inverted triangles)



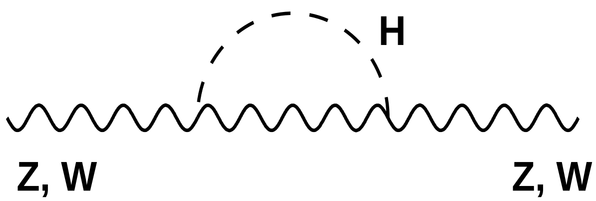
- Compare to latest PDG compilation (2005):

$$m_t = 172.7 \pm 2.9 \text{ (direct observation)},$$

$$m_t = 178.1^{+10.4}_{-8.3} \text{ (SM EW fits)}.$$

Precision Indirect Higgs Mass

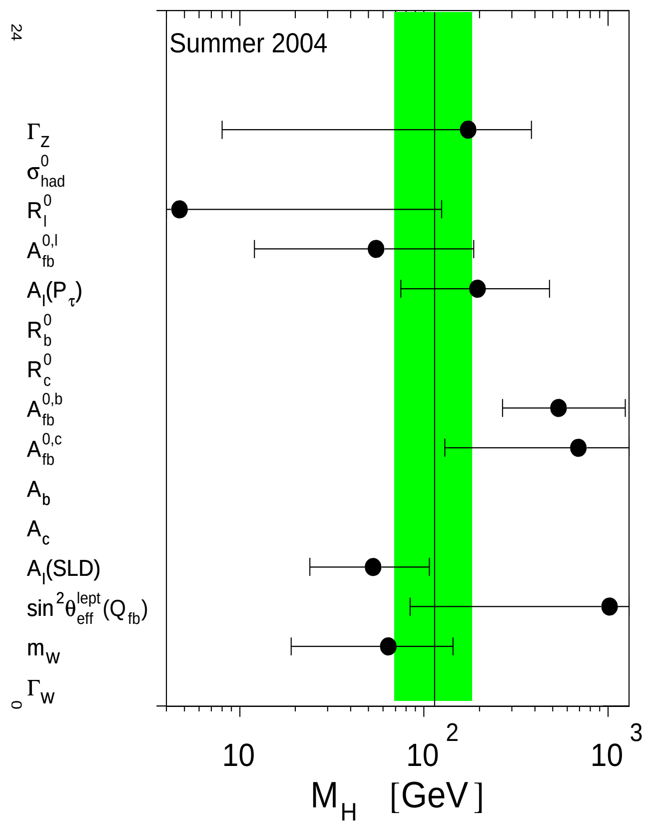
Try same trick to find Higgs mass:



$$\delta M_W^2 \sim \ln \frac{M_H}{M_W}$$

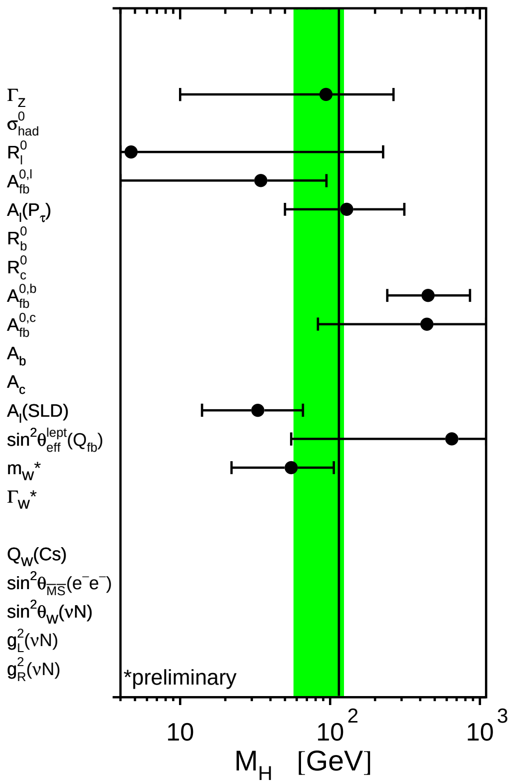
Much weaker dependence on M_H than on m_t.

➥Task is harder and requires as much EW precision data as you can get your hands on...

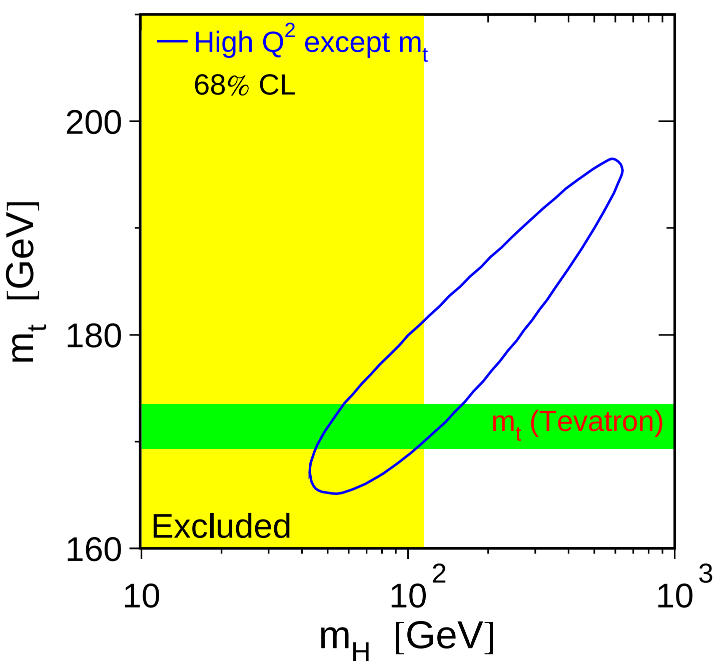


LEP+SLD EW Working Group '04 (adapted)

• Sensitivity to M_H only logarithmic, still limits available.



• Allowed region in $m_t - M_H$ plane:



• A SM Higgs boson with mass $M_{\text{Higgs}} < 114$ GeV is already excluded from direct LEP searches at 95% CL (see later).

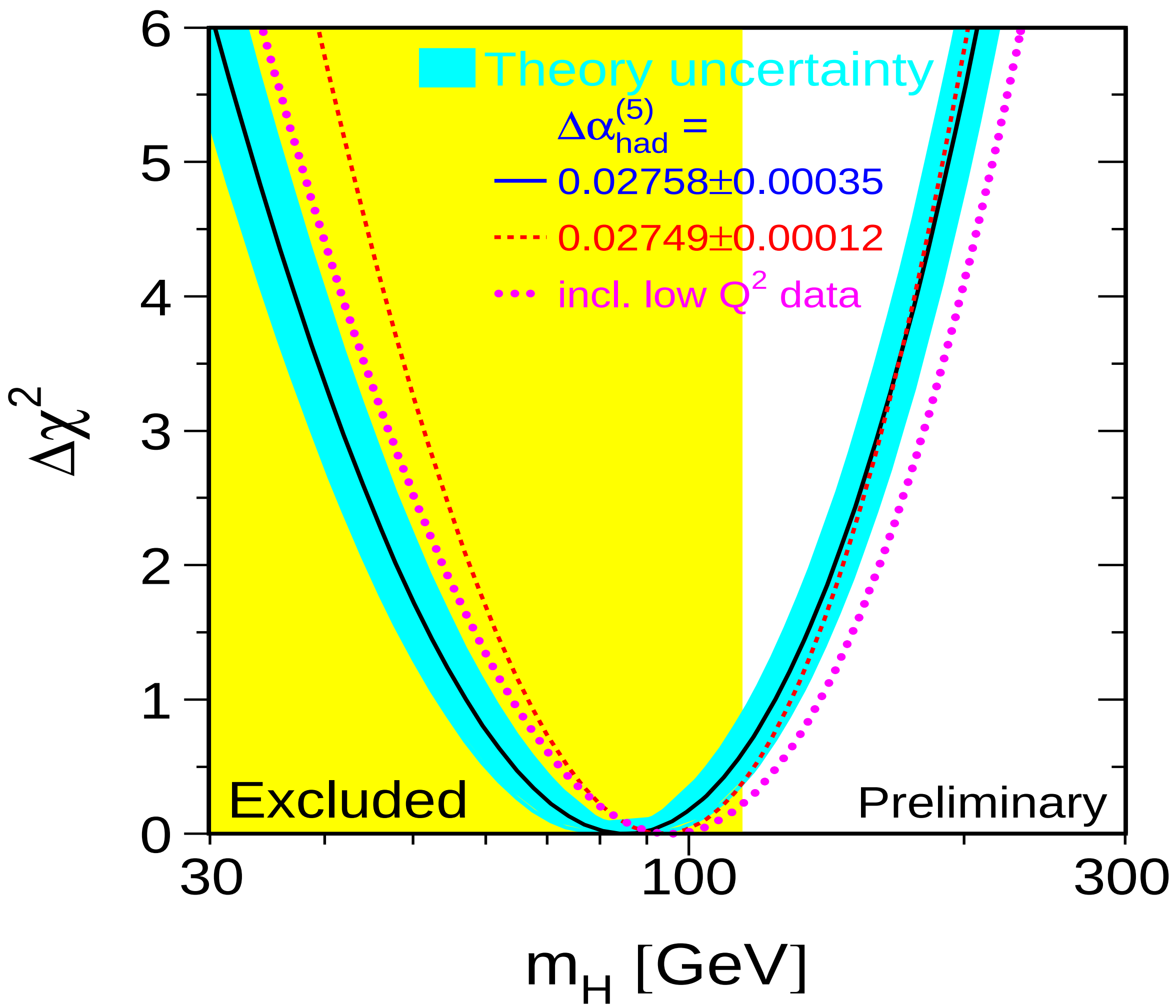
• Possible LEP evidence of SM Higgs at 115 GeV (see later).

• Fixing m_t to experimental value from direct measurement at Tevatron, precision data lead to:

$$M_H^{\rm SM} < 166 \text{ GeV (approx) at } 95\% \text{ CL } (\Delta\chi^2 = 2.7)$$

$$\text{Best fit : } M_H = 85\pm^{39}_{28} \text{ GeV}$$

(Summer 2006 combination for $m_t = 171.4 \pm 2.1$ GeV)



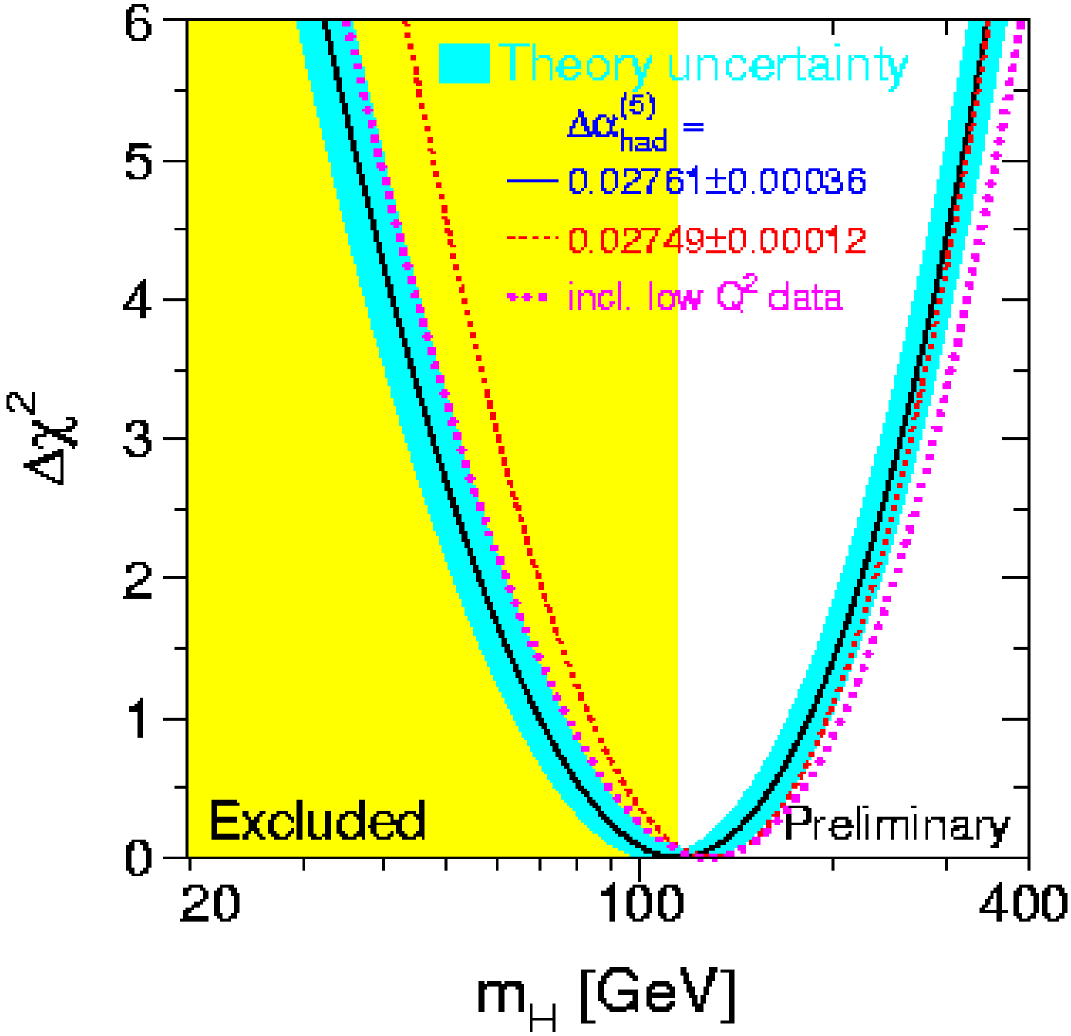
• Minimal impact of NuTeV anomaly.

• Recall: $M_W^2 = M_Z^2 \cos^2\theta_W + \mathcal{O}(m_t^2) - \mathcal{O}\left(\log \frac{M_H^2}{M_W^2}\right)$.

- Compare to 2004 plot (larger m_t, limited Run 2 stats)

$$M_H^{\rm SM} < 260 \text{ GeV} \text{ at } 95\% \, \text{CL } (\Delta\chi^2 = 2.7)$$

(LEPEWWG+LEPHWG Winter'04)



- Dramatic shift downwards of best M_H fit and upper limit !

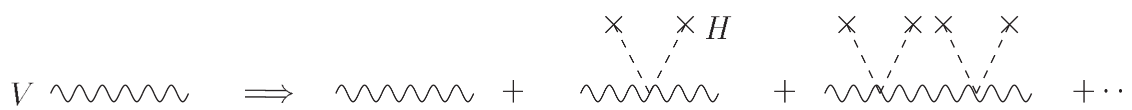
Higgs Boson search

Outline

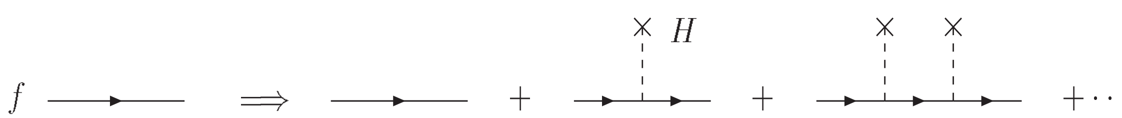
- The Higgs mechanism
- The Higgs picture
- The Higgs profile
- Collider searches

The Higgs mechanism

- Unitarity of EW interactions requires existent of a scalar Higgs field which couples to other particles proportional to their mass.
- In Higgs sector of theory, scalar fields interact with each other in such a way that ground state acquires a non-zero field strength, breaking EW symmetry spontaneously.
- Masses of gauge bosons V and fermions f build up by (infinitely) repeated interactions with the background Higgs field.
- Such interactions masses from zero to finite values:

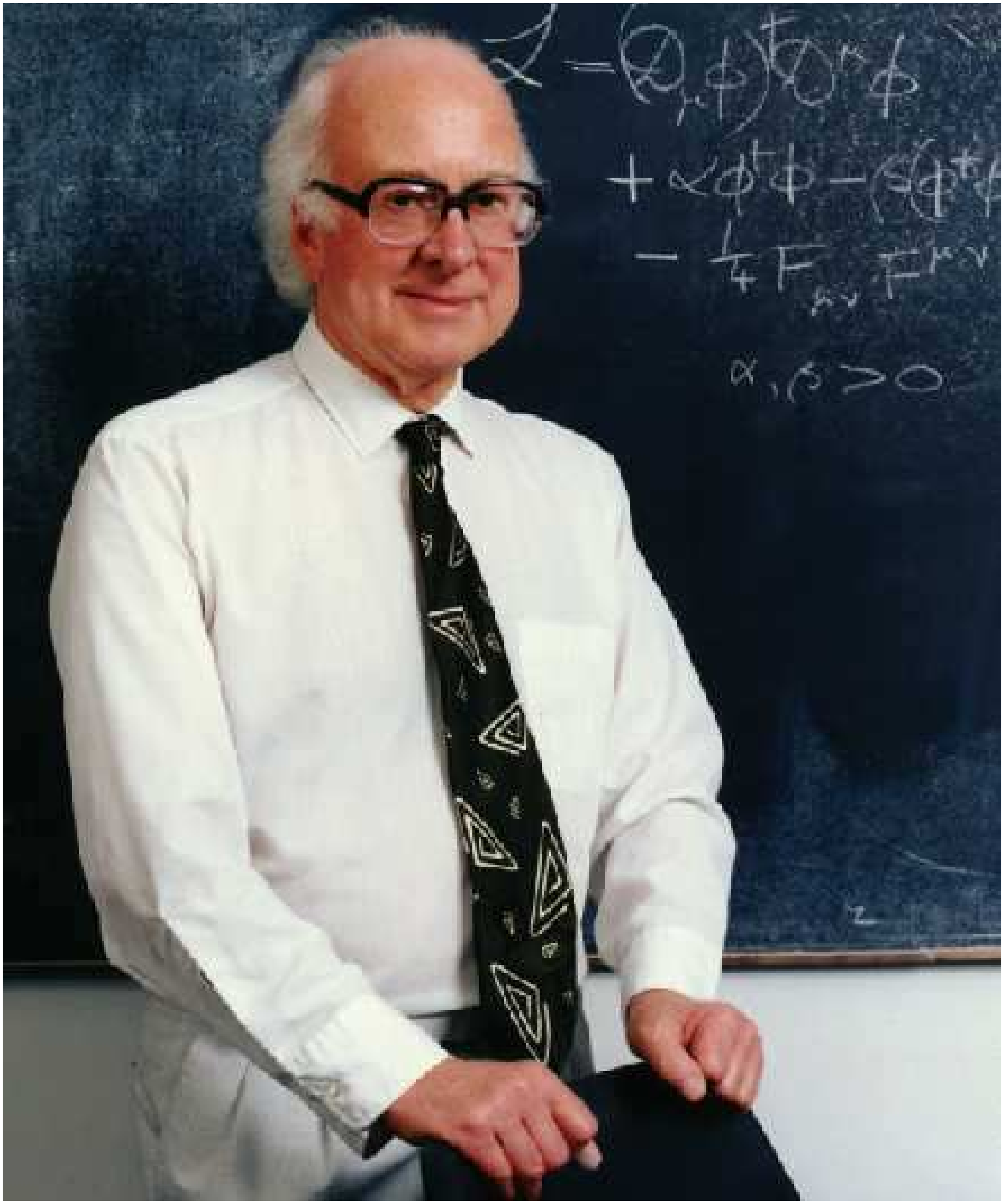


$$\frac{1}{q^2} \Rightarrow \frac{1}{q^2} + \sum_j \frac{1}{q^2}\left[\left(\frac{g_W v}{\sqrt{2}}\right)^2 \frac{1}{q^2}\right]^j = \frac{1}{q^2 - M_V^2} \;:\; M_V^2 = g_W^2 \frac{v^2}{4}$$



$$\frac{1}{\not{q}} \Rightarrow \frac{1}{\not{q}} + \sum_j \frac{1}{\not{q}}\left[\frac{g_f v}{\sqrt{2}} \frac{1}{\not{q}}\right]^j = \frac{1}{\not{q} - m_f} \quad : \; m_f = g_f \frac{v}{\sqrt{2}}$$

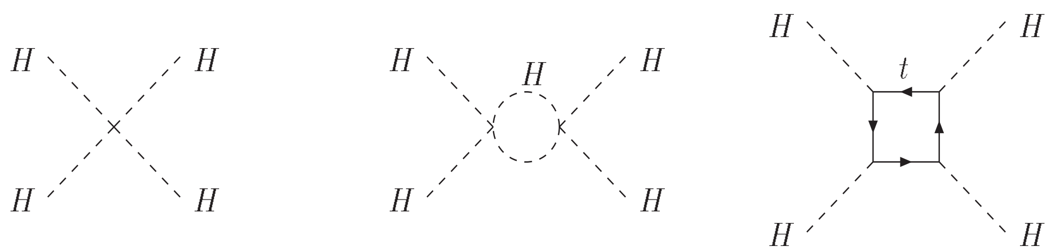
The Higgs picture



The Higgs profile

- M_H from curvature of self-energy potential V, $M_H^2 = \lambda v^2$.
- SM cannot predicted it since quartic coupling λ is unknown.
- Nevertheless, restrictive bounds on M_H follow from hypothetical assumptions on energy scale Λ up to which SM is valid before NP emerges.

$\rightarrow$ quantum fluctuations introduce energy dependence $\lambda(\mu)$.



- $\lambda(\mu)$ running from renormalisation group equation (RGE):

$$\frac{d\lambda}{d\ln \mu^2} = \frac{3}{8\pi^2}[\lambda^2 + \lambda g_t^2 - g_t^4]$$

with $\lambda(v^2) = M_H^2/v^2$ and $g_t(v^2) = \sqrt{2} m_t/v$.

- For moderate m_t large M_H,

$$d\lambda/d\ln \mu^2 \sim +\lambda^2,$$

and becomes strong shortly before Landau pole:

$$\lambda(\mu^2) = \frac{\lambda(v^2)}{1 - \frac{3\lambda(v^2)}{8\pi^2} \ln \frac{\mu^2}{v^2}}.$$

- Requirement SM be perturbative up to a scale Λ: $\lambda(\Lambda) < \infty$.

- Can be translated into upper bound on M_H

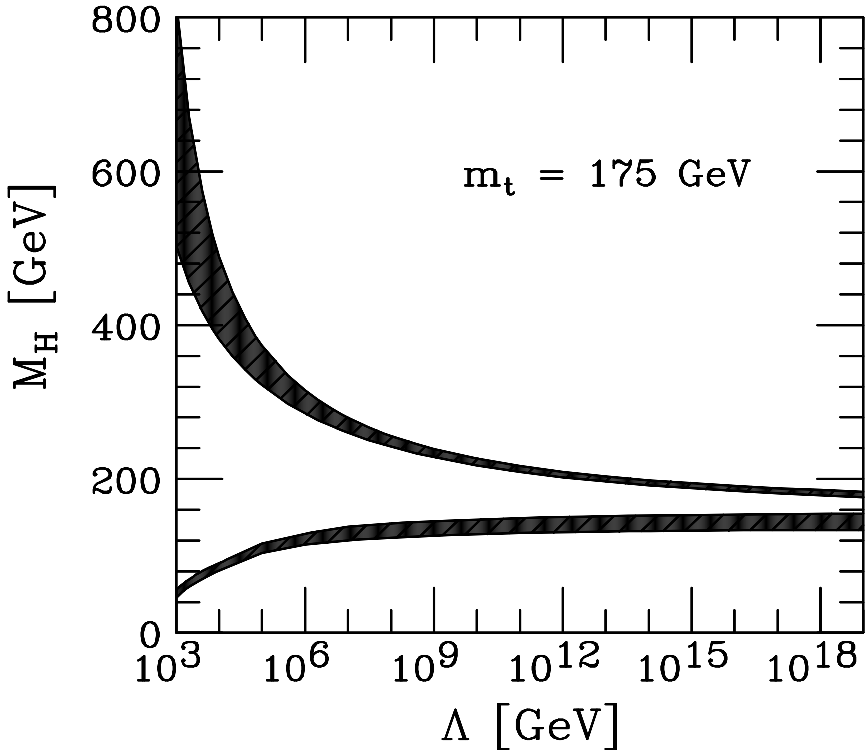
$$M_H^2 \lesssim \frac{8\pi^2 v^2}{3 \ln (\Lambda^2 / v^2)}$$

- Lower bound on M_H derived from requirement of vacuum stability:
$\rightarrow$ top-loop corrections reduce λ for increasing m_t, λ becomes negative if m_t too large.

- Self-energy potential would become deeply negative and ground state would no longer be stable.

- To avoid instability, M_H must exceed minimal value for given m_t to balance negative contribution.

- Lower bound depends on cut-off value Λ.



• For $m_t = 175$ GeV:

Λ	M_H
1 TeV	55 GeV $\leq M_H \leq$ 700 GeV
10^{19} GeV	130 GeV $\leq M_H \leq$ 190 GeV

• If SM valid up to grand unification theory (GUT) scale, 130 GeV $< M_H <$ 190 GeV !

• Observation of M_H outside this range would demand a new strong interaction scale below GUT scale.

Unitarity: longitudinal gauge boson scattering cross section at high energy grows with M_H.

Electroweak Equivalence Theorem:
in the high energy limit ($s \gg M_V^2$)

$$\mathcal{A}(V_L^1 \ldots V_L^n \to V_L^1 \ldots V_L^m) = (i)^n(-i)^m \mathcal{A}(\omega^1 \ldots \omega^n \to \omega^1 \ldots \omega^m) + O\left(\frac{M_V^2}{s}\right)$$

(V_L^i=longitudinal weak gauge boson; ω^i=associated Goldstone boson).

Example: $\boxed{W_L^+ W_L^- \to W_L^+ W_L^-}$

$$\mathcal{A}(W_L^+ W_L^- \to W_L^+ W_L^-) \sim -\frac{1}{v^2}\left(-s - t + \frac{s^2}{s - M_H^2} + \frac{t^2}{t - M_H^2}\right)$$

$$\mathcal{A}(\omega^+ \omega^- \to \omega^+ \omega^-) = -\frac{M_H^2}{v^2}\left(\frac{s}{s - M_H^2} + \frac{t}{t - M_H^2}\right)$$

$$\Downarrow$$

$$\mathcal{A}(W_L^+ W_L^- \to W_L^+ W_L^-) = \mathcal{A}(\omega^+ \omega^- \to \omega^+ \omega^-) + O\left(\frac{M_W^2}{s}\right)$$

Using partial wave decomposition:

$$\mathcal{A} = 16\pi \sum_{l=0}^{\infty} (2l+1) P_l(\cos\theta) a_l$$

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2 s} |\mathcal{A}^2| \quad \longrightarrow \quad \sigma = \frac{16\pi}{s} \sum_{l=0}^{\infty} (2l+1)|a_l|^2 = \frac{1}{s} \mathrm{Im}\left[\mathcal{A}(\theta=0)\right]$$

$$\Downarrow$$

$$\boxed{|a_l|^2 = \mathrm{Im}(a_l)} \quad \longrightarrow \quad \boxed{|\mathrm{Re}(a_l)| \leq \tfrac{1}{2}}$$

Most constraining condition for $W_L^+ W_L^- \to W_L^+ W_L^-$ from

$$a_0(\omega^+\omega^- \to \omega^+\omega^-) = -\frac{M_H^2}{16\pi v^2}\left[2 + \frac{M_H^2}{s - M_H^2} - \frac{M_H^2}{s}\log\left(1 + \frac{s}{M_H^2}\right)\right) \xrightarrow{s \gg M_H^2} -\frac{M_H^2}{8\pi v^2}$$

$$\boxed{|\mathrm{Re}(a_0)| < \tfrac{1}{2}} \quad \longrightarrow \quad \boxed{M_H < 870 \text{ GeV}}$$

Best constraint from coupled channels ($2W_L^+ W_L^- + Z_L Z_L$):

$$a_0 \xrightarrow{s \gg M_H^2} -\frac{5M_H^2}{32\pi v^2} \quad \longrightarrow \quad \boxed{M_H < 780 \text{ GeV}}$$

Observe that: if there is no Higgs boson, i.e. $M_H \gg s$:

$$a_0(\omega^+\omega^- \to \omega^+\omega^-) \xrightarrow{M_H^2 \gg s} -\frac{s}{32\pi v^2}$$

Imposing the unitarity constraint $\longrightarrow$ $\boxed{\sqrt{s_c} < 1.8 \text{ TeV}}$

Most restrictive constraint $\longrightarrow$ $\boxed{\sqrt{s_c} < 1.2 \text{ TeV}}$

$$\Downarrow$$

New physics expected at the TeV scale

Exciting !!

this is the range of energies of both Tevatron and LHC

Triviality: a $\lambda\phi^4$ theory cannot be perturbative at all scales unless $\lambda = 0$.

In the SM the scale evolution of λ is more complicated:

$$32\pi^2 \frac{d\lambda}{dt} = 24\lambda^2 - (3g'^2 + 9g^2 - 24y_t^2)\lambda + \frac{3}{8}g'^4 + \frac{3}{4}g'^2 g^2 + \frac{9}{8}g^4 - 24y_t^4 + \cdots$$

($t = \ln(Q^2/Q_0^2)$, $y_t = m_t/v \to$ top quark Yukawa coupling).

Still, for large λ ($\leftrightarrow$ large M_H) the first term dominates and (at 1-loop):

$$\lambda(Q) = \frac{\lambda(Q_0)}{1 - \frac{3}{4\pi^2}\lambda(Q_0)\ln\left(\frac{Q^2}{Q_0^2}\right)}$$

$$\boxed{\text{when } Q \text{ grows}} \quad \longrightarrow \quad \boxed{\lambda(Q) \text{ hits a pole} \to \text{triviality}}$$

Imposing that $\lambda(Q)$ is finite, gives a scale dependent bound on M_H:

$$\boxed{\frac{1}{\lambda(\Lambda)} > 0 \quad \longrightarrow \quad M_H^2 < \frac{8\pi^2 v^2}{3\log\left(\frac{\Lambda^2}{v^2}\right)}}$$

where we have set $Q \to \Lambda$ and $Q_0 \to v$.

Vacuum stability: $\boxed{\lambda(Q) > 0}$

For small λ ($\leftrightarrow$ small M_H) the last term in $d\lambda/dt = \ldots$ dominates and:

$$\lambda(\Lambda) = \lambda(v) - \frac{3}{4\pi^2} y_t^2 \log\left(\frac{\Lambda^2}{v^2}\right)$$

from where a first rough lower bound is derived:

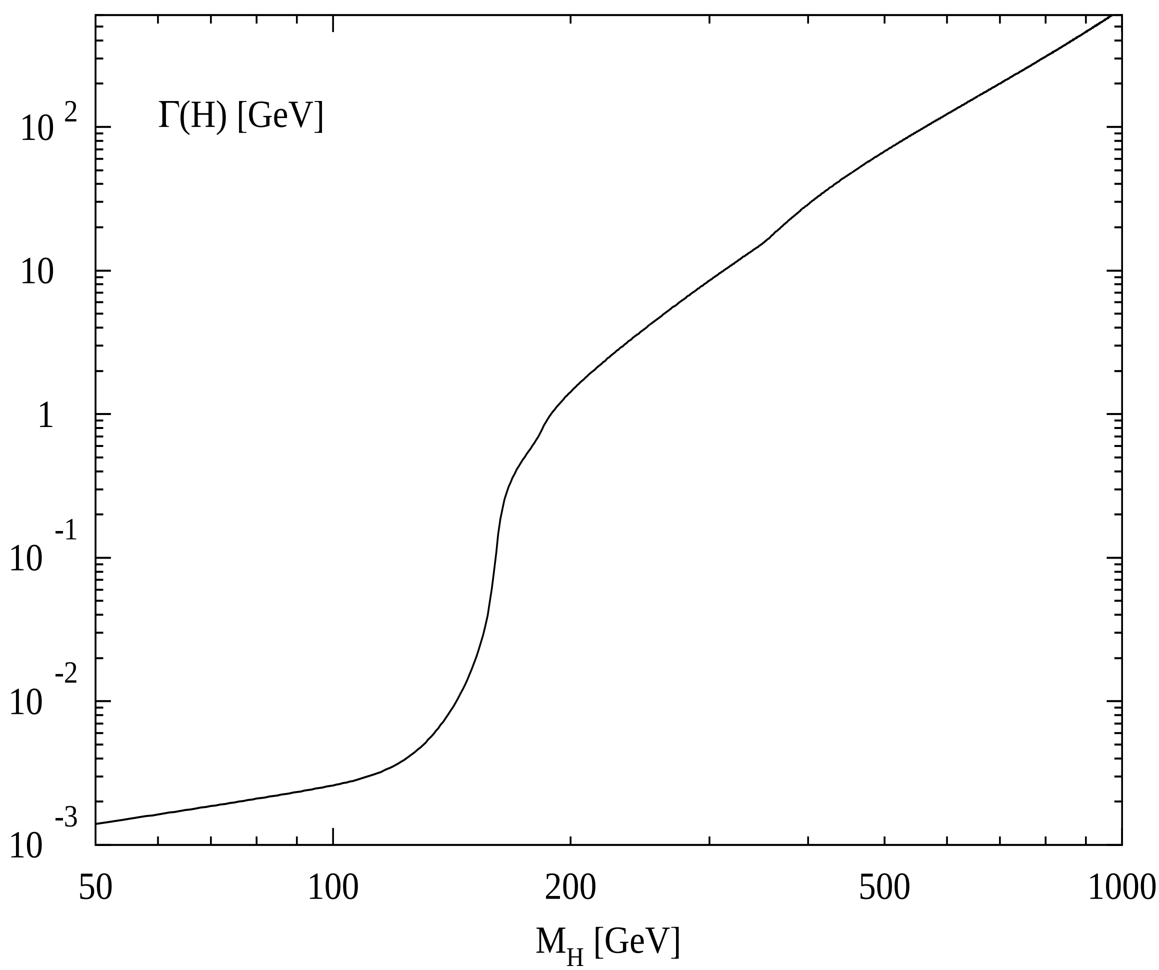
$$\boxed{\lambda(\Lambda) > 0 \quad \longrightarrow \quad M_H^2 > \frac{3v^2}{2\pi^2} y_t^2 \log\left(\frac{\Lambda^2}{v^2}\right)}$$

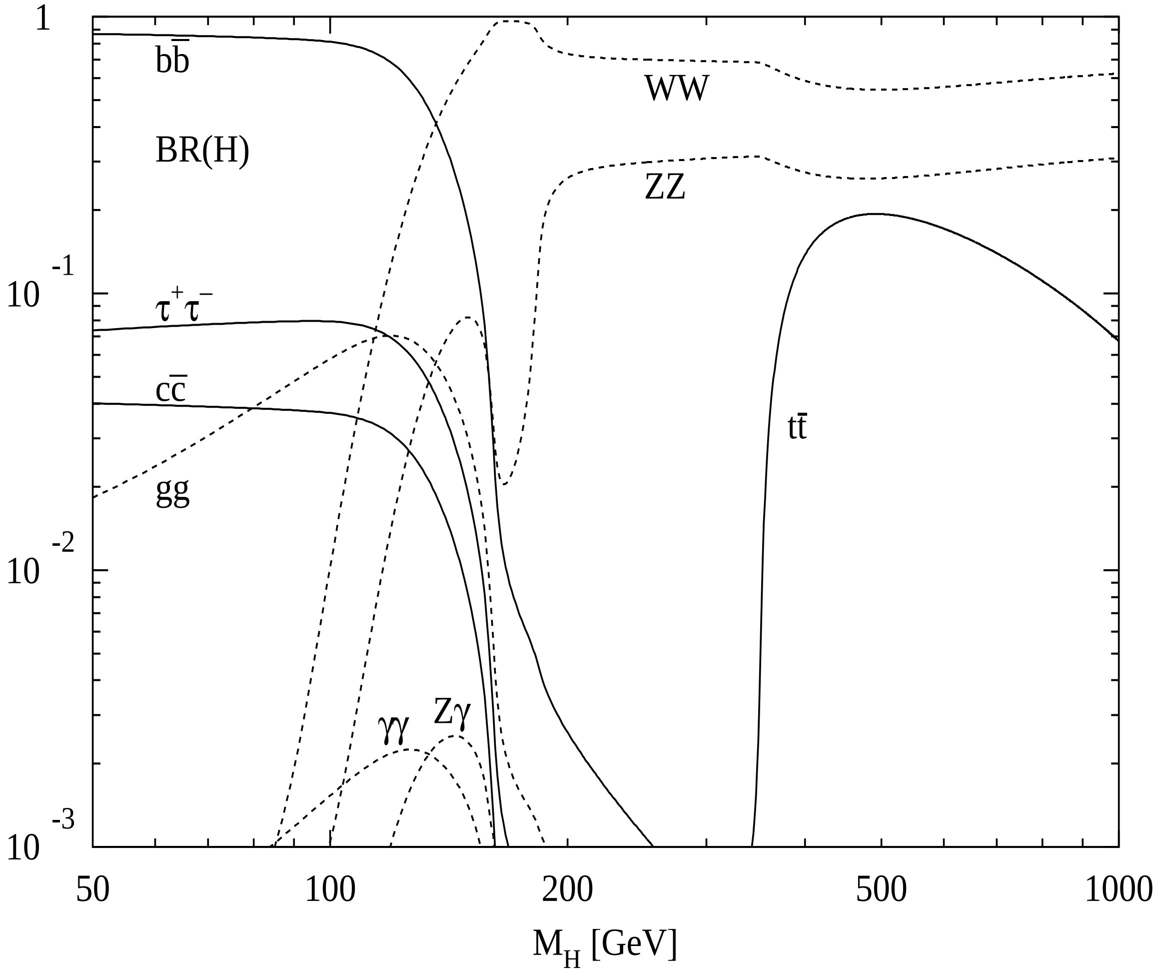
More accurate analyses use 2-loop renormalization group improved V_{eff}.

- H couplings to EW gauge bosons and fermions:

$$\begin{aligned} g_{ffH} &= \left[\sqrt{2}G_F\right]^{1/2} m_f \ , \\ g_{VVH} &= 2\left[\sqrt{2}G_F\right]^{1/2} M_V^2 \ . \end{aligned}$$

- ? $_H$ and Branching Ratios (BRs) determined by these:





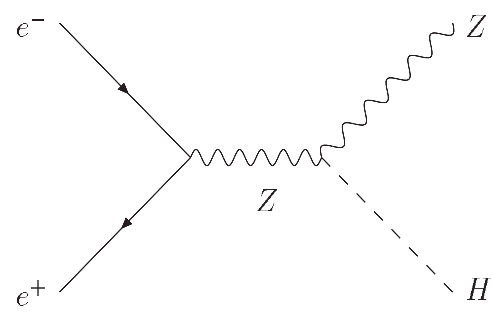
- $M_H \geq 250$ GeV: Higgs too wide to resolve experimentally.
- Best decay channel depends on collider environment:

  1. leptonic/photonic decays needed at hadron colliders !
  2. can also use hadronic decays at lepton colliders !

- (Muon colliders could scan resonance, like LEP with Z).

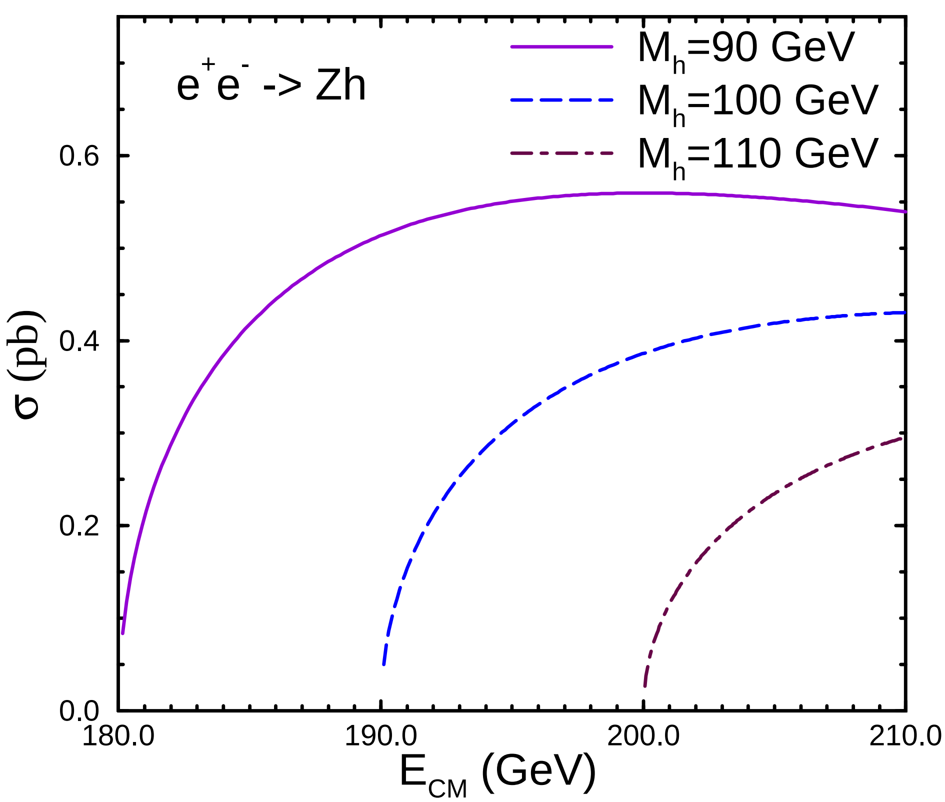
Collider searches

- LEP, SLC:



Higgs-strahlung

- Inclusive Higgs cross section (LEP2):



- Look for $H \to b\bar{b}$ (use b-tagging) and $Z \to X$: no evidence,

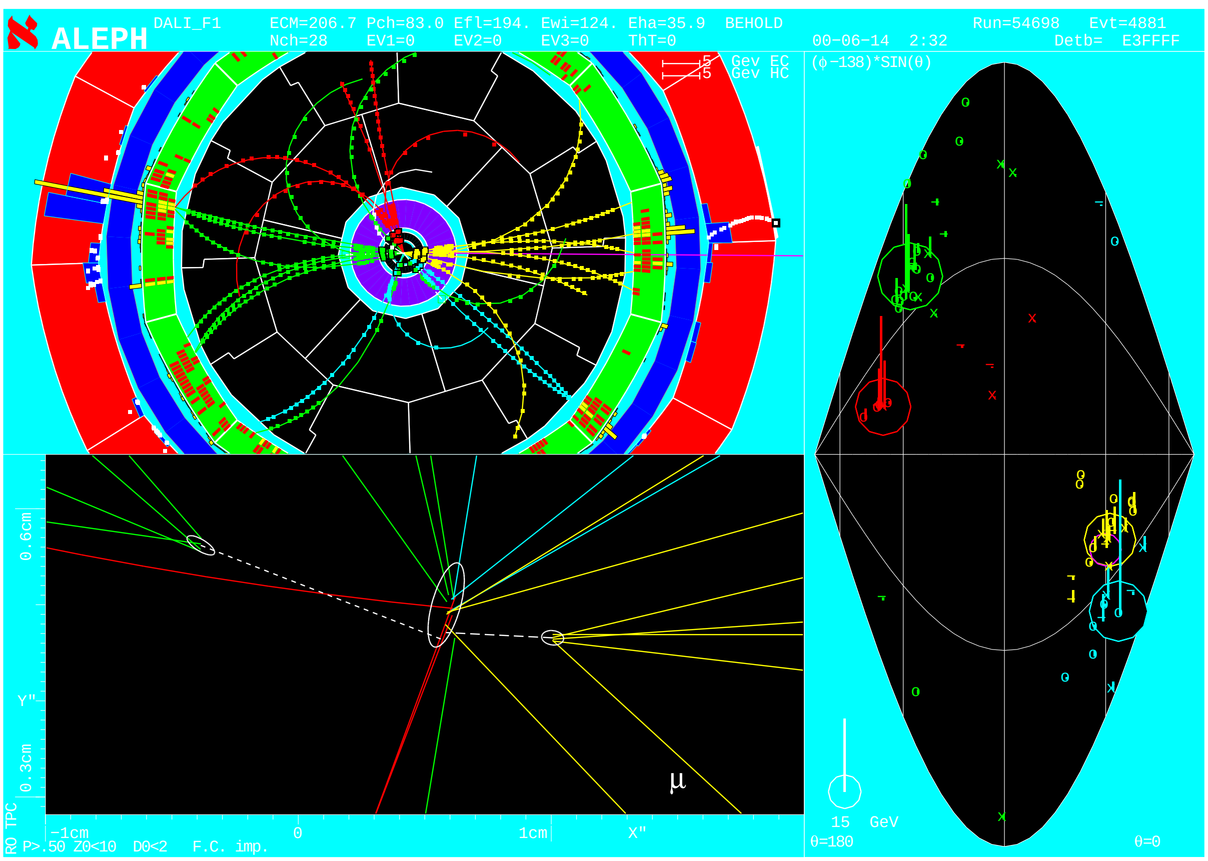
$$M_H < 114.4 \text{ GeV} \quad \text{at } 95\% \text{ CL} \to \text{ fig}$$

- Small excess can be interpreted as production of a SM Higgs boson with $M_H \approx 115$ GeV. $\to$ fig

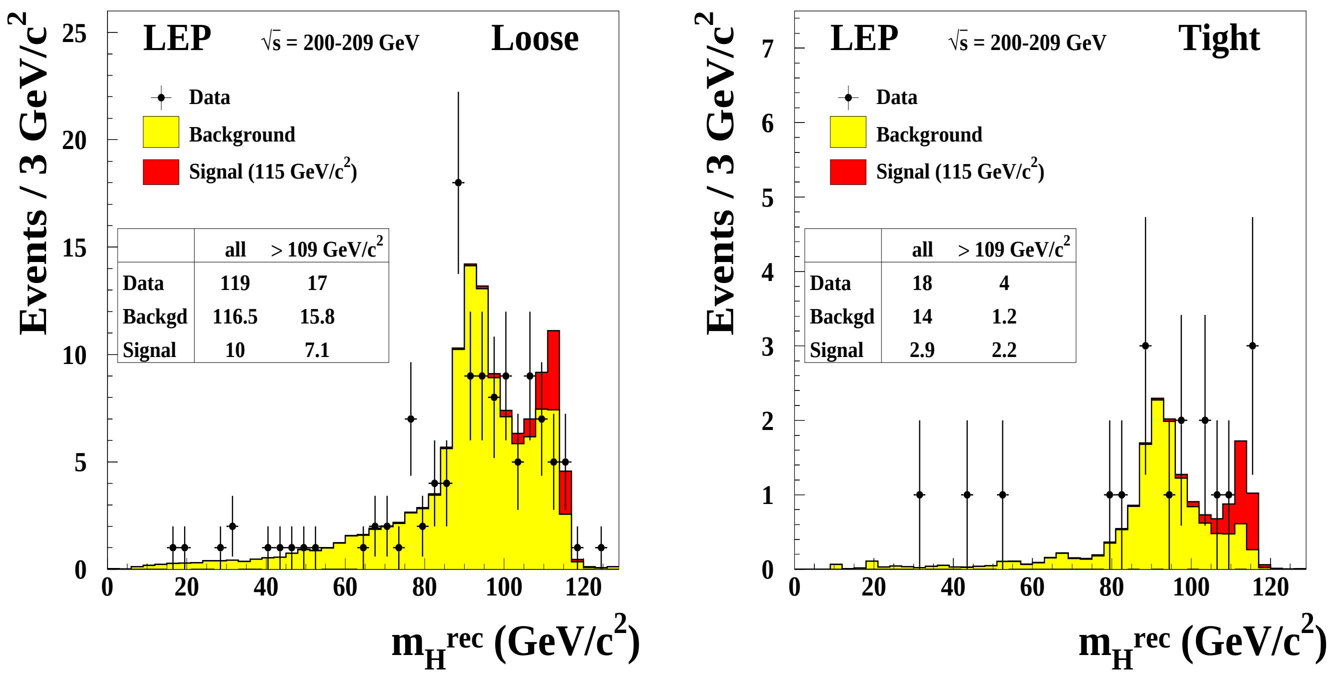
- Significance insufficient to claim Higgs observation/discovery.

- Most candidates from four-jet final states:

$$e^+e^- \to Z(\to q\bar{q})H(\to b\bar{b}).$$



Data v. expected signal & background

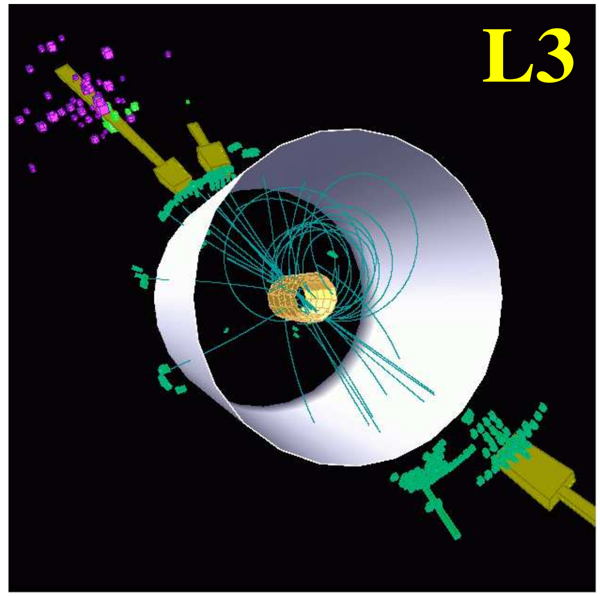


LEP Higgs WG conclusions:

statistical analysis: signal at 1.7 standard dev., corresponding to $M_H \simeq 116$ GeV

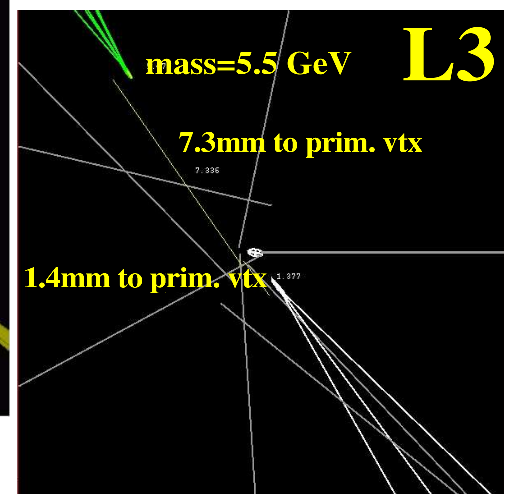
Missing energy channel

most significant H$\nu\nu$ candidate



measured H mass=115 GeV
H mass resolution ~3 GeV

Secondary vtx's view



• Hadron colliders, Tevatron and Large Hadron Collider (LHC):

(a) gluon fusion	:	$gg \to H$
(b) WW, ZZ fusion	:	$W^+W^-, ZZ \to H$
(c) Higgs-strahlung off W, Z	:	$q\bar{q} \to W, Z \to W, Z + H$
(d) Higgs bremsstrahlung off b, t	:	$q\bar{q}, gg \to (b\bar{b})t\bar{t} + H$

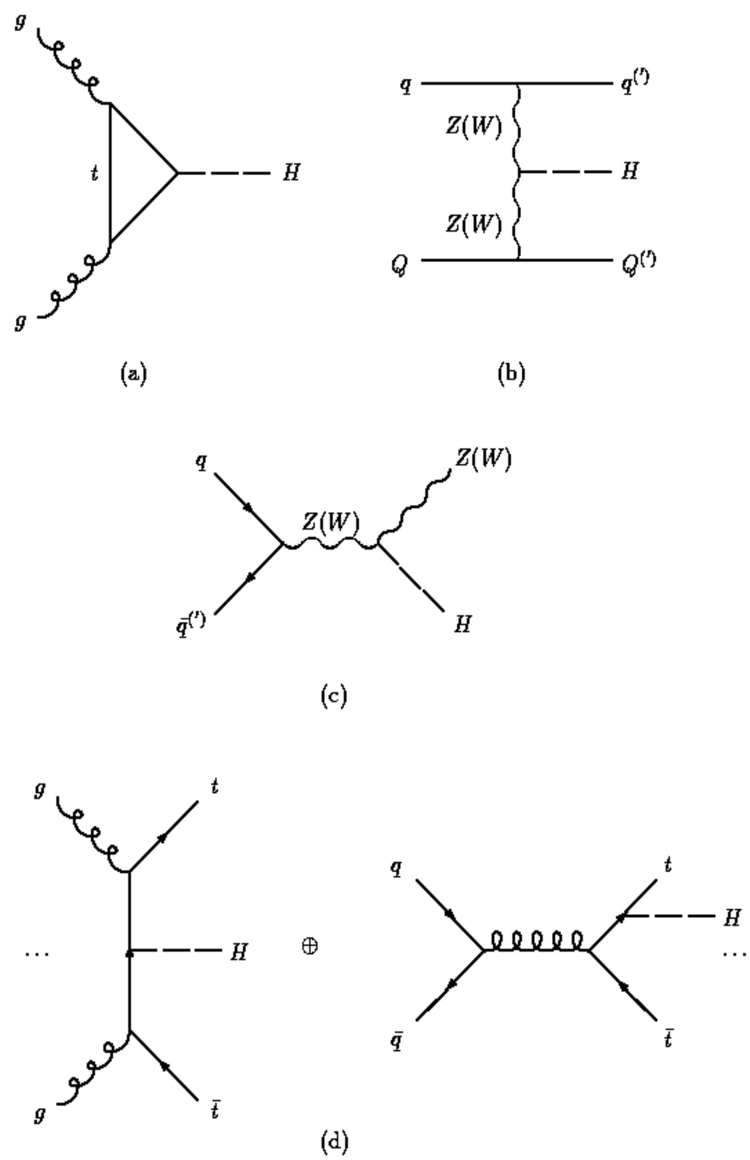
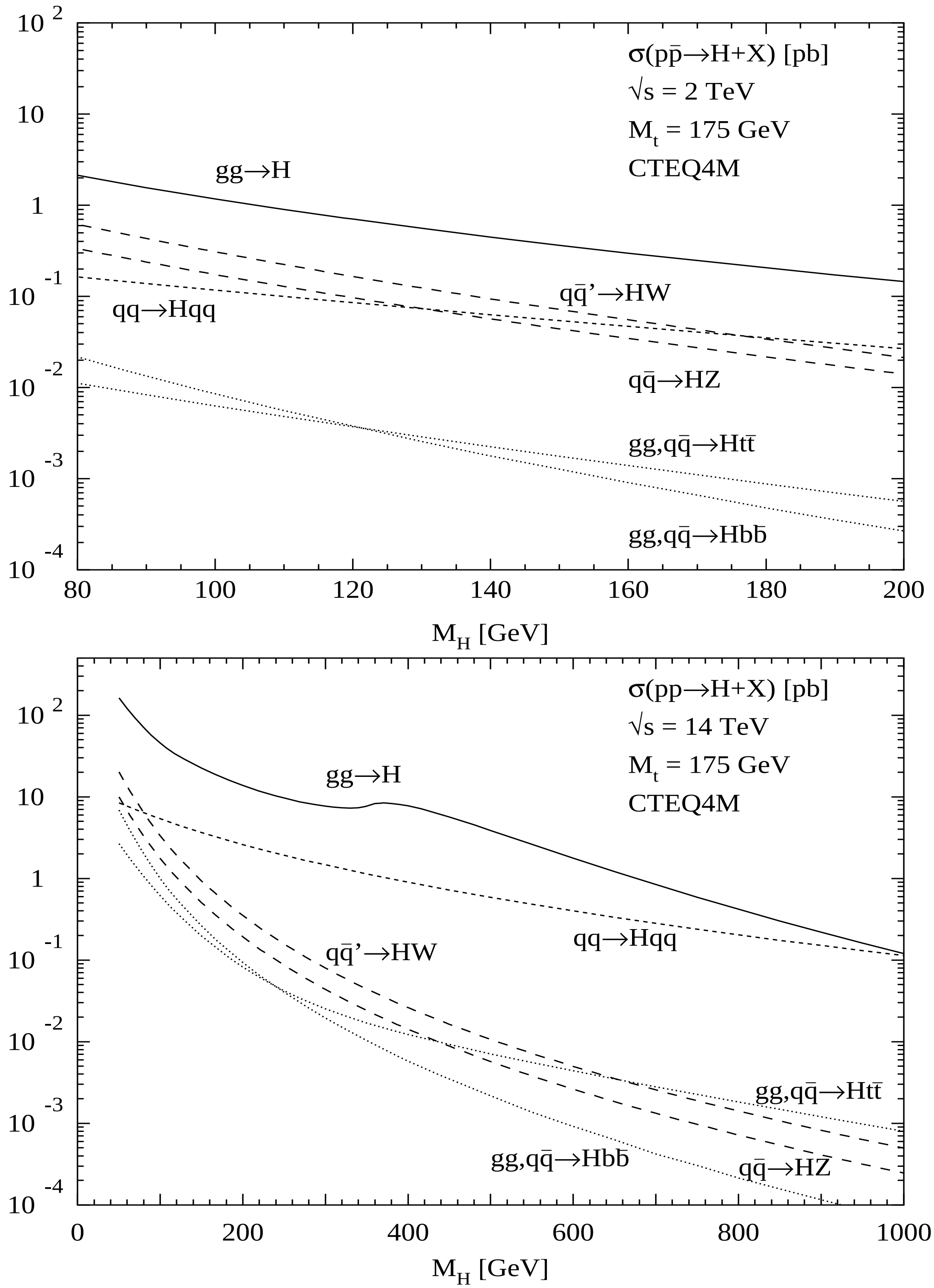


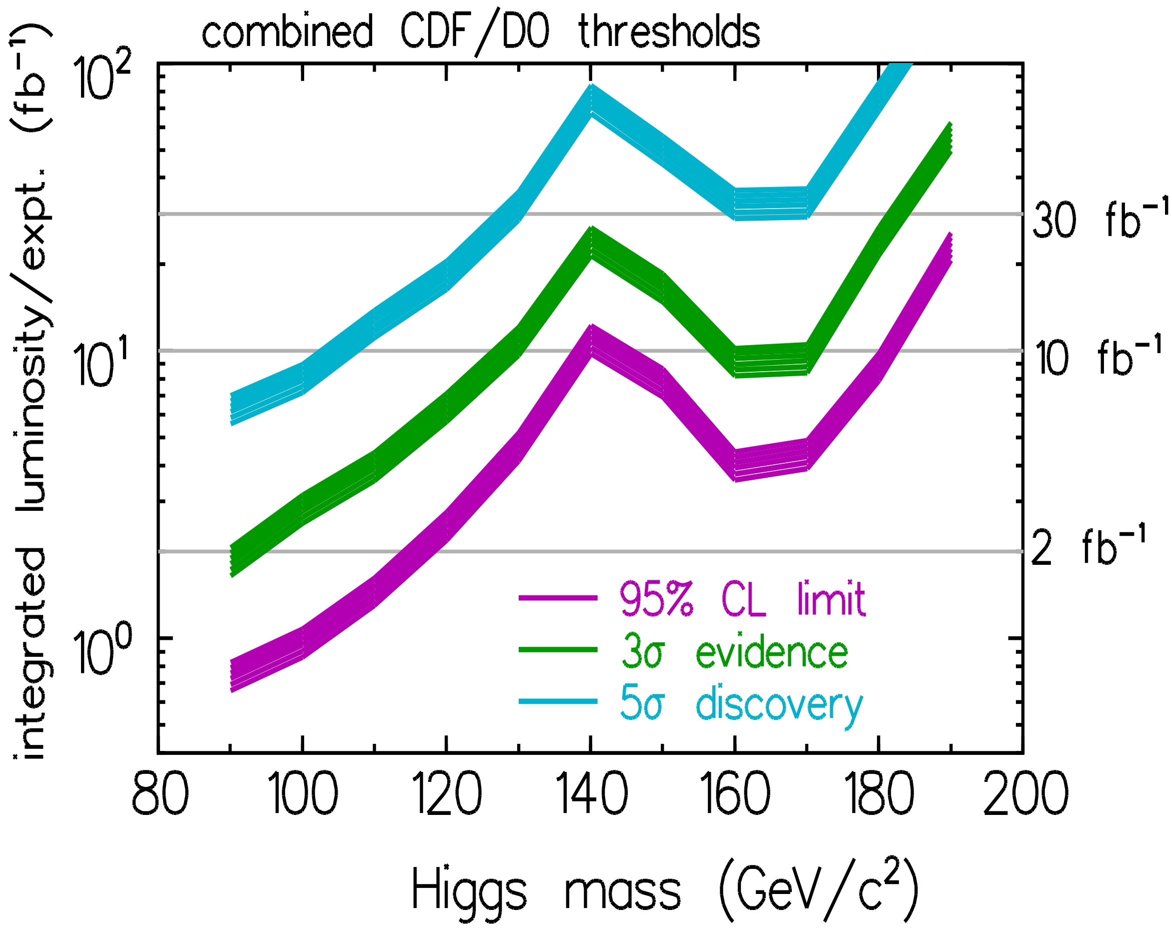
Fig. 4

• Inclusive production cross sections:



• Sensitivity for Higgs searches at Tevatron best for Higgs-strahlung with $H \to b\bar{b}$ and $Z \to \ell^+\ell^-$ (other channels too). $\to$ figs

• Integrated luminosity per experiment required for SM Higgs boson exclusion and evidence, as function of M_H.



(Note: legends in reverse order with respect to curves.)

• Can do better than LEP ? (Luminosity problems ?)

Final State Modes and Backgrounds

Signal Production and Final State:	Primary Background Processes:
$gg \to H \to b\bar{b}$	QCD Dijet Background…Huge
$p\bar{p} \to WH \to q\bar{q}'b\bar{b}$	QCD Jet Background/W+jets
$p\bar{p} \to WH \to \ell\nu b\bar{b}$	W+$b\bar{b}$/$c\bar{c}$, Single top, $t\bar{t}$
$p\bar{p} \to ZH \to q\bar{q}b\bar{b}$	QCD Jet Background/W+jets
$p\bar{p} \to ZH \to \ell^+\ell^-b\bar{b}$	W/Z+$b\bar{b}$/$c\bar{c}$, $t\bar{t}$ (Poor BR)
$p\bar{p} \to ZH \to \nu\bar{\nu}b\bar{b}$	W/Z+$b\bar{b}$/$c\bar{c}$, $t\bar{t}$, QCD Jets

Essentials:

Lepton Acceptance, b-tagging eff/Acceptance, dijet Mass Resolution

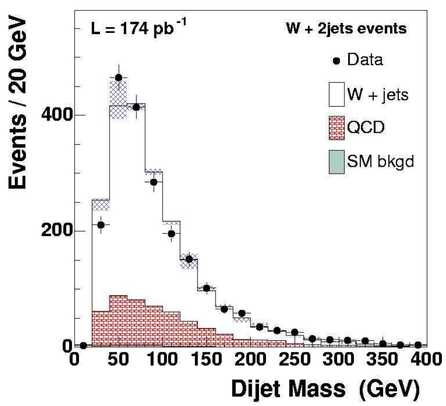
Event Rates/fb^{-1}

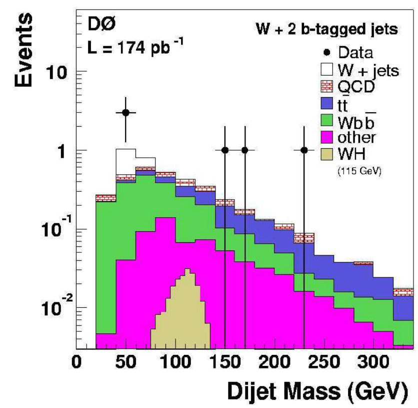
Rates determined from a combination of MC and data.

Missed Chg Lepton

	No Mass Window	Mass Window
WH Signal(115)	1.7	1.5
ZH Signal(115)	2.5	2.3
Total Signal	4.2	3.8
tt	8.8	2.2
t(W*)	3.3	0.7
t(Wg)	2.4	0.5
W/Z bb	22.3	3.3
WZ/ZZ	16.5	2.7
QCD	61.2	10.2
Total Bkg	114	19.6
$S/\sqrt{B}$	0.39	0.85
S/B	0.037	0.19

Leptons + jets: WH→eνbb



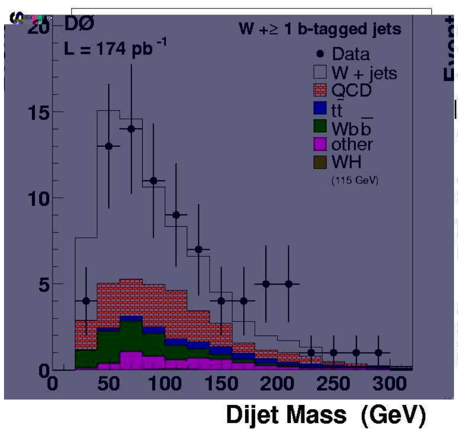


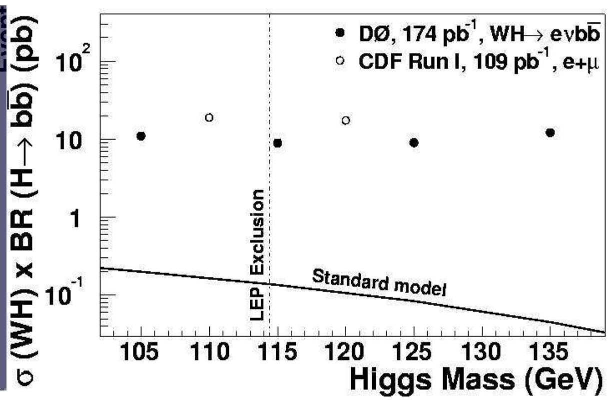
DØ : 6 double b-tag events in 174 pb⁻¹

spectra well described by SM

- σ(Wbb) < 6.6 pb (for P_T(b)>20 GeV, ΔR(bb) > 0.75)
- σ(WH) < 9.0-12.2 pb (for M_H=105-135 GeV)

hep-ex/0410062



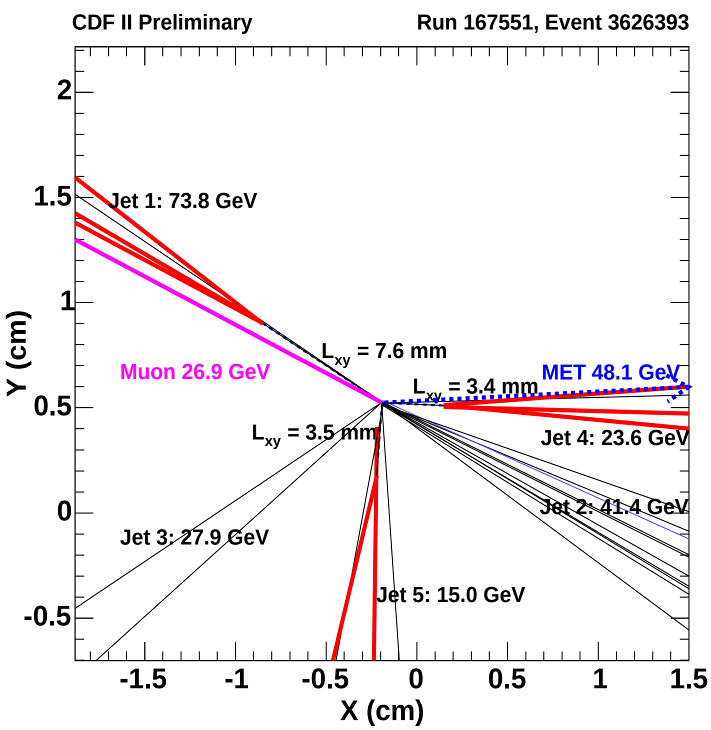


- Now CDF can do also:

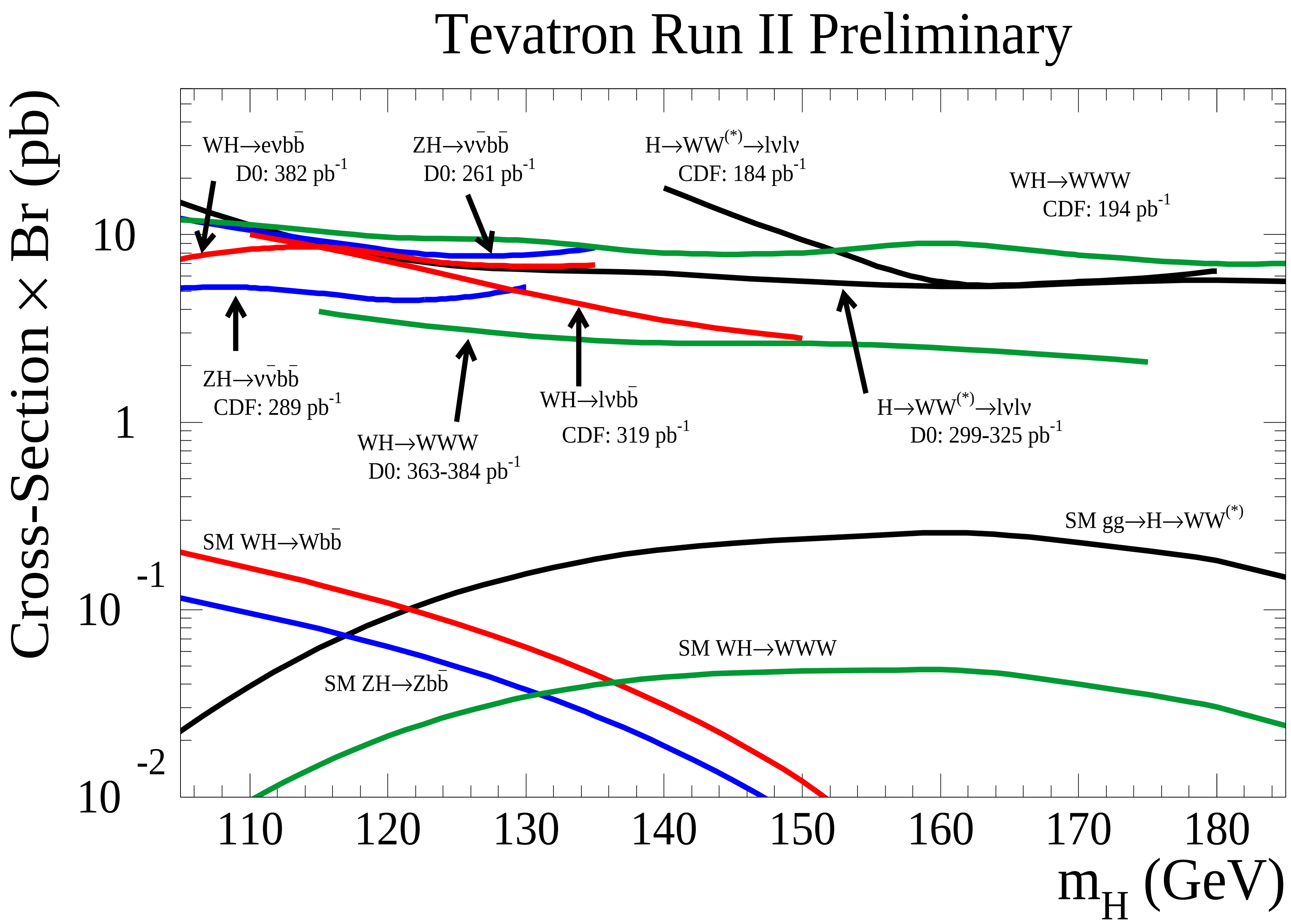
$ttH \rightarrow lvjjbbbb$ (lepton, missing E_T, ≥ 5 jets with ≥ 3 b-tag)

Observed Event - Kinematics

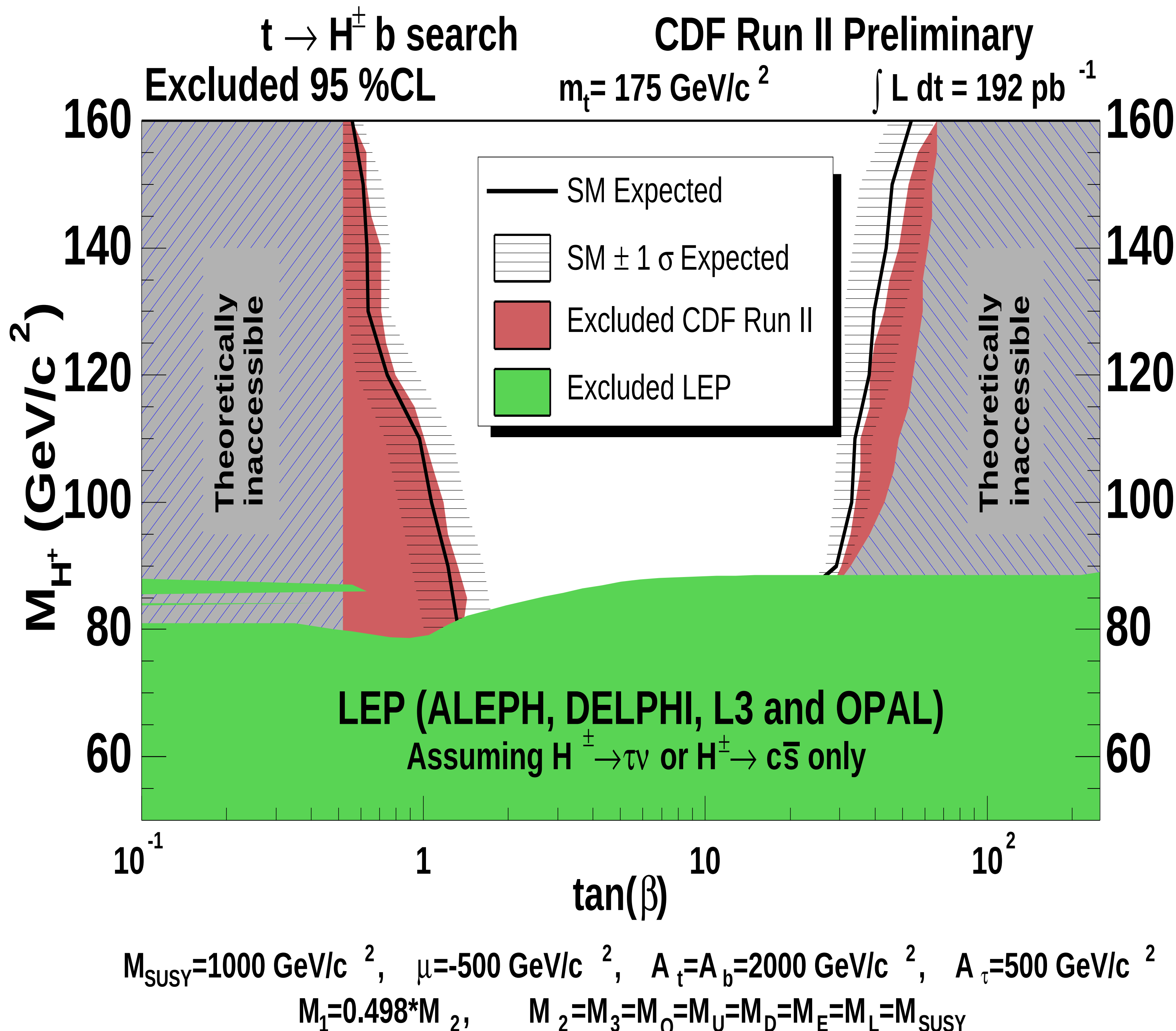
Event with an identified, central muon and three b-tagged jets:



(APS06)

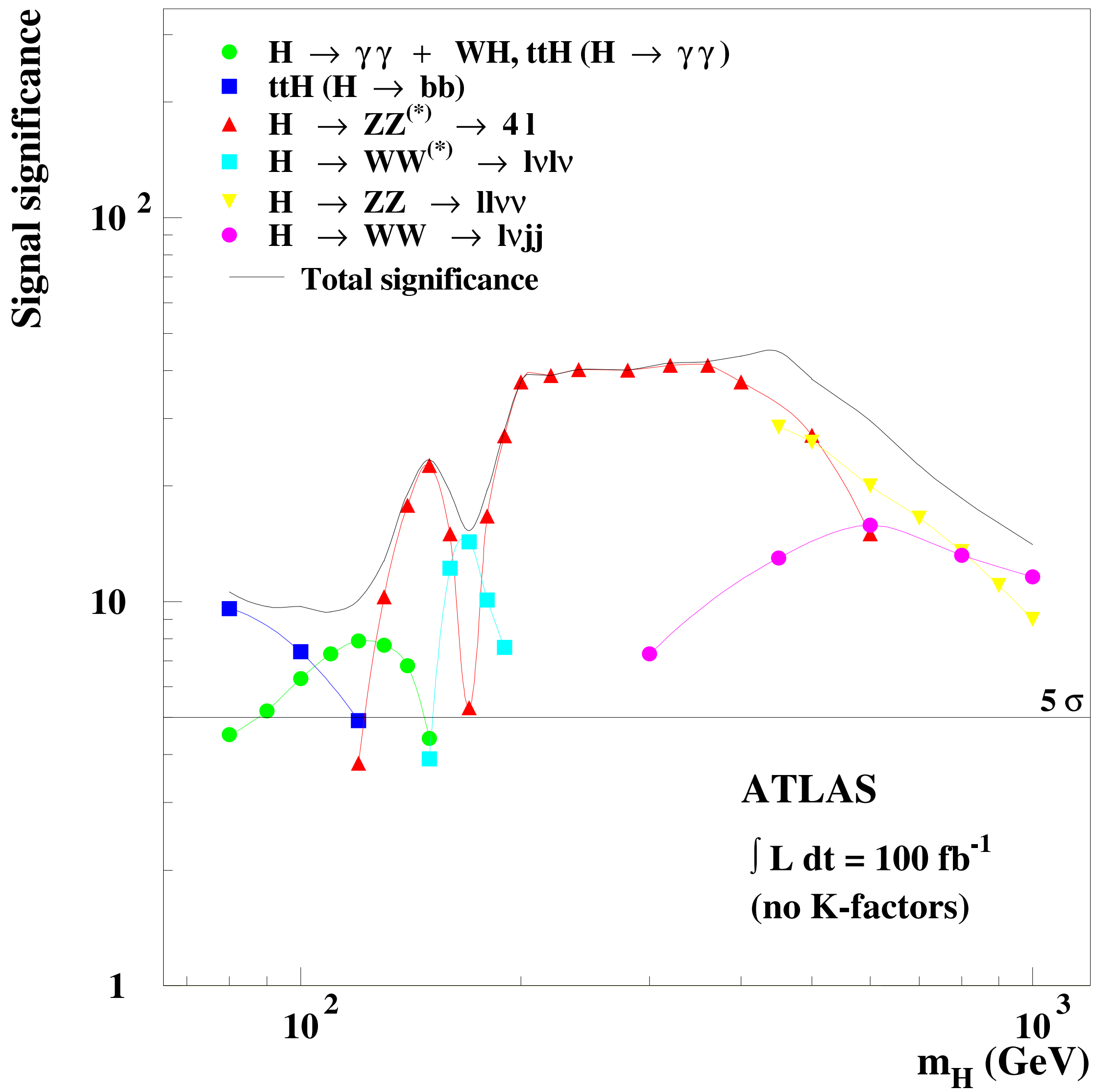


Tevatron Run II Preliminary
Cross-Section × Br (pb)
m_H (GeV)
10
1
10^-1
10^-2
110
120
130
140
150
160
170
180
WH→eνbb̄
D0: 382 pb^-1
ZH→νν̄bb̄
D0: 261 pb^-1
H→WW^(*)→lνlν
CDF: 184 pb^-1
WH→WWW
CDF: 194 pb^-1
ZH→νν̄bb̄
CDF: 289 pb^-1
WH→WWW
D0: 363-384 pb^-1
WH→lνbb̄
CDF: 319 pb^-1
H→WW^(*)→lνlν
D0: 299-325 pb^-1
SM gg→H→WW^(*)
SM WH→Wbb̄
SM WH→WWW
SM ZH→Zbb̄

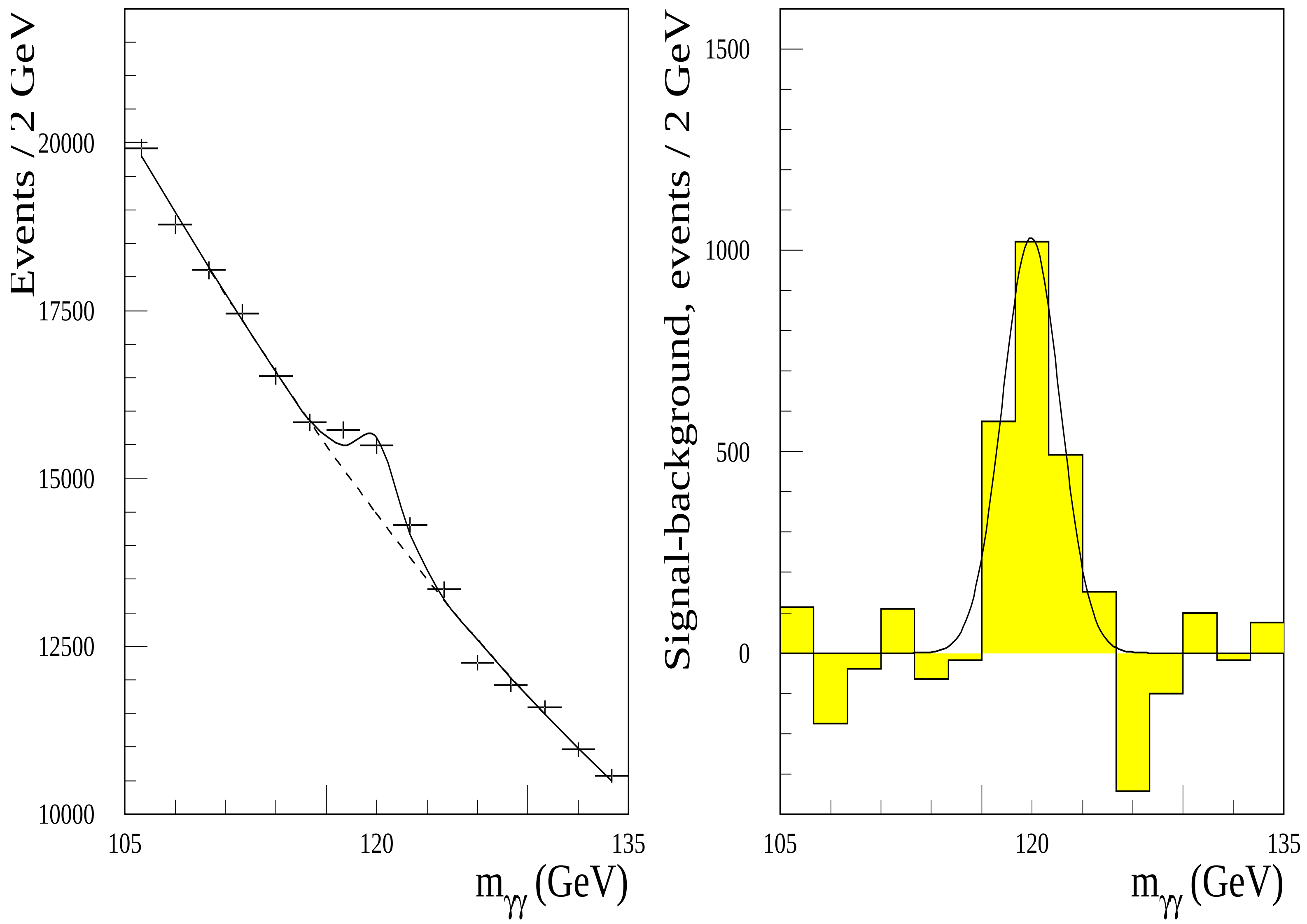

t → H± b search
CDF Run II Preliminary
Excluded 95 %CL
mt= 175 GeV/c 2
∫ L dt = 192 pb -1
SM Expected
SM ± 1 σ Expected
Excluded CDF Run II
Excluded LEP
Theoretically inaccessible
Theoretically inaccessible
LEP (ALEPH, DELPHI, L3 and OPAL)
Assuming H± → τν or H± → cs̄ only
MH+ (GeV/c2)
160
140
120
100
80
60
10-1
1
10
102
tan(β)
MSUSY=1000 GeV/c2, μ=-500 GeV/c2, At=Ab=2000 GeV/c2, Aτ=500 GeV/c2
M1=0.498*M2, M2=M3=MQ=MU=MD=ME=ML=MSUSY

• Huge QCD background at LHC requires triggering on leptonic decays of W, Z and t and exploiting $H \to \gamma\gamma$ and $H \to ZZ \to 4\ell^{\pm}$ resonances. → fig

• Expected 5σ discovery limits from ATLAS:



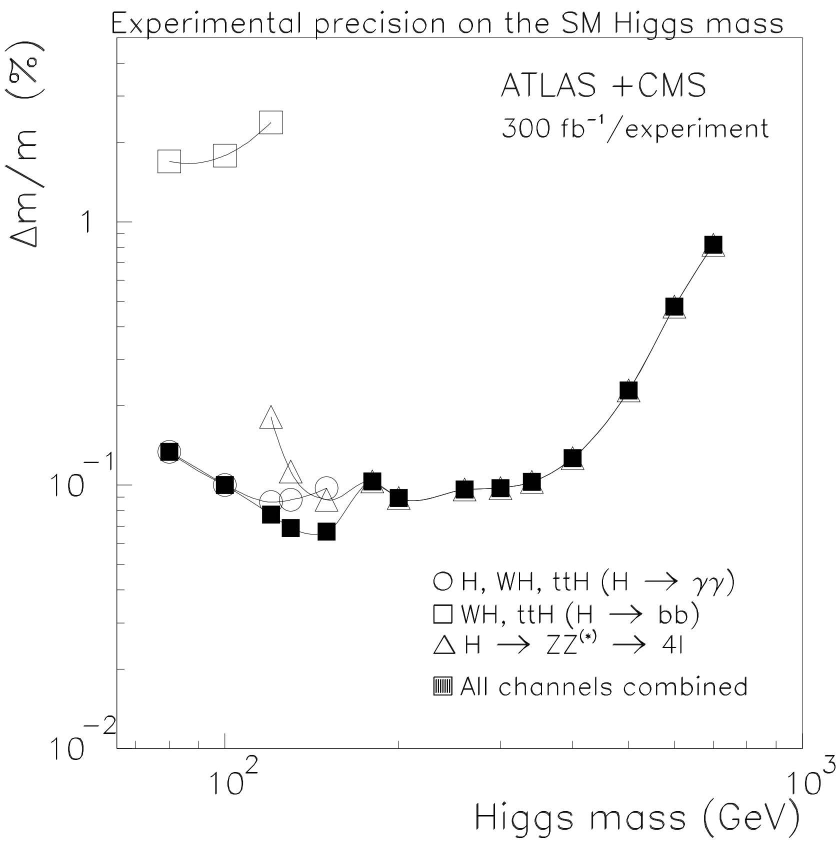
• LHC expected to cover entire canonical SM Higgs mass range: $M_H \lesssim 700$ GeV !



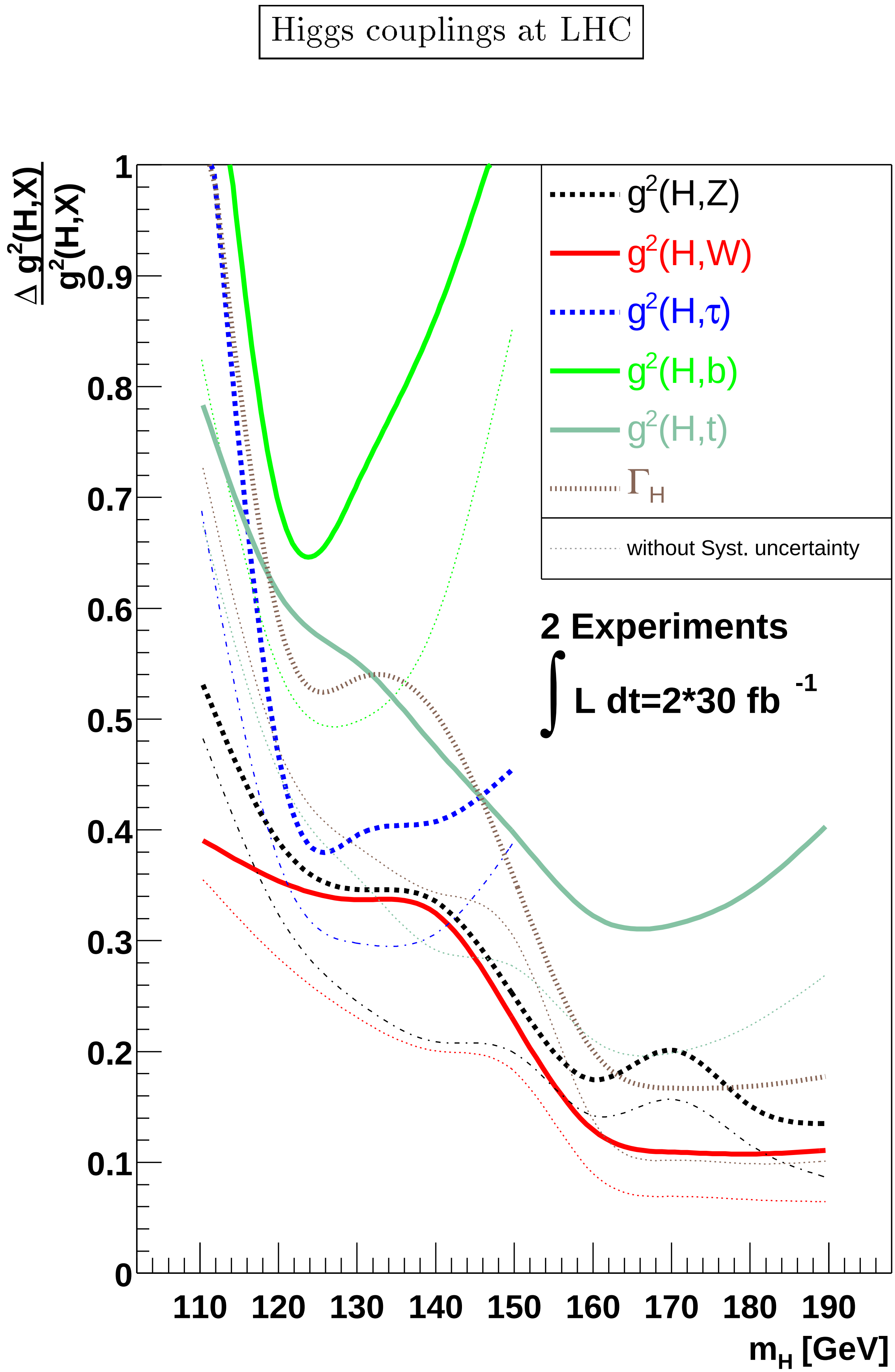
Events / 2 GeV
20000
17500
15000
12500
10000
105
120
135
$m_{\gamma\gamma}$ (GeV)
Signal-background, events / 2 GeV
1500
1000
500
0
105
120
135
$m_{\gamma\gamma}$ (GeV)

- To firmly establish Higgs nature need measuring:

1. mass:

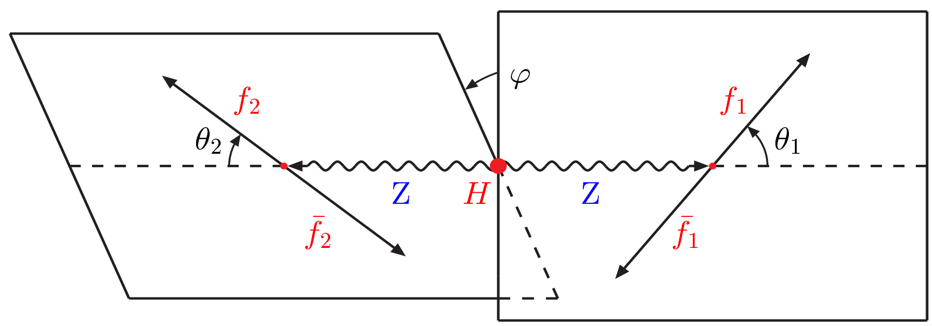


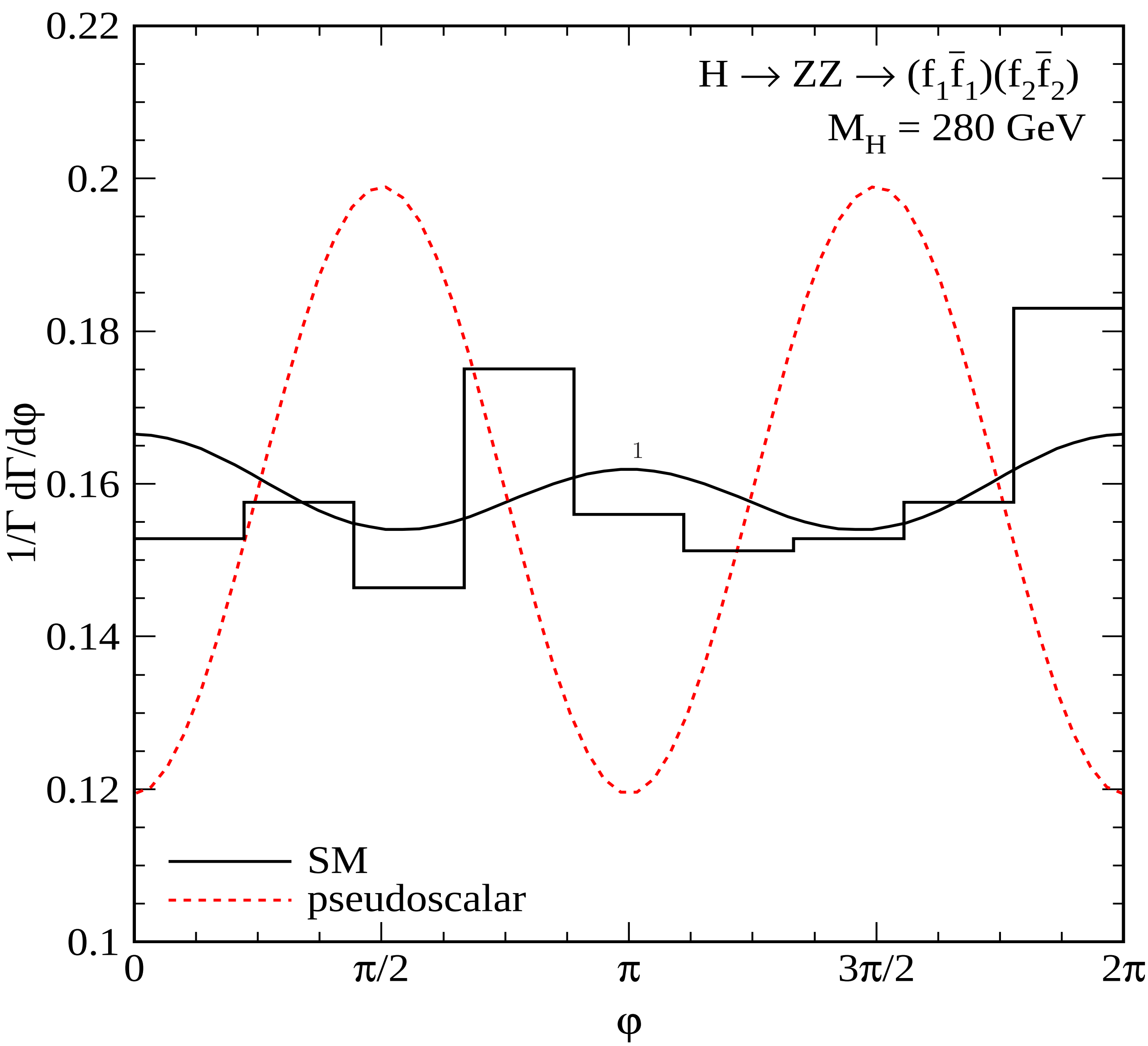
2. possibly lifetime/width
3. couplings to gauge bosons and fermions → fig
4. Higgs self-couplings
5. spin/parity quantum numbers (e.g., $H \to ZZ \to 4\ell$) → fig



Higgs couplings at LHC
Δ g²(H,X) / g²(H,X)
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
110
120
130
140
150
160
170
180
190
m_H [GeV]
g²(H,Z)
g²(H,W)
g²(H,τ)
g²(H,b)
g²(H,t)
Γ_H
without Syst. uncertainty
2 Experiments
∫ L dt=2*30 fb -1

Higgs Parity at LHC

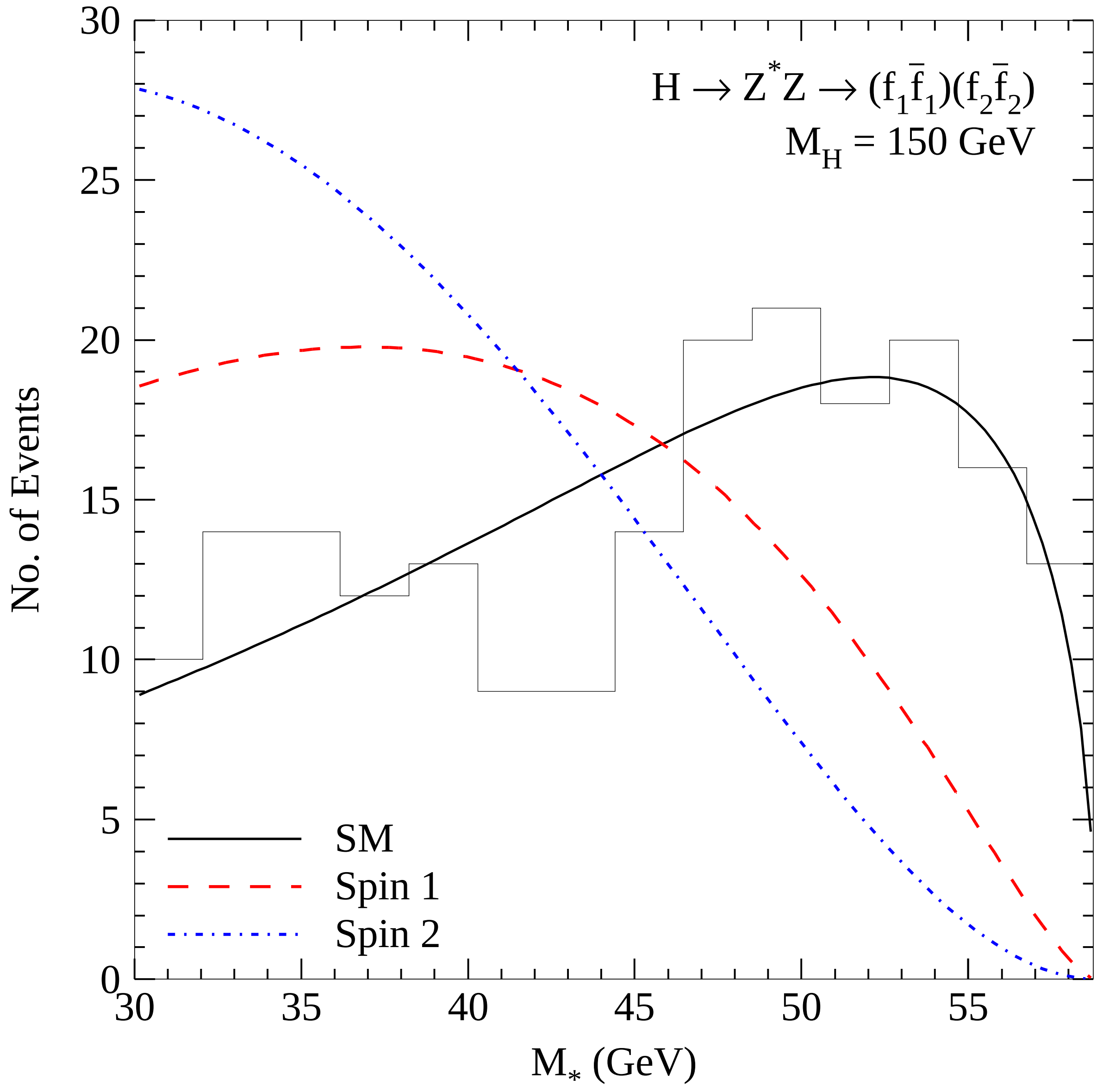




Higgs Spin at LHC

$$\frac{d?_H}{dM_*^2} = \frac{3G_F^2 M_Z^4 \delta_Z}{16\pi^3 M_H} \frac{12 M_*^2 M_Z^2 + M_H^4 \beta^2}{(M_*^2 - M_Z^2)^2 + M_Z^2 ?_Z^2} \beta$$

where β is the Z^*/Z three-momentum in H rest frame in units of Higgs particle mass M_H.



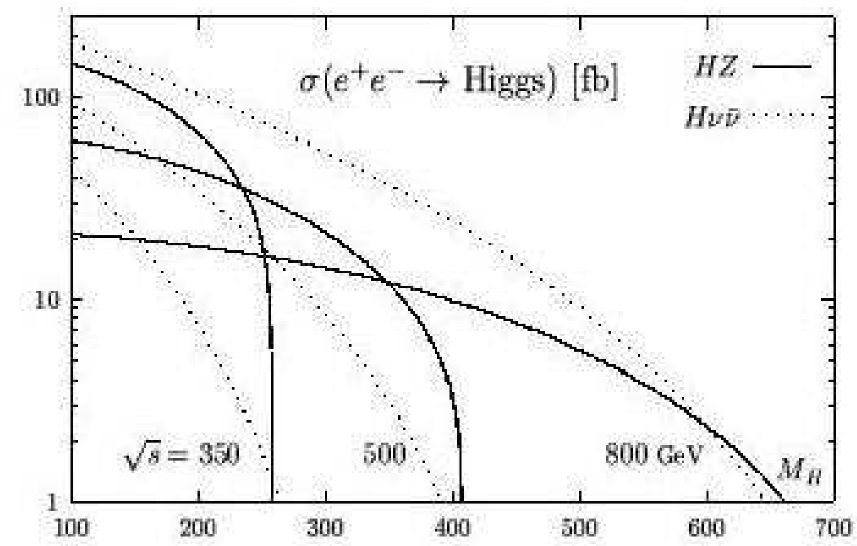
Example of Precision of Higgs Measurements at The next Linear Collider

For M_H = 140 GeV, 500 fb^{-1} @ 500 GeV

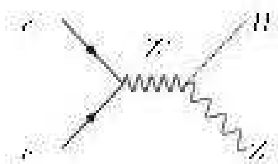
Mass Measurement	$\delta M_H \approx 60$ MeV $\approx 5 \times 10^{-4} M_H$
Total width	$\delta \Gamma_H / \Gamma_H \approx 3$ %
Particle couplings	
tt	(needs higher $\sqrt{s}$ for 140 GeV, except through $H \to gg$)
bb	$\delta g_{Hbb} / g_{Hbb} \approx 2$ %
cc	$\delta g_{Hcc} / g_{Hcc} \approx 22.5$ %
$\tau^+\tau^-$	$\delta g_{H\tau\tau} / g_{H\tau\tau} \approx 5$ %
WW*	$\delta g_{Hww} / g_{Hww} \approx 2$ %
ZZ*	**??**
gg	$\delta g_{Hgg} / g_{Hgg} \approx 12.5$ %
$\gamma\gamma$	$\delta g_{H\gamma\gamma} / g_{H\gamma\gamma} \approx 10$ %
Spin-parity-charge conjugation	
	establish $J^{PC} = 0^{++}$
Self-coupling	
	$\delta\lambda_{HHH} / \lambda_{HHH} \approx 32$ % (statistics limited)

If Higgs is lighter, precision is often better

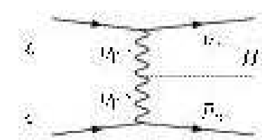
The Higgs Production Cross section at The next Linear Collider



Higgs-strahlung



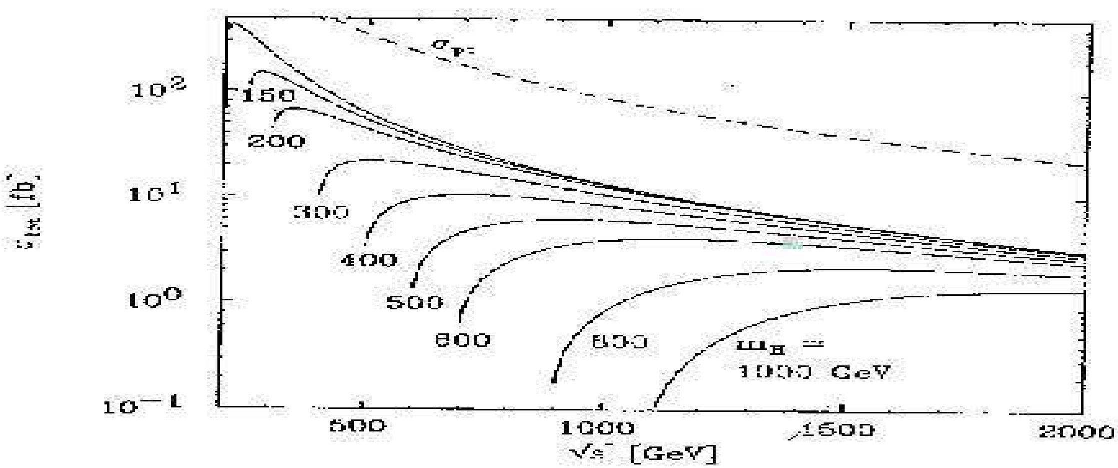
WW fusion



Recall, σ_{pt} = **87** nb / $(E_{cm})^2$ ~ **350** fb @ 500 GeV

The Higgs Production Cross section at The next Linear Collider

Higgs-strahlung



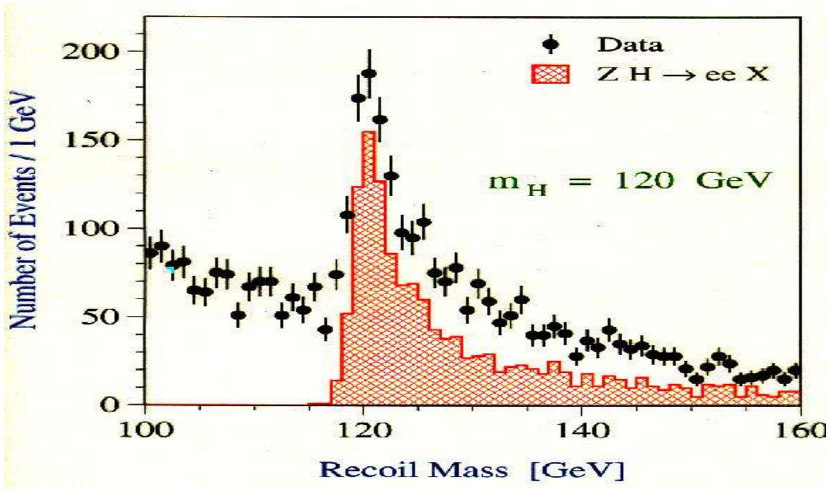
$e^+e^- \to ZH \to l^+ l^- X$
@ 500 GeV

M_H (GeV)	events/ 500 fb^{-1}
120	2020
140	1910
160	1780
180	1650
200	1500
250	1110

Higgs studies – The power of simple reactions

The LC can produce the Higgs recoiling from a Z, with known CM energy⇓, which provides a powerful channel for unbiassed tagging of Higgs events, allowing measurement of even invisible decays (⇓ - some beamstrahlung)

- Tag $Z \rightarrow l^+ l^-$
- Select $M_{recoil} = M_{Higgs}$



500 fb^{-1} @ 500 GeV, TESLA TDR, Fig 2.1.4

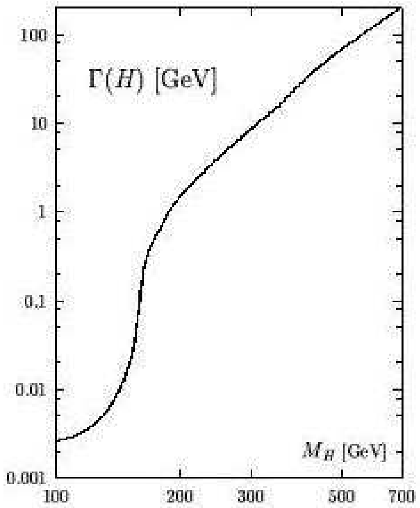
Invisible decays are included

Higgs studies – The Mass Measurement

M_H	δM_H (Recoil)	δM_H (Recon & fit)
120 GeV		40 MeV (3.3×10^{-4})
150 GeV	90 MeV	70 MeV (2×10^{-4})
180 GeV	100 MeV	80 MeV (4×10^{-4})

500 fb^{-1} @ 350 GeV, TESLA TDR, Table 2.2.1

Total width



$\Gamma_{TOT} = \Gamma_X / BR(H \rightarrow X)$

- $BR(H \rightarrow WW^*) = \Gamma_{WW} / \Gamma_{TOT}$
- Γ_{WW} from WW fusion cross section

M_H	WW fusion	Higgs-strahlung
120 GeV	6.1%	5.6%
140 GeV	4.5%	3.7%
160 GeV	13.4%	3.6%

500 fb^{-1} @ 350 GeV, TESLA TDR, Table 2.2.4

Γ_{TOT} to few%

Higgs Couplings - the branching ratios

M_H	$H \to b\bar{b}$	$H \to c\bar{c}$	$H \to gg$	$H \to \tau^+\tau^-$
120 GeV	2.9 %	39 %	18 %	7.9 %
140 GeV	4.1 %	45 %	23 %	10 %

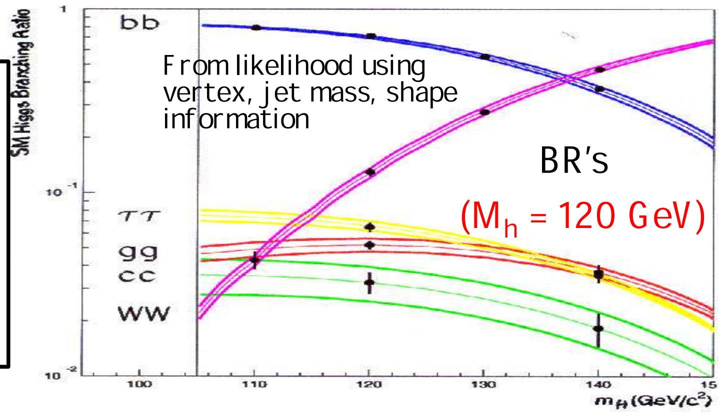
(through Higgs-strahlung, only)

500 fb^{-1} @ 500 GeV, LC Physics Resource Book, Table 3.1

At lower energy, including $e^+e^- \to H\nu\nu$, along with $e^+e^- \to ZH$

M_H	$H \to bb$	$H \to cc$	$H \to gg$	$H \to \tau^+\tau^-$
120 GeV	2.4 %	8.3 %	5.5 %	5.0 %
140 GeV	2.6 %	19.0 %	14.0 %	8.0 %
160 GeV	6.5 %			

500 fb^{-1} @ 350 GeV, TESLA TDR, Table 2.2.5



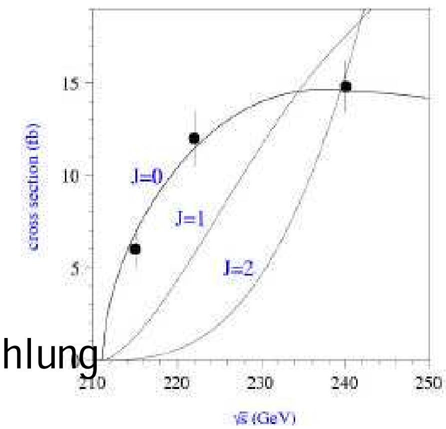
Measurement of BR's is powerful indicator of new physics

e.g. in MSSM, these differ from the SM in a characteristic way. Higgs BR must agree with MSSM parameters from many other measurements.

Higgs spin parity and charge conjugation (J^{PC})

$H \to \gamma\gamma$ or $\gamma\gamma \to H$ rules out J =1 and indicates C=+1

Threshold cross section ($e^+e^- \to Z\ H$) for J =0
$\sigma \sim \beta$, while for J >0, generally higher power of β (assuming $n = (-1)^J\ P$)

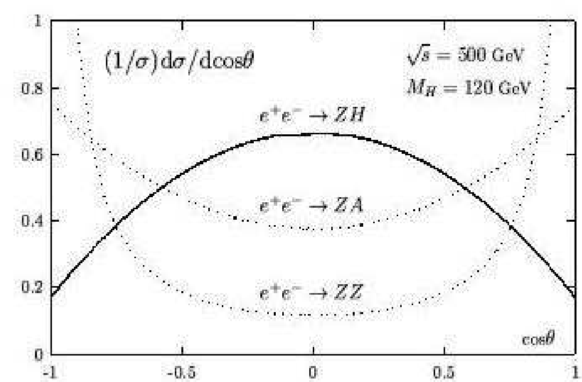


LC Physics Resource Book, Fig 3.23(a)

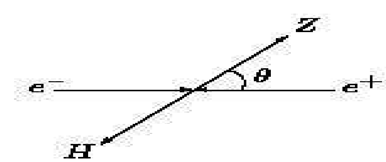
Production angle (θ) and Z decay angle in Higgs-strahlung reveals J^P ($e^+e^- \to Z\ H \to f\bar{f}H$)

	$J^P = 0^+$	$J^P = 0^-$
$d\sigma/d\cos\theta$	$\sin^2\theta$	$(1 - \sin^2\theta)$
$d\sigma/d\cos\phi$	$\sin^2\phi$	$(1 +/- \cos\phi)^2$

ϕ is angle of the fermion, relative to the Z direction of flight, in Z rest frame



TESLA TDR, Fig 2.2.8



Higgs self couplings

Measures Higgs potential λ

$V(\Phi)=\lambda(\Phi^2-\frac{1}{2}v^2)^2$ $\quad v \sim$ 246 GeV

$m_h^2=4\lambda v^2$

double Higgs-strahlung: $e^+e^- \rightarrow Zhh$

Study ZHH production and decay to 6 jets (4 b's). Cross section is small; premium on very good jet energy resolution. Can enhance x5 with positron polarization.

m_h (GeV/c²)	σ_{hhZ} (fb)	N_{hhZ}^{500}	ϵ_{hhZ}	$\mathcal{L}=$ 500 fb⁻¹	1000 fb⁻¹	2000 fb⁻¹
120	0.186	93.	43%	24.1%	17.3%	11.6%
130	0.149	74.	43%	26.6%	19%	17.7%
140	0.115	57.	39%	32%	23 %	17%

$\Delta\lambda/\lambda$ error 36% → 18%

Supersymmetry

Outline

- Why Supersymmetry ?
- The hierarchy problem and gauge coupling unification
- The Minimal Supersymmetric Standard Model (MSSM)
- Indirect Searches: $g - 2$
- Collider Searches

Why supersymmetry?

- No direct experimental evidence of SUSY exists to date ...
- Nonetheless, prime candidate for BSM physics !

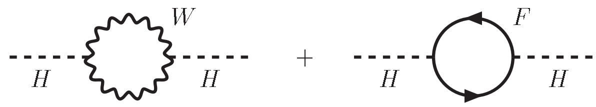
1. SUSY is a generalisation of space-time symmetries of QFT transforming fermions into bosons and vice versa.
2. It also provides a framework for unification of particle physics and gravity, at a scale $M_{\text{Planck}} \approx 10^{19}$ GeV.
3. If SUSY were an exact symmetry of Nature, particles and superpartners would be degenerate in mass.
4. Thus, SUSY cannot be an exact symmetry of nature: if it is realised, it must be broken.
5. Crucial question for phenomenology: at what scale might SUSY be broken ? Is there a reason why masses of superpartners should not be as heavy as M_{Planck} ?

- Most compelling motivation for existence of TeV scale SUSY particles at TeV scale is linked to so-called hierarchy problem: instability of Higgs mass under quadratically divergent radiative corrections.

- Additional support for TeV scale SUSY comes from unification of gauge couplings in SUSY GUTs.

• <u>Hierarchy problem</u>: why is Higgs mass so much smaller than Planck scale ?

Quantum corrections to Higgs mass are quadratically divergent as internal momentum in loops becomes very large.



$\rightarrow \delta m_H^2 \sim \frac{\alpha}{\pi}(\Lambda^2 + m_F^2)$

• Cutoff Λ represents scale up to which SM remains valid.

• If $\Lambda \sim M_{\mathrm{Planck}}$ extreme fine-tuning between bare Higgs mass and quantum fluctuations δM_H^2 would be needed to generate a physical Higgs mass of order $\mathcal{O}(100)$ GeV.

• Most elegant solution is to introduce additional symmetry that transforms fermions into bosons: SUSY !

• <u>Pauli's principle</u>: additional Higher Order (HO) corrections due to superpartners enter δM_H^2 with sign opposite to SM contributions

$\rightarrow$ divergent terms cancel:

$\rightarrow \delta m_H^2 \sim -\frac{\alpha}{\pi}(\Lambda^2 + \widetilde{m}_F^2)$

• Since $\delta m_H^2 \sim \frac{\alpha}{\pi}(m_F^2 - \widetilde{m}_F^2)$, any fine-tuning is avoided for SUSY particle (sparticle) masses $\widetilde{m} \lesssim \mathcal{O}(1$ TeV$)$.

• Argument is qualitative and does not tell where SUSY is !

• In renormalisable theories all infinities can be absorbed in bare parameters: one might not need worry about SM fine-tuning.

• Argument tells that large hierarchy is intrinsically unstable:

$\rightarrow$ SUSY very plausible way of stabilising it !

Coupling unification

- Additional support for SUSY $\tilde{m} \lesssim \mathcal{O}(1\ \mathrm{TeV})$ follows from gauge coupling unification.
- GUTs seek gauge group including $SU(3)$, $SU(2)$ and $U(1)$.
- Apparent obstacle is $\alpha_s \gg \alpha$ at EW scale, yet quantum corrections introduce energy dependence:

$$\frac{d\alpha_i(\mu)}{d\ln\mu^2} = \beta_i(\alpha_i(\mu)) \qquad \beta_i = -\beta_{i,0}\alpha_i^2 + \mathcal{O}(\alpha_i^3).$$

- β-functions depend on gauge group and matter multiplets to which gauge bosons couple.
- Only particles with mass $< \mu$ contribute to β_i and to coupling evolution at $Q \leq \mu$.
- SM couplings evolve with μ according to

$$\begin{aligned} SU(3) &: \beta_{3,0} = (33 - 4n_g)/(12\pi) \\ SU(2) &: \beta_{2,0} = (22 - 4n_g - n_h/2)/(12\pi) \\ U(1) &: \beta_{1,0} = (-4n_g - 3n_h/10)/(12\pi) \end{aligned}$$

($n_g = 3$ quark/lepton generations; $n_h = 1$ Higgs doublets).

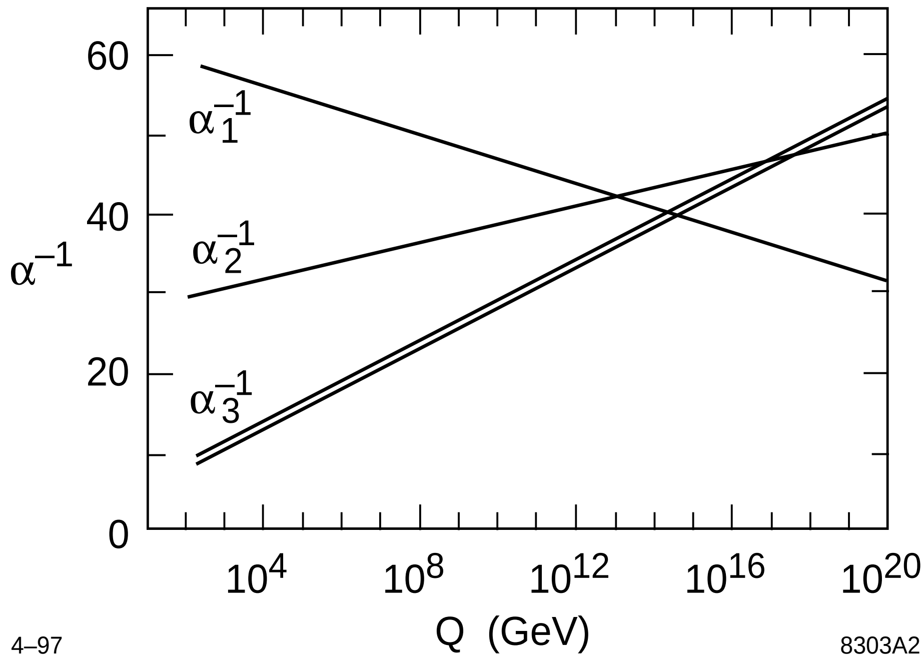
- $SU(3)$, $SU(2)$ β-functions negative ($\beta_0 > 0 \to \beta < 0$):

$\to$ α_3 and α_2 decrease as μ increases (asymptotic freedom).

- $U(1)$ β-function negative and α_1 increases with μ:

$\to$ extrapolated to high energy, couplings must converge !

• SM coupling evolution:



• Couplings do not come to a common value at any scale.

• Loop contributions of superpartners change β-functions hence evolution of gauge couplings in SUSY.

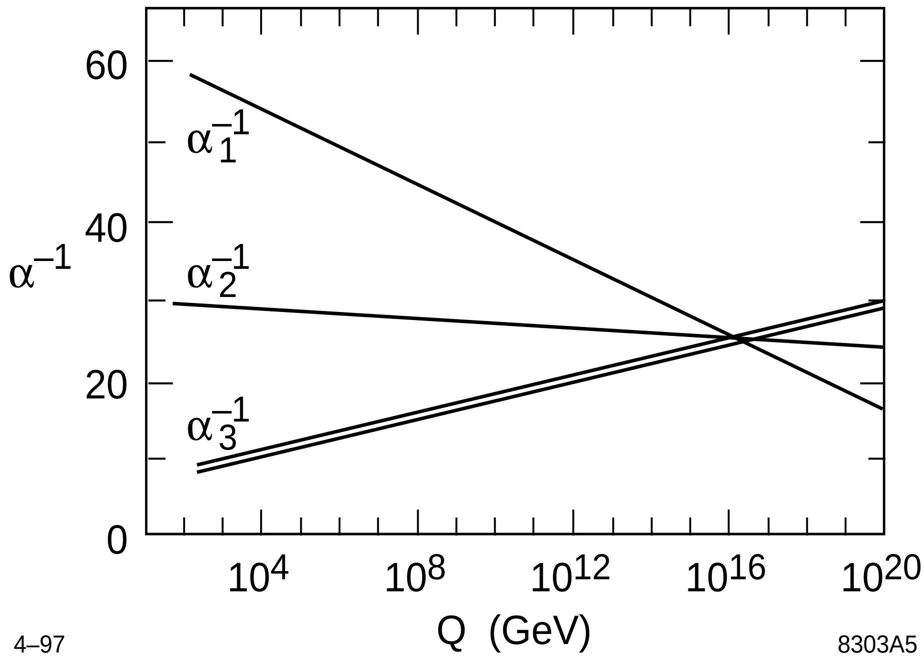
• Most economical model (MSSM):

$$
\begin{aligned}
SU(3) &: \quad \beta_{3,0}^{\mathrm{SUSY}} = (27 - 6n_g)/(12\pi) \\
SU(2) &: \quad \beta_{2,0}^{\mathrm{SUSY}} = (18 - 6n_g - 3n_h/2)/(12\pi) \\
U(1) &: \quad \beta_{1,0}^{\mathrm{SUSY}} = (-6n_g - 9n_h/10)/(12\pi)
\end{aligned}
$$

• Couplings evolution changes also because SUSY requires two Higgs doublets:

$\rightarrow n_h = 2$ in above equations above ...

• After including SUSY:



• Coupling implies SUSY masses $\widetilde{m} \lesssim \mathcal{O}(1 \text{ TeV})$.

• Theoretical uncertainties (e.g. model-dependent thresholds) are such that one cannot constrain $\widetilde{m}$ very tightly ...

• Higgs mechanism: Supergravity realisations of SUSY with universal scalar masses at GUT scale, one mass evolves down to negative values inducing EWSB !

- MSSM based upon:

1. minimal particle content;
2. Poincare invariance;
3. gauge invariance;
4. SUSY.

- MSSM particle content:

Gauge bosons $S=1$	Gauginos $S=1/2$
gluon, $W^{\pm}, Z, \gamma$	gluino, $\widetilde{W}, \widetilde{Z}, \widetilde{\gamma}$
Fermions $S=1/2$	Sfermions $S=0$
$\begin{pmatrix} u_L \\ d_L \end{pmatrix} \begin{pmatrix} \nu_L^e \\ e_L \end{pmatrix}$	$\begin{pmatrix} \widetilde{u}_L \\ \widetilde{d}_L \end{pmatrix} \begin{pmatrix} \widetilde{\nu}_L^e \\ \widetilde{e}_L \end{pmatrix}$
u_R, d_R, e_R	$\widetilde{u}_R, \widetilde{d}_R, \widetilde{e}_R$
Higgs	Higgsinos
$\begin{pmatrix} H_2^0 \\ H_2^- \end{pmatrix} \begin{pmatrix} H_1^+ \\ H_1^0 \end{pmatrix}$	$\begin{pmatrix} \widetilde{H}_2^0 \\ \widetilde{H}_2^- \end{pmatrix} \begin{pmatrix} \widetilde{H}_1^+ \\ \widetilde{H}_1^0 \end{pmatrix}$

- Charged and neutral higgsinos mix with non-coloured gauginos to form physical mass eigenstates, so-called charginos $\widetilde{\chi}_{1,2}^{\pm}$ neutralinos $\widetilde{\chi}_{1,\ldots 4}^{0}$.
- Mixing also expected in b, t and τ sfermion sector.

• Introduce additional discrete symmetry, R-parity:

→ forbid $B - L$ violating interactions (i.e., no proton decay).

• SM particles are R-even while SUSY ones are R-odd !

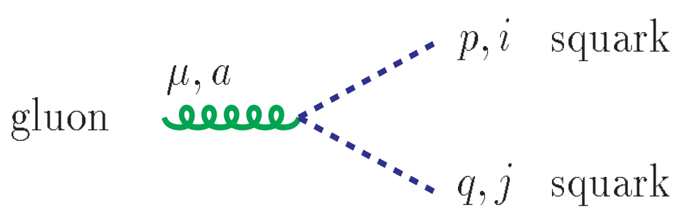
• MSSM with R-parity conservation:

→ SUSY in pairs and Lightest SUSY Particle (LSP) is stable.

• Sparticles interactions fixed by gauge symmetries and SUSY:

→ no adjustable parameters, i.e. predictive !

• $q, g, \tilde{q}, \tilde{g}$ interactions determined by α_s only:



$$= -i\, g_s\, (T_a)_{ij} (p+q)^\mu$$

• SUSY is <u>non-exact symmetry of nature</u>:

→ SM particle and SUSY sparticles non-degenerate masses !

• Mechanism of SUSY breaking not understood:

$$\mathcal{L} = \mathcal{L}(\text{SUSY}) + \mathcal{L}(\text{SUSY-breaking}).$$

(see Sacha's course).

• (<u>Soft-breaking terms terms</u> are consistent with Poincare and SM gauge invariance) and <u>do not reintroduce</u> quadratic divergences for scalar particles !)

• $>$ 100 free parameters to parametrise SUSY breaking: $\rightarrow$ assume universality of parameters at Plank scale

• Introduce SUSY-breaking framework, e.g. Supergravity (mSUGRA):

$\rightarrow$ SUSY breaking parameters become related and number of MSSM free parameters in Supergravity models is reduced to only five (mSUGRA)

• At $\mathcal{O}(M_{\rm Planck})$:

1. scalars (Higgs bosons, $\tilde{\ell}$ and $\tilde{q}$) have common mass, $\widetilde{M}_0$;
2. gauginos ($\tilde{B}$, $\tilde{W}$, and $\tilde{g}$) have common mass, $\widetilde{M}_{1/2}$;
3. trilinear couplings have common value, A_0;

plus

4. Ratio of two Higgs vacuum expectation values, $\tan\beta$,
5. Higgs mass parameter in Superpotential, sign(μ).

• Evolving universal masses from $M_{\rm Planck}$ to EW scale (RGEs):

$\rightarrow$ entire spectrum of SUSY particles can be generated !

- Two-doublet Higgs models are anomaly-free, e.g. MSSM.

- SUSY structure also requires (at least) two Higgs doublets to generate masses for both "up"-type and "down"-type quarks and charged leptons.

- MSSM Higgs sector consists of five physical Higgs particles:

$\rightarrow$ two CP-even neutral Higgses, h^0 and H^0 $(M_{h^0} \leq M_{H^0})$

$\rightarrow$ one CP-odd neutral Higgs boson, A^0

$\rightarrow$ a charged Higgs boson pair, $H^\pm$.

- AT LO, $\tan\beta$ (ratio of VEVs) and one Higgs mass (M_{A^0}) completely determine MSSM Higgs sector.

- E.g. at LO: $M_{h^0} < M_Z$, $M_{A^0} < M_{H^0}$ and $M_W < M_{H^\pm}$!

- Sparticle (virtual) effects enter in higher orders via

$$\epsilon = \frac{3G_F}{\sqrt{2}\pi^2}\frac{M_t^4}{\sin^2\beta}\log\left(1+\frac{M_S^2}{M_t^2}\right).$$

- When radiative corrections are included (NLO):

$$M_{h^0}^2 \leq M_Z^2\cos^2 2\beta + \epsilon\sin^2\beta\,.$$

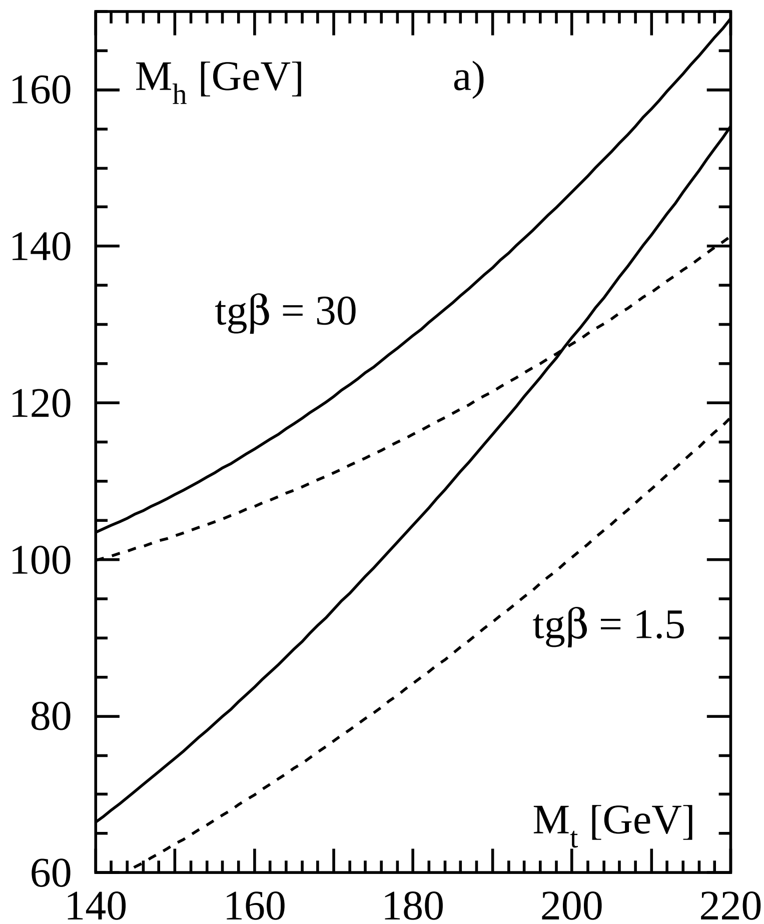
- NNLO (almost): $\rightarrow$ fig

$$M_{h^0} \lesssim 130 \text{ GeV !} \qquad \text{(MSSM)}$$

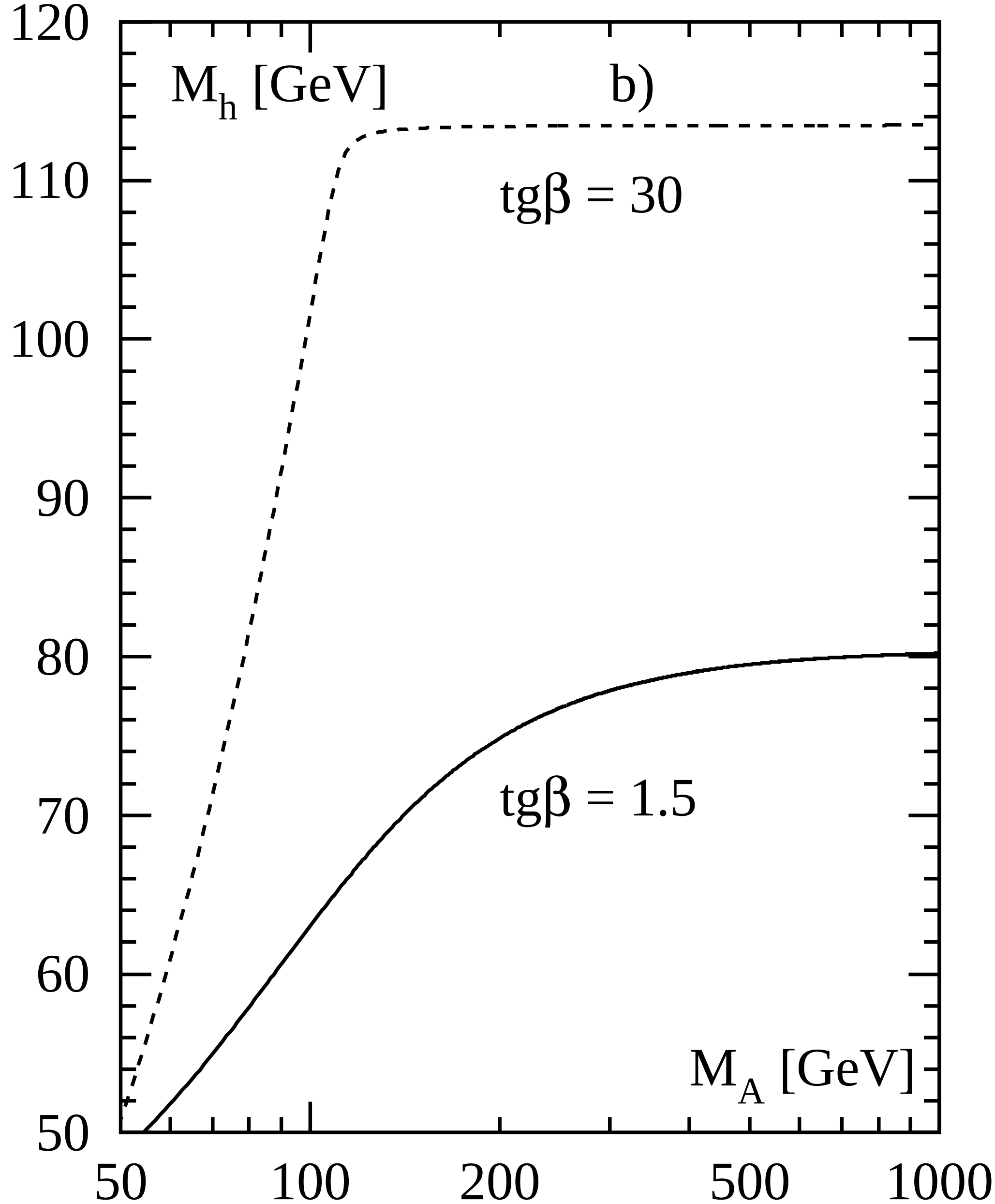
- Generalise: existence of light Higgs boson with $M_{h^0} \lesssim 200$ GeV is generic prediction of SUSY and its search is <u>most important</u> test of SUSY theories.

- Assume two scenarios:

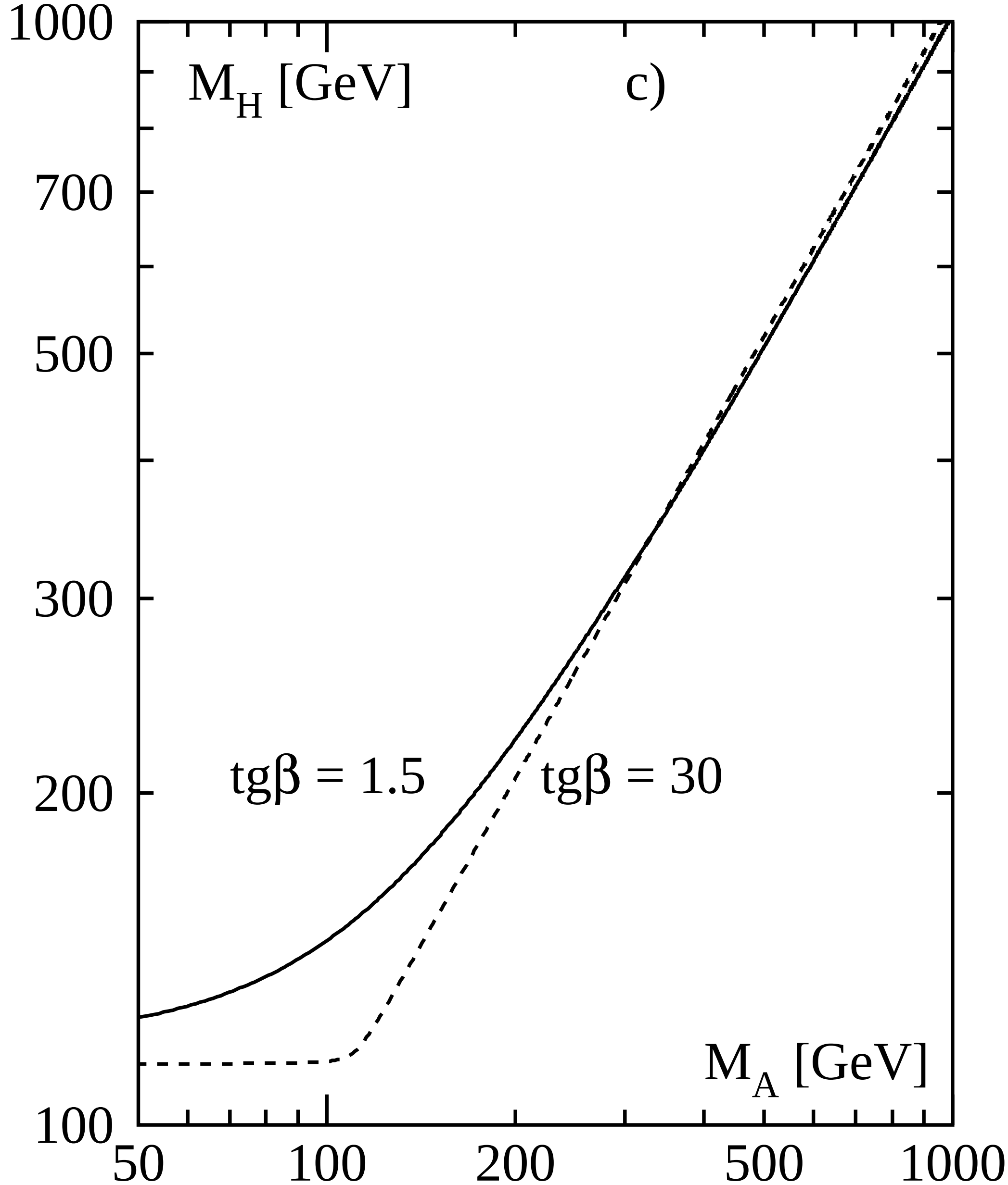
  1. Minimal mixing $\mu = A_t = A_b = 0$ (dash);
  2. Maximal mixing $\mu = 0$, $A_b = 0$, $A_t = \sqrt{6} M_{\rm SUSY}$ (solid).



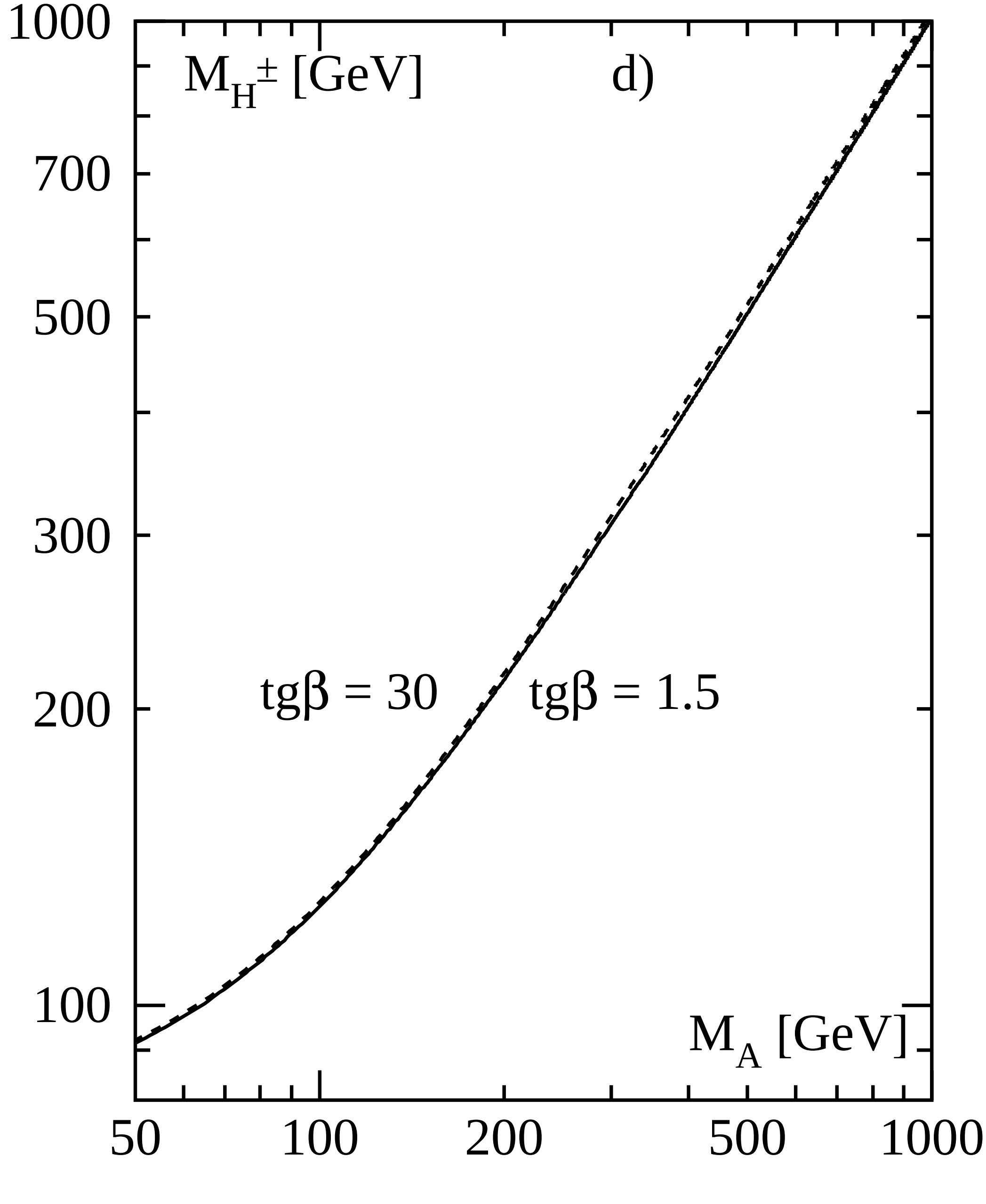
- M_{h^0} rather sensitive to top mass:



120
110
100
90
80
70
60
50
M_h [GeV]
b)
tgβ = 30
tgβ = 1.5
M_A [GeV]
50
100
200
500
1000



1000
700
500
300
200
100
M_H [GeV]
c)
tgβ = 1.5
tgβ = 30
M_A [GeV]
50
100
200
500
1000



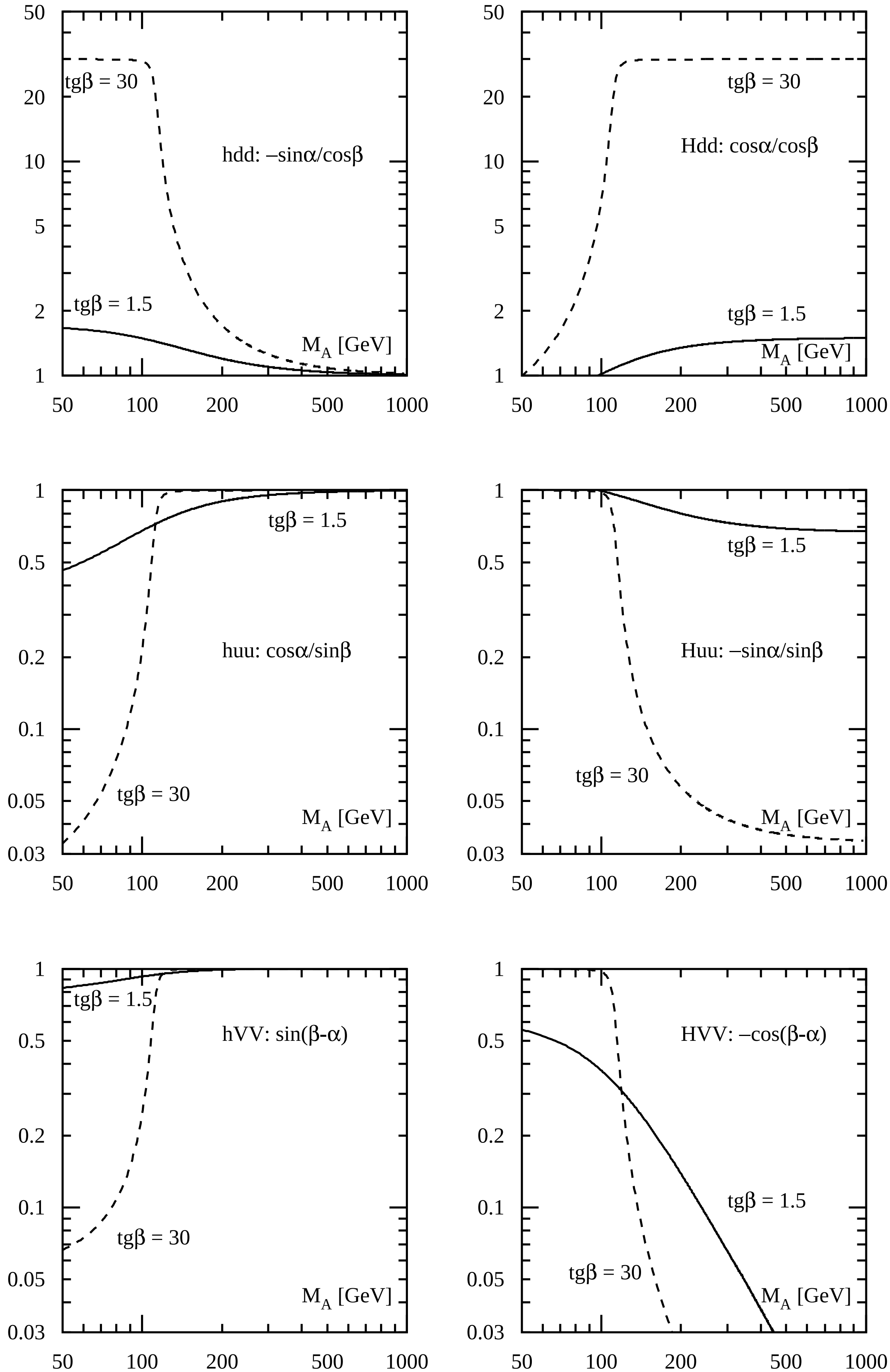
1000
700
500
300
200
100
$M_{H^\pm}$ [GeV]
d)
tgβ = 30
tgβ = 1.5
M_A [GeV]
50
100
200
500
1000

- Higgs mixing parameter α is fixed by $\tan\beta$ and M_{A^0},

$$\tan 2\alpha = \tan 2\beta \frac{M_{A^0}^2 + M_Z^2}{M_{A^0}^2 - M_Z^2 + \epsilon / \cos 2\beta} \quad \text{with} \quad -\frac{\pi}{2} < \alpha < 0 \,.$$

- Higgs couplings to ordinary matter: → fig

Φ		g_u^Φ	g_d^Φ	g_V^Φ
SM	H	1	1	1
MSSM	h	$\cos\alpha / \sin\beta$	$-\sin\alpha / \cos\beta$	$\sin(\beta-\alpha)$
	H	$\sin\alpha / \sin\beta$	$\cos\alpha / \cos\beta$	$\cos(\beta-\alpha)$
	A	$1 / \tan\beta$	$\tan\beta$	0



hdd: –sinα/cosβ
tgβ = 30
tgβ = 1.5
M_A [GeV]
50
20
10
5
2
1
50
100
200
500
1000
Hdd: cosα/cosβ
tgβ = 30
tgβ = 1.5
M_A [GeV]
huu: cosα/sinβ
tgβ = 1.5
tgβ = 30
M_A [GeV]
1
0.5
0.2
0.1
0.05
0.03
Huu: –sinα/sinβ
tgβ = 1.5
tgβ = 30
M_A [GeV]
hVV: sin(β-α)
tgβ = 1.5
tgβ = 30
M_A [GeV]
HVV: –cos(β-α)
tgβ = 1.5
tgβ = 30
M_A [GeV]

Indirect searches: $g-2$

- SUSY can appear via HO effects in precision observables:

$\rightarrow$ μ magnetic moment provides limits on BSM physics !

- Dirac equation for μ in EM field A is given by

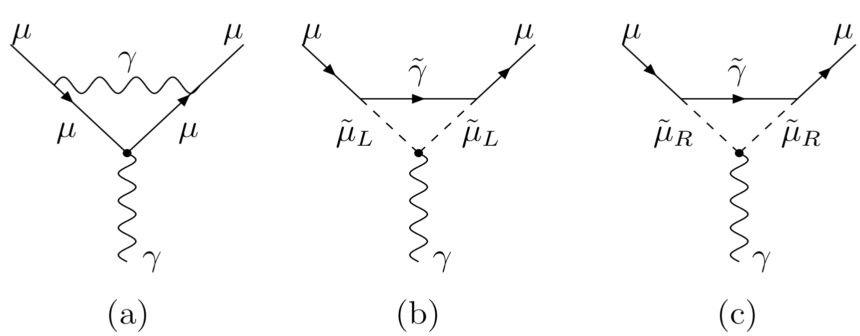
$$(\not\partial - e\,\not A - m_\mu)\psi_\mu = 0.$$

- For external magnetic field, Hamiltonian for a μ is given by

$$\mathcal{H} = \frac{e}{m}\vec{S}(\mu)\cdot\vec{B},$$

($\vec{S} \rightarrow$ spin vector; $\vec{B} \rightarrow$ magnetic field)

- Bohr magneton of electron $\mu_B = \frac{e}{2m_e}$ and magnetic moment of muon $\mu_\mu \equiv g_\mu \mu_B (m_e \rightarrow m_\mu)$.
- Dirac equation predicts $g_\mu = 2$ (tree − level).
- One-loop corrections from SM (QED) and SUSY (MSSM)



(a) photon, (b) photino+left-handed smuon, (c) photino+right-handed smuon.

- SM QED at one-loop:

$$\Delta\left(\frac{g_\mu - 2}{2}\right) = \frac{\alpha_{QED}(Q^2=0)}{2\pi} = 0.0011614.$$

- SUSY QED at one-loop

$$\left(\frac{g-2}{2}\right)^{susy}_{\mu} = -\frac{m_\mu^2 e^2}{8\pi^2}\int_0^1 dx \frac{x^2(1-x)}{m_\mu^2 x^2 + (m^2_{\tilde{\mu}_L} - M^2_{\tilde{\gamma}} - m_\mu^2)x + M^2_{\tilde{\gamma}}}.$$

- Latter is quadratic in m_μ^2 !

- Consider integral in limit $m_{\tilde{\mu}_L} = m_\mu$ and $M_{\tilde{\gamma}} = 0$,

$$\left(\frac{g-2}{2}\right)^{\rm susy}_{\mu} = -\frac{\alpha_{\rm em}}{6\pi}.$$

- SM and SUSY contribution have opposite sign !

- Assume light photino and heavy smuon ($m_{\tilde{\mu}_L} \gg M_{\tilde{\gamma}}, m_\mu$):

$$\left(\frac{g-2}{2}\right)^{\rm susy}_{\mu} = -\frac{\alpha_{\rm em}}{6\pi}\frac{m_\mu^2}{m^2_{\tilde{\mu}_L}}.$$

- Realistic scenario for SUSY breaking

- SUSY contribution decouples rapidly !

- SUSY correction $\propto$ fermion mass squared,

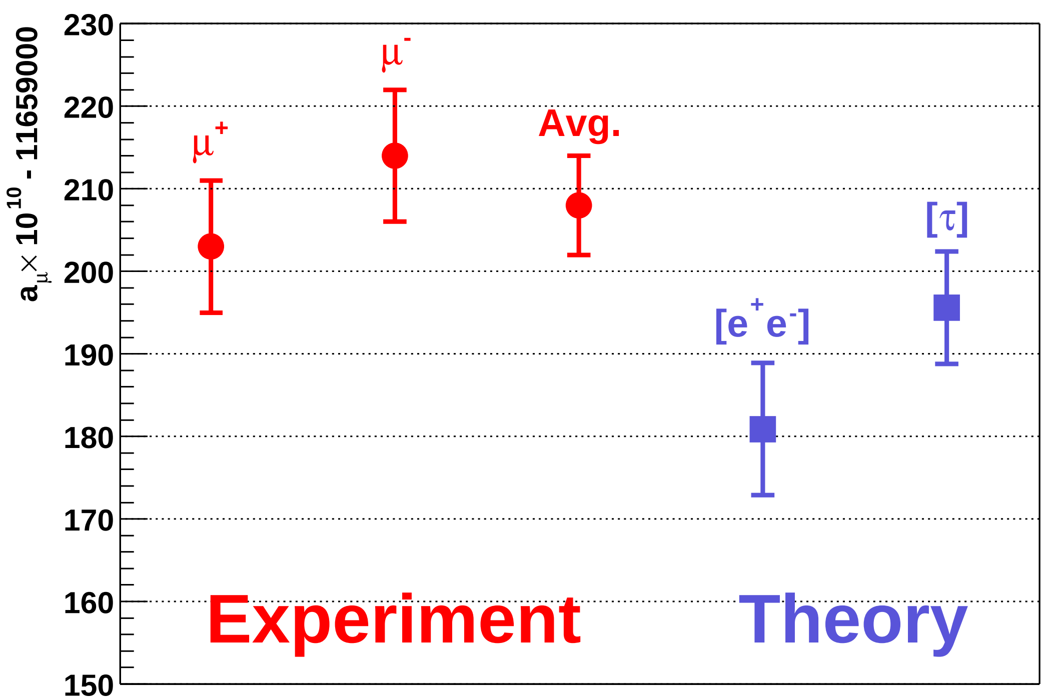
$\rightarrow$ suppressed for electron !

• BNL E821 experiment has reported measurement of

$$a_\mu \equiv \frac{1}{2}(g_\mu - 2) =$$

$\rightarrow$ 2.7σ from SM based $e^+e^- \rightarrow$ hadrons data.

• Discrepancy is $\approx 0.8\sigma$ if one uses $\tau \rightarrow$ hadrons SM data:



(http://www.g-2.bnl.gov/index.shtml)

• Though situation is not conclusive, a_μ does provide a significant constraint on low-energy SUSY.

MSSM Higgs collider searches

- There exist limits from LEP2 and Tevatron, again assume:

  1. Minimal mixing scenario → fig
  2. Maximal mixing scenario → fig

- MSSM parameter space available is reducing !

- LHC prospects from ATLAS and CMS: → figs

→ no-lose theorem but large decoupling area !

- Ought to observe second Higgs state or make precision measurements of BRs, ? s, etc. → fig

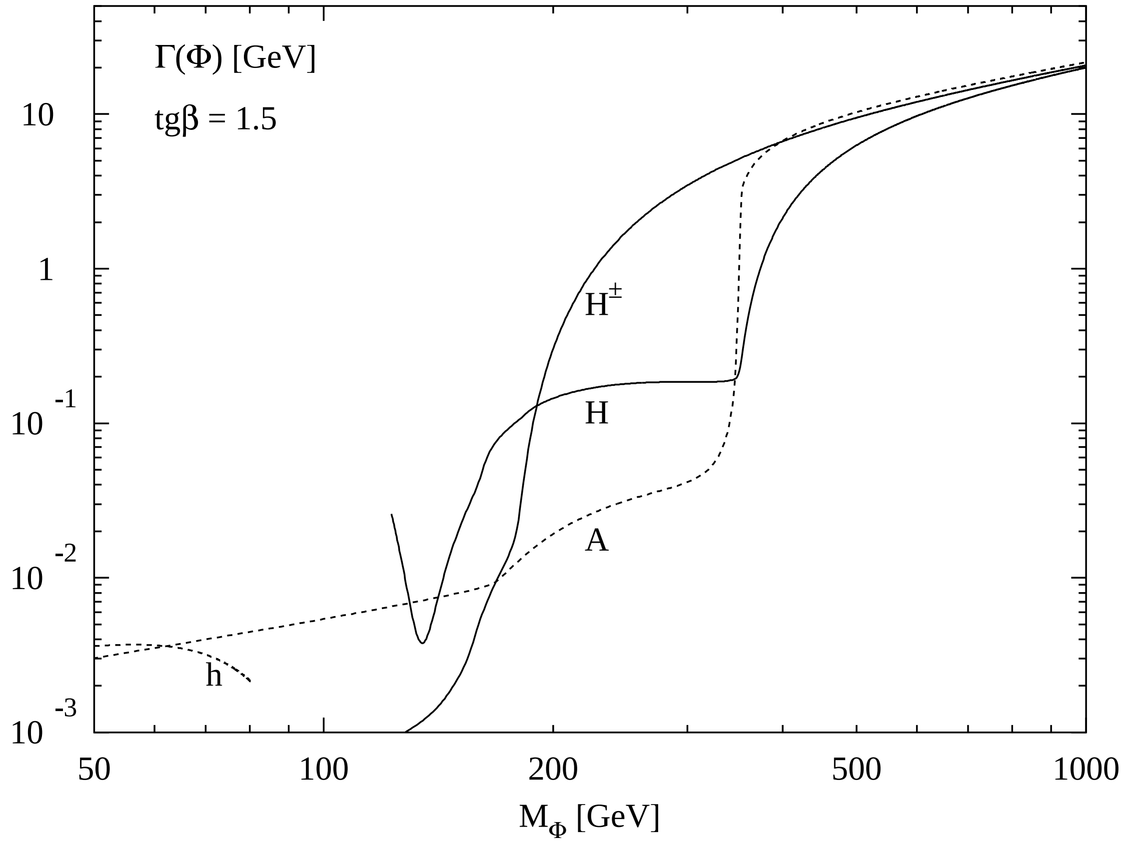
- Can construct modifications of MSSM: e.g.

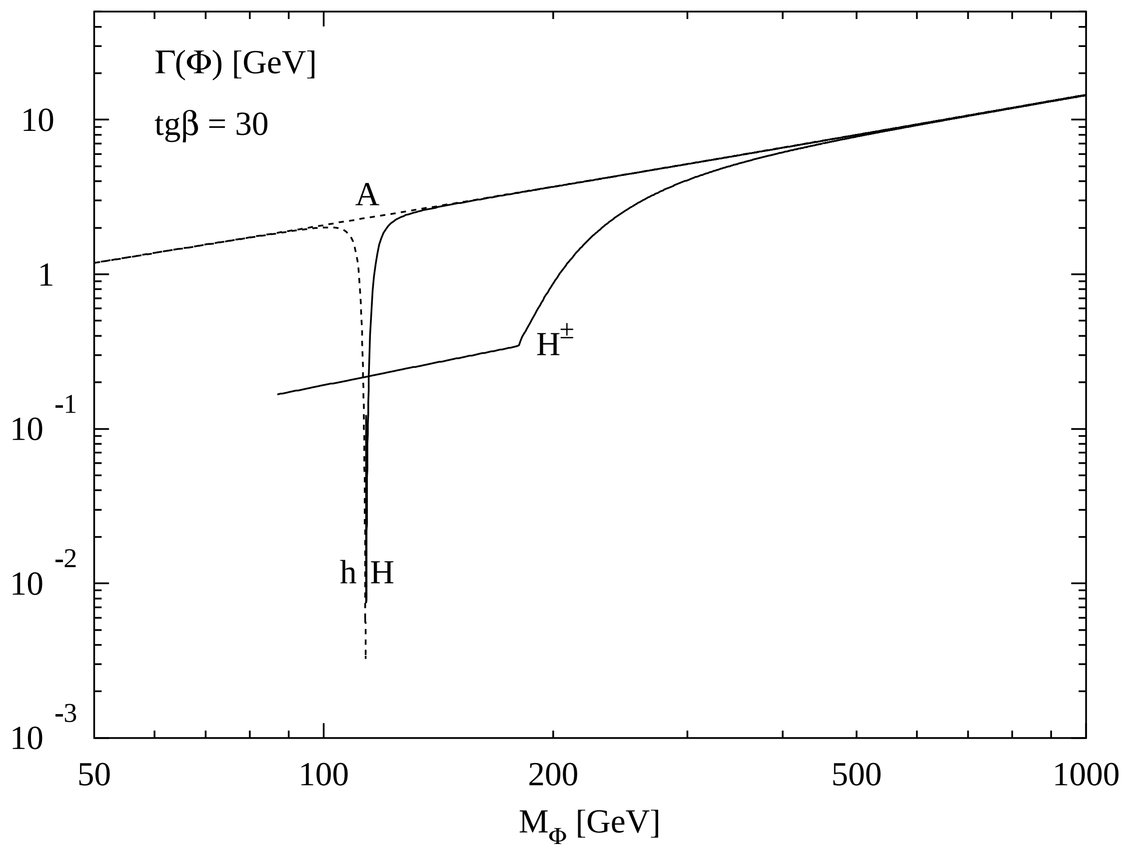
  1. Next-to-MSSM (NMSSM) → fig

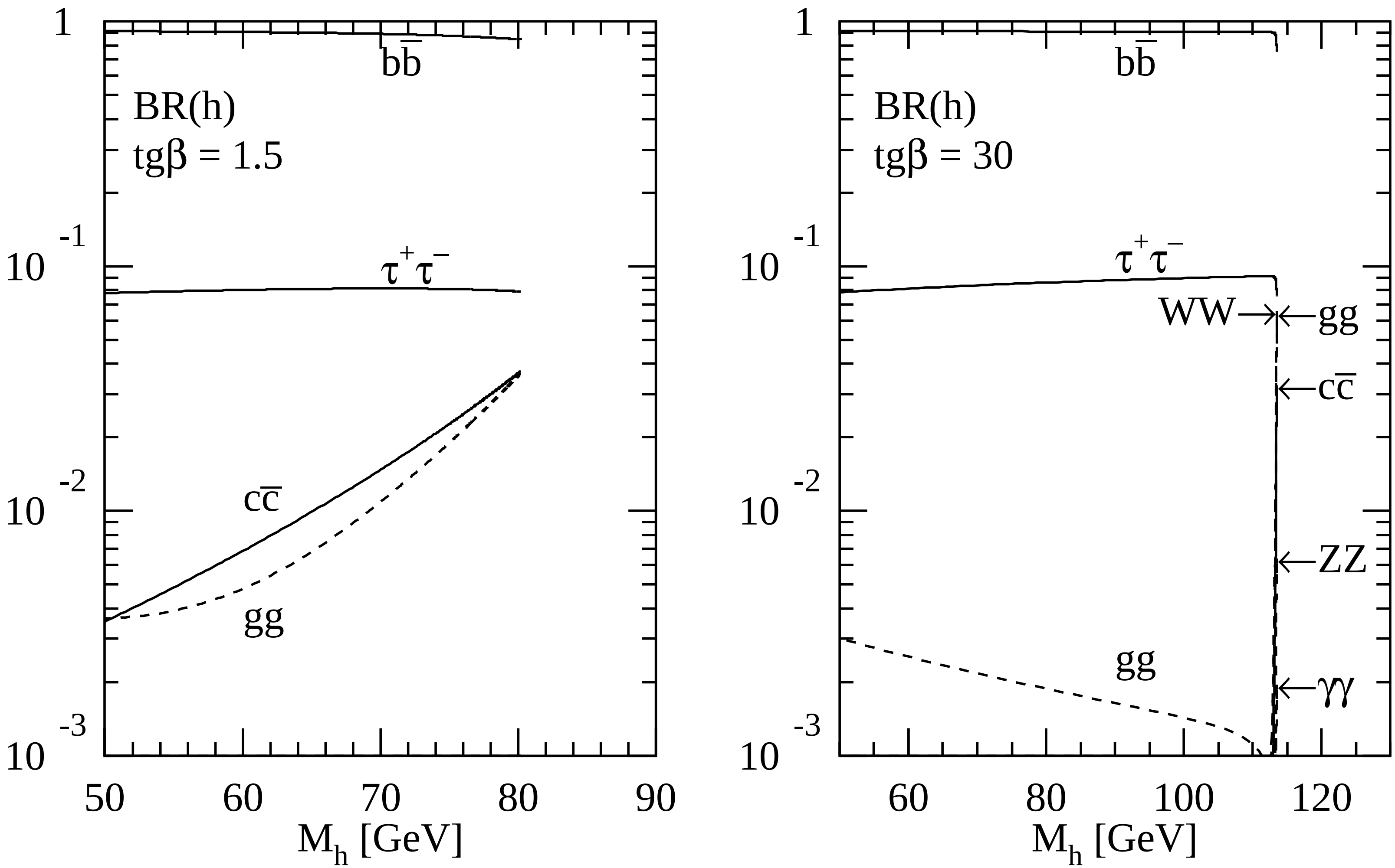
- Ought to include interaction between Higgs/SUSY sectors:

→ Higgs → SUSY and SUSY → Higgs signatures ! → figs

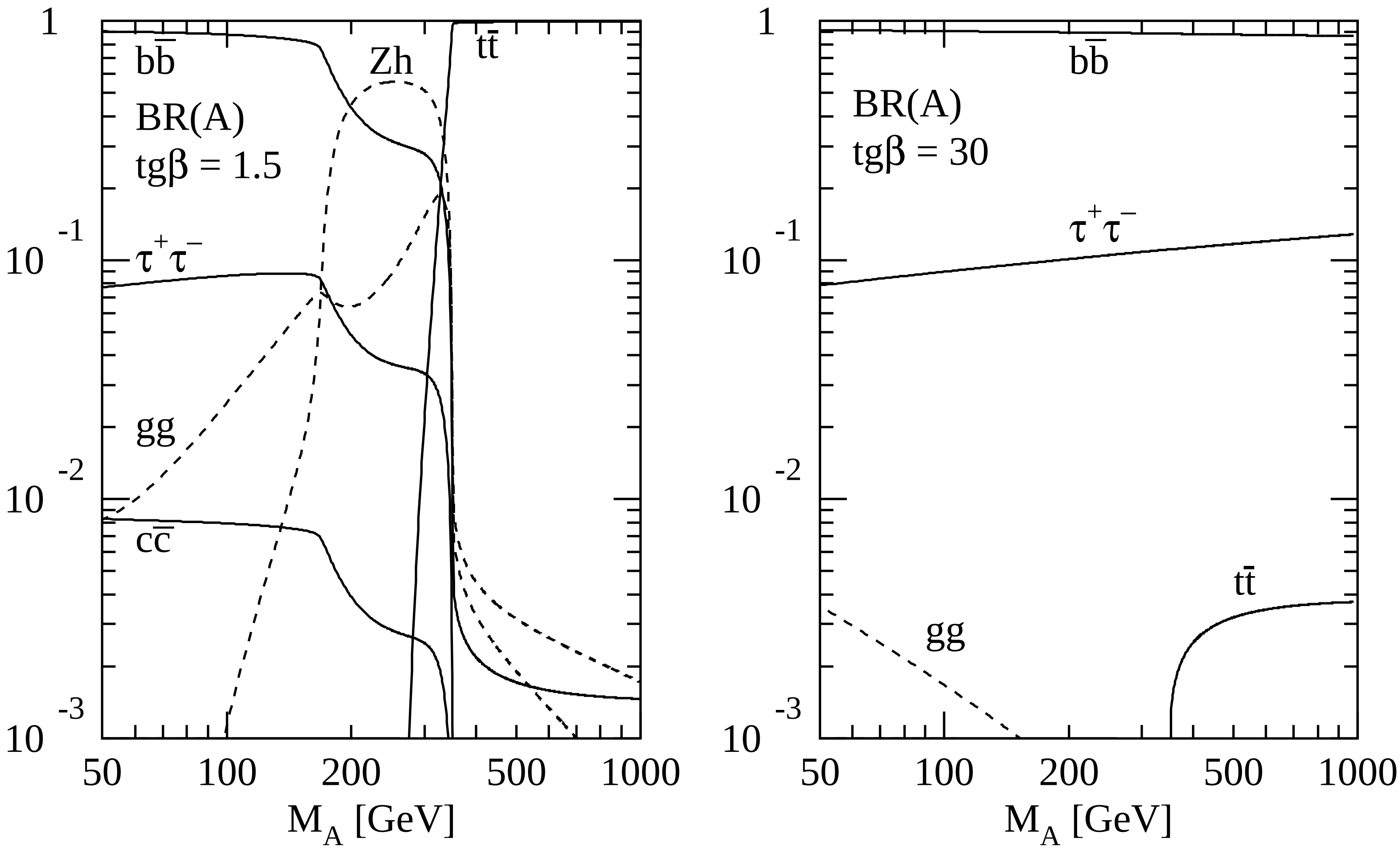
• MSSM Higgs decay rates:





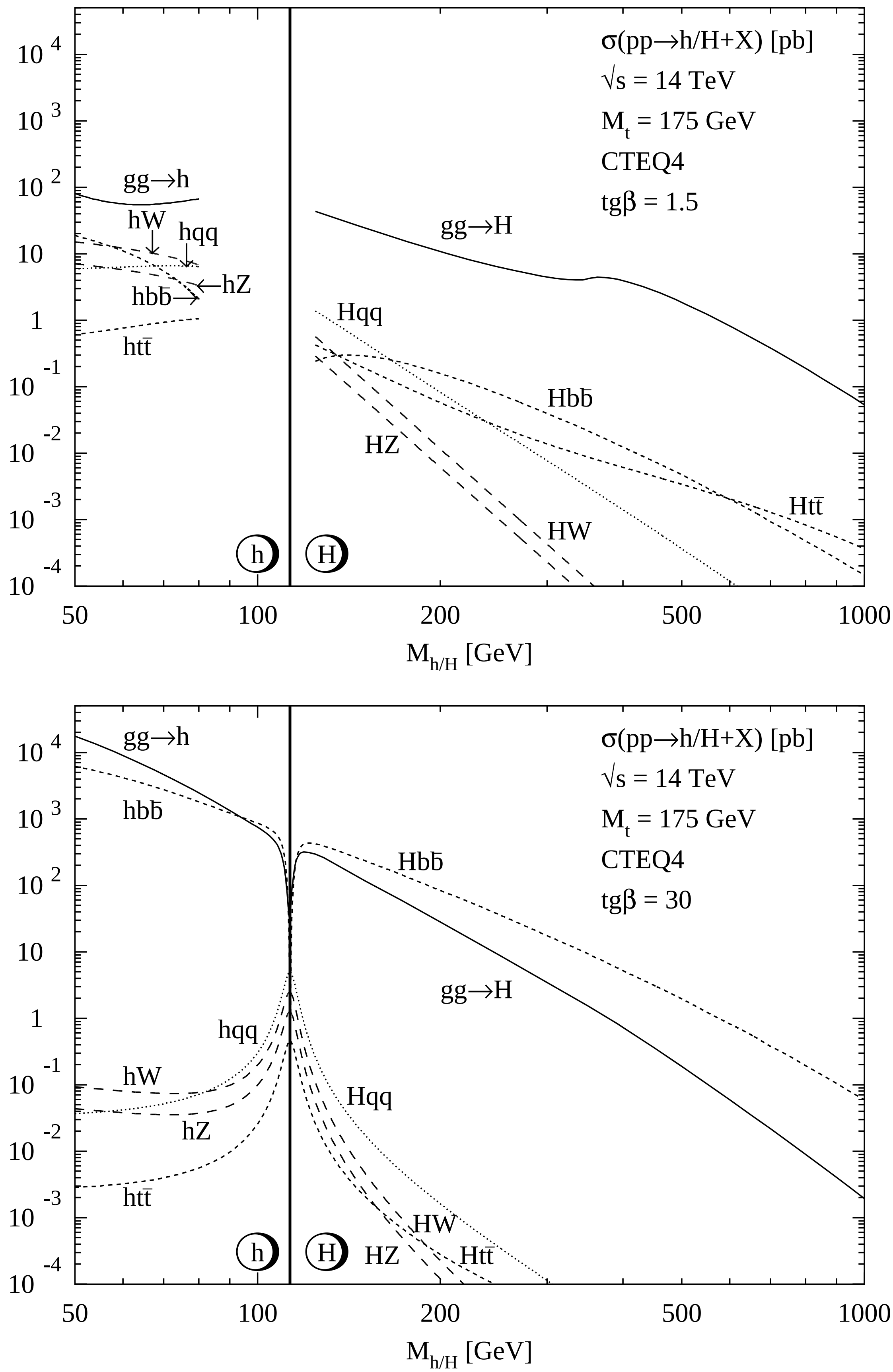


BR(h)
tgβ = 1.5
bb̄
τ+τ−
cc̄
gg
1
10-1
10-2
10-3
50
60
70
80
90
Mh [GeV]
BR(h)
tgβ = 30
bb̄
τ+τ−
WW
gg
cc̄
ZZ
γγ
gg
1
10-1
10-2
10-3
60
80
100
120
Mh [GeV]

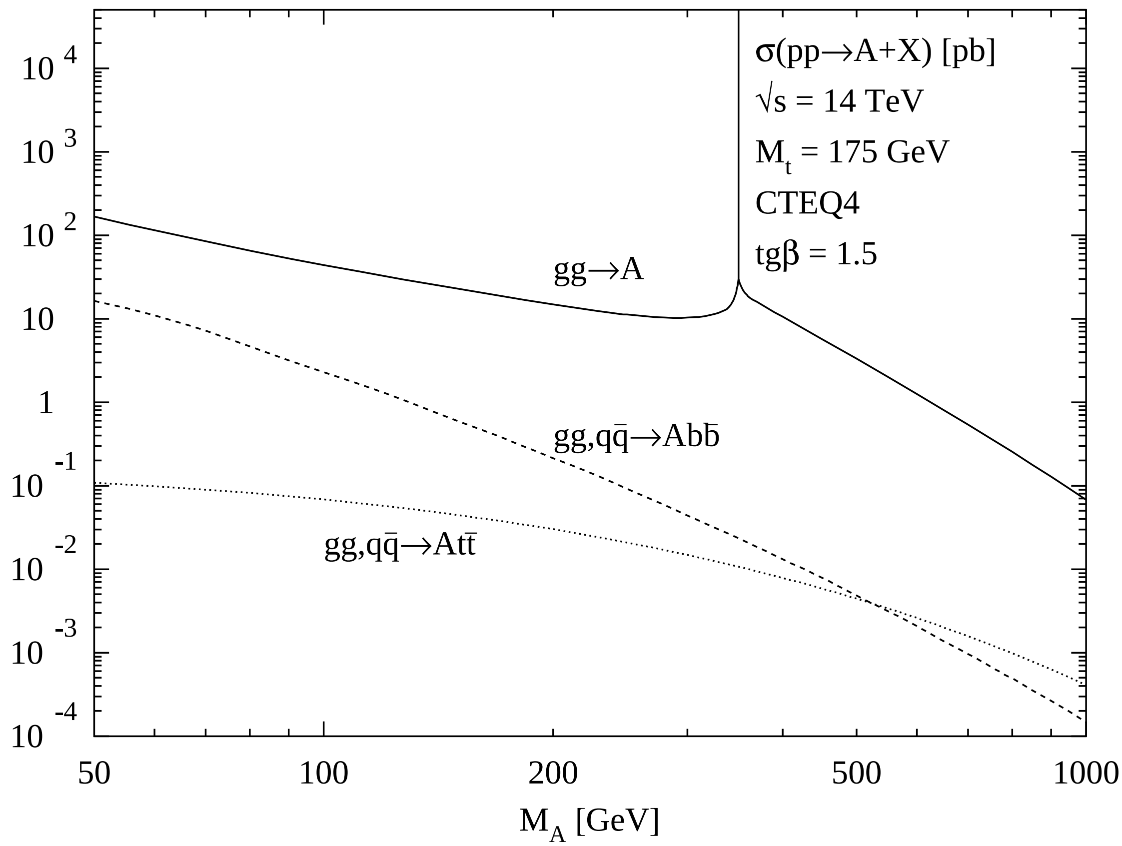


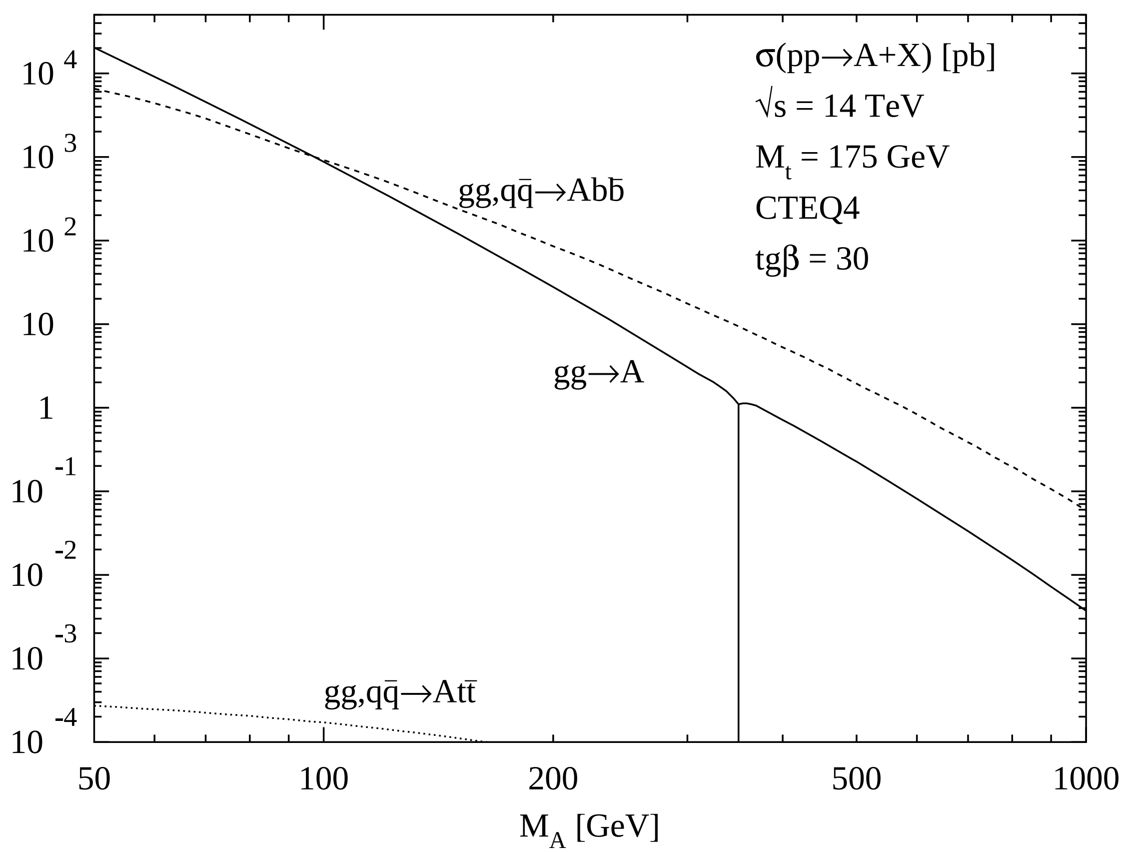
1
10^{-1}
10^{-2}
10^{-3}
$b\bar{b}$
BR(A)
tgβ = 1.5
Zh
$t\bar{t}$
$\tau^+\tau^-$
gg
$c\bar{c}$
50
100
200
500
1000
M_A [GeV]
BR(A)
tgβ = 30

• MSSM Higgs production rates at LHC (h^0, H^0):



• MSSM Higgs production rates at LHC (A^0):





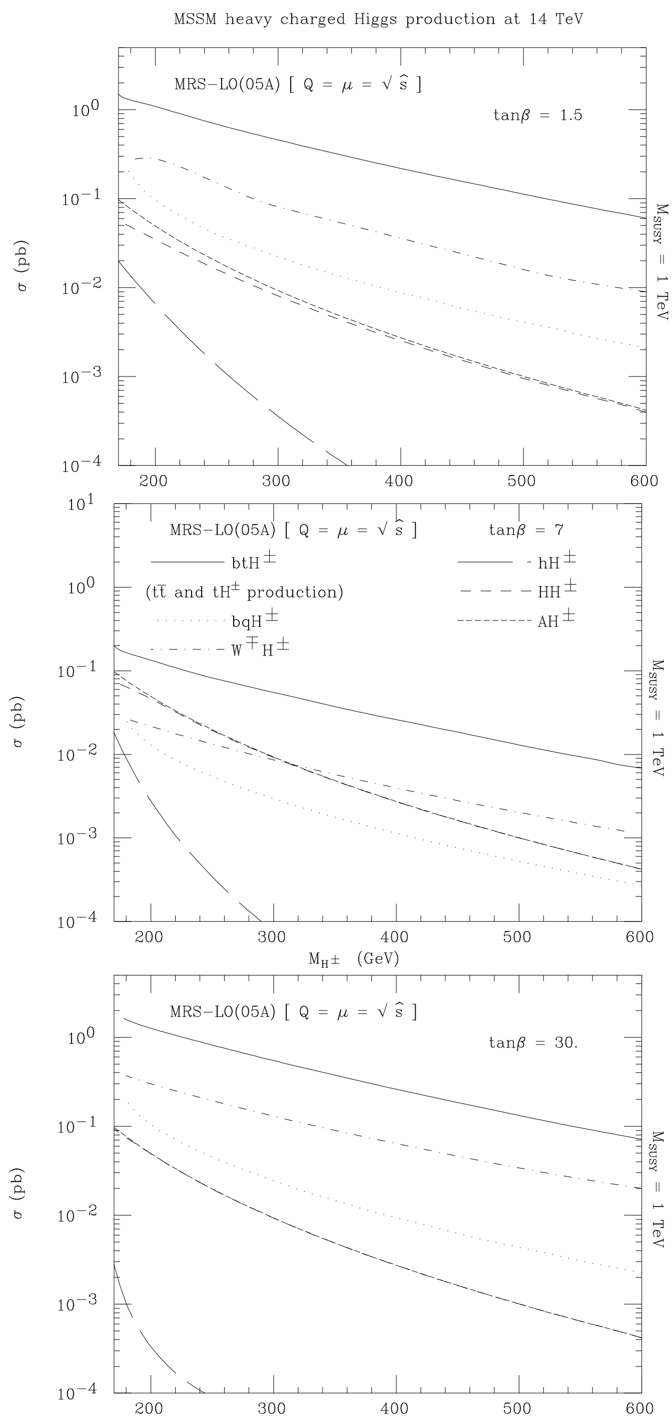
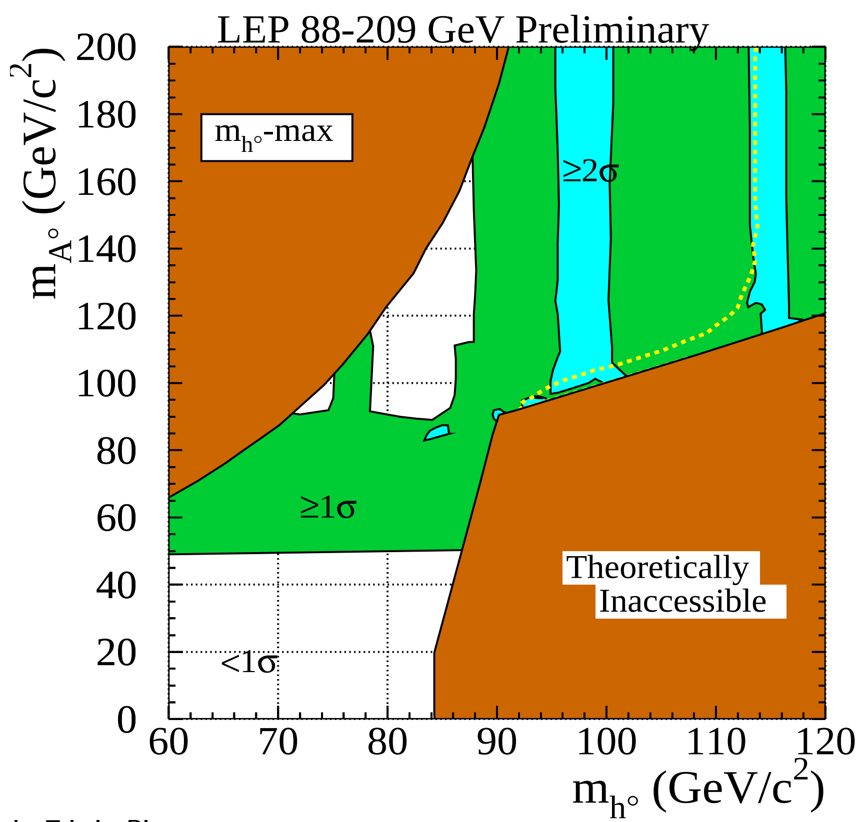


Figure 5: Cross sections in picobarns at the LHC for the production mechanisms of a single charged Higgs boson, for $\tan\beta = 1.5$ (top), 7 (middle) and 30 (bottom). (The $q\bar{q}' \to \Phi H^{\pm}$ rates, with $\Phi = h, A$, visually coincide for $\tan\beta = 30$.)

MSSM: m_{h}-max benchmark

$1 - \mathrm{CL_b}$ in $(m_{\mathsf{h}}, m_{\mathsf{A}})$:



➜ $e^+e^- \to$ hA searches:

$\sim 2\sigma$ excess for

$(m_{\mathsf{h}}, m_{\mathsf{A}}) \approx$ (83,83), (93,93) G

➜ $e^+e^- \to$ hZ searches:

$\geq 2\sigma$ excess for

$m_{\mathsf{h}} =$ 97 & 115 GeV

excluded at 95% CL (- - -):

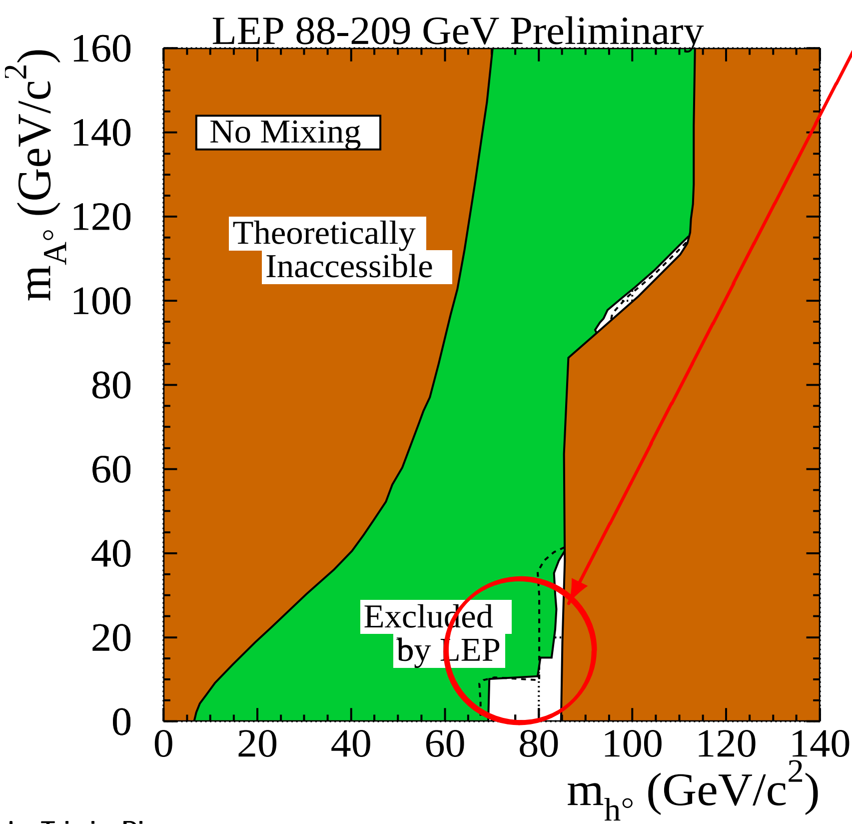
$m_{\mathsf{h}} <$ 91.0 (95.0 exp.) GeV

$m_{\mathsf{A}} <$ 91.9 (94.6 exp.) GeV

$0.5 < \tan\beta < 2.4$

MSSM: no-mixing benchmark

(m_h, m_A) **95% CL exclusion:**



- $e^+e^- \to hZ \to AAZ$, **but no** $A \to b\bar{b}$ **!**

  occurs for

  $\tan\beta <$ **0.7 &** $m_{H^\pm} <$ **74 GeV**

- **Use** $H^\pm$ **direct searches to exclude this .**

  ... but: $B(H^\pm \to W^\pm A) \neq 0$

  $\Rightarrow$ **under investigation**

excluded at 95% CL (for $\tan\beta >$**0.7):**

$m_h <$ **91.5 (95.0 exp.) GeV**

$m_A <$ **92.2 (95.3 exp.) GeV**

0.7 $< \tan\beta <$ **10.5**

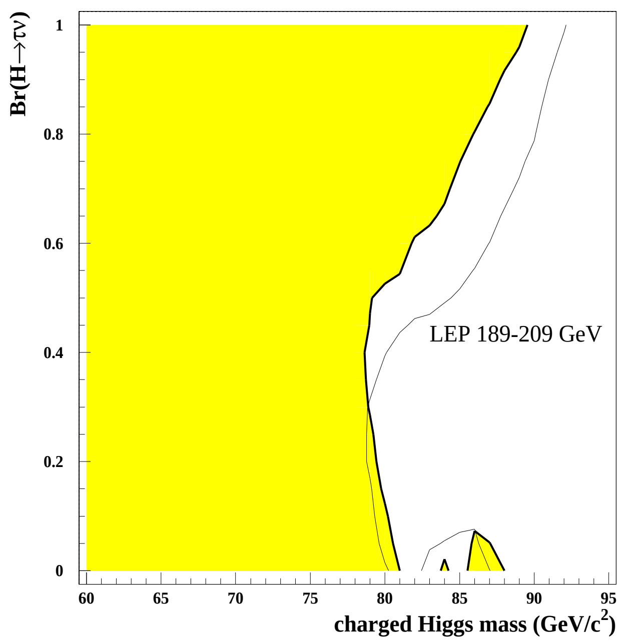
Charged Higgs, $\mathrm{H}^{\pm}$

Assume $\mathrm{B}(\mathrm{H}^+ \rightarrow \mathrm{c}\bar{\mathrm{s}}) + \mathrm{B}(\mathrm{H}^+ \rightarrow \tau^+\nu) \doteq 1$

➜ $\mathrm{e}^+\mathrm{e}^- \rightarrow \mathrm{H}^+\mathrm{H}^- \rightarrow \mathrm{c}\bar{\mathrm{s}}\mathrm{s}\bar{\mathrm{c}},\ \mathrm{c}\bar{\mathrm{s}}\tau^-\bar{\nu},\ \tau^+\nu\tau^-\bar{\nu}$

- L3 observe a large excess in the 4-jets channel

➜ compatibility with ALEPH, DELPHI, OPAL is being investigated.



LEP combined search excludes (95%CL)

$m_{\mathrm{H}^{\pm}} <$ **78.6 (78.8 exp.) GeV**

for any $\mathrm{B}(\mathrm{H}^+ \rightarrow \tau^+\nu)$

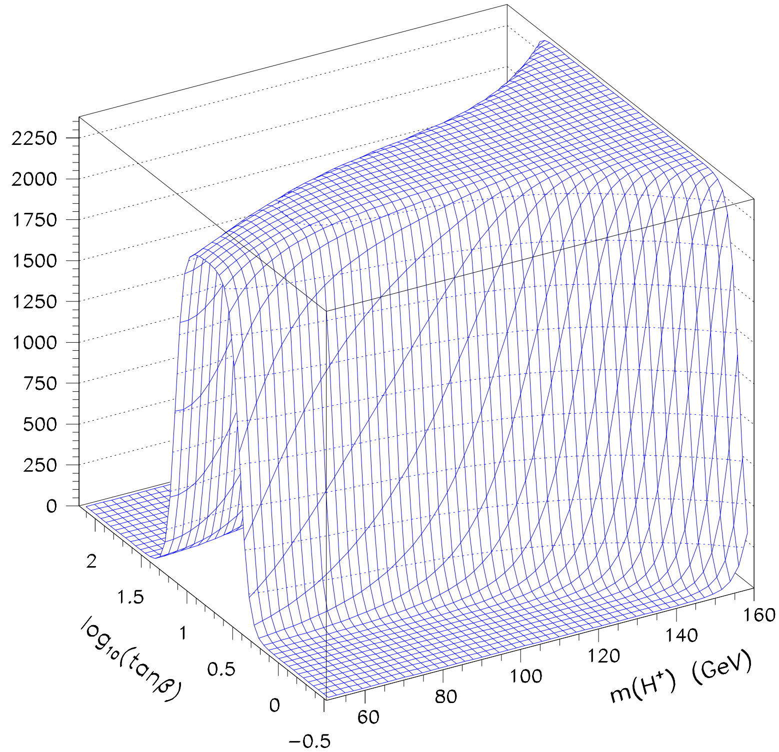


FIGURE 101. The likelihood of $n_{obs} = 600$ (in arbitrary units), as a function of m_{H^+} and $\tan\beta$ (assuming $m_t = 175$ GeV and parameters given in Table 50).

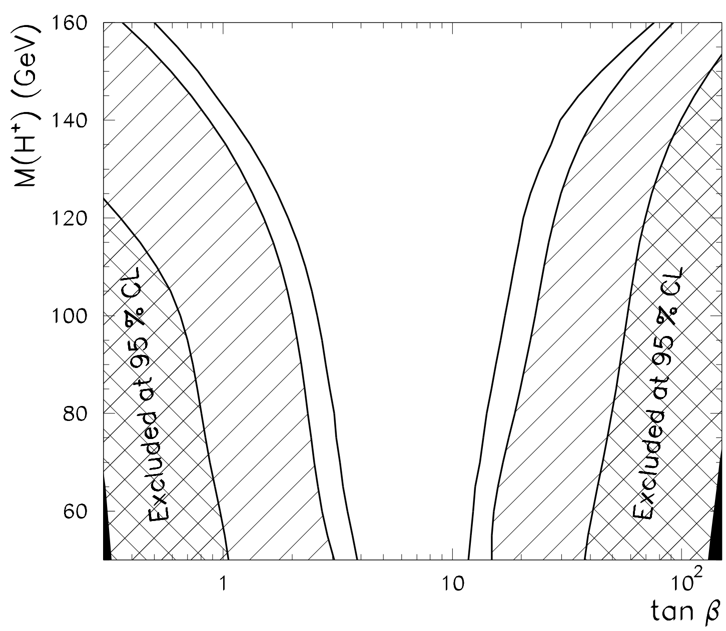


FIGURE 102. The 95% CL exclusion boundaries in the $[m_{H^+}, \tan\beta]$ plane for $m_t = 175$ GeV and several values of the integrated luminosity: 0.1 fb^{-1} (at $\sqrt{s} = 1.8$ TeV, cross-hatched), 2.0 fb^{-1} (at $\sqrt{s} = 2.0$ TeV, single-hatched), and 10 fb^{-1} (at $\sqrt{s} = 2.0$ TeV, hollow), if the n_{obs} continues to be where the SM-based prediction peaks.

LHC search channels for MSSM Higgses

Typical discovery channels are:

- $h \to \gamma\gamma$, inclusive production and production in association with an isolated lepton in Wh^0 and $t\bar{t}h^0$ final states

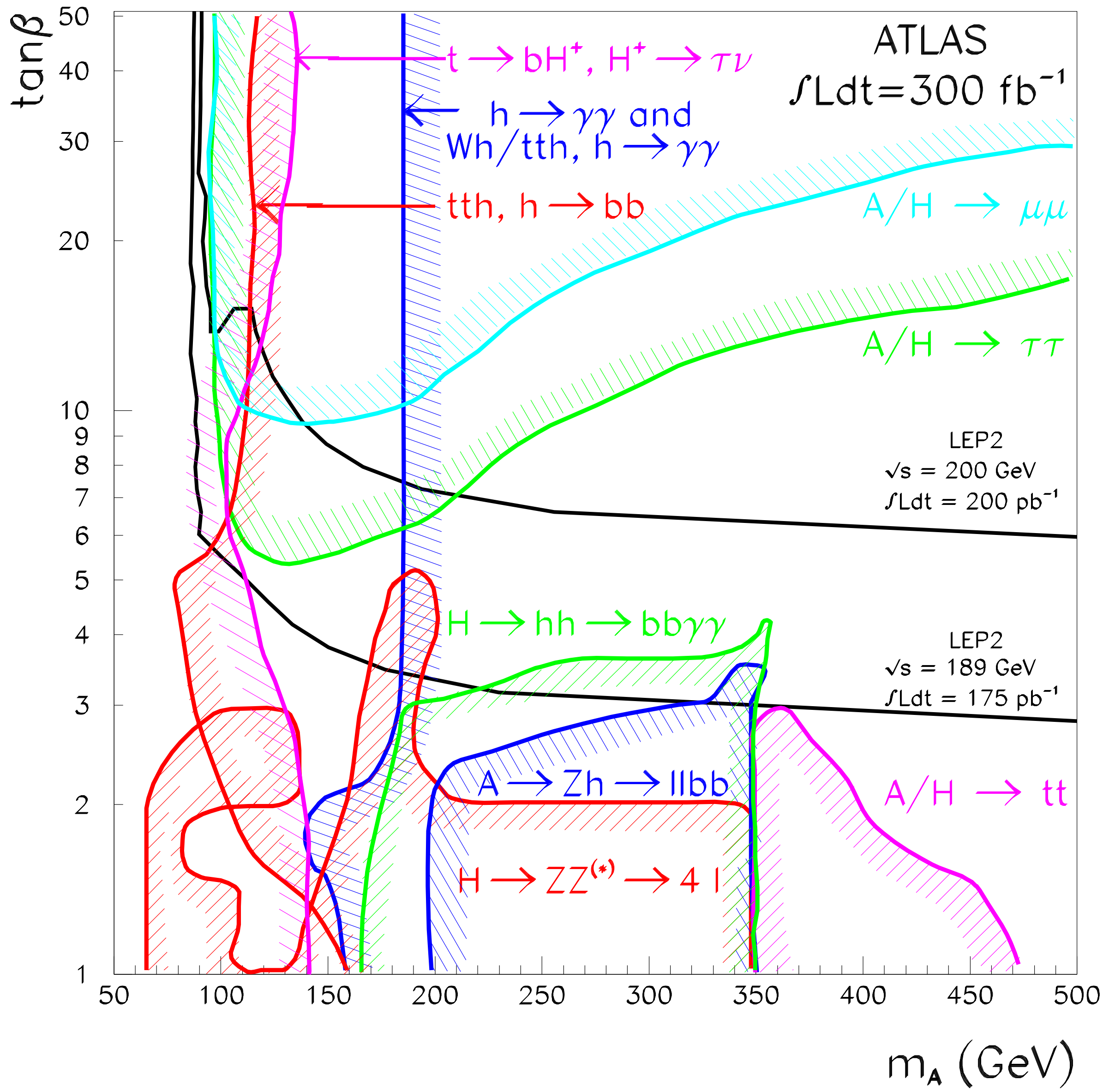
- $h \to b\bar{b}$ in association with an isolated lepton and b-jets in Wh^0 and $t\bar{t}h^0$

- $A^0, H^0 \to \mu\mu$, inclusive and in $b\bar{b}H^0/A^0$ final states

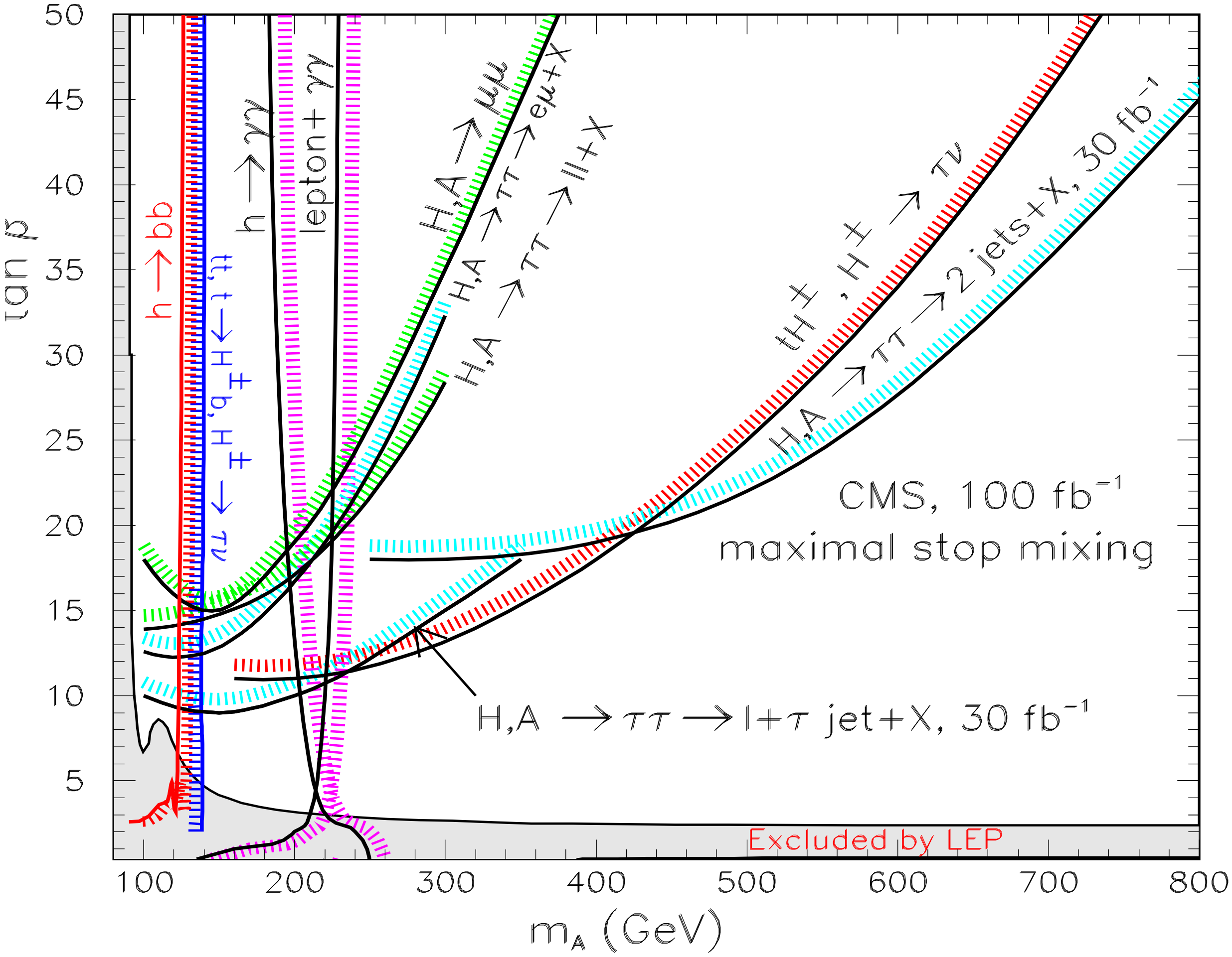
- $A^0, H^0 \to \tau\tau$ with 2ℓ, $\ell + \tau$-jet and 2τ-jet final states

- $H^\pm \to \tau\nu$ in $gb \to tH^\pm$ and in $q\overline{q}' \to H^\pm$

- $H^\pm \to tb$ in $gb \to tH^\pm$



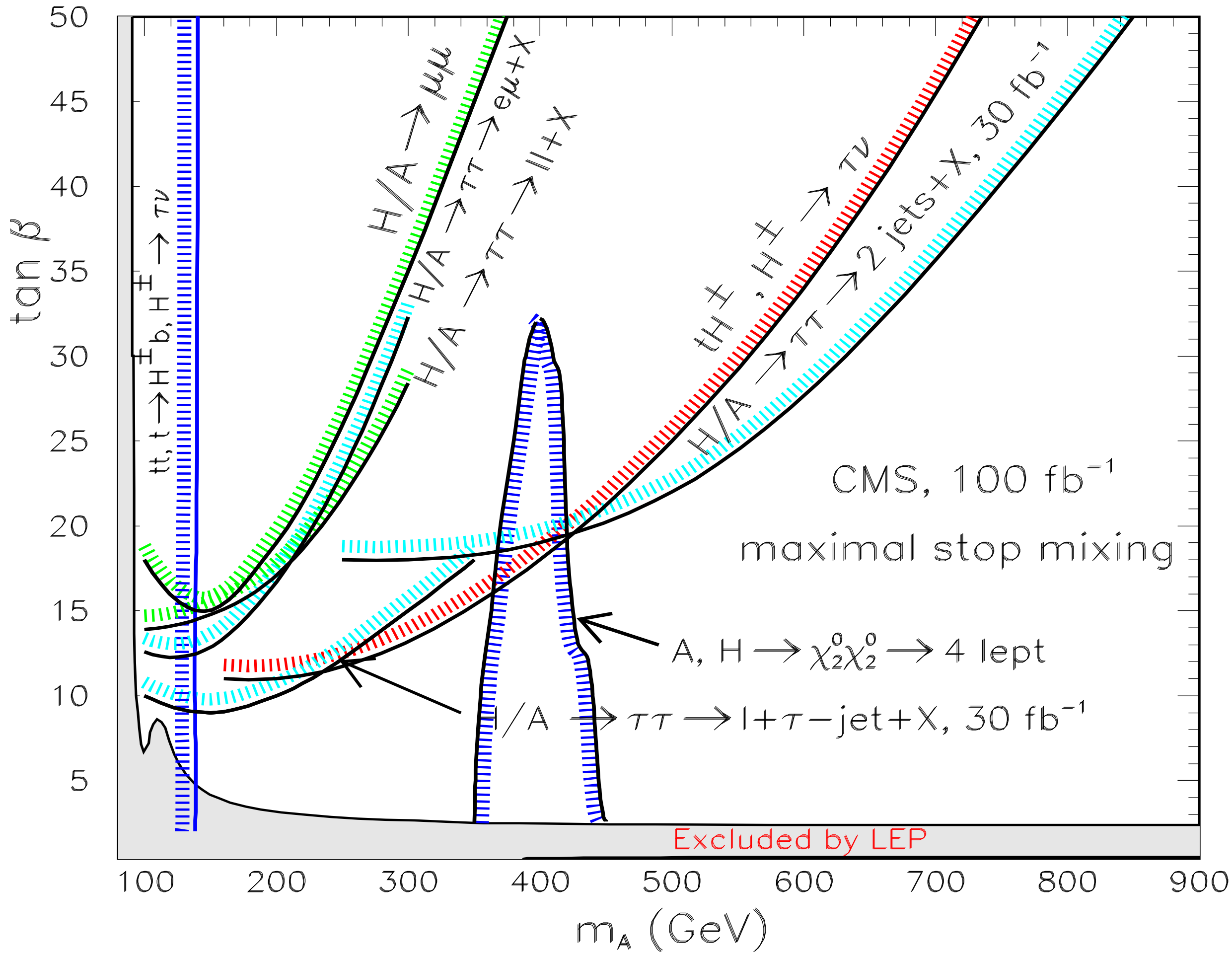
ATLAS after 300 fb^{-1} of luminosity (MaxMix)



CMS after 100 fb^{-1} of luminosity (MaxMix)

Higgses → SUSY

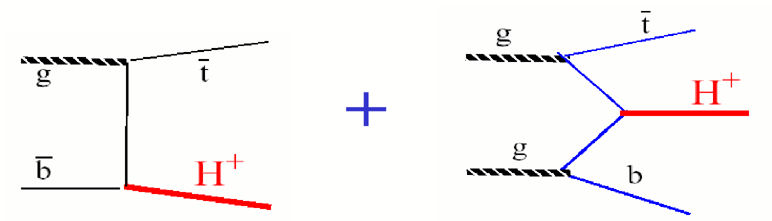
- Clean channel: $A, H \to \chi_2^0 \chi_2^0 \to 4\ell^\pm + X$

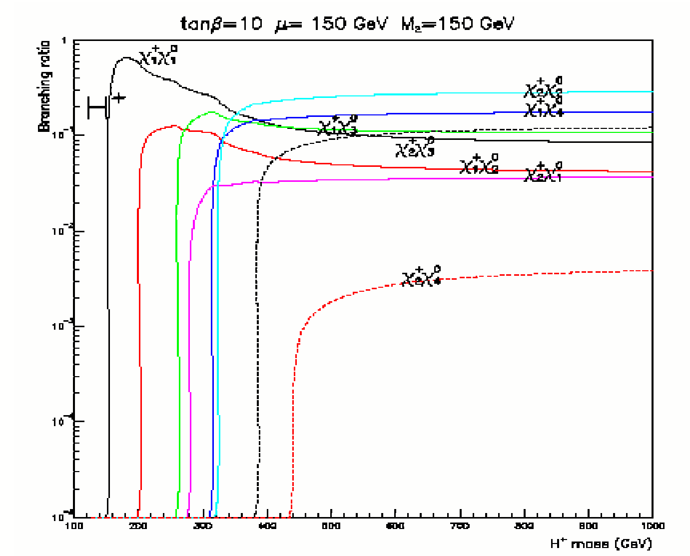


CMS after 100 fb^{-1} assuming $M_1 = 90$ GeV, $M_2 = 180$ GeV, $\mu = 500$ GeV, $M_{\tilde{\ell}} = 250$ GeV, $M_{\tilde{q},\tilde{g}} = 1000$ GeV.

Charged Higgs to sparticles

Analogue production mechanism for $H^{\pm}$:



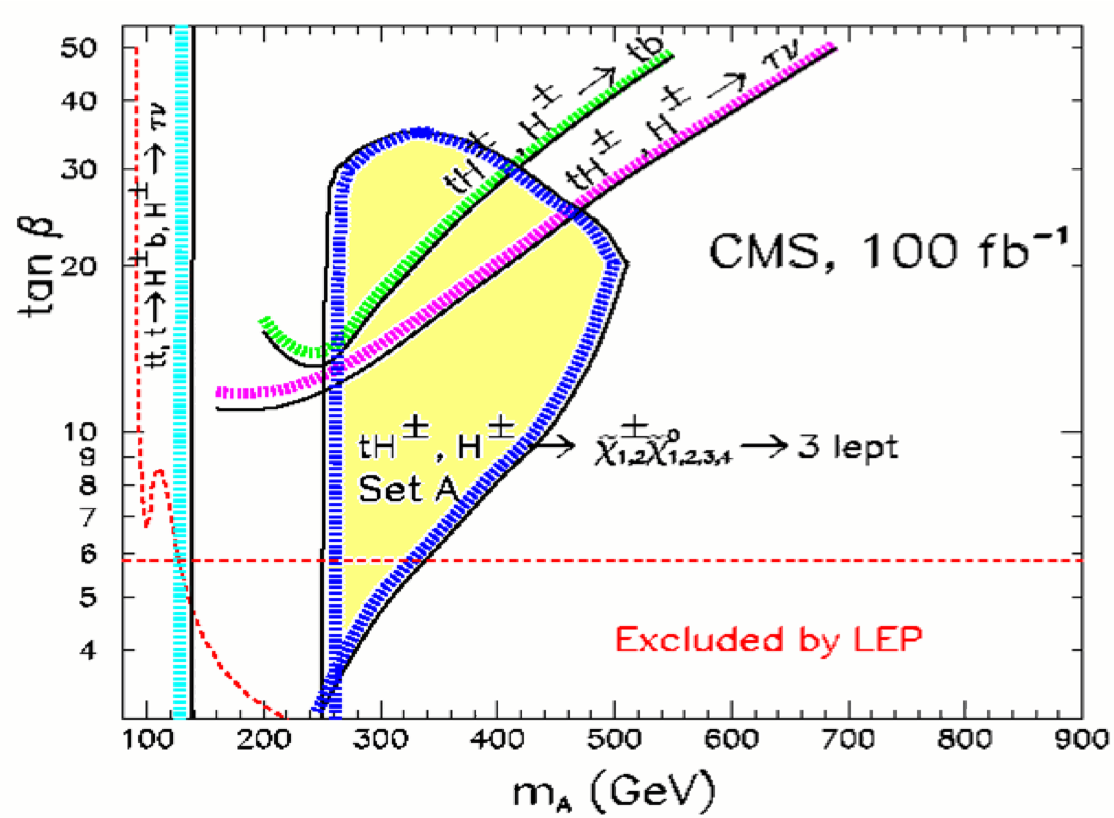


Analogue decay mode:

$$H^{\pm} \rightarrow \chi_{2,3}^{0}\, \chi_{1,2}^{\pm} \rightarrow 3l + E_{T}^{miss}$$

$\rightarrow$ only 3 leptons, need to reconstruct additional top ($t \rightarrow bjj$)

Discovery Reach



MSSM parameters:

$M_2 = 210$ GeV,
$\mu = 135$ GeV,
$M_{sleptons} = 110$ GeV,
$M_{squark,\ gluino} = 1$TeV

SUSY → Higgses

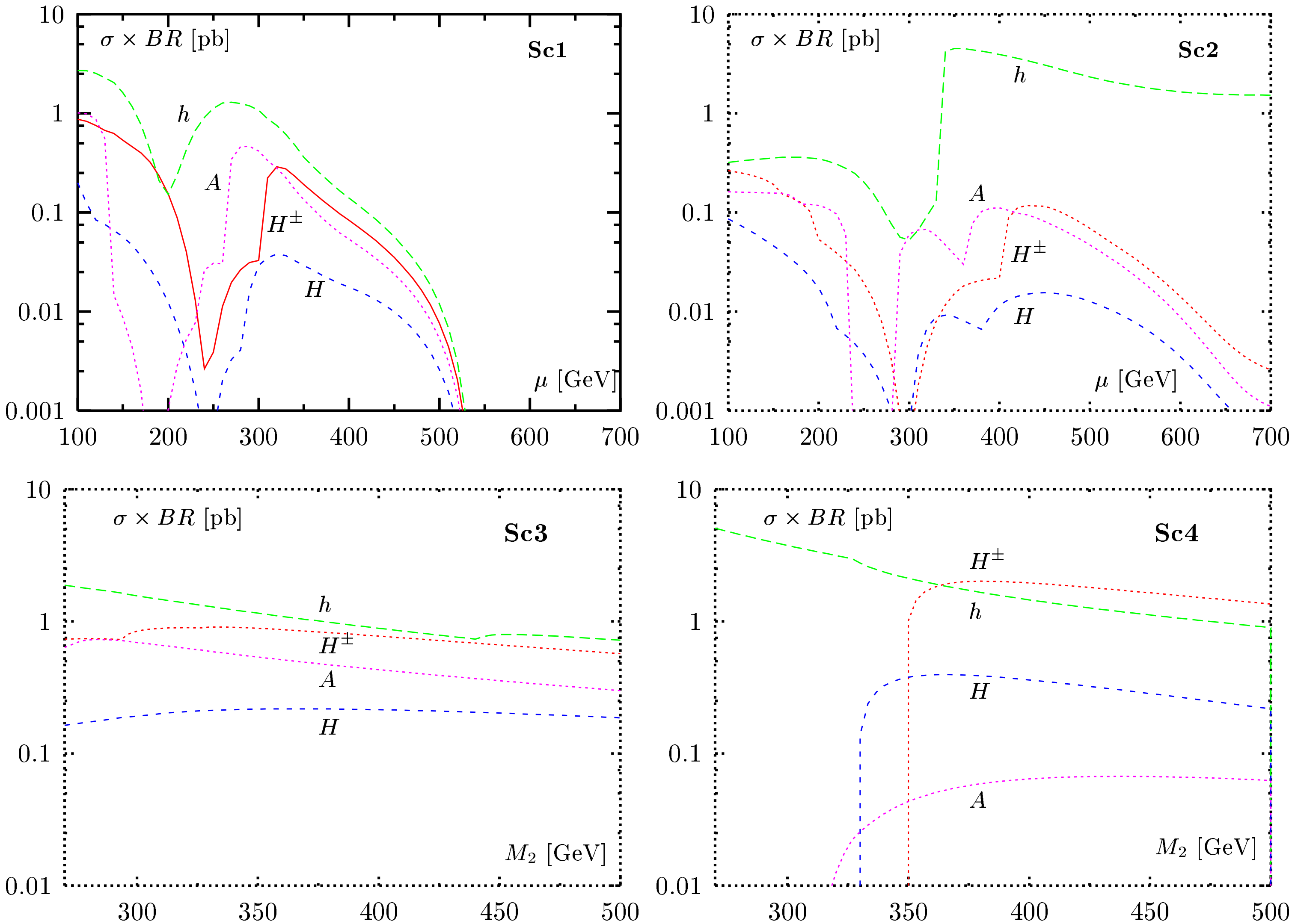
- Possible channels:

$$pp \to \tilde{g}\tilde{g}, \tilde{q}\tilde{q}, \tilde{q}\tilde{q}^*, \tilde{q}\tilde{g} \quad \to \quad \chi_2^\pm, \chi_3^0, \chi_4^0 X$$
$$\to \quad \chi_1^\pm, \chi_2^0, \chi_1^0 h^0, H^0, A^0, H^\pm \; X \quad (1)$$

$$pp \to \tilde{g}\tilde{g}, \tilde{q}\tilde{q}, \tilde{q}\tilde{q}^*, \tilde{q}\tilde{g} \quad \to \quad \chi_1^\pm, \chi_2^0 X$$
$$\to \quad \chi_1^0 H^\pm, h^0, H^0, A^0 \; X \qquad (2)$$

$$pp \quad \to \quad \tilde{t}_2\tilde{t}_2^*, \tilde{b}_2\tilde{b}_2^* \;\; \text{with} \;\; \tilde{t}_2(\tilde{b}_2) \to \tilde{t}_1(\tilde{b}_1) h^0, H^0, A^0 \text{ or } \tilde{b}_1(\tilde{t}_1) H^\pm$$

$$pp \to \tilde{g}\tilde{g}, \tilde{q}\tilde{q}, \tilde{q}\tilde{q}^*, \tilde{q}\tilde{g} \quad \to \quad t/\bar{t}X \to H^\pm X$$



σ × BR [pb]
Sc1
h
A
H±
H
μ [GeV]
σ × BR [pb]
Sc2
h
A
H±
H
μ [GeV]
σ × BR [pb]
Sc3
h
H±
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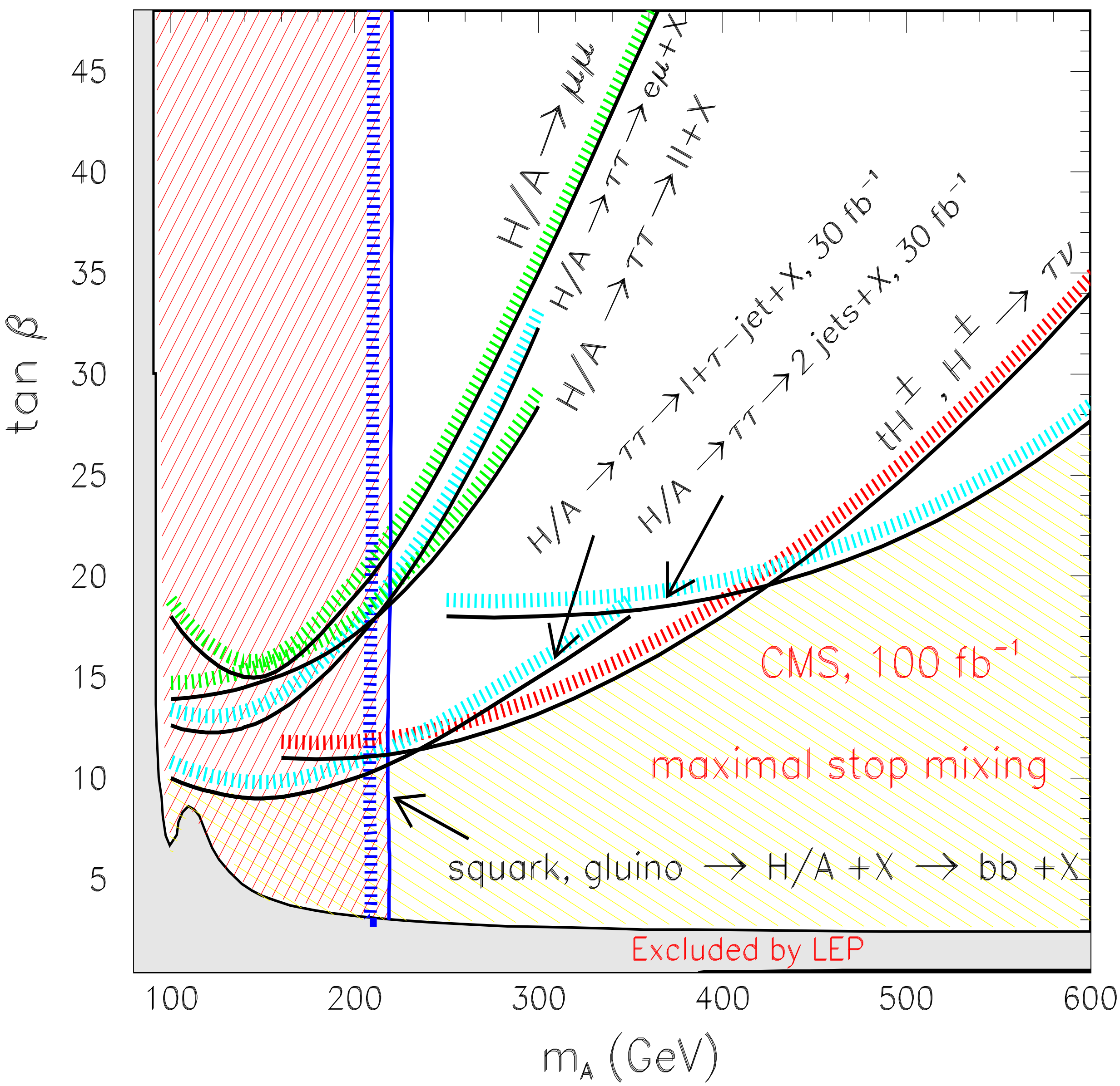
H
M2 [GeV]
σ × BR [pb]
Sc4
H±
h
H
A
M2 [GeV]



CMS after 100 fb^{-1} assuming $M_2 = 175$ GeV, $M_2 = 350$ GeV, $\mu = 150$ GeV, $M_{\tilde{\ell}} = 175$ GeV, $M_{\tilde{q},\tilde{g}} = 800$ GeV

MSSM sparticle collider searches

- To date no experimental evidence for SUSY.

- Assuming R-parity conservation and LSP (e.g. neutralino $\tilde{\chi}_1^0$) identification, SUSY production signatures:

→ multi-jets/leptons plus missing transverse momentum.

- Limits from e^+e^- and $p\bar{p}$ colliders. → figs

- SUSY searches at Tevatron and LHC: → fig

- At LHC $\tilde{q}, \tilde{g}$ up to masses $\lesssim 2.5$ TeV can be discovered !

- Since $M_{\rm SUSY} \sim 1$ TeV:

→ LHC will either confirm or disprove (low-energy) SUSY ?!

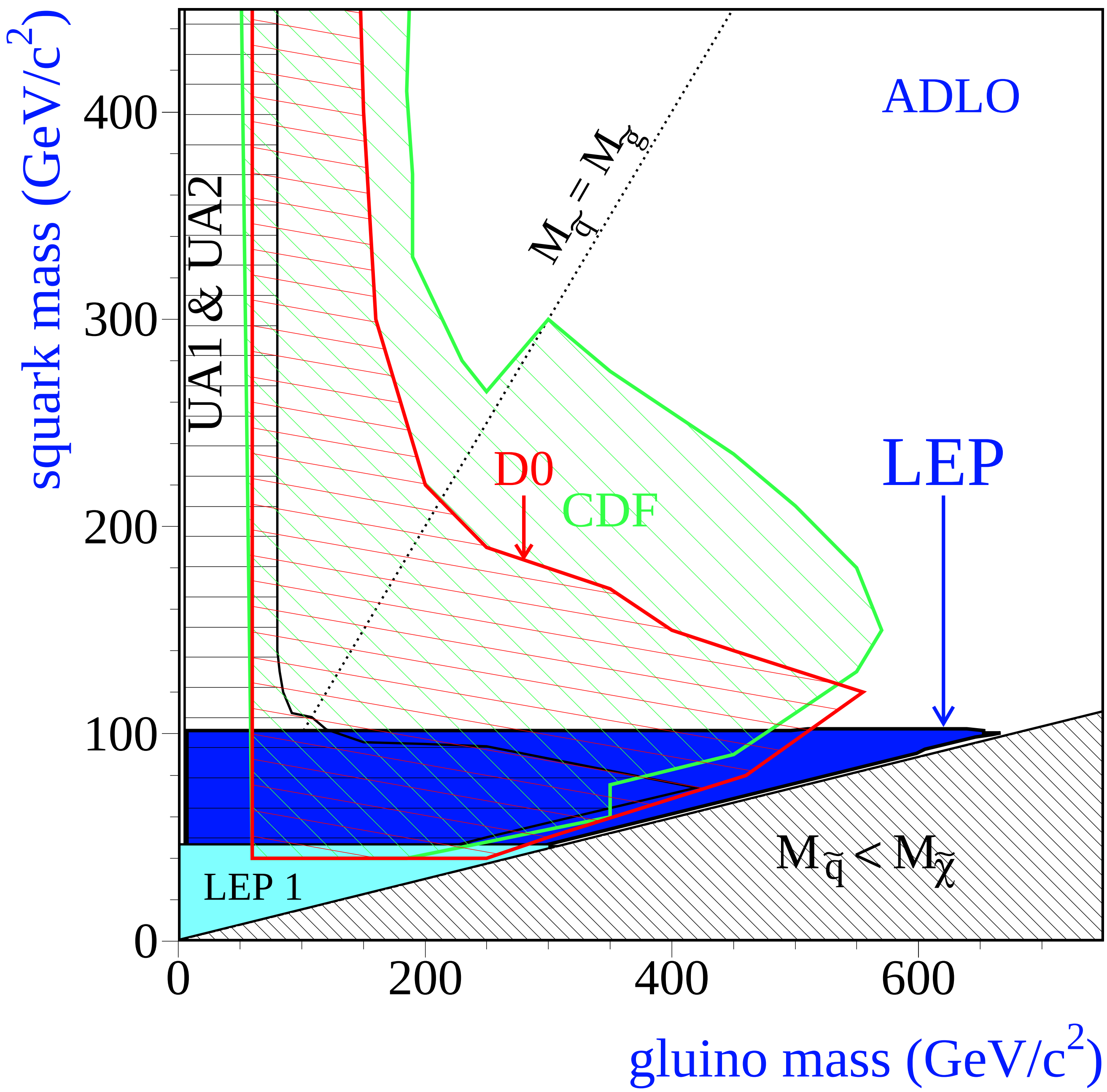
- Measurements of gross features of SUSY particle production will allow to determine typical mass scale of coloured SUSY particles at the LHC.

- SUSY cascade decays with favorable BRs can be exploited to determine mass differences of sparticles. → fig

- It is in general difficult to observe heavy weakly interacting particles such as $\tilde{\chi}_i^0, \tilde{\chi}_i^\pm, \tilde{\ell}$ at LHC: need an International Linear Collider (ILC).

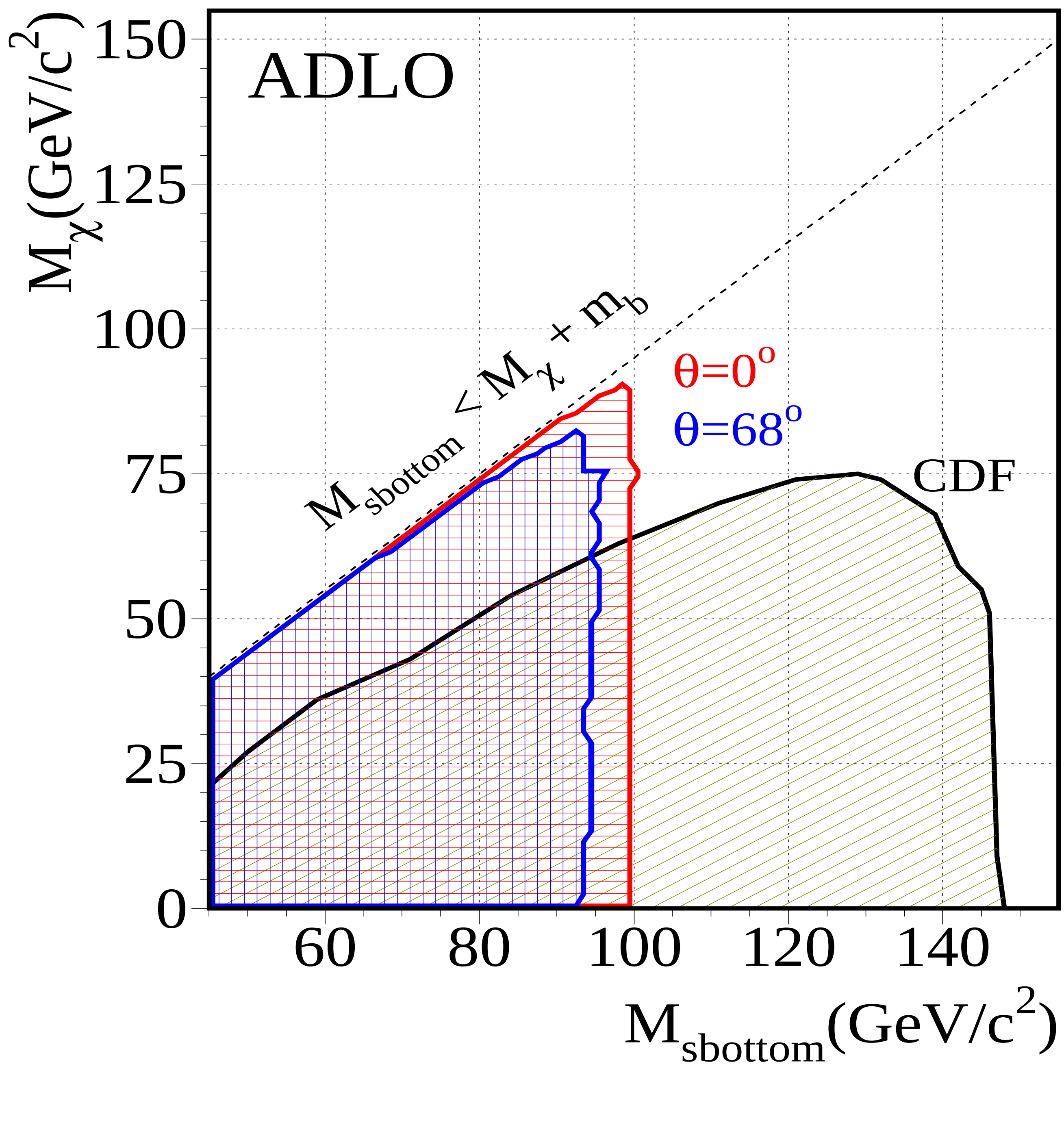
- Future LCs tool for precise measurements of masses and couplings of SUSY particles (especially non-coloured ones). → fig

• Gluino mass from $\tilde{g} \to q\chi$ (LEP & Tevatron combined)



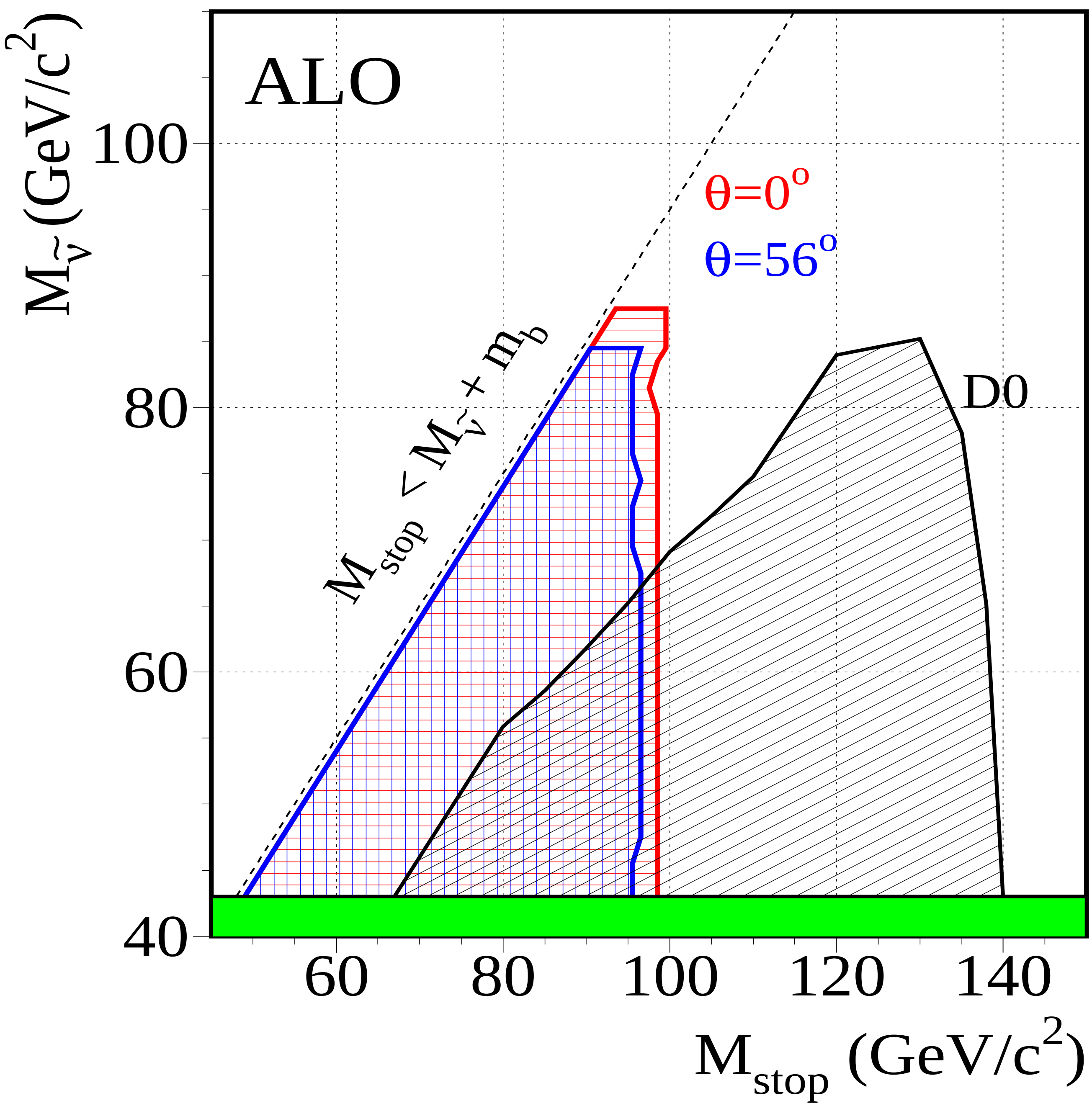
• Sbottom mass from $\tilde{b} \to b\chi$ (LEP & Tevatron combined)

($\theta = 0(68)$ horizontal(vertical) hatching)

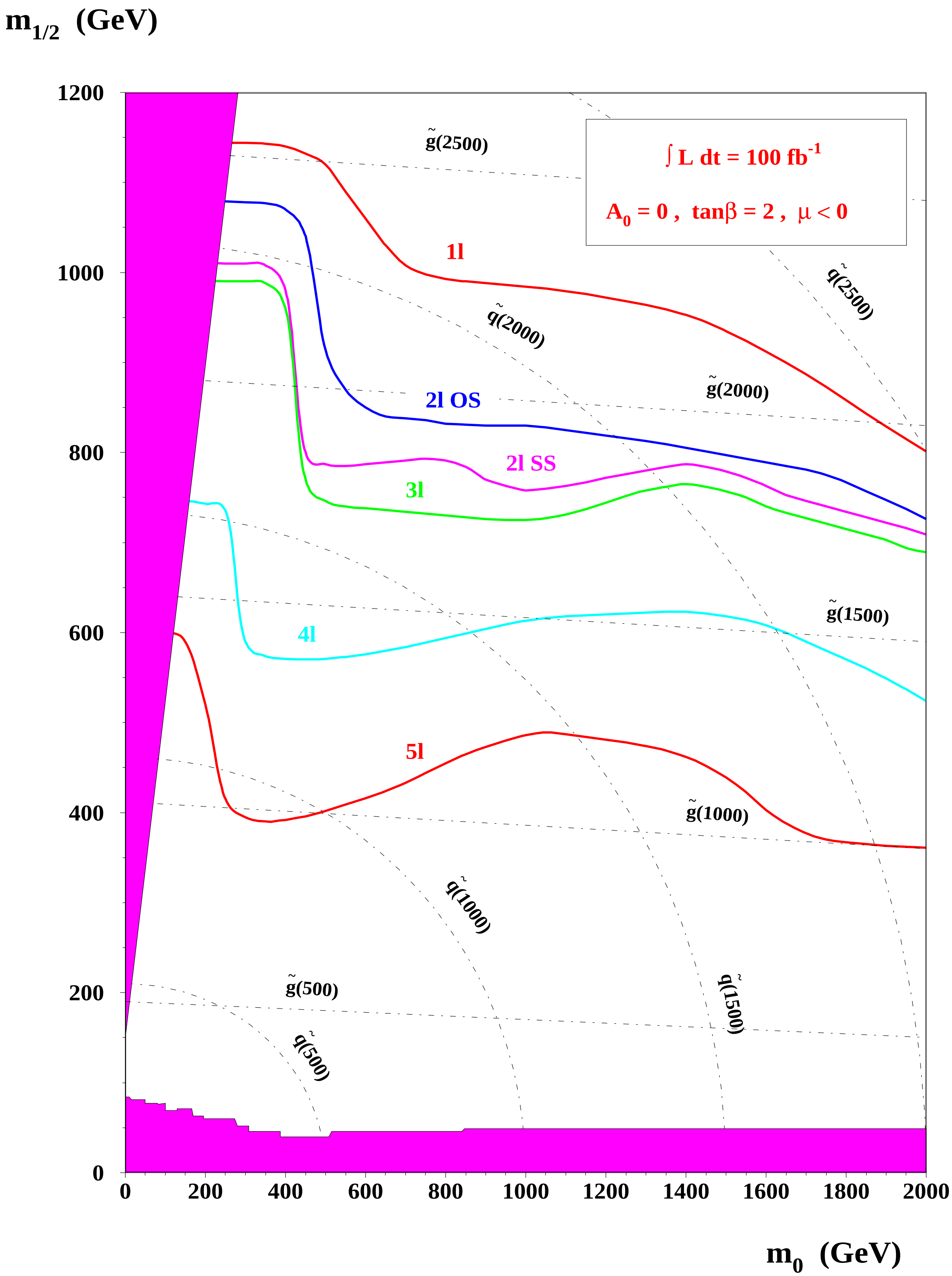


• Stop mass from $\tilde{t} \to b\ell\tilde{\nu}$ (LEP & Tevatron combined)

($\theta = 0(56)$ horizontal(vertical) hatching)



• SUSY searches via ≥ 2 jets, $n\ell^{\pm}$ + missing energy (LHC):



• SUSY processes reconstruction at LHC:

→ kinematically solves for neutralino momenta and masses of heavier sparticles using measured jet and lepton momenta and a few mass inputs.

• E.g.:

$$\tilde{g} \to \tilde{b} b \to \tilde{\chi}_2^0 bb \to \tilde{\ell} bb\ell \to \tilde{\chi}_1^0 bb\ell\ell.$$

(Decay chain basically free from SM background after cuts.)

• Five mass shell conditions:

$$\begin{aligned} m^2_{\tilde{\chi}_1^0} &= p^2_{\tilde{\chi}_1^0}, \\ m^2_{\tilde{\ell}} &= (p_{\tilde{\chi}_1^0} + p_{\ell_1})^2, \\ m^2_{\tilde{\chi}_2^0} &= (p_{\tilde{\chi}_1^0} + p_{\ell_1} + p_{\ell_2})^2, \\ m^2_{\tilde{b}} &= (p_{\tilde{\chi}_1^0} + p_{\ell_1} + p_{\ell_2} + p_{b_1})^2, \\ m^2_{\tilde{g}} &= (p_{\tilde{\chi}_1^0} + p_{\ell_1} + p_{\ell_2} + p_{b_1} + p_{b_2})^2. \end{aligned}$$

• Assume $m_{\tilde{\chi}_1^0}$,$m_{\tilde{\chi}_2^0}$ and $m_{\tilde{\ell}}$ measured at LHC using first generation squark cascade decays with $\sim 10\%$ accuracy.

• For two $bb\ell\ell$ events, we have ten equations and ten unknowns (two neutralino four momenta, $m_{\tilde{g}}$ and $m_{\tilde{\chi}_1^0}$) !

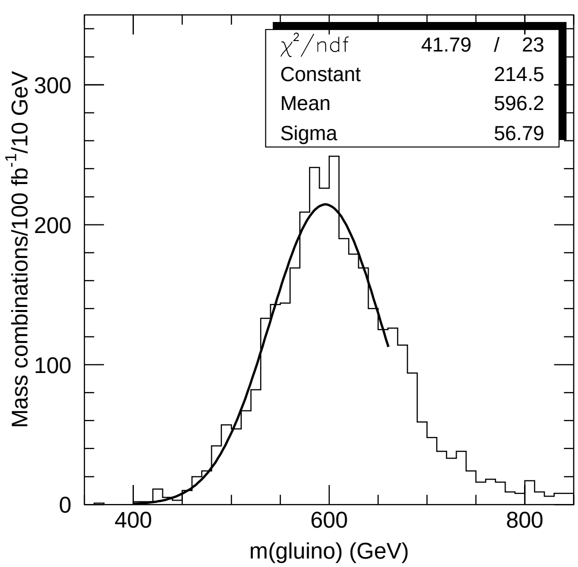
• Solution of above equations can be written:

$$\begin{aligned} m^2_{\tilde{g}} &= F_0 + F_1 m^2_{\tilde{b}} \pm F_2 D, \\ \text{where} \quad D^2 &\equiv D_0 + D_1 m^2_{\tilde{b}} + D_2 m^4_{\tilde{b}}. \end{aligned}$$

where F_i and D_i depend upon p_{ℓ_i}, p_{b_i}, $\tilde{\chi}_1^0$ and $\tilde{\ell}$ masses.

- Procedure:
  - In the event, there are two b jets: assume highest p_T b-jet from $\tilde{b}$ decay.
  - Two leptons must come from $\tilde{\chi}_2^0$ and $\tilde{\ell}$ decay.
  - Four sets of gluino and sbottom mass solutions together with two lepton assignments for each decay, because cannot determine from which decay the lepton originates.
  - To reduce combinatorics take event pair which satisfies:

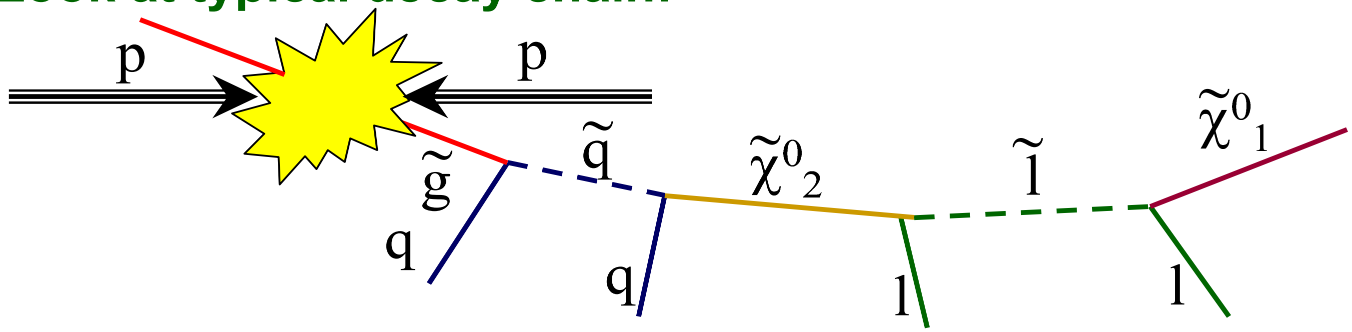
1. Only one lepton assignment has solution to equations.
2. For a pair of events there are only two solutions and there is a difference $\sim$ 100 GeV between two gluino mass solutions.



- (Input was $m_{\tilde{g}} = 595.2$ GeV.)

SUSY Signatures

- Q: What do we expect SUSY events @ LHC to look like?
- A: Look at typical decay chain:



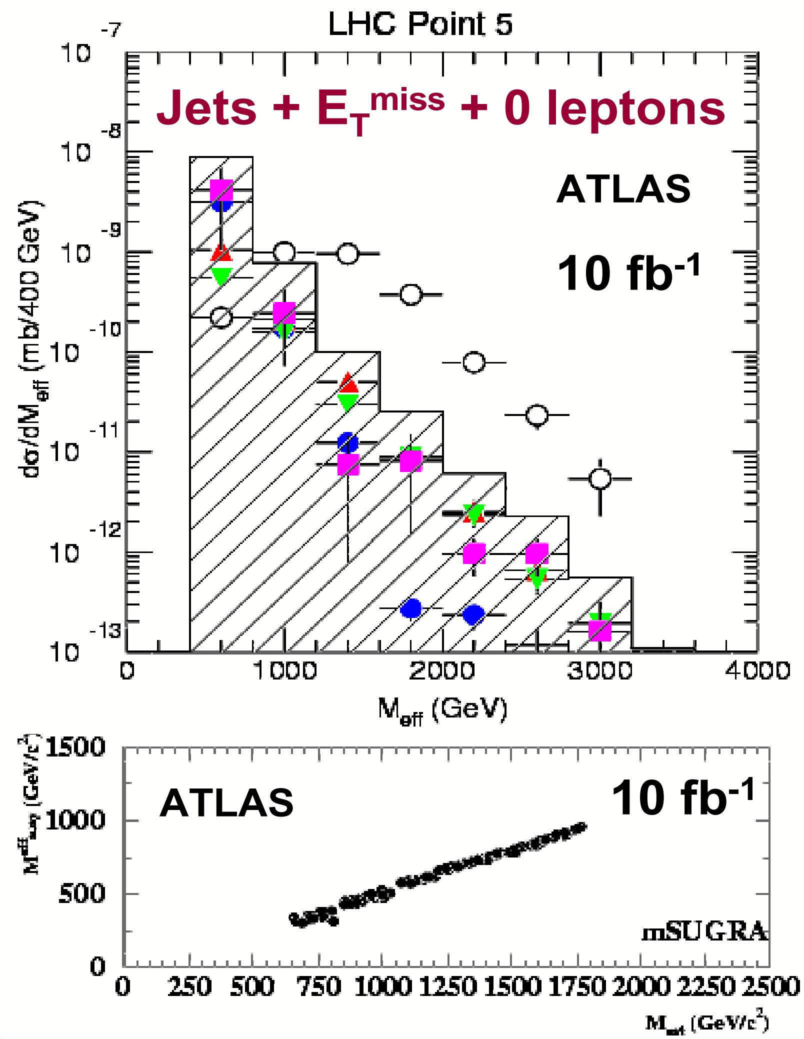
- Strongly interacting sparticles (squarks, gluinos) dominate production.
- Heavier than sleptons, gauginos etc. → cascade decays to LSP.
- Long decay chains and large mass differences between SUSY states
  - Many high p_T objects observed (leptons, jets, b-jets).
- If R-Parity conserved LSP (lightest neutralino in mSUGRA) stable and sparticles pair produced.
  - Large E_T^{miss} signature (c.f. $W \rightarrow l\nu$).
- Closest equivalent SM signature $t \rightarrow Wb$.

SUSY Mass Scale

- First measured SUSY parameter likely to be mass scale:
  – Defined as weighted mean of masses of initial sparticles.
- Calculate distribution of 'effective mass' variable defined as scalar sum of masses of all jets (or four hardest) and E_T^{miss}:

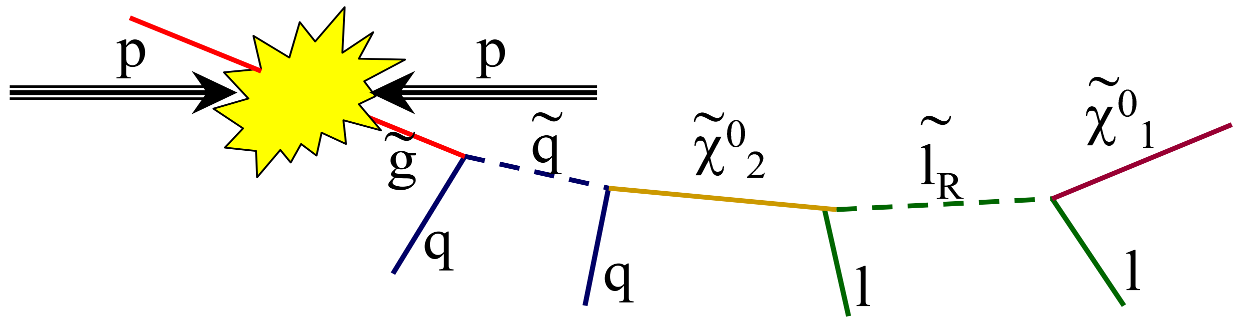
$$M_{eff}=\Sigma|p_T^i| + E_T^{miss}.$$

- Distribution peaked at ~ twice SUSY mass scale for signal events.
- Pseudo 'model-independent' measurement.
- Typical measurement error (syst+stat) ~10% for mSUGRA models for 10 fb^{-1}.



Exclusive Studies

- With more data will attempt to measure weak scale SUSY parameters (masses etc.) using exclusive channels.
- Different philosophy to TeV Run II (better S/B, longer decay chains) → aim to use model-independent measures.



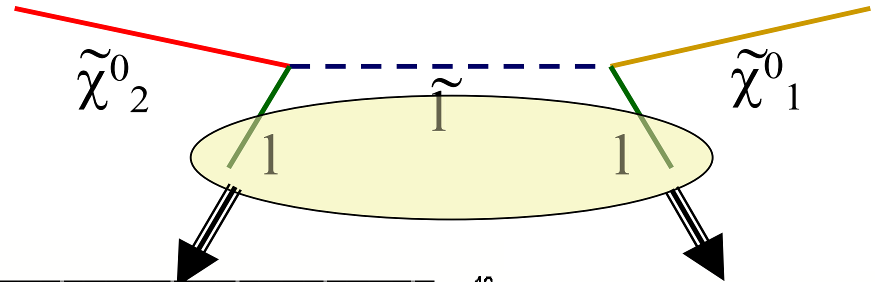
- Two neutral LSPs escape from each event
  - Impossible to measure mass of each sparticle using one channel alone
- Use kinematic end-points to measure combinations of masses.
- Old technique used many times before (ν mass from β decay spectrum, W (transverse) mass in $W \rightarrow l\nu$).
- Difference here is we don't know mass of neutral final state particles.

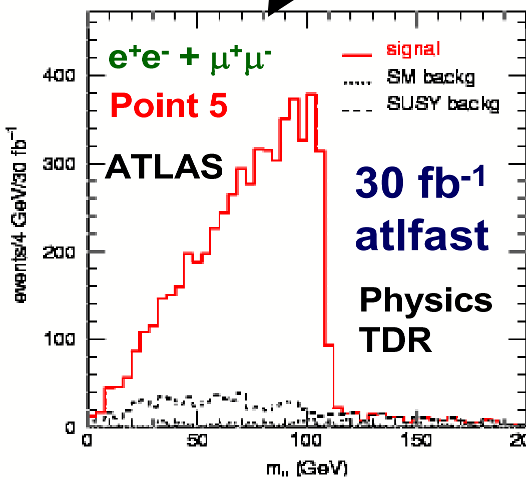
Dilepton Edge Measurements

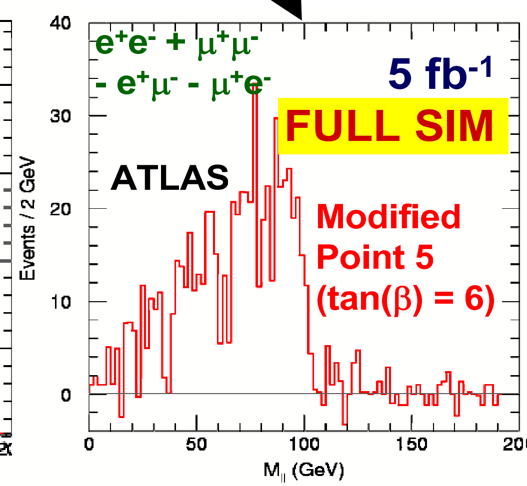
- When kinematically accessible $\tilde{\chi}^0_2$ can undergo sequential two-body decay to $\tilde{\chi}^0_1$ via a right-slepton (e.g. LHC Point 5).
- Results in sharp OS SF dilepton invariant mass edge sensitive to combination of masses of sparticles.
- Can perform SM & SUSY background subtraction using OF distribution

  $e^+e^- + \mu^+\mu^- - e^+\mu^- - \mu^+e^-$

- Position of edge measured with **precision ~ 0.5%** (30 fb^{-1}).



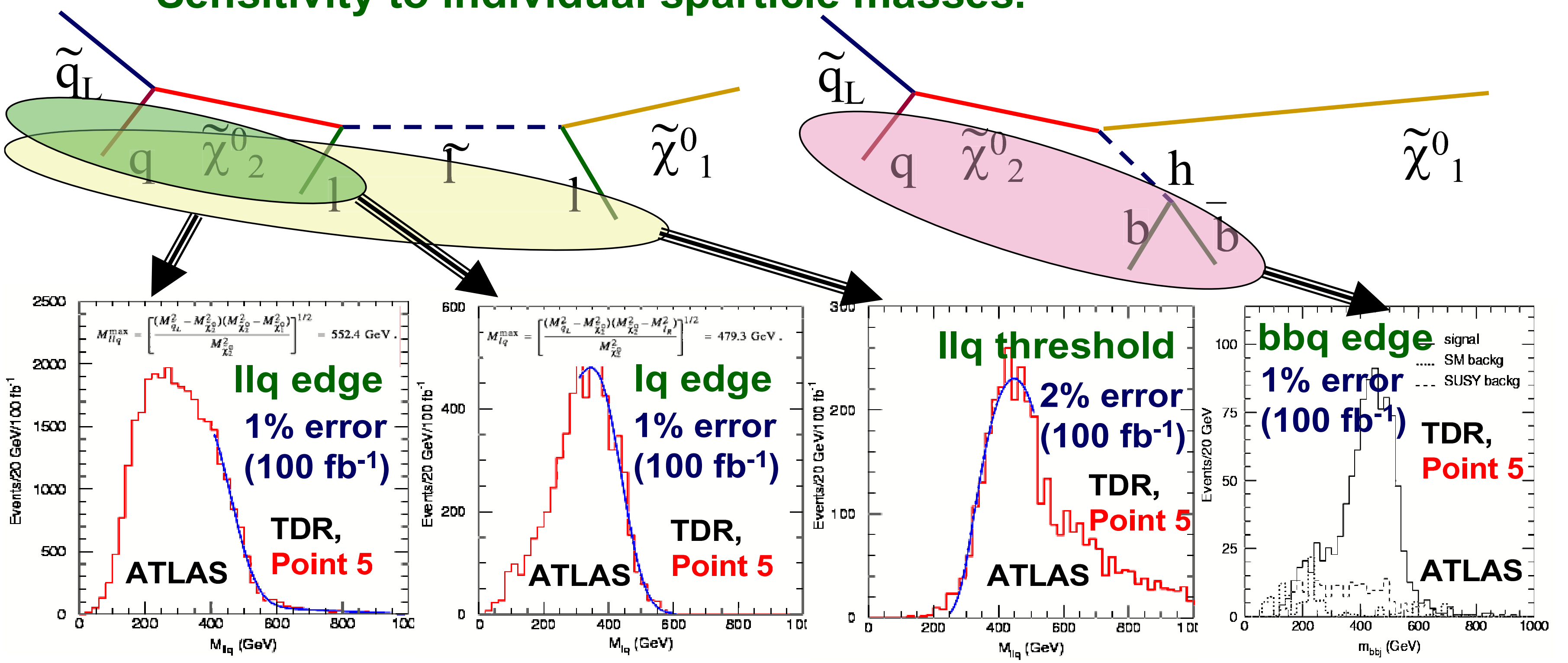




$$M_{ll}^{\max} = M(\tilde{\chi}^0_2)\sqrt{1-\frac{M^2(\tilde{l}_R)}{M^2(\tilde{\chi}^0_2)}}\sqrt{1-\frac{M^2(\tilde{\chi}^0_1)}{M^2(\tilde{l}_R)}} = 108.93\ \mathrm{GeV}$$

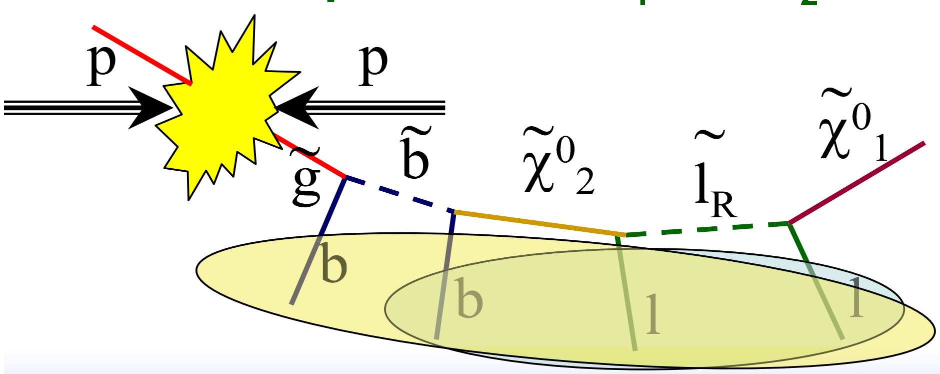
Measurements With Squarks

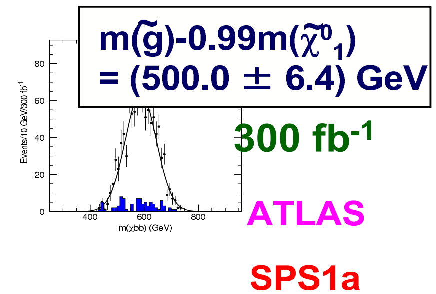
- Dilepton edge starting point for reconstruction of decay chain.
- Make invariant mass combinations of leptons and jets.
- Gives multiple constraints on combinations of four masses.
- Sensitivity to individual sparticle masses.

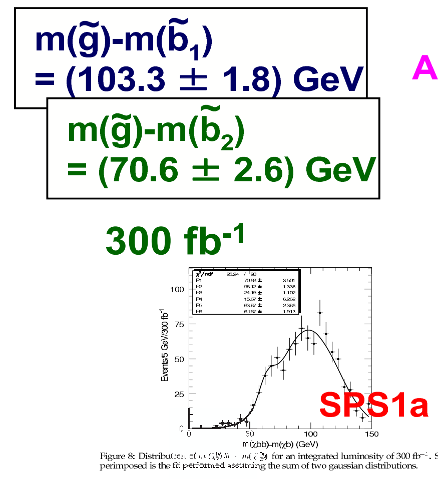


Sbottom/Gluino Mass

- Following measurement of squark, slepton and neutralino masses move up decay chain and study alternative chains.
- One possibility: require b-tagged jet in addition to dileptons.
- Give sensitivity to sbottom mass (actually two peaks) and gluino mass.
- Problem with large error on input $\tilde{\chi}^0_1$ mass remains → reconstruct difference of gluino and sbottom masses.
- Allows separation of $\tilde{b}_1$ and $\tilde{b}_2$ with 300 fb^{-1}.

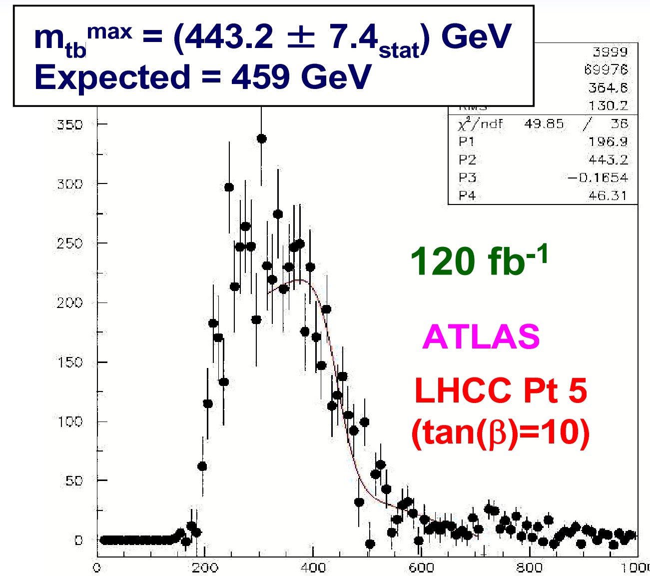


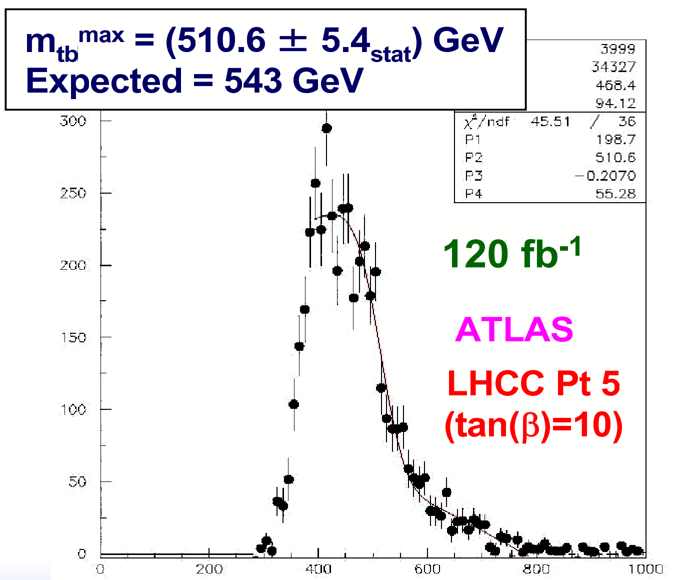




Stop Mass

- Look at edge in tb mass distribution.
- Contains contributions from
  - $\tilde{g} \rightarrow t\tilde{t}_1 \rightarrow tb\tilde{\chi}^+_1$
  - $\tilde{g} \rightarrow b\tilde{b}_1 \rightarrow bt\tilde{\chi}^+_1$
  - SUSY backgrounds
- Measures weighted mean of end-points
- Require m(jj) ~ m(W), m(jjb) ~ m(t)

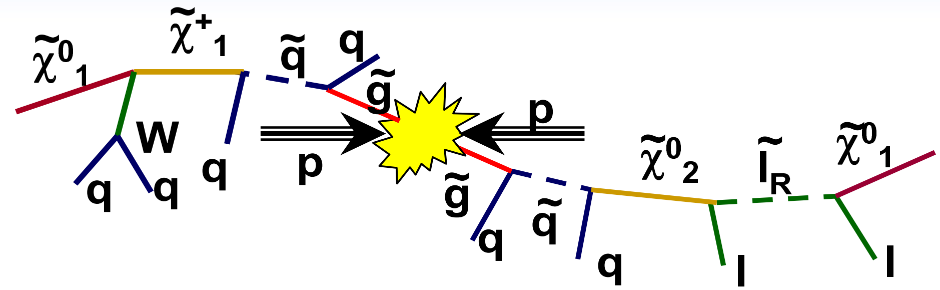


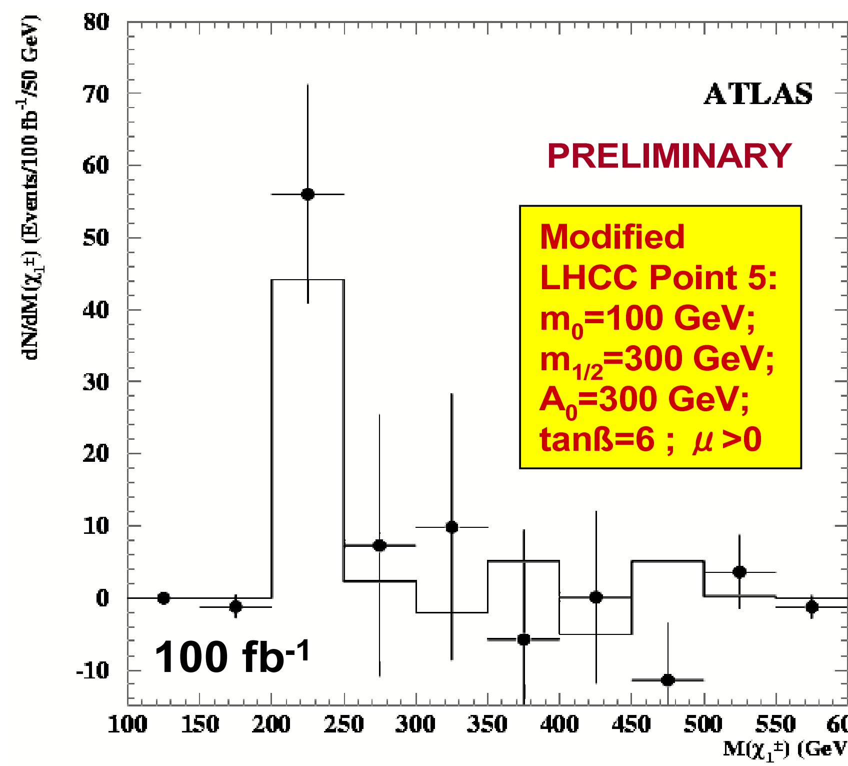


- Subtract sidebands from m(jj) distribution
- Can use similar approach with $\tilde{g} \rightarrow t\tilde{t}_1 \rightarrow tt\tilde{\chi}^0_i$
  - Di-top selection with sideband subtraction
- Also use 'standard' bbll analyses (previous slide)

Chargino Mass Measurement

- Mass of lightest chargino very difficult to measure as does not participate in standard dilepton SUSY decay chain.
- Decay process via ν+slepton gives too many extra degrees of freedom - concentrate instead on decay $\tilde{\chi}^+_1 \rightarrow W \tilde{\chi}^0_1$.
- Require dilepton $\tilde{\chi}^0_2$ decay chain on other 'leg' of event and use kinematics to calculate chargino mass analytically.
- Using sideband subtraction technique obtain clear peak at true chargino mass (218 GeV).
- ~ 3 σ significance for 100 fb^{-1}.





Measuring Model Parameters

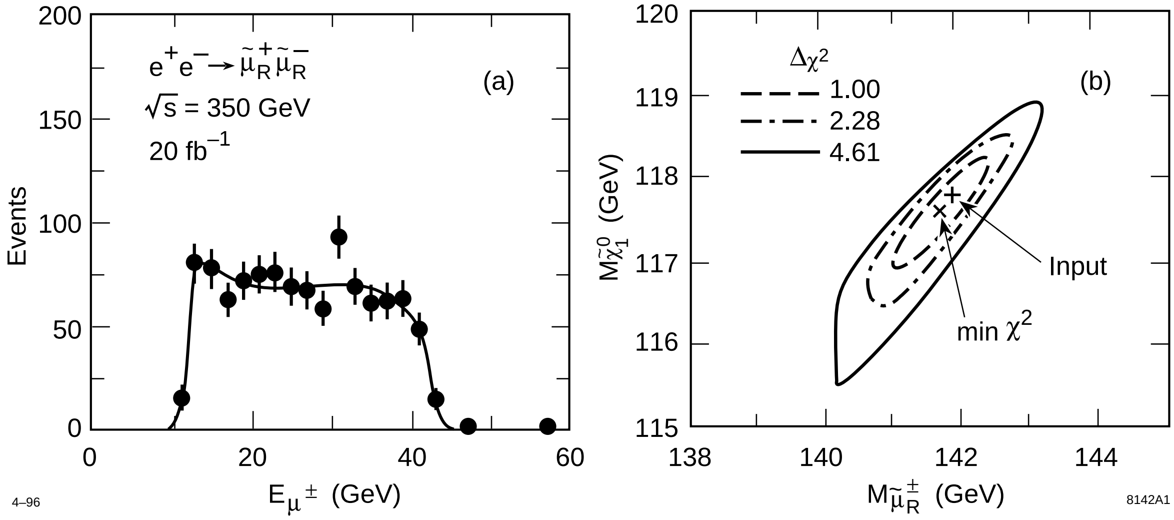
- Alternative use for SUSY observables (invariant mass end-points, thresholds etc.).
- Here assume mSUGRA/CMSSM model and perform global fit of model parameters to observables
  - So far mostly private codes but e.g. SFITTER, FITTINO now on the market;
  - c.f. global EW fits at LEP, ZFITTER, TOPAZ0 etc.

Point	m_0	$m_{1/2}$	A_0	$\tan(\beta)$	$\mathrm{sign}(\mu)$
LHC Point 5	100	300	300	2	+1
SPS1a	100	250	-100	10	+1

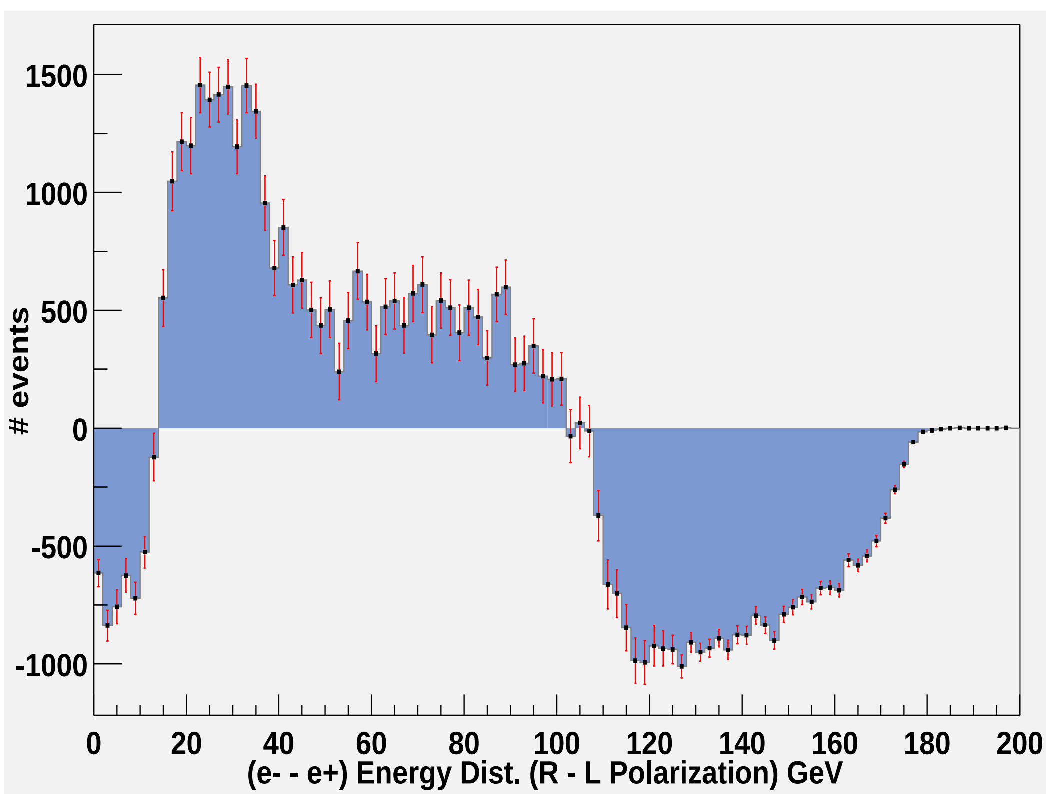
Variable	Value (GeV)	Errors Stat. (GeV)	Errors Scale (GeV)	Errors Total
$m_{\ell\ell}^{max}$	77.07	0.03	0.08	0.08
$m_{\ell\ell q}^{max}$	428.5	1.4	4.3	4.5
$m_{\ell q}^{low}$	300.3	0.9	3.0	3.1
$m_{\ell q}^{high}$	378.0	1.0	3.8	3.9
$m_{\ell\ell q}^{min}$	201.9	1.6	2.0	2.6
$m_{\ell\ell b}^{min}$	183.1	3.6	1.8	4.1
$m(\ell_L) - m(\tilde{\chi}_1^0)$	106.1	1.6	0.1	1.6
$m_{\ell\ell}^{max}(\tilde{\chi}_4^0)$	280.9	2.3	0.3	2.3
$m_{\tau\tau}^{max}$	80.6	5.0	0.8	5.1
$m(\tilde{g}) - 0.99 \times m(\tilde{\chi}_1^0)$	500.0	2.3	6.0	6.4
$m(\tilde{q}_R) - m(\tilde{\chi}_1^0)$	424.2	10.0	4.2	10.9
$m(\tilde{g}) - m(\tilde{b}_1)$	103.3	1.5	1.0	1.8
$m(\tilde{g}) - m(\tilde{b}_2)$	70.6	2.5	0.7	2.6

Parameter	Expected precision (300 fb^{-1})
m_0	± 2%
$m_{1/2}$	± 0.6%
$\tan(\beta)$	± 9%
A_0	± 16%

ILC Potential in the non-coloured SUSY sector



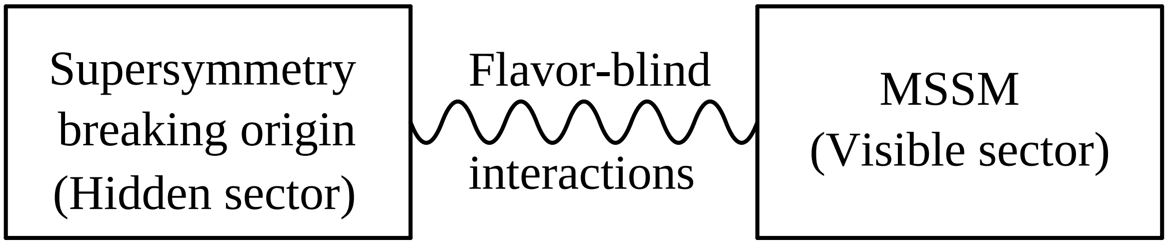
An example of the possible accuracy of the determination of the smuon mass and neutralino mass, for an e^+e^- linear collider with $\sqrt{s} = 500$ GeV and $\mathcal{L} = 100$ fb^{-1}.



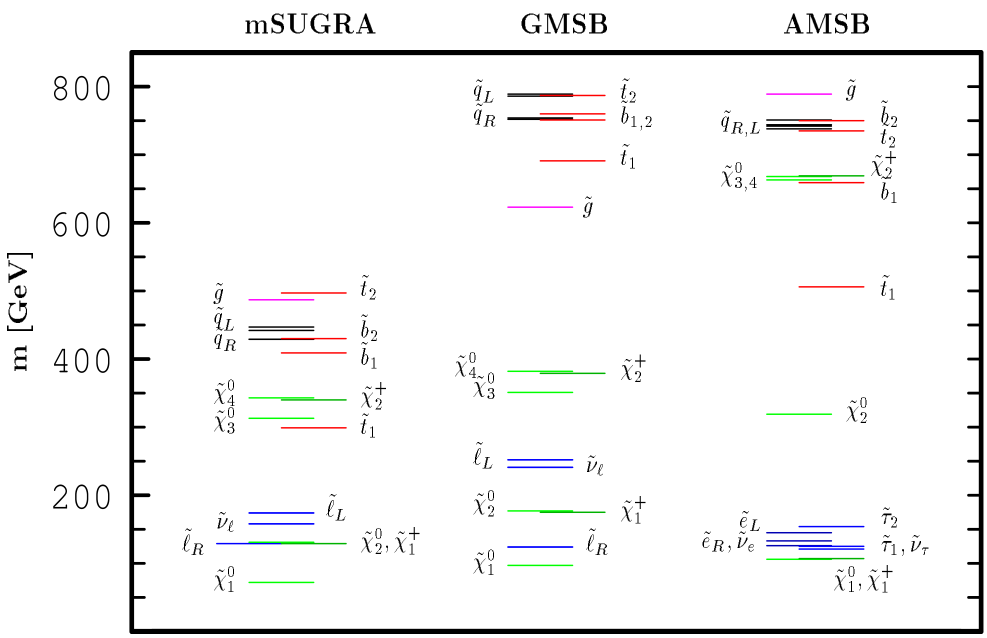
Energy spectrum $(E_{e^-} - E_{e^+})_{e^-_R} - (E_{e^-} - E_{e^+})_{e^-_L}$ for $e^-_{R,L}e^+ \to \tilde{e}_R\tilde{e}_L$ in the model SPS 1a at $\sqrt{s}$=500 GeV, $\mathcal{L} = 500$ fb^{-1}.

• SUSY discovery beginning of extensive and exciting experimental research program.

• Crucial to make accurate measurements of SUSY masses and couplings to constrain SUSY-breaking mechanism.



• Different models of SUSY-breaking can give rise to very different sparticle mass spectra:



• (GMSB: Gauge Mediated SUSY Breaking; AMSB: Anomaly Mediated SUSY Breaking)

• Understanding SUSY-breaking mechanism will open window onto physics at Planck or GUT scales.

Epilogue

The theoretical speculations about physics beyond the Standard Model discussed in this lecture course have centered around supersymmetry and grand unified theories. However, even much more radical changes could happen as we approach a new frontier of high-energy physics:

- qualitatively new degrees of freedom could appear, like strings or extra dimensions.
- symmetries of the Standard Model, like baryon and lepton number conservation could be broken.
- "sacred principles" like locality, micro-causality or CPT invariance could be violated.
- theoretical frameworks like general relativity and quantum mechanics might break down under certain conditions.

Physics beyond the Standard Model will be complex and maybe confusing, with new interactions and a rich phenomenological structure. The accelerators and experiments planned and envisaged for the next generation will provide the tools required to unravel the structure of fundamental physics at the TeV scale and – hopefully – beyond.

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