QCD meets Compact Stars

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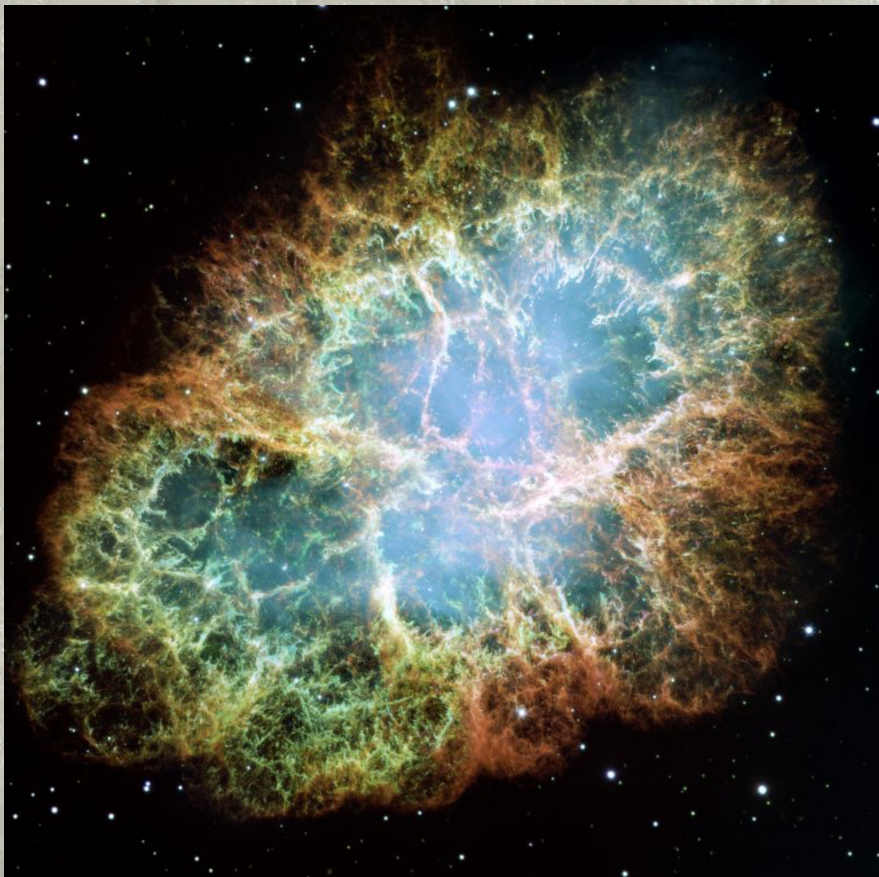
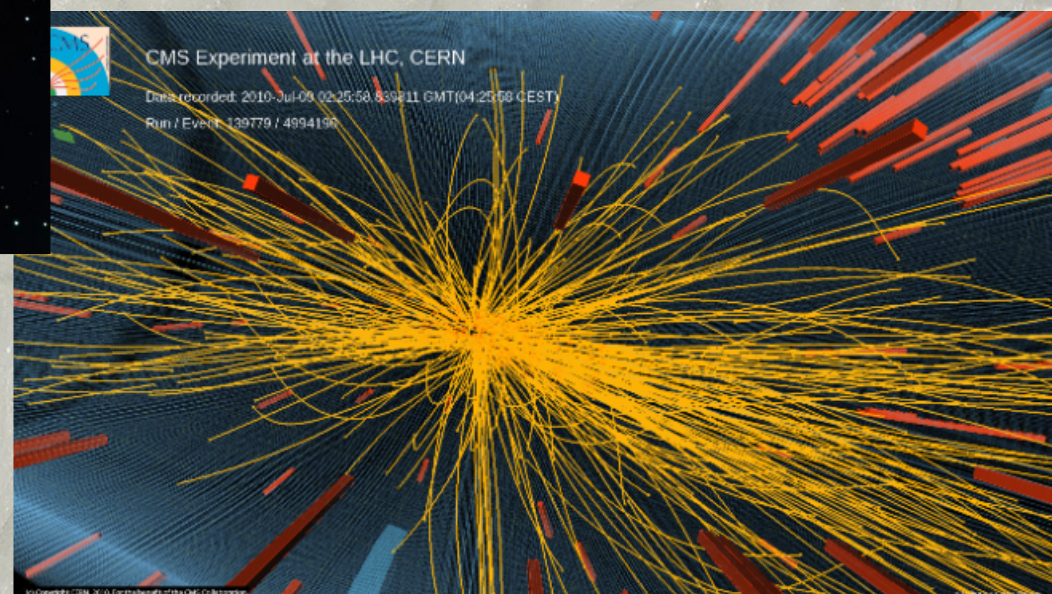
Compact stars & particle physics ??!!

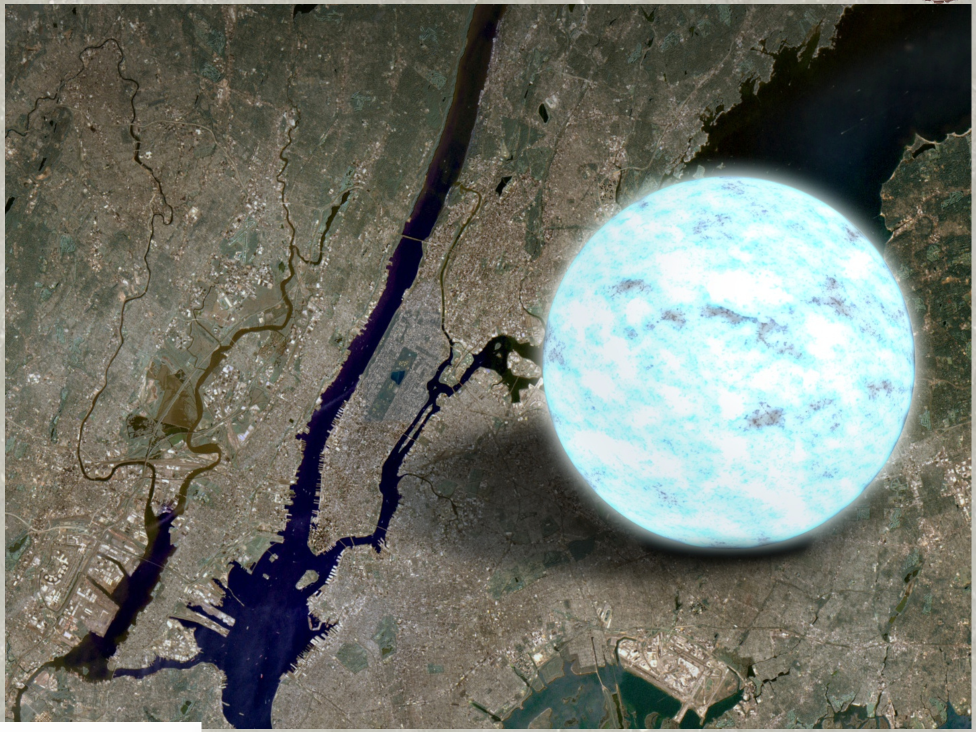


[NASA, ESA, J. Hester and A. Loll (Arizona State University)]

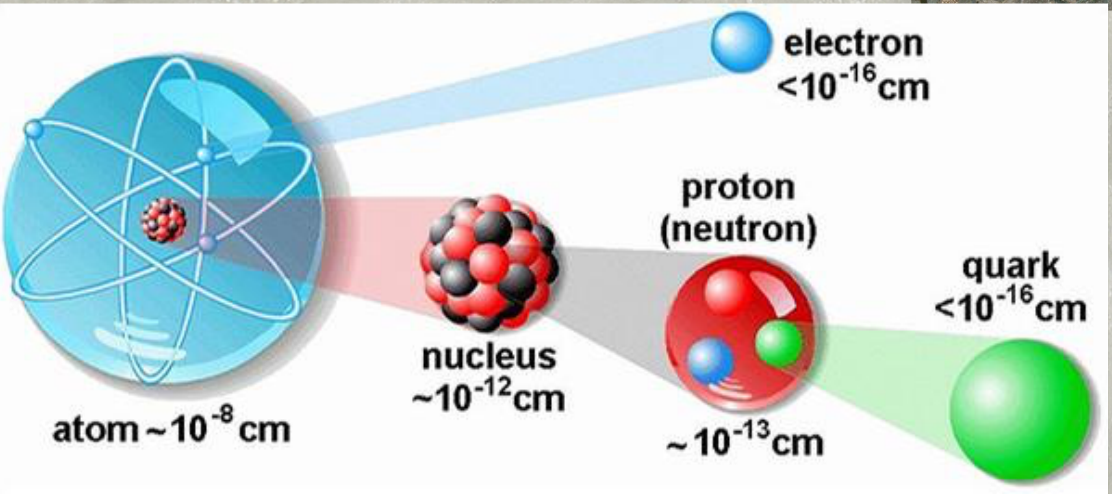
★ A new (nearby) future, one that deeply connects high-energy nuclear & particle physics under extreme conditions to astrophysics of compact stars!



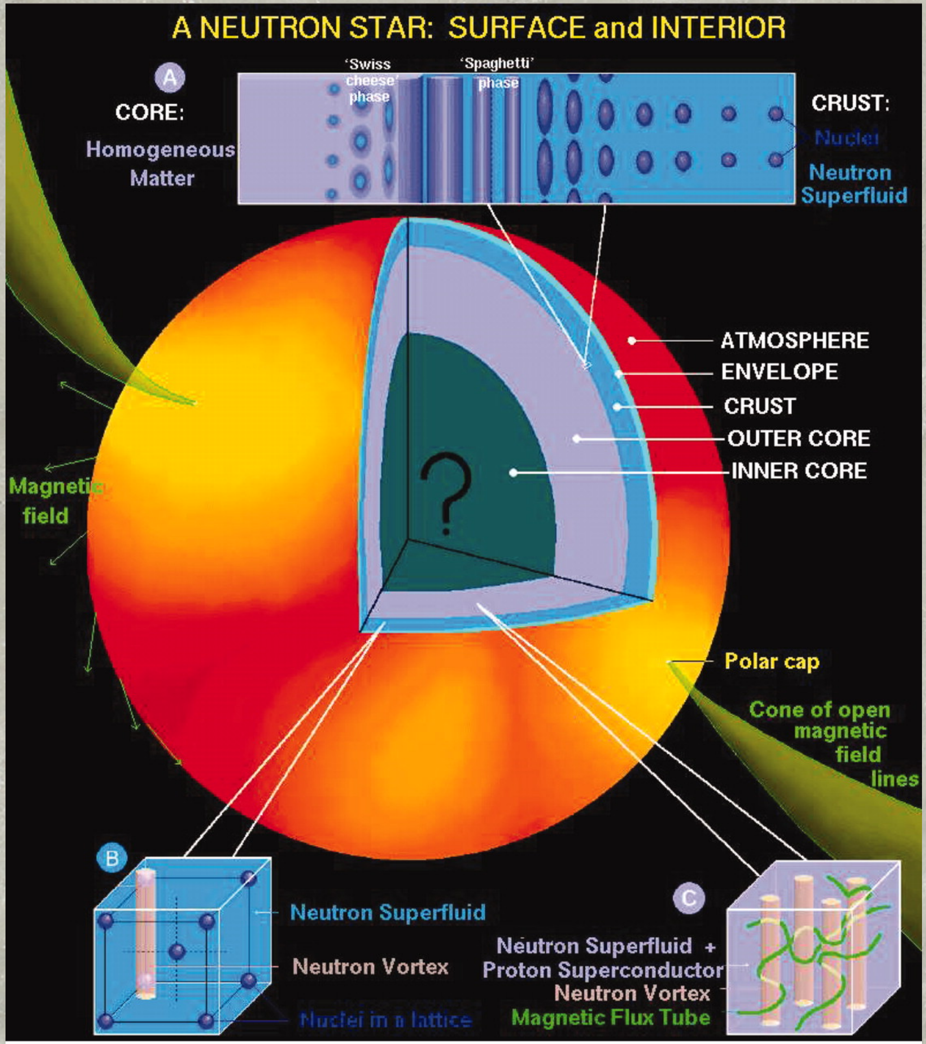
Very different scales, yet somehow connected?



[NASA’s Goddard Space Flight Center]

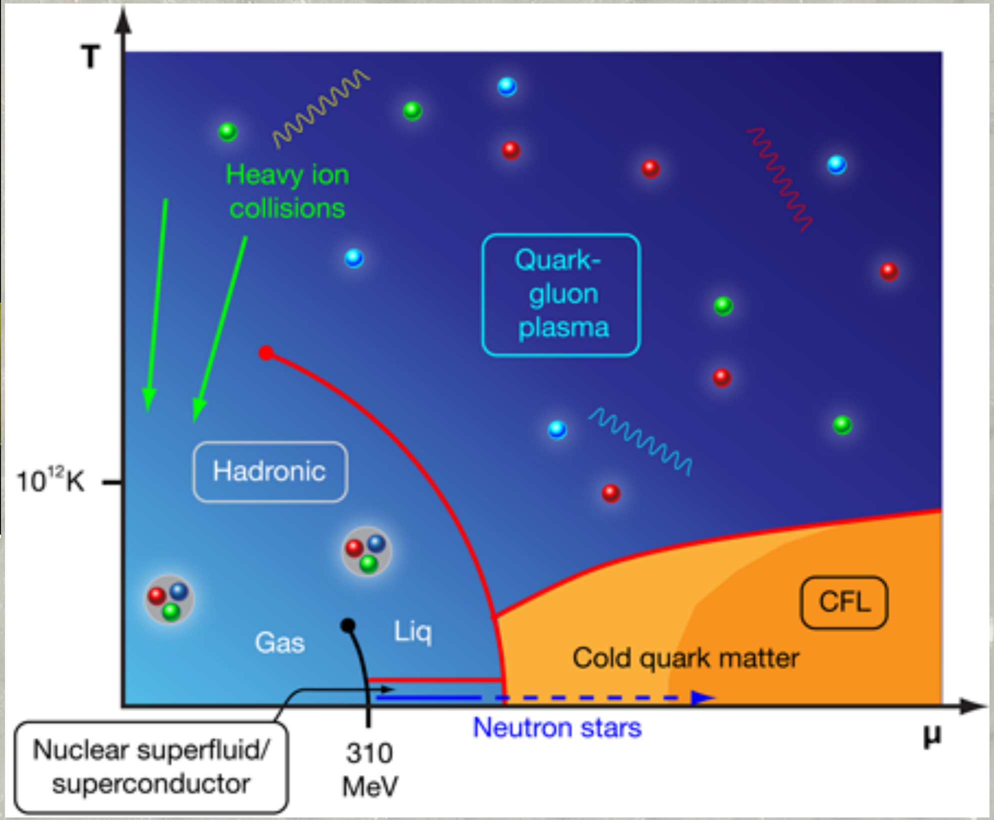


[ETH Zürich]



[Lattimer & Prakash,

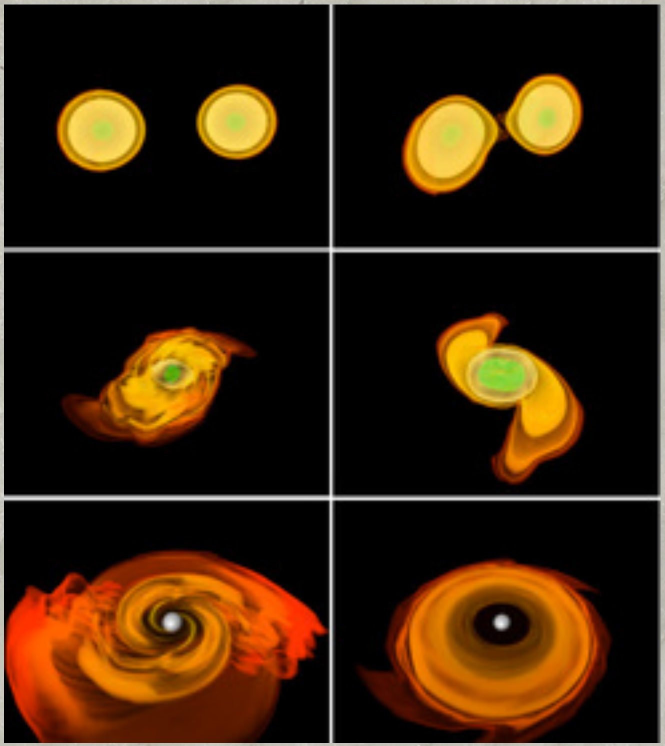
★ Large domains of the phase diagram of strong interactions not reachable by lab experiments: great challenges!



[Alford, 2010]

★ Compact stars provide a new window to the Standard Model, complementary to accelerators.

[NASA]



[Rezzolla et al., 2008]

★ Observations of compact objects entered a new era after the first direct observation of gravitational waves from binary mergers. Higher-precision mass and radius measurements are hopefully at the corner.

★ Hence, yes, compact stars & particle physics! :-)

Compact stars & QCD (and the Standard Model)

★ Main question in the background of this talk:

Can first-principle results from QCD, as well as controllable approximations of the theory of strong interactions, bring some hints to the physics of compact stars and vice-versa?

★ The answer is yes, and we discuss two examples:

- ❖ The equation of state for neutron star matter.
- ❖ The role of isospin in a possible new class of stars.

First part - EoS for neutron star matter from QCD

⋆ One expects that NS will exhibit new states of matter.

⋆ Some questions: Is there quark matter in NS? What is the formation mechanism? Can we make predictions from first principles?

⋆ To address these questions one needs to deal with the structure of neutron stars.

⋆ For a much broader coverage, please see:

EMMI Rapid Reaction Task Force Meeting on
"Quark Matter in Compact Star"

Michael Buballa,[1] Veronica Dexheimer,[2] Alessandro Drago,[3] Eduardo Fraga,[4,5,6] Pawel Haensel,[7] Igor Mishustin,[5] Giuseppe Pagliara,[3] Jürgen Schaffner-Bielich,[4] Stefan Schramm,[5] Armen Sedrakian,[4] and Fridolin Weber[8,9]

Michael Buballa et al 2014 J. Phys. G: Nucl. Part. Phys. 41 123001

Hydrostatic equilibrium in stars - compact stars:

★ Tolman-Oppenheimer-Volkov (TOV) equations: Einstein's GR field equations + spherical symmetry + hydrostatic equilibrium.

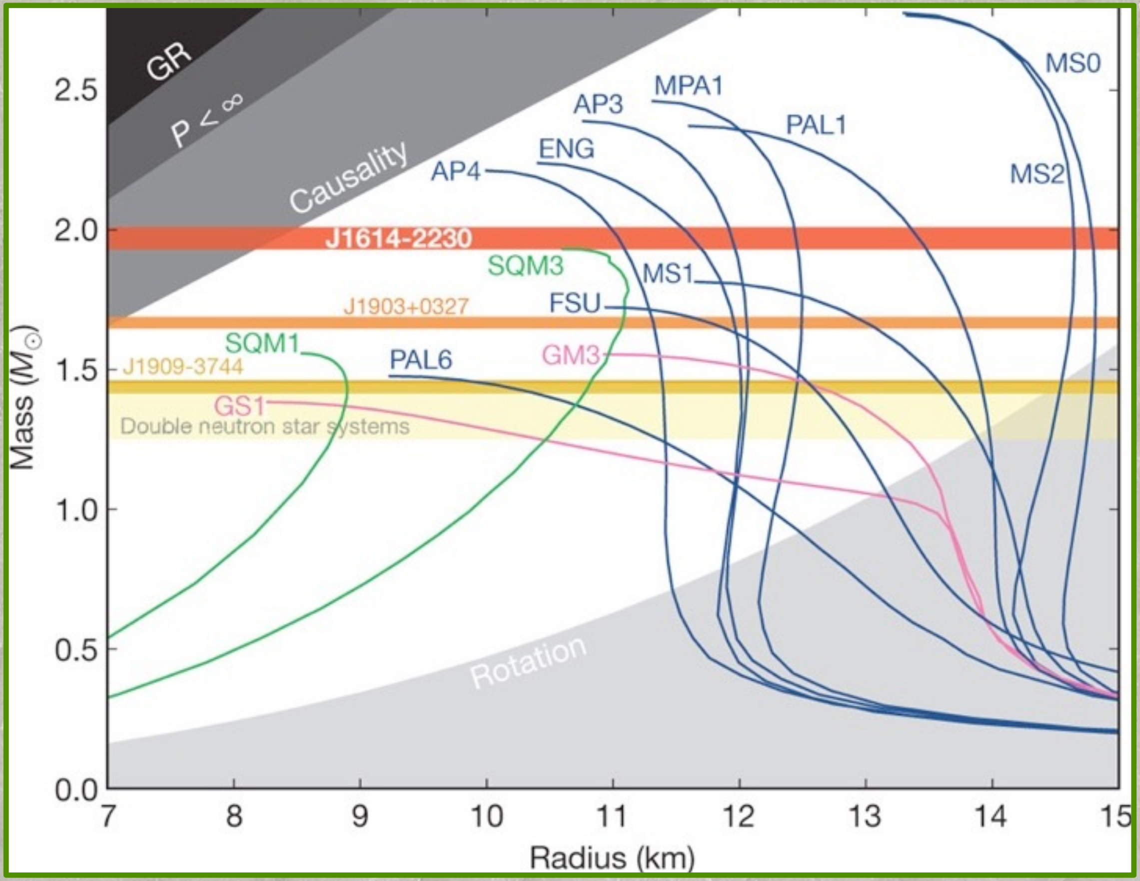
★ In GR, energy density and pressure also contribute to gravity!

$$\frac{dp}{dr} = -\frac{G\mathcal{M}(r)\epsilon(r)}{r^2\left[1-\frac{2G\mathcal{M}(r)}{r}\right]}\left[1+\frac{p(r)}{\epsilon(r)}\right]\left[1+\frac{4\pi r^3 p(r)}{\mathcal{M}(r)}\right]$$

$$\frac{d\mathcal{M}}{dr} = 4\pi r^2\epsilon(r) \;\; ; \quad \mathcal{M}(R) = M$$

★ One equation missing in the system: the equation of state!

★ Equation of state: different EoSs define different types of stars (neutron stars, strange stars, hybrid stars).



[Demorest et al (2010)]

★ So, we need the EoS for the kind of matter we expect to find inside neutron stars, which goes way beyond neutrons!

What kind of matter do we expect to find in neutron stars?

★ Tough question! So far observations cannot tell much in this respect...

★ Some numbers can give a hint, though:

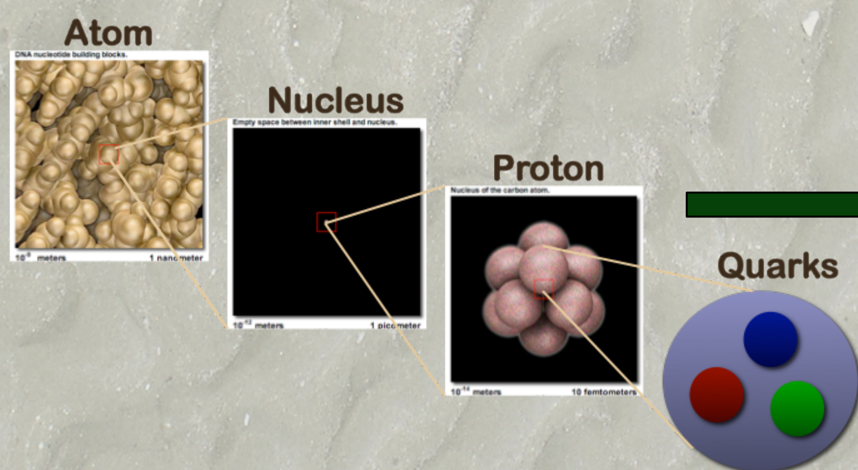
$n_0 = 3 \times 10^{14}$ g/cm^3 = 0.16 fm^{-3}

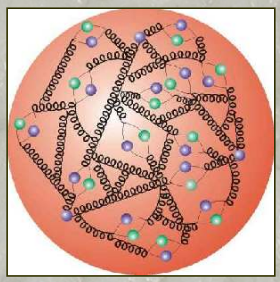
$n_{core} \approx (4 - 15)\, n_0$ [$\langle n_{Earth} \rangle \approx 5.5$ g/cm^3]

$M \approx 1 - 2$ solar masses [$M_S \approx 2 \times 10^{30}$ kg $\approx 10^{57}$ GeV]

$R \approx 6 - 16$ Km [$R_S \approx 7 \times 10^5$ km]

⟹ QCD at "high" density + General Relativity

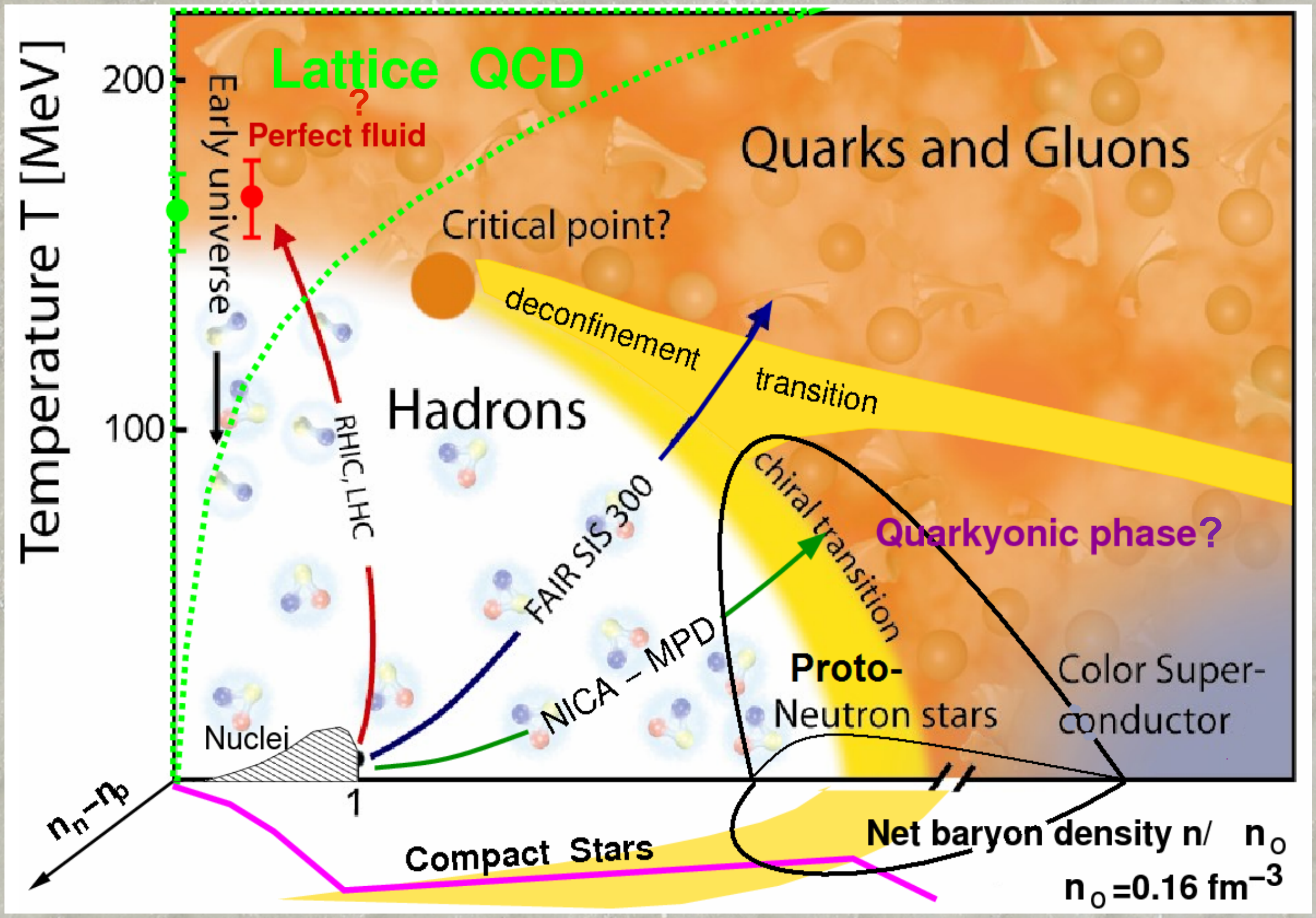




(Univ. Oxford)

Real origin of mass...
Sorry, Higgs!

Then, neutron stars are intimately connected to strong interactions under extreme conditions!



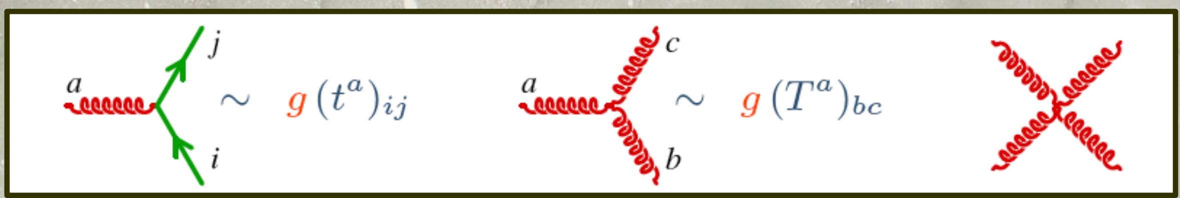
We need the EoS in this region !!

Theory (QCD)

The theory is, in principle, given:

$$\boxed{\mathcal{L} = -\frac{1}{4} F^a_{\mu\nu} F_a^{\mu\nu} + \sum_f \overline{\psi}_f (i\gamma^\mu \partial_\mu - m_f - ig\gamma^\mu A^a_\mu \tau^a)\psi_f}$$

$$\boxed{F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^c_\nu}$$



Then: compute the thermodynamic potential and then the EoS.

Well, it's not so simple...

Equation of state - naïve field map

hadrons
low temp.
and density

……………………………………………………

???

$T \sim T_c$; $n \sim n_c$

quarks
high temp.
and density

hadronic models,
effective field theory

pQCD at $T>0$ & $\mu>0$
asymptotic freedom

where all the things that matter happen…

there is no appropriate formalism yet…

Some serious difficulties:

⋆ Differently from the hot and dilute case, the cold and dense physical setup suffers from the Sign Problem* -> no Lattice QCD guidance…

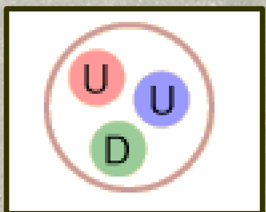
⋆ We are left with the following possibilities, then:

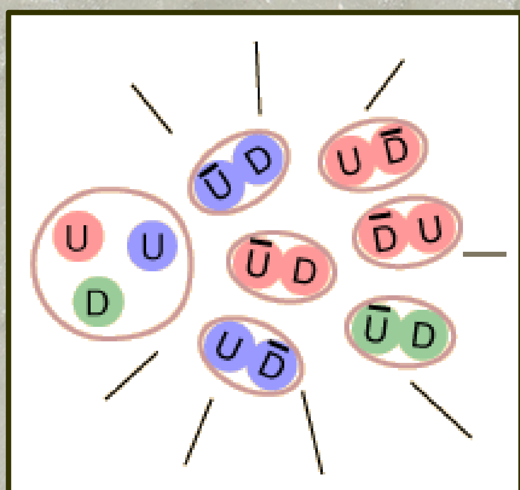
- moving up from effective field theory (EFT)
- models (NJL, LSM, etc and extensions)
- moving down from pQCD & imposing constraints
- the MIT bag model (a long story…)

*Sign Problem: in QCD with $\mu \neq 0$, the effective S_{eucl} is complex, so that one does not have a well-defined weight factor any more… Monte Carlo calculations hindered…

★ Let us start with a discussion of an important but also in a way "bad start": the MIT bag model, perhaps the most popular approach to quark matter in NS.

★ The MIT bag model (70's) is something REALLY simple:





Asymptotic freedom + confinement in the simplest and crudest fashion: bubbles (bags) of perturbative vacuum in a confining medium.
+ eventual corrections ~ α_s

- Asymptotic freedom: free quarks and gluons inside color singlet bags
- Confinement: vector current vanishes on the boundary

★ This makes it easy to use and dangerous...

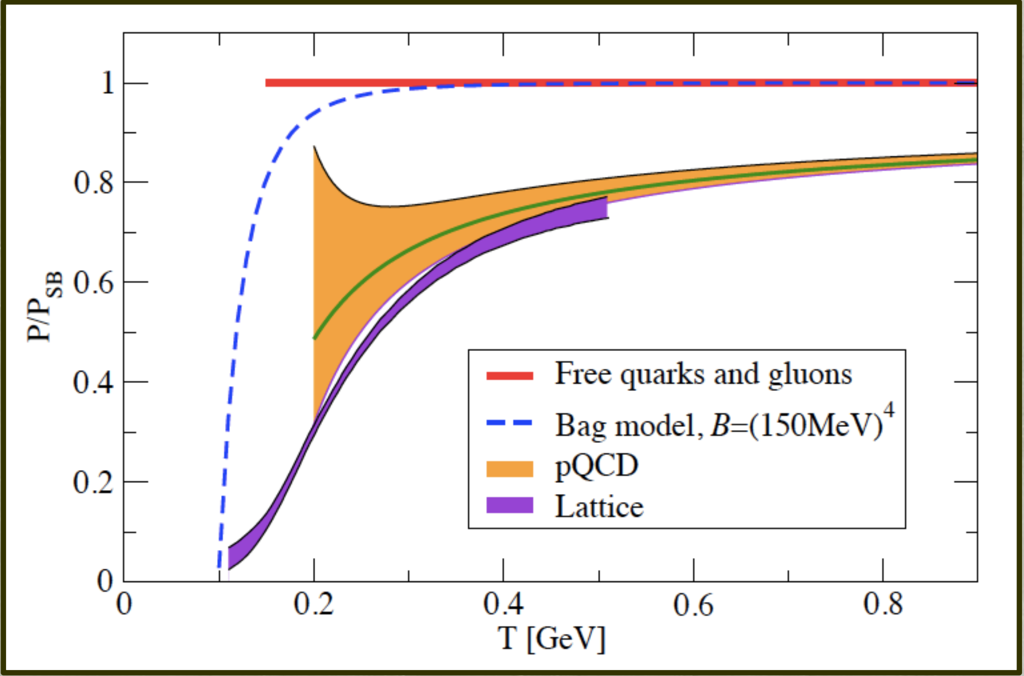
For instance, at finite temperature the pressure has the form:

$$p = \left(\nu_b + \frac{7}{4} \nu_f \right) \frac{\pi^2 T^4}{90} - B$$

★ B: phenomenological constant to be adjusted (for instance, originally to hadron masses) — very unconstrained...

★ Counting of Stefan-Boltzmann degrees of freedom.

Comparison to lattice QCD and pQCD:



[ESF, Kurkela & Vuorinen (2013)]

★ Gives false impression of precision and accuracy. In fact, very crude!

★ Comparison to lattice shows that it misses: the nature of the phase transition, relevant scales (quantitative), general behavior of the EoS, ...

[1 GeV ~ 10^{13} K ~ 10^6 T_{sun} (core)]

Cold quark matter via pQCD, also an old story…

Freedman & McLerran, 1977-1978
Baluni, 1978 ; Toimela, 1980's

Kajantie et al, 2001 ; Peshier et al, 1999-2003
Blaizot, Iancu & Rebhan, 1999-2003
ESF, Pisarski & Schaffner-Bielich, 2001
Andersen & Strickland, 2002
Rebhan & Romatschke, 2003
Vuorinen, 2004-2007

Quark stars & hybrid stars with large masses from pQCD (~20 years ago…)

[sorry for unavoidable omissions…]

Including the strange quark mass:

Freedman & McLerran, 1977-1978; Baluni, 1978
(considered irrelevant for over 20 years…)

ESF and Romatschke, 2005
Kurkela, Romatschke & Vuorinen, 2010

NLO, NNLO: vacuum diagrams & ring resummation (the state of the art)

ESF, Kurkela & Vuorinen (2014, 2016)
Kurkela, ESF, Schaffner-Bielich & Vuorinen (2014)

New T & μ state of the art:
Cool quark matter
Kurkela & Vuorinen, 2016

Higher order cold QM:
Gorda et al, 2018

Caveats for the pQCD EoS

★ Validity is assured only for very high densities. Strictly speaking, way beyond the relevant region for NS.

★ However, this is the theory of strong interactions there.

★ There is always a dependence on the renormalization scale. This is unfortunate, of course, but it also provides a measure of our uncertainty (which is a good thing!).

★ Nevertheless, pQCD gives us something that no model can deliver: the correct behavior at large enough densities and robust constraints. And so we can ask different questions...

Asking questions we can answer using controlled approximations:

[Kurkela, ESF, Schaffner-Bielich & Vuorinen (2014)]

★ A very important and challenging question: is there deconfined quark matter in the core of compact stars?

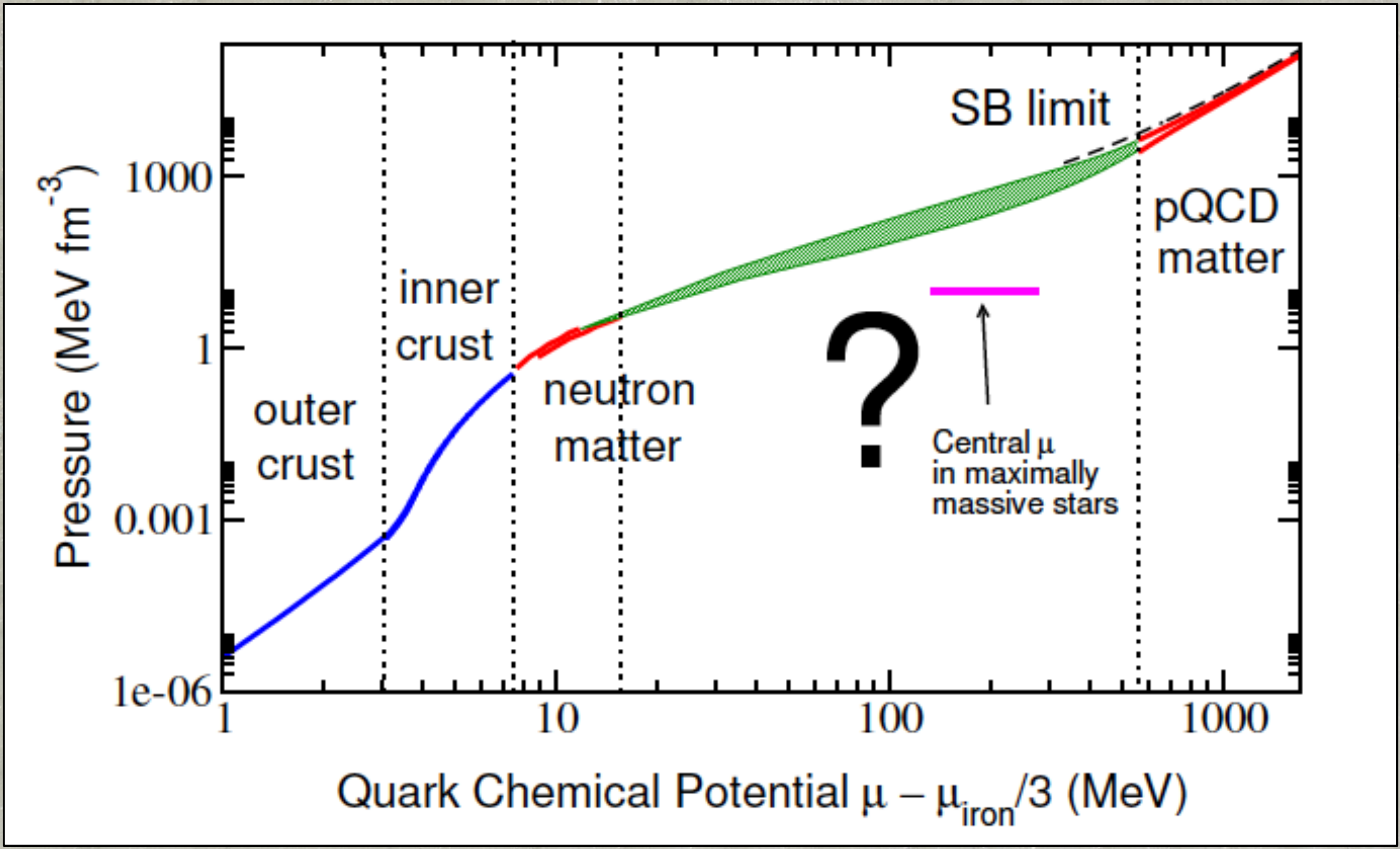
Very hard to answer in a definite (model-independent) way.

★ A related but totally different question: does the fact that at asymptotically high densities one must have deconfined quark matter constrain the equation of state for compact stars?

Yes!

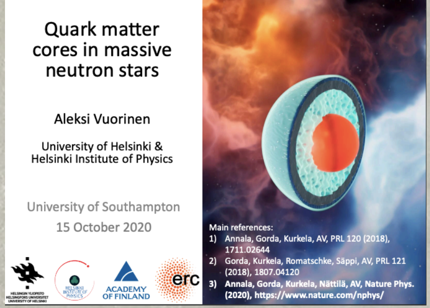
★ Main message: even if there is no deconfined quark matter in the core of neutron stars, the form of the QCD EoS at very large densities (which is known perturbatively) affects dramatically the EoS for compact stars!

In fact, now we know that the EoS is constrained to:



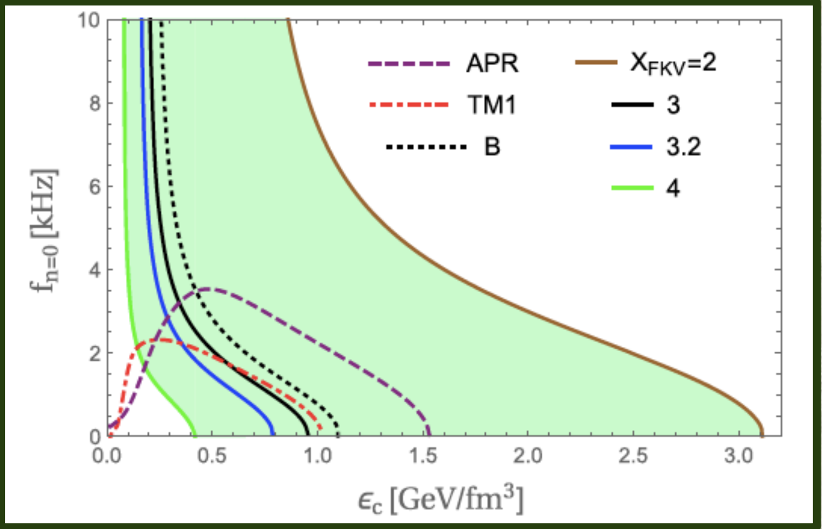
[Kurkela, ESF, Schaffner-Bielich & Vuorinen (2014)]

How to get to this plot & consequences covered by Aleksi Vuorinen in his talk 5 weeks ago! So, let me skip this discussion and give you a couple of other results from pQCD...



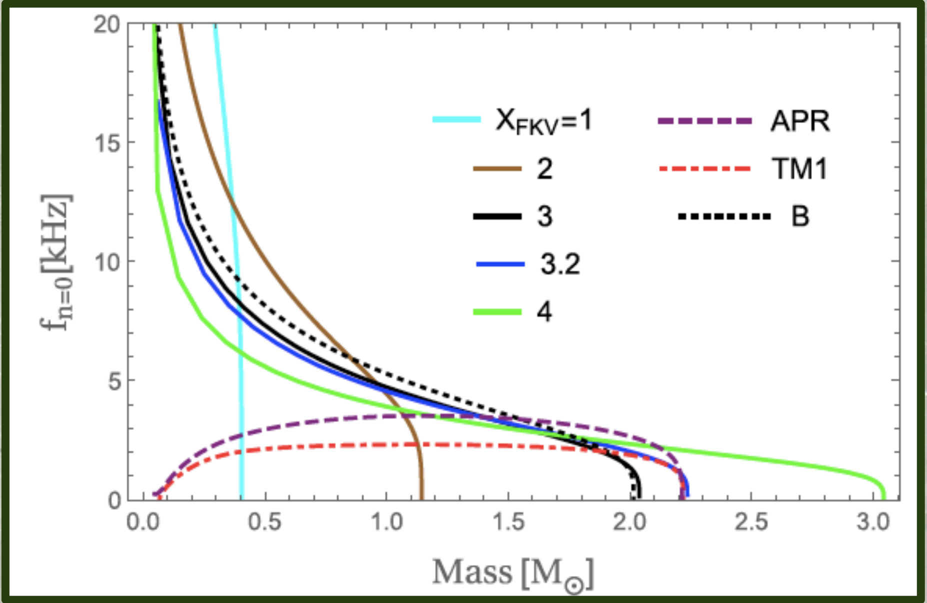
Radial oscillations of quark stars

[Jiménez & ESF (2019)]



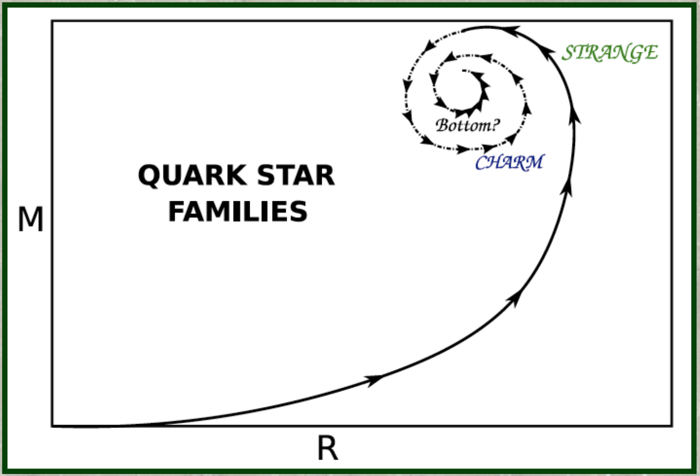
[1 GeV/fm^3 ~ 2 x 10^{15} g/cm^3]

★ Fundamental & 1st excited modes of nucleonic and quark stars not distinguishable near the two-solar mass limit.

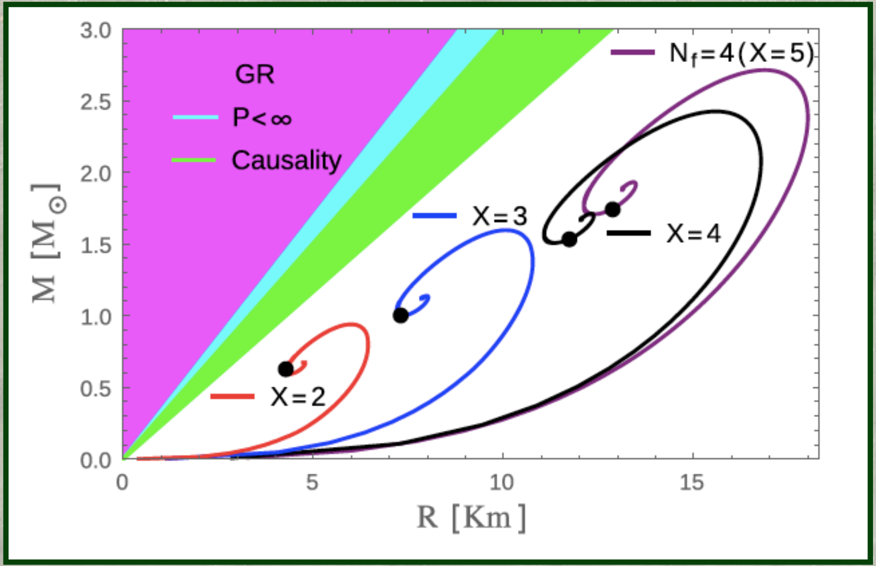


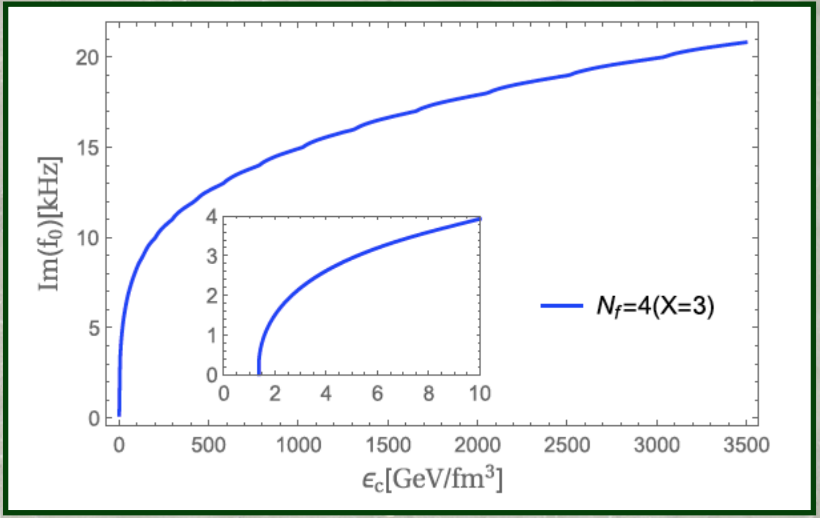
★ But: different curves -> discriminate between hadronic & quark stars by comparing their nonradial pulsation modes (correlated to the radial modes in gravitational waves)?

Charm stars? (results from N_f=2+1+1 pQCD) [Jiménez & ESF (2020)]



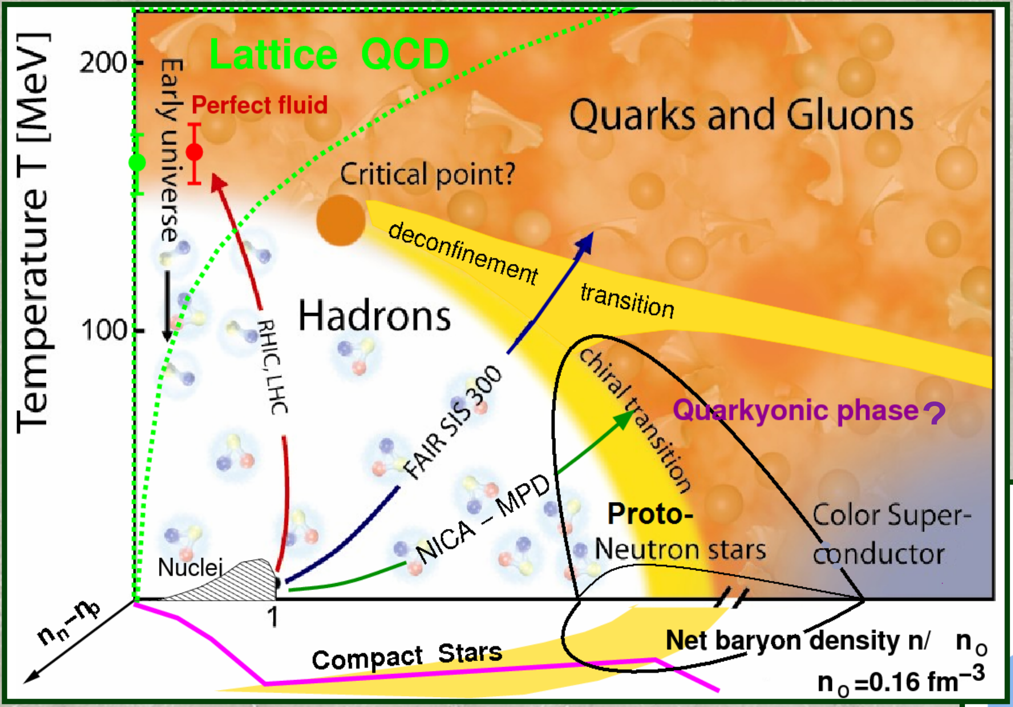
- ★ Effects of heavy quarks on the EoS.
- ★ RG scale is the only free parameter.
- ★ pQCD more reliable at higher densities.
- ★ Charm matter (charge neutral; β equilibrium).





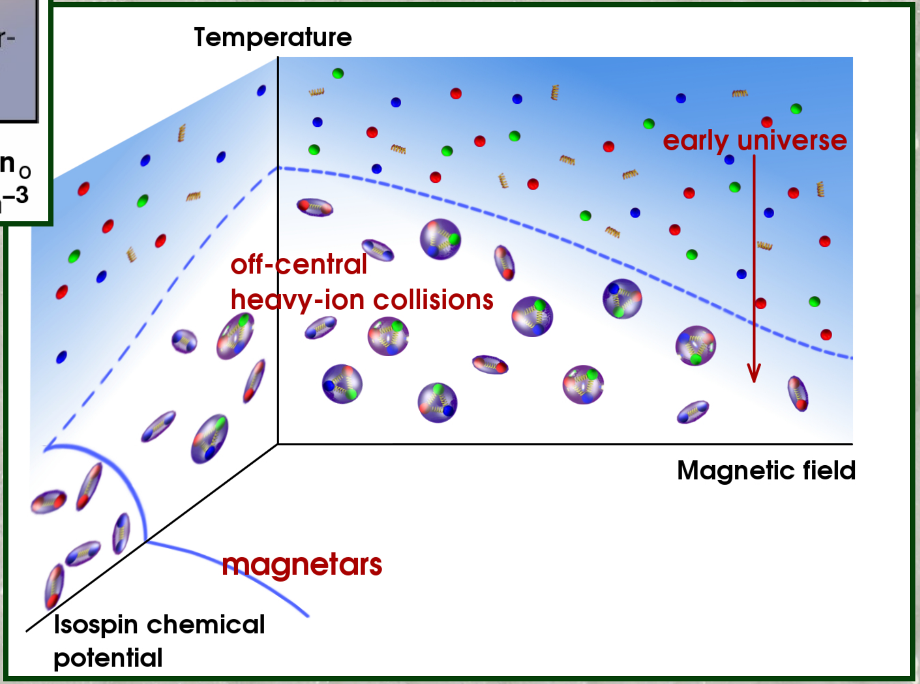
★ Unstable charm stars...

Second part - Why QCD at finite isospin ??



★ Physical direction in the phase diagram, if you are interested in compact stars & heavy-ion collisions.

★ Even combined with magnetic fields can be simulated on the Lattice (relevant for magnetars & heavy ions).



[G. Endrodi (ITP-Frankfurt)]

So, QCD at finite isospin has...

Strong motivation: in-medium strong interactions exhibiting appreciable isospin asymmetry are:

★ of experimental relevance

➡ heavy ion collisions, neutron stars, boson stars (?) ...

★ rich in new phenomenology

➡ pion condensation, ...

★ amenable to lattice simulations (no Sign Problem for $m_u = m_d$!): new open channel for comparisons!

➡ model constraining, tests for pQCD, χPT and effective models, ...

Particle physics 101: isospin symmetry recap

★ Isospin SU(2) symmetry [Heisenberg, 1932]: $[I_3, \mathcal{H}_{\text{strong}}] \approx 0$

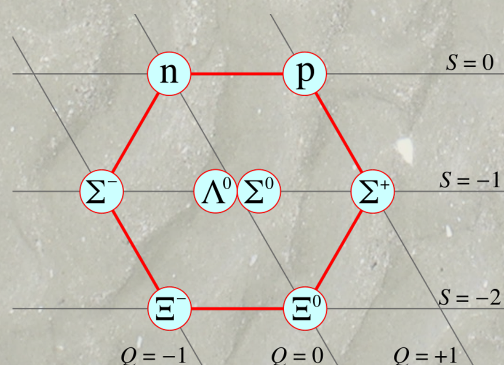
- Strong interactions approx. invariant (p $\longleftrightarrow$ n, etc.)
- Differences of 2 - 3% in a multiplet:

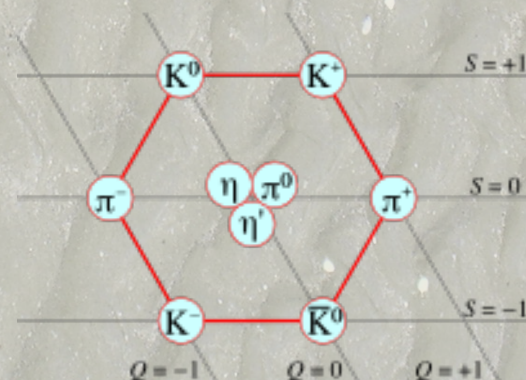
$$\frac{m_n - m_p}{m_n + m_p} \approx 0.7 \times 10^{-3} \qquad \frac{m_\pi^+ - m_\pi^0}{m_\pi^+ + m_\pi^0} \approx 1.7 \times 10^{-2}$$

- Algebra:

$$[I_i, I_j] = i\epsilon_{ijk} I_k \quad ; \quad I_k = \frac{\sigma_k}{2} \qquad I_3|p\rangle = \frac{1}{2}|p\rangle \quad ; \quad I_3|n\rangle = -\frac{1}{2}|n\rangle$$

- Multiplets:





...

★ At the quark level, up and down quarks form an isospin doublet:

$$I_3|u\rangle = \frac{1}{2}|u\rangle \quad ; \quad I_3|d\rangle = -\frac{1}{2}|d\rangle$$

- For equal quark masses $m_u = m_d$, QCD exhibits isospin SU(2) symmetry

  → Noether: conserved three-component isospin current connected with the electric charge

- One can introduce an isospin chemical potential: $\mu_I = \mu_u - \mu_d$ (*)

- If $\mu_u > \mu_d$, we have an excess of neutrons over protons and of π^- over π^+ (and vice-versa).

- To avoid the Sign Problem, one can consider the scenario with nonzero μ_I, $m_u = m_d$ and vanishing baryon chemical potential, μ_B. Then:

$$\boxed{\mathcal{D} = \gamma(\partial + iA) + \tfrac{1}{2}\mu_I \gamma_0 \tau_3 + m}$$

$$\boxed{\tau_1 \gamma_5 \mathcal{D} \gamma_5 \tau_1 = \mathcal{D}^\dagger}$$

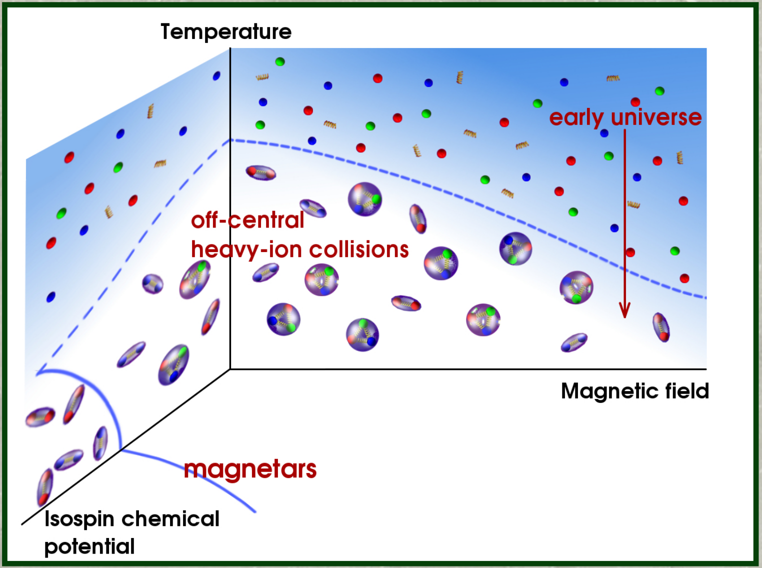
(*) NB: Definitions vary by factors of 2 [including here!].

Is this an academic problem?

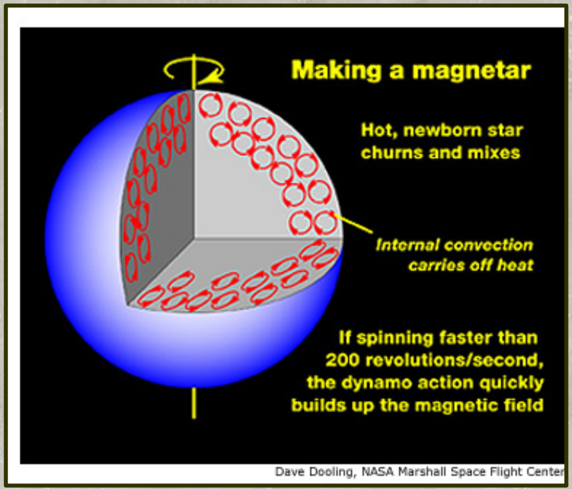
★ It is not clear yet...

★ However, even if it is academic, we can still learn important things, since we will be able to test pQCD & χPT in this scenario and constrain effective models from which one can make predictions.

★ In fact, one can "double" this scenario by including an external magnetic field on the lattice. And:



[G. Endrodi (ITP-Frankfurt)]



"Magnetars": B ~ 10^{14}–10^{15} G at the surface, much higher in the core [Duncan & Thompson (1992/1993)]

⋆ Perhaps not entirely academic…

⋆ Even if in neutron stars one has large baryon densities, boson stars (with no baryons!) are hypothetically possible! [Liebling & Palenzuela (2017)]

⋆ Boson stars have been considered candidates for Dark Matter for over 3 decades. [Colpi, Shapiro & Wasserman (1986)]

⋆ A major issue is that the absence of repulsive interactions would lead directly to gravitational collapse. There is no degenerate Fermi pressure there!

⋆ Well, QCD at low energies ≈ pions.

⋆ Pions, as composite bosons, could in principle form boson stars (one also needs leptons to balance electric charge). New compact stars? Yet to be seen.

Pion condensation and phase diagram, an old story...

⋆ At high densities, pion condensation (and also kaon condensation) might happen. [Migdal, 1971] [Sawyer, 1972 ; Scalapino, 1972] [Kaplan & Nelson, 1986]

⋆ Does it happen before the transition to quark matter? [Son & Stephanov, 2001]

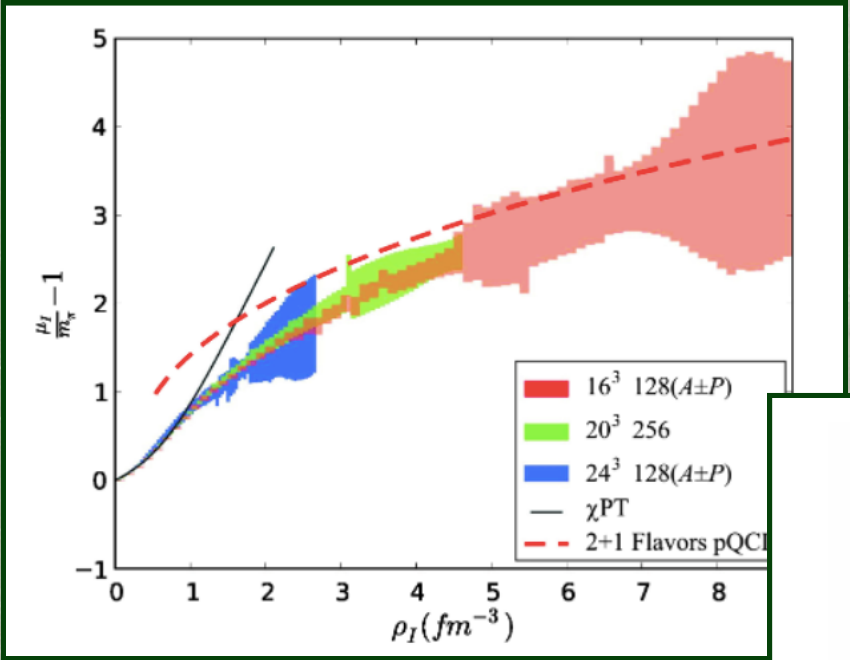
⋆ At the level of charged pions: $\mu_\pi = 2\mu_I$. At zero temperature:

$\mu_\pi < m_\pi$	vacuum state
$\mu_\pi = m_\pi$	Bose-Einstein condensation
$\mu_\pi > m_\pi$	

⋆ Pion condensation at low μ_I; superfluid/conducting phase; breaking of U(1) symmetry → Nambu-Goldstone boson.

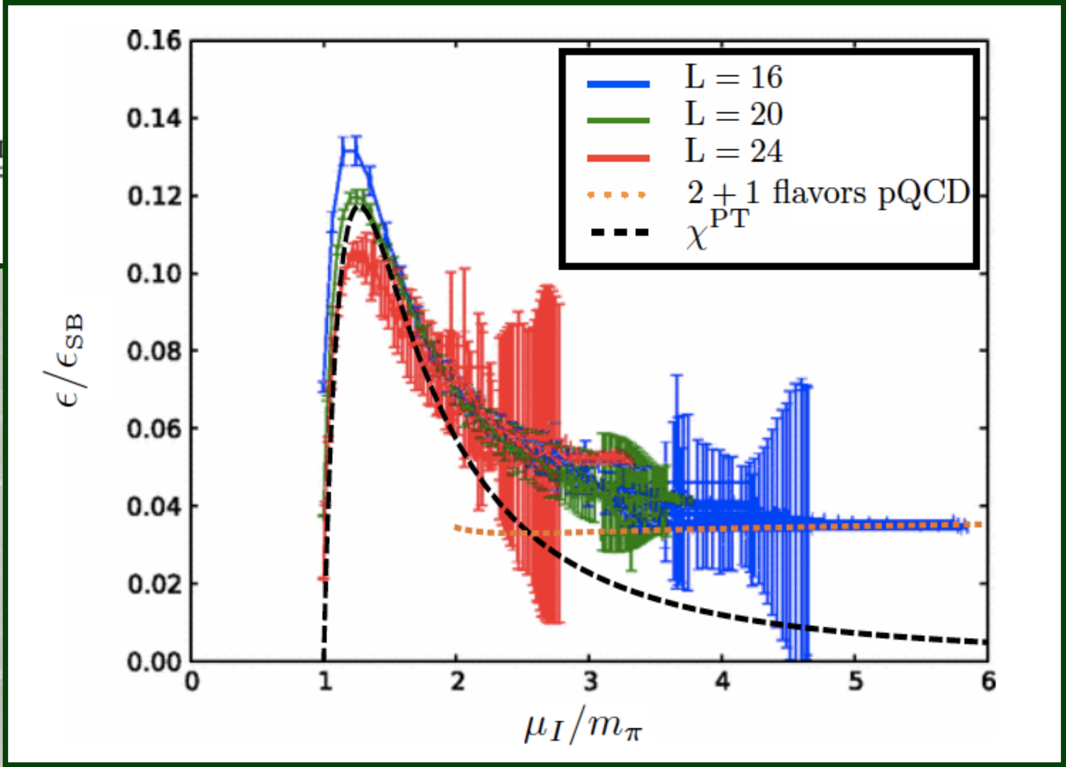
⋆ At the level of quarks: pairs with pion quantum numbers. Cross-over to perturbative regime? Lattice simulations & chiral effective model!

Thermodynamics from different approaches:



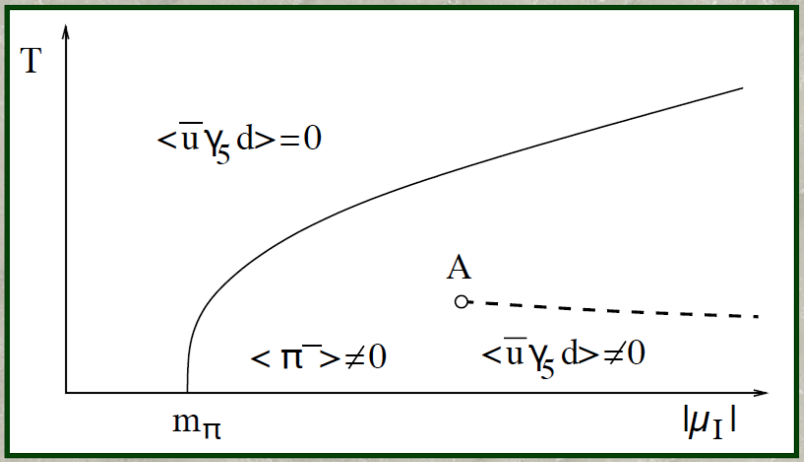
[Graf, Schaffner-Bielich & ESF (2016)]

- Lattice [m_π=390MeV]: Detmold, Orginos & Shi (2012)
- pQCD: Graf, Schaffner-Bielich & ESF (2016)
- χPT: Carignano, Mammarella & Mannarelli (2016)



[Carignano, Mammarella & Mannarelli (2016)]

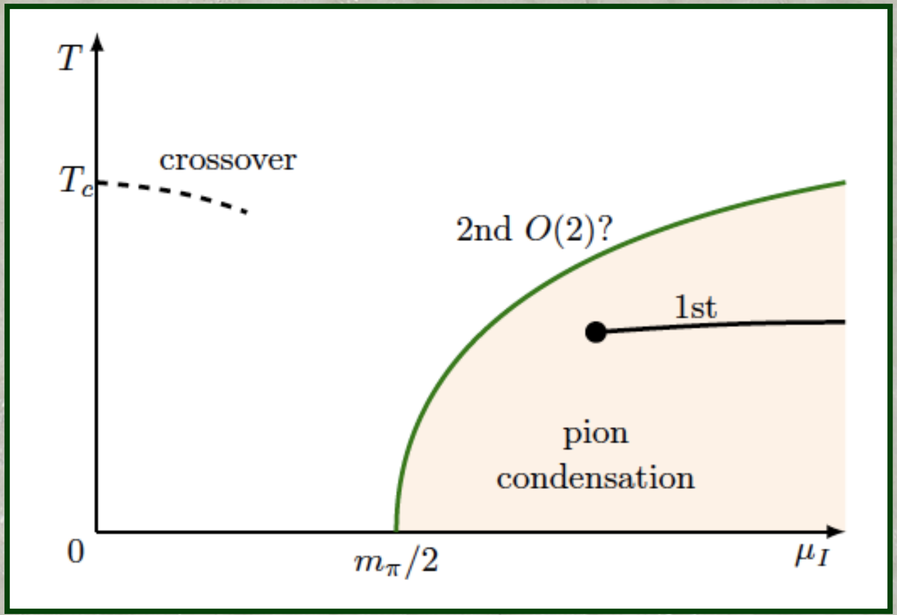
Phase diagram (old predictions & recent lattice results)



[Son & Stephanov (2001)]

★ Pion condensation & confinement determine this phase diagram.

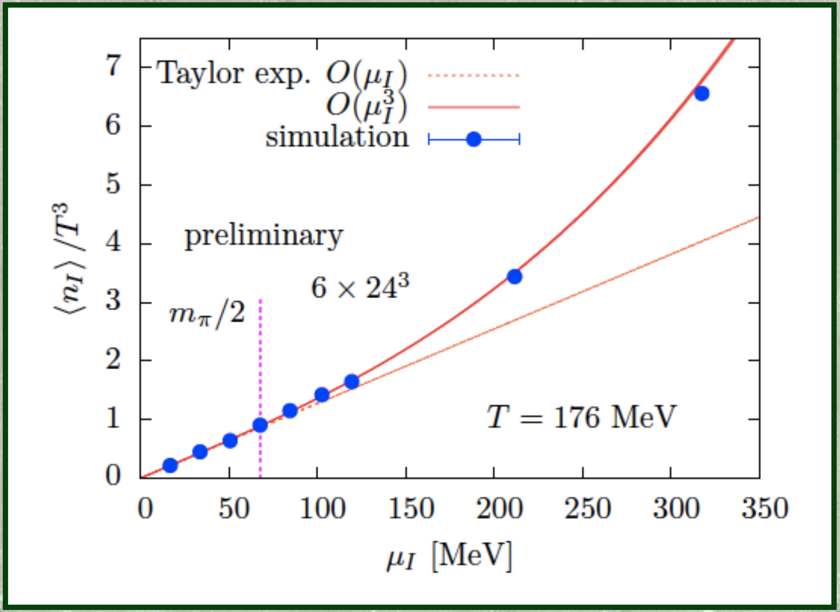
★ Deconfinement line just for illustration, but should end since we have a crossover for vanishing isospin.



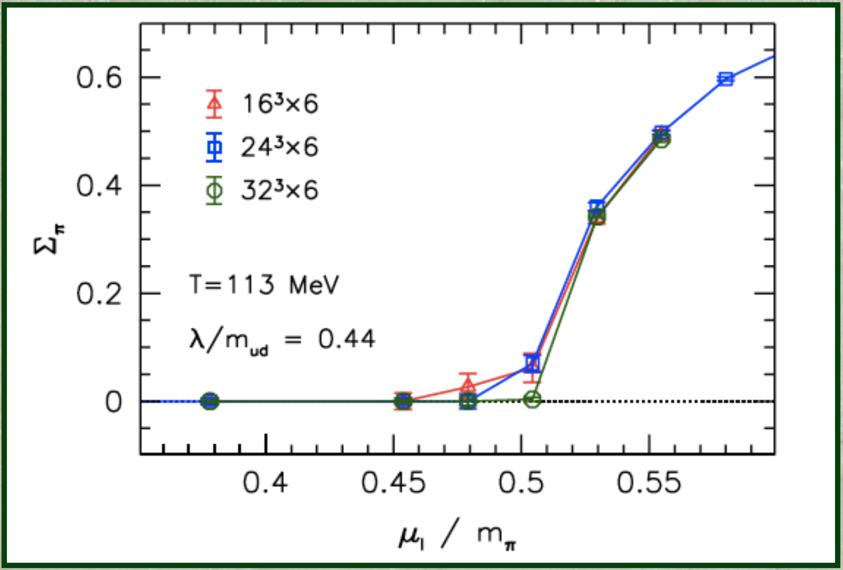
[Brandt, Endrodi & Schmalzbauer (2017)]

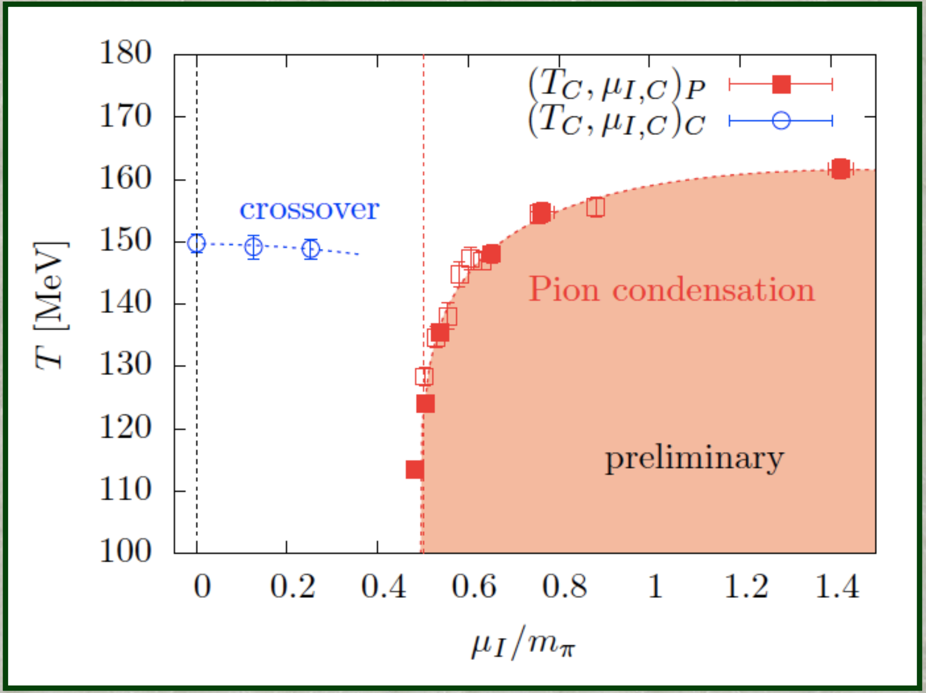
Recent lattice results

[Brandt, Endrodi & Schmalzbauer (2017)]



- Equation of state.
- Order of transition: 2nd order.
- Preliminary phase diagram.





Pion stars?

[Brandt, Endrodi, ESF, Hippert, Schaffner-Bielich & Schmalzbauer (2018)]

★ Assuming a metastable condensate is given, one can obtain the EoS for π^+ directly from the Lattice with a physical pion mass (or using a model).

★ Then, we can impose hydrostatic equilibrium assuming a compact star configuration:

→ TOV equations: Einstein's GR field equations + spherical symmetry + hydrostatic equilibrium.

$$\frac{dp}{dr} = -\frac{G\mathcal{M}(r)\epsilon(r)}{r^2\left[1 - \frac{2G\mathcal{M}(r)}{r}\right]}\left[1 + \frac{p(r)}{\epsilon(r)}\right]\left[1 + \frac{4\pi r^3 p(r)}{\mathcal{M}(r)}\right]$$

$$\frac{d\mathcal{M}}{dr} = 4\pi r^2 \epsilon(r) \ ; \quad \mathcal{M}(R) = M$$

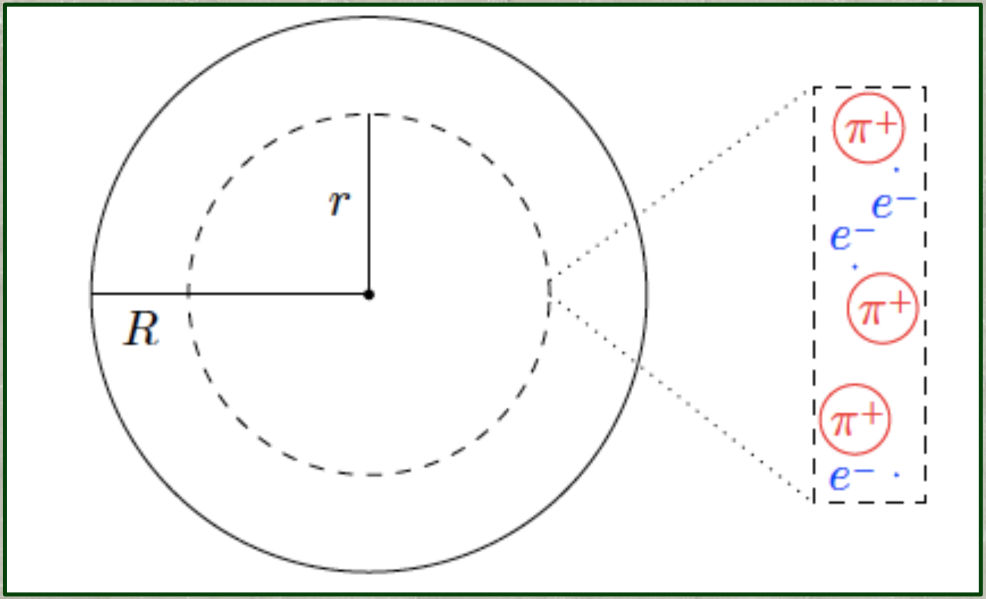
★ NB: a proper description should include EM interactions in the EoS & a modification of the external metric to Reissner-Nordström instead of Schwarzschild.

⋆ However, a pure π^+ star (which we can build for fun) is mechanically unstable, since

$$\frac{F_e}{F_G} = \frac{e^2/(4\pi\epsilon_0)}{Gm_{\pi^2}} \approx 10^{38}$$

⋆ One should add leptons (electrons or muons) to neutralize the total charge, i.e.:

$$Q = 4\pi \int_0^R \mathrm{d}r\, r^2 \left[n_\pi(r) - n_e(r)\right] = 0$$



[Fig. from Schmalzbauer]

- Assuming equal densities:

$$\mathbf{n}_\pi(\mathbf{r}) = \mathbf{n}_\mathbf{e}(\mathbf{r})$$

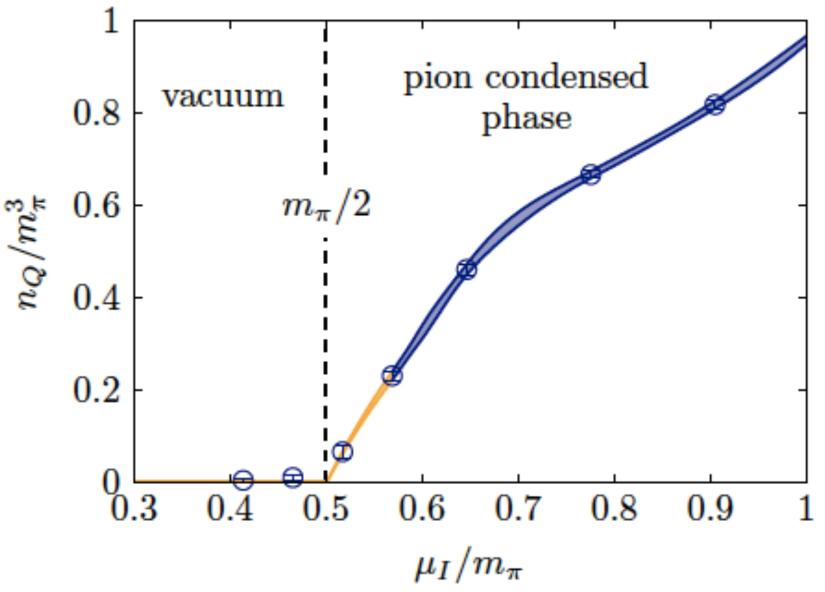
- and free relativistic electrons:

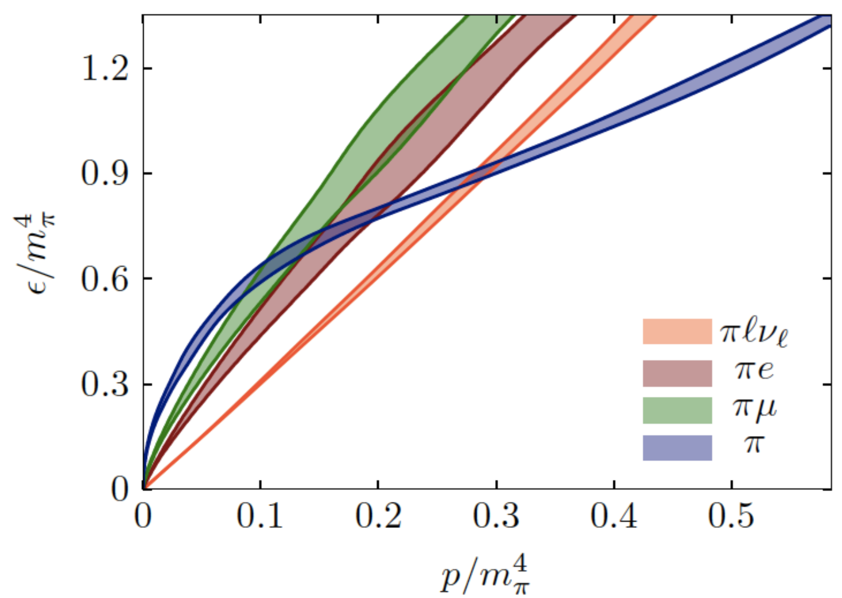
$$n_e(\mu_e > m_e) = \frac{1}{3\pi^2}\left(\mu_e^2 - m_e^2\right)^{3/2}$$

- Then:

$$p = p_\pi + p_e$$
$$\epsilon = \epsilon_\pi + \epsilon_e$$

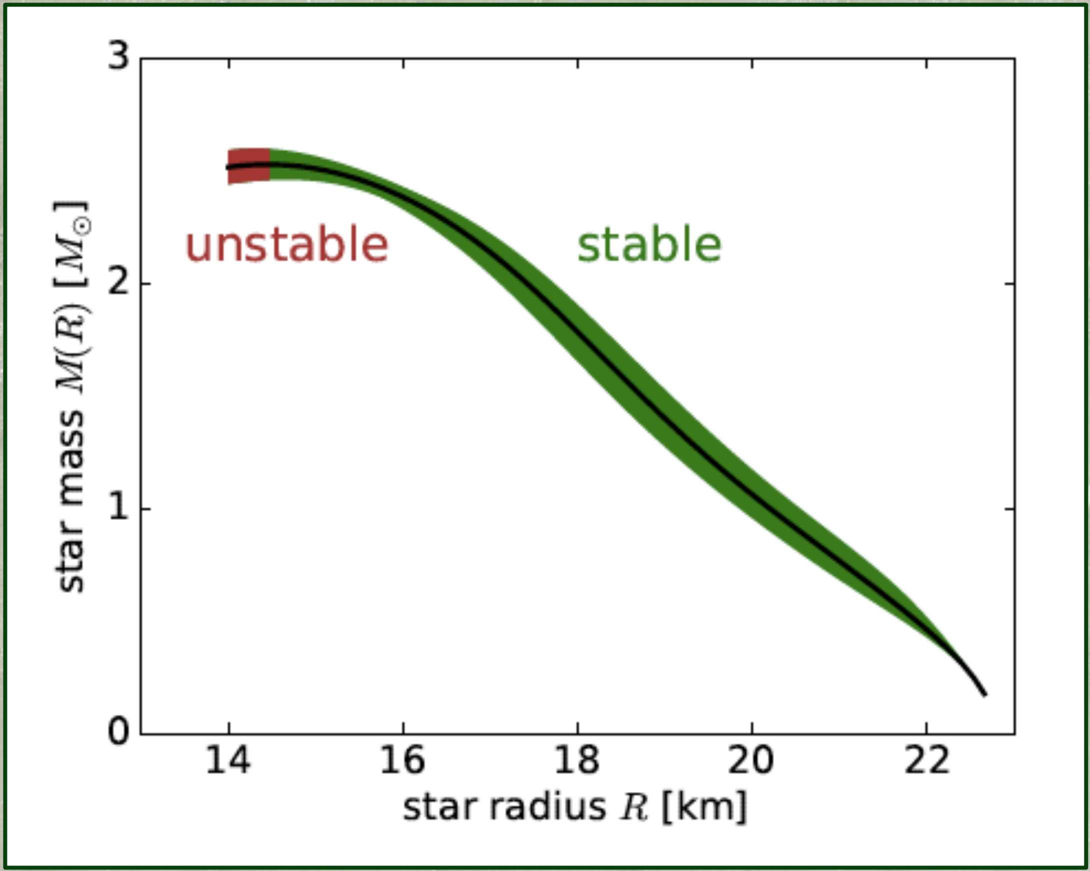
★ Phase transition to the condensed phase & EoS from the Lattice





Results for (possible?) pion stars (from SM physics!)

★ π^+ stars (gravitationally stable, EM unstable):



[Brandt, Endrodi & Schmalzbauer (2017)]

- Max. central energy density:

$$\boxed{\epsilon_c = 850\ \mathrm{MeV}/\mathrm{fm}^3}$$

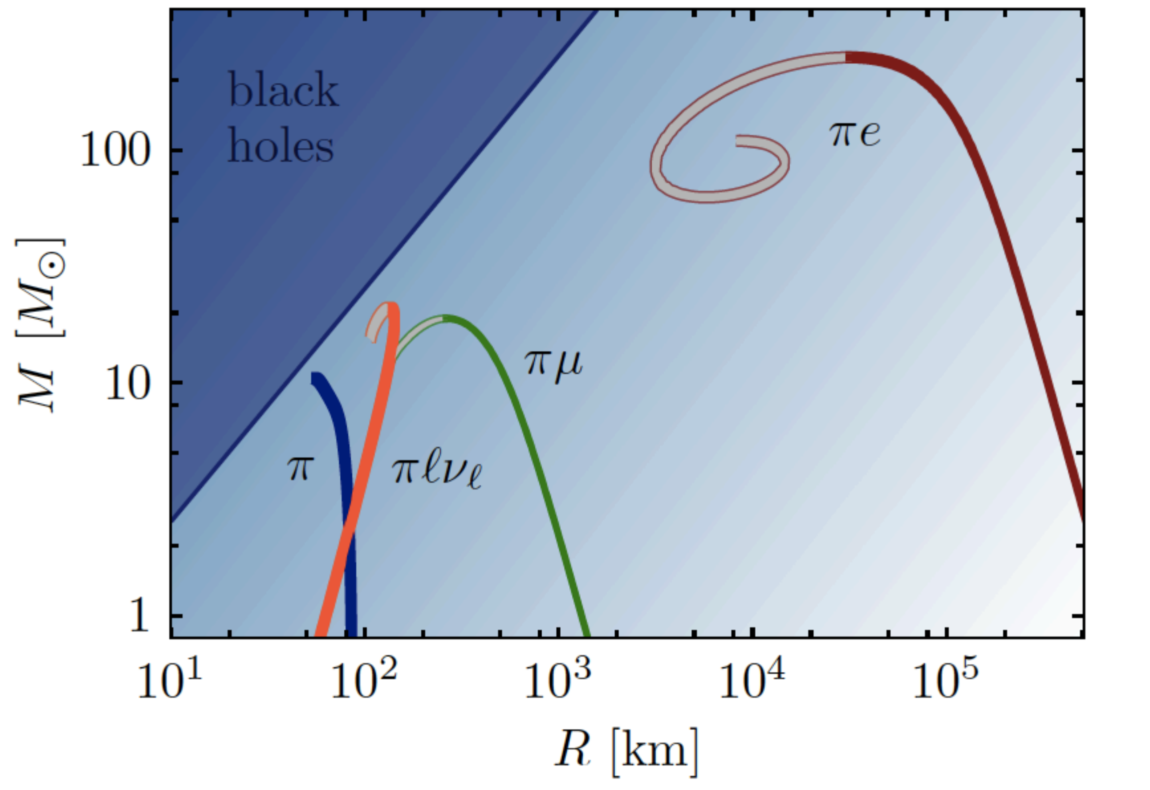
- Mass & radius increase as $\propto m_\pi^{-2}$.

- Physical pion mass (139.6 MeV).

- Max. M & R close to NS.

★ (π^+ + lepton$^-$ & π^+ + lepton$^-$ + neutrino) stars:

[Brandt, Endrodi, ESF, Hippert, Schaffner-Bielich & Schmalzbauer (2018)]



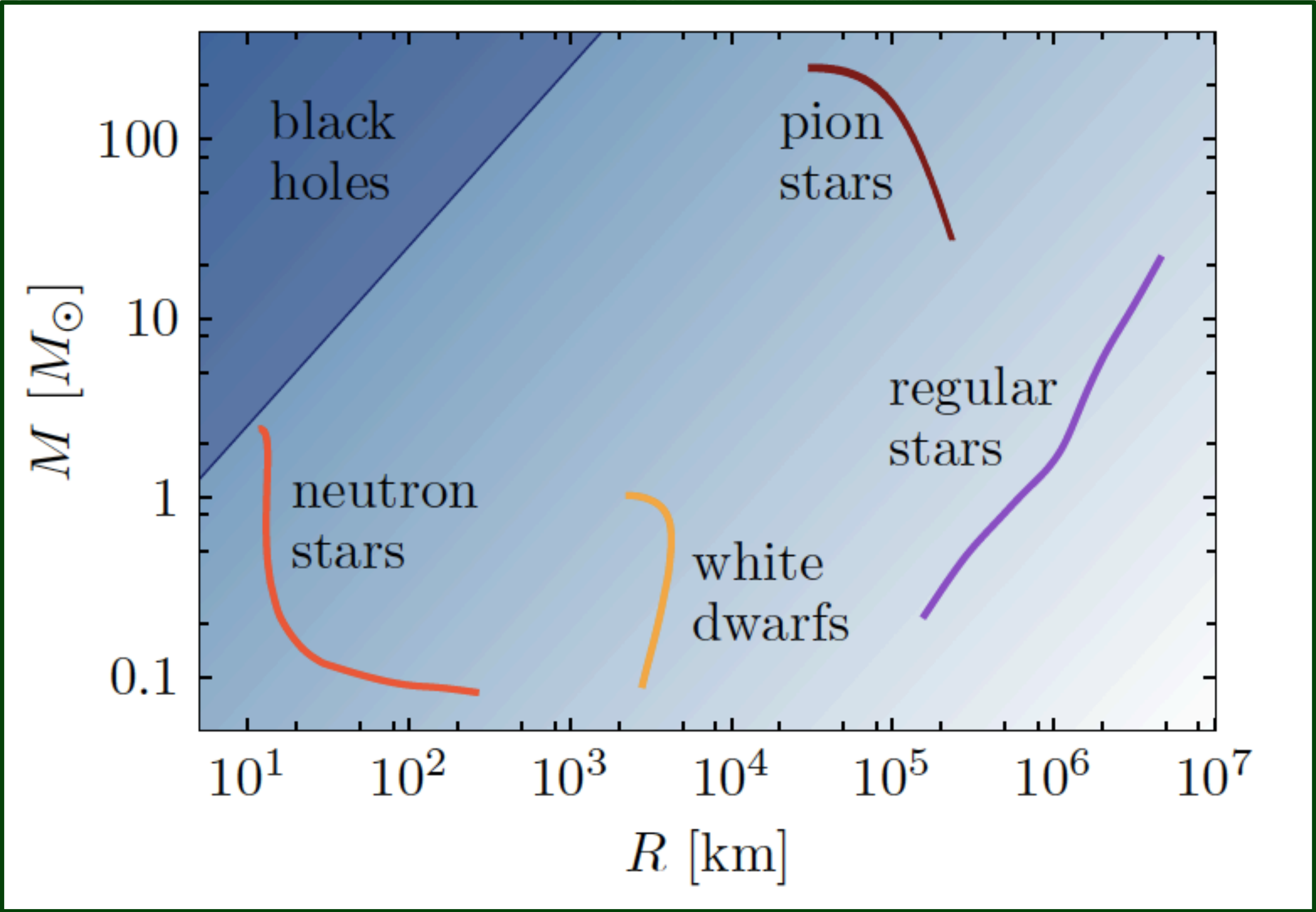
- Large max. mass and radius.
- R ~ constant for pure pion stars — a signature for interaction-dominated EoS.
- Adding leptons, MR^3 ~ constant, similarly to stars made of fermions.

TABLE I. Relevant properties of possible pion star compositions at their respective maximum mass. The last row correponds to the scenario with minimal neutrino pressure and $\mu_{\ell,c}/m_\ell$ is the relative electron chemical potential in this case.

composition	$M^{\max}\ [M_\odot]$	$R^{\max}$ [km]	$\epsilon_c^{\max}$ [MeV/fm^3]	$p_c^{\max}$ [MeV/fm^3]	$\mu_{I,c}^{\max}/m_\pi - 1$	$\mu_{\ell,c}^{\max}/m_\ell - 1$
π	10.5(5)	55(3)	57(5)	25(3)	1.068(4)	-
πe	250(10)	$3.3(2) \times 10^4$	$4.5(4) \times 10^{-5}$	$3.5(4) \times 10^{-7}$	$5.59(2) \times 10^{-7}$	7.4(2)
$\pi\mu$	18.9(4)	267(8)	2.7(3)	0.22(3)	$1.623(5) \times 10^{-2}$	0.58(3)
$\pi\ell\nu_\ell\vert_{\mu_\mu=\mu_e}$	20.8(9)	137(6)	7.5(7)	2.3(2)	$4.13(2) \times 10^{-3}$	160(4)
$\pi\ell\nu_\ell\vert_{\text{min. } p_{\nu_\ell}}$	28(2)	193(8)	3.7(4)	1.1(1)	$3.77(2) \times 10^{-3}$	155(4)

★ Comparison to other branches of compact stars:

[Brandt, Endrodi, ESF, Hippert, Schaffner-Bielich & Schmalzbauer (2018)]



⋆ Stable, unstable or metastable?

➡ In vacuum, charged pions decay weakly into leptons (mostly muons), with a characteristic lifetime of 10^{-8} s. But we don't have excitations around the vacuum here...

➡ The BEC of interacting pions is <u>not</u> like a bunch of pions (a tensor product state). It is a coherent classical state that can only be destroyed by the creation of excitations.

➡ The other possibility would be producing lepton excitations in the presence of the condensate (background field). [can be blocked by the presence of neutrinos].

➡ Surface effects, where the isospin density is small, will certainly induce evaporation.

Final remarks

★ Although strictly valid for very large densities, pQCD provides a controllable approximation for the EoS that can be systematically improved.

★ Even if there is no deconfined quark matter in the core of neutron stars, the form of the QCD EoS at very large densities affects dramatically the EoS for compact stars!

★ QCD at finite isospin is a fantastic playground to develop and test effective models & controllable approximations coming from chiral descriptions, pQCD, holography, and so on. There we have the Lattice benchmark, then one can go beyond!

★ Boson stars could provide an exciting possibility for a new class of compact stars and even contribute to the Dark Matter content of the universe.

➡ Unlike boson stars considered previously, pion stars have no need for any beyond Standard Model constituents. And no specific model to solve TOV.

➡ Probably the pion star will evaporate, but one still has to estimate its rate. Their formation mechanism is hard to guess. But, if it can be there, maybe Nature has found a way…

Back up slides

★ Naturally, a high-energy physicist, especially one that likes QCD, will ask questions such as:

- Is there quark matter in NS?
- Can it reach an equilibrium configuration?
- What is the formation mechanism?
- Can we make predictions from first principles?

★ To address these questions one needs to deal with the structure of neutron stars.

★ For a much broader coverage, please see:

EMMI Rapid Reaction Task Force Meeting on

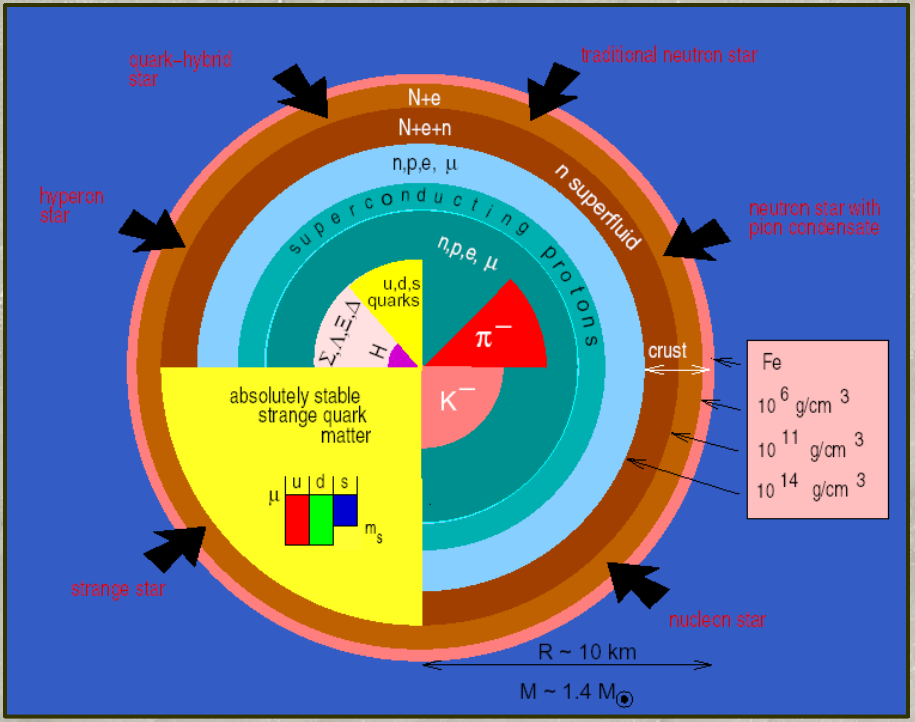
"Quark Matter in Compact Star"

Michael Buballa,[1] Veronica Dexheimer,[2] Alessandro Drago,[3] Eduardo Fraga,[4,5,6] Pawel Haensel,[7] Igor Mishustin,[5] Giuseppe Pagliara,[3] Jürgen Schaffner-Bielich,[4] Stefan Schramm,[5] Armen Sedrakian,[4] and Fridolin Weber[8,9]

Michael Buballa et al 2014 J. Phys. G: Nucl. Part. Phys. 41 123001

★ So, the kind of matter we expect inside NS is strongly interacting matter (QCD matter) under extreme conditions.

★ This brings several exciting possibilities!

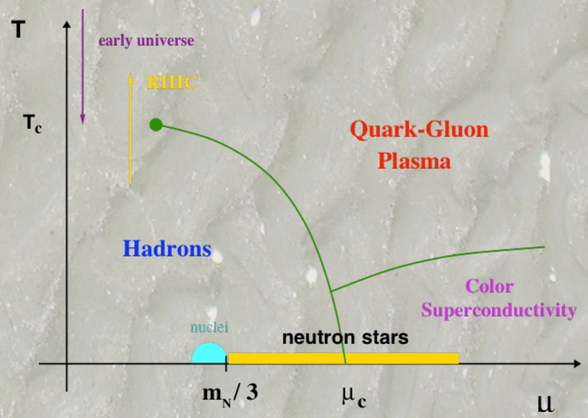


[F. Weber, 2000]

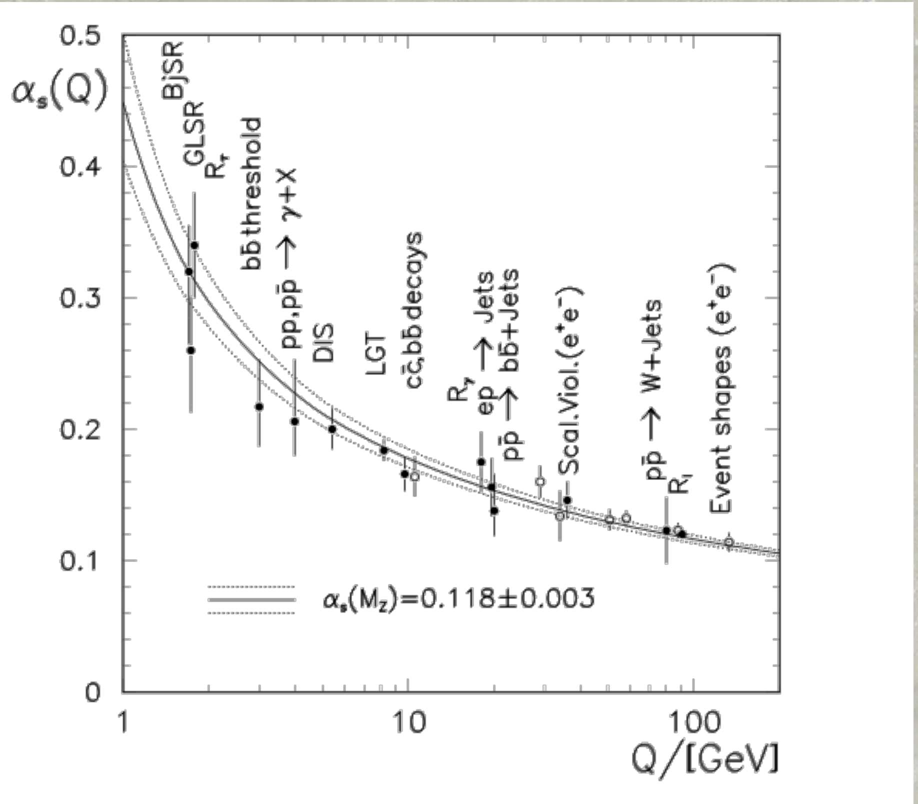
• New phases, condensates, color superconductivity, etc in the core.

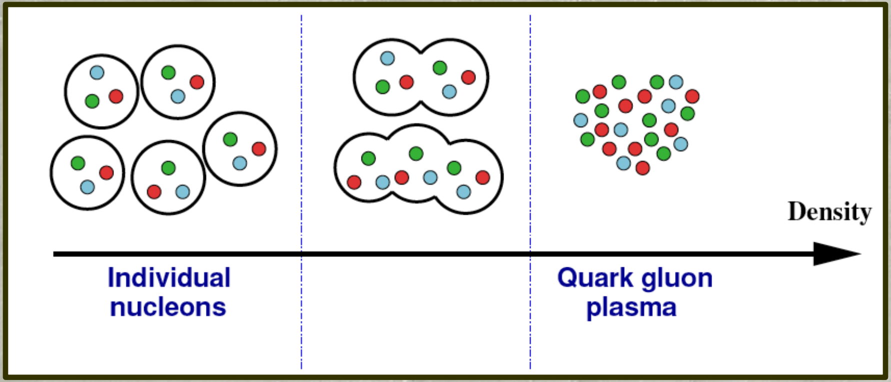
• Deconfinement and chiral symmetry partial restoration (might affect SN explosion mechanism via EoS).

• Strange matter (strange stars?).



Asymptotic freedom x the vacuum of QCD





- Matter becomes simpler at very high densities (baryon chemical potential μ as energy scale), but very complicated in the opposite limit...

- μ is not high enough in the interesting cases...

- In-medium pQCD needs optimization to produce sensible results.

Ok, let's not desperate… There are several possibilities…

Some popular examples:

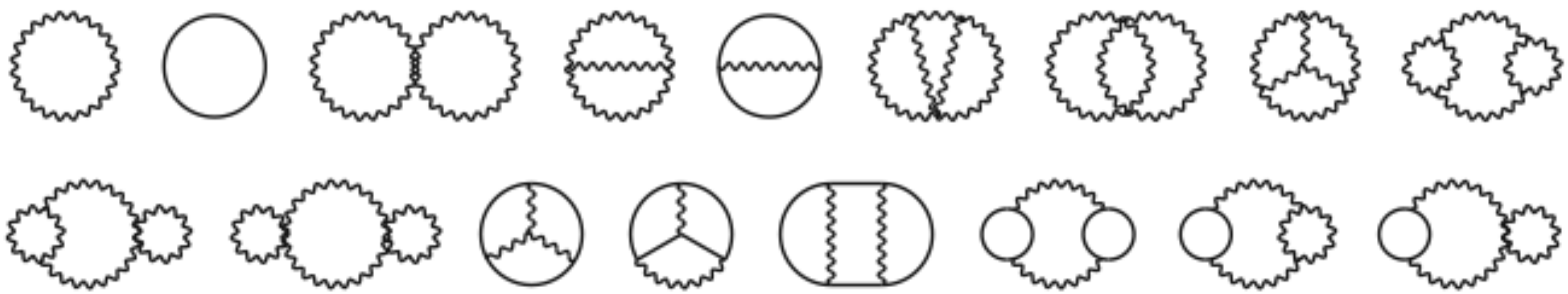
- Very intelligent and sophisticated brute force: lattice QCD
- Intensive use of symmetries: effective field theory models
- Redefining degrees of freedom: quasiparticle models
- "Moving down" from high-energy pQCD
- "Moving up" from hadronic low-energy (nuclear) models

Very high densities:

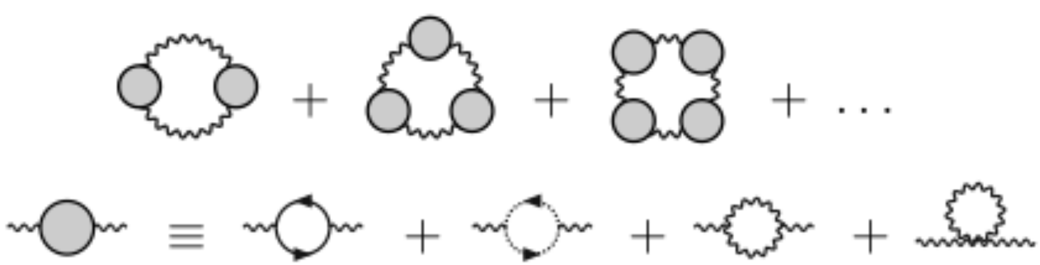
Pressure from pQCD (diagrammatically speaking…)

(Vuorinen, 2004)

Up to three loops (g^4) the diagrams to be evaluated are (without ghosts)

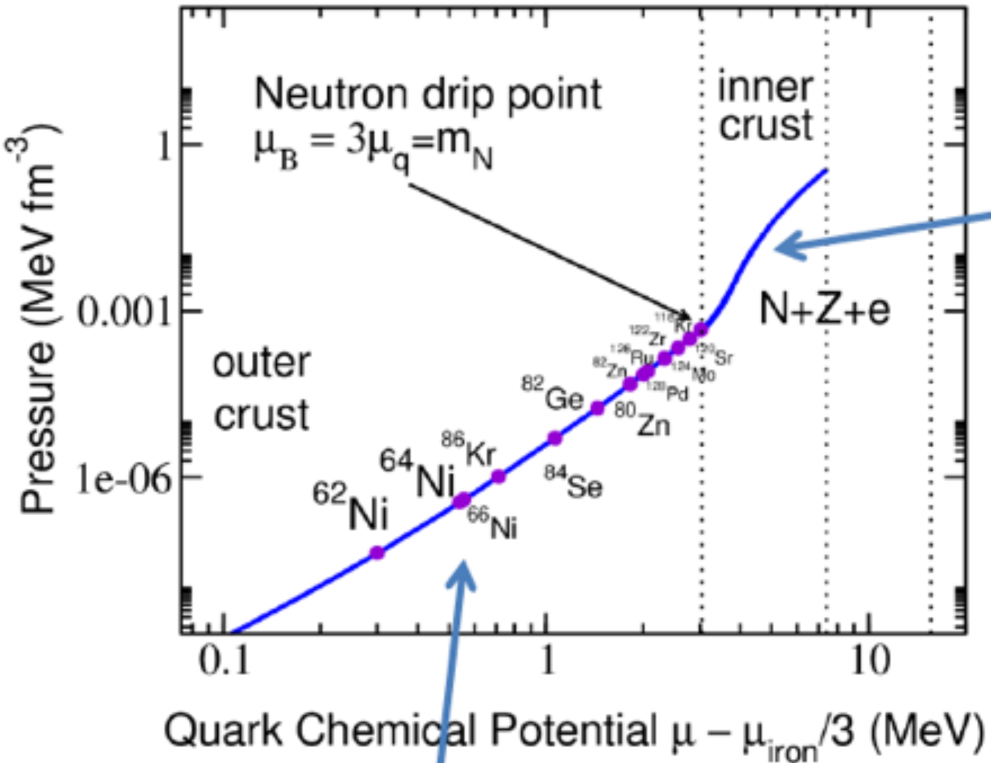


Also at low T the first IR problems (for the pressure) appear at 3-loop order: on top of 2PI graphs must resum



Low densities:

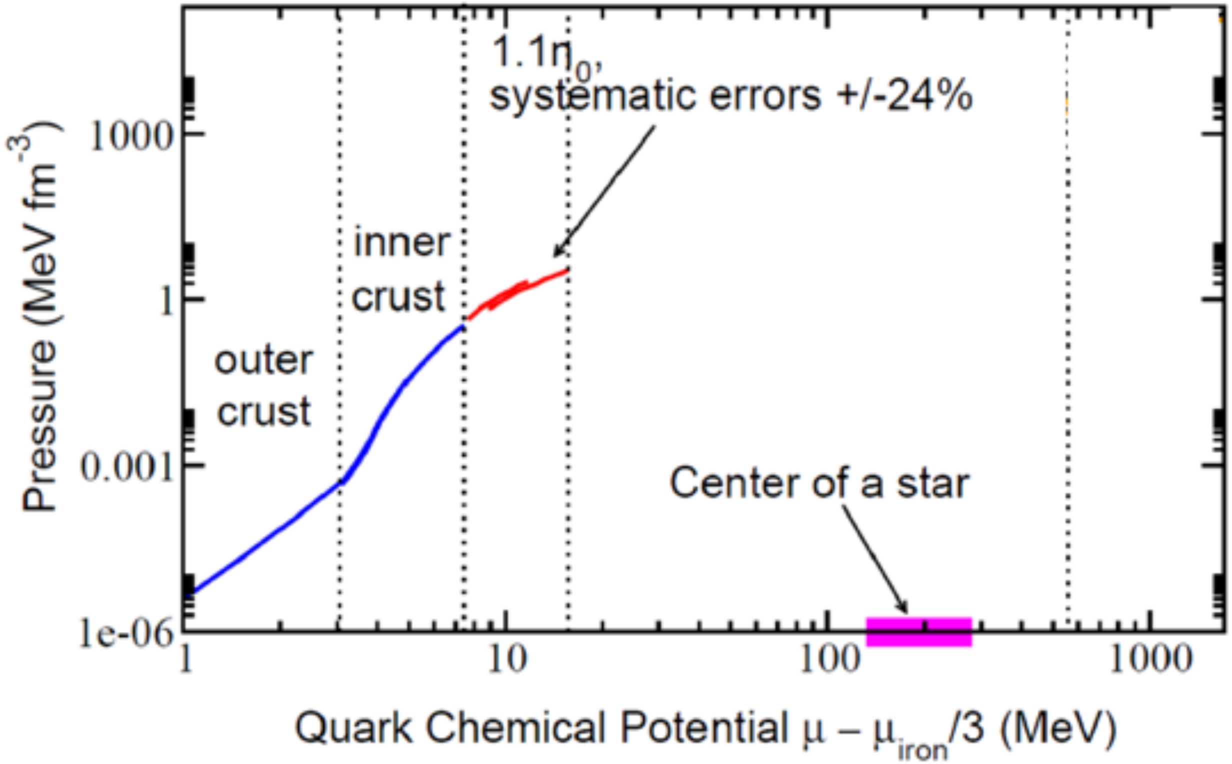
[From talks by Kurkela & Vuorinen]



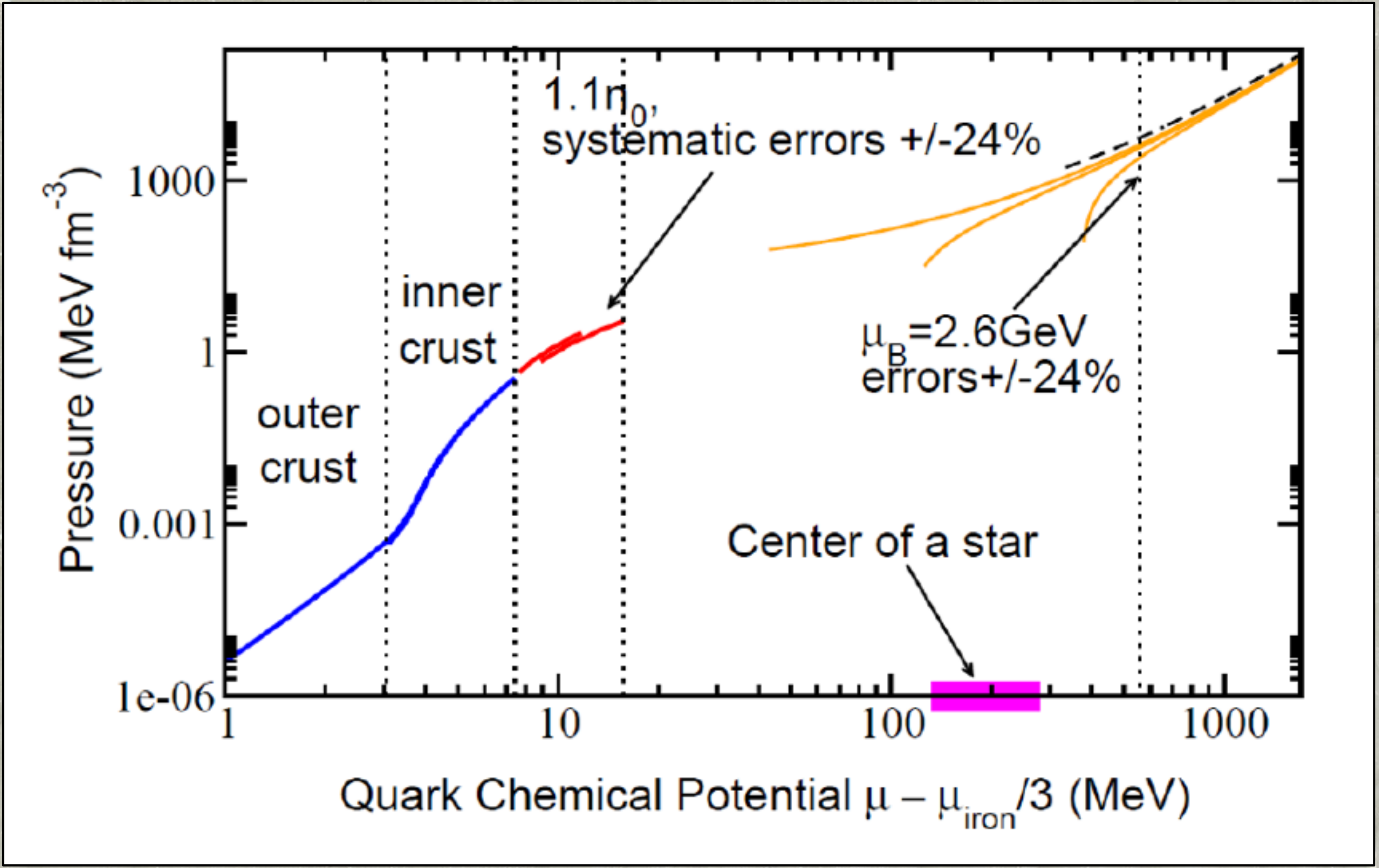
- Neutron gas with nuclei and electrons
- NN interactions important for collective properties; modeled via phenom. potential models
- Eventually need 3N interactions, boost corrections,...

- Lattice of increasingly neutron rich nuclei in electron sea; pressure dominated by that of the electron gas
- At zero pressure nuclear ground state ^{56}Fe

Closer to saturation density:



- Closer to saturation density n_s, need many-body calculations within Chiral Effective Theory, including 3N and 4N interactions
- At $1.1 n_s$, errors $\pm 24\%$ - mostly due to uncertainties in effective theory parameters
- State-of-the-art NNNLO Tews et al., PRL 110 (2013), Hebeler et al., APJ 772 (2013)

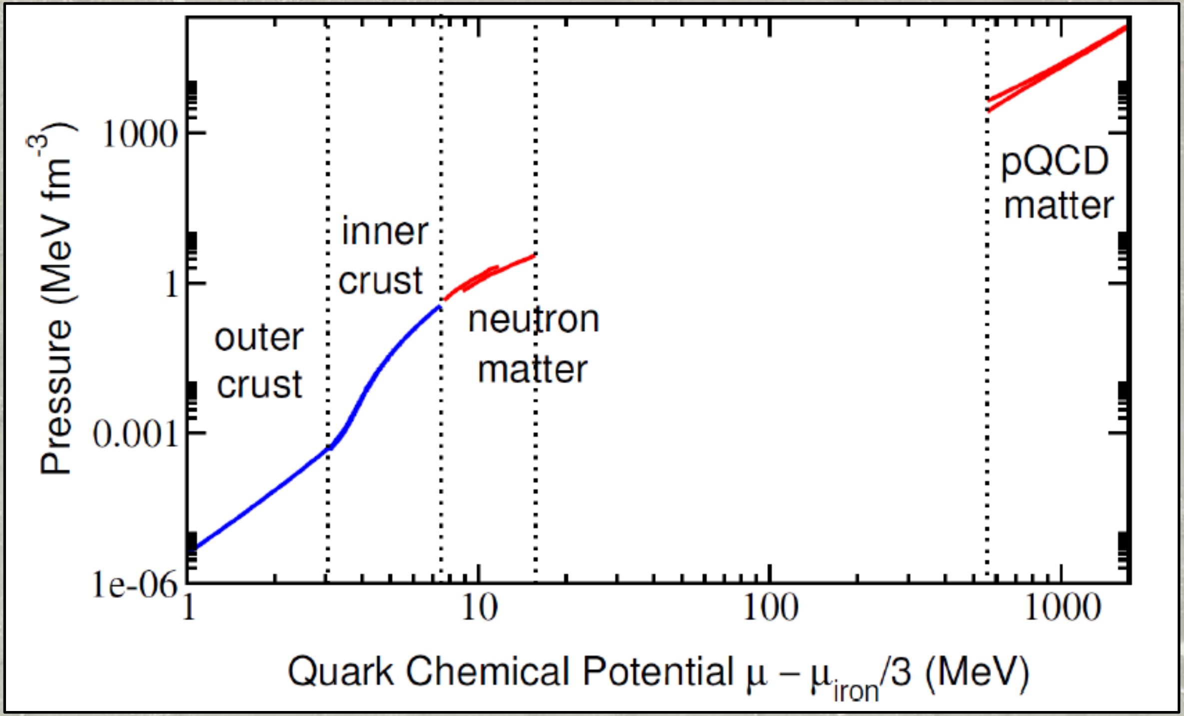


Uncertainties mostly from renormalization scale dependence, running of α_s & value of the strange quark mass.

The two limits are given:

Any interpolation in between is bound to satisfying the extrema

strong constraints!



★ One can use different phenomenological models to study this region, satisfying the "boundary" constraints.

★ We chose, instead, to use a multiple piecewise polytropic parametrization for the EoS: $\boxed{p_i(n) = \kappa_i n^{\gamma_i}}$

[Hebeler, Lattimer, Pethick & Schwenk (2013)]

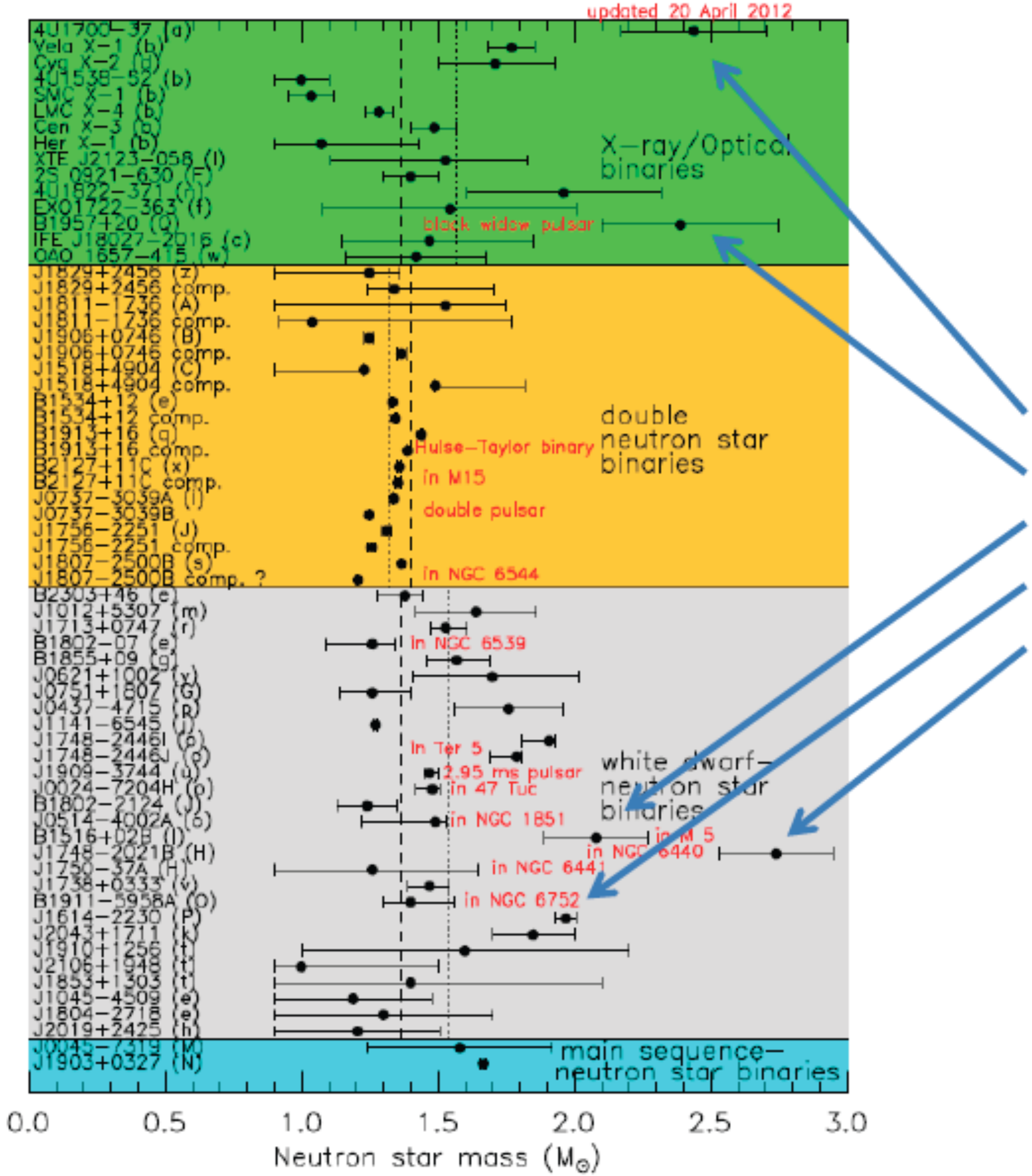
So, by this parametrization, we quantify our ignorance by varying all parameters requiring the following:

⋆ a smooth matching to nuclear and quark matter EoSs

⋆ smoothness: continuity of pressure & density when matching monotropes (can be relaxed)

⋆ causality: $c_s \leq 1$ (asymptotically equivalent to $\gamma \leq 2$)

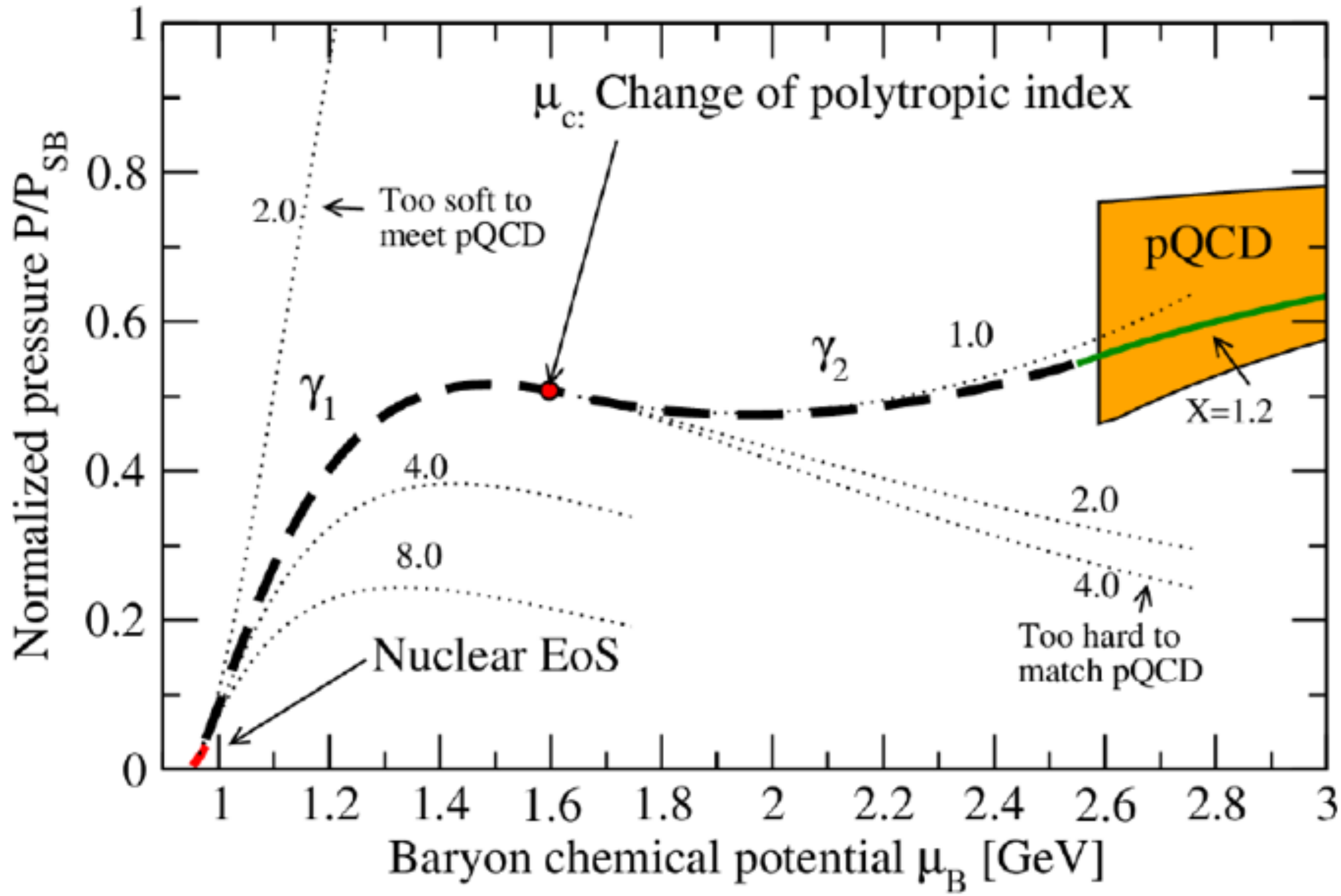
⋆ possibility to support a two solar mass star



!!

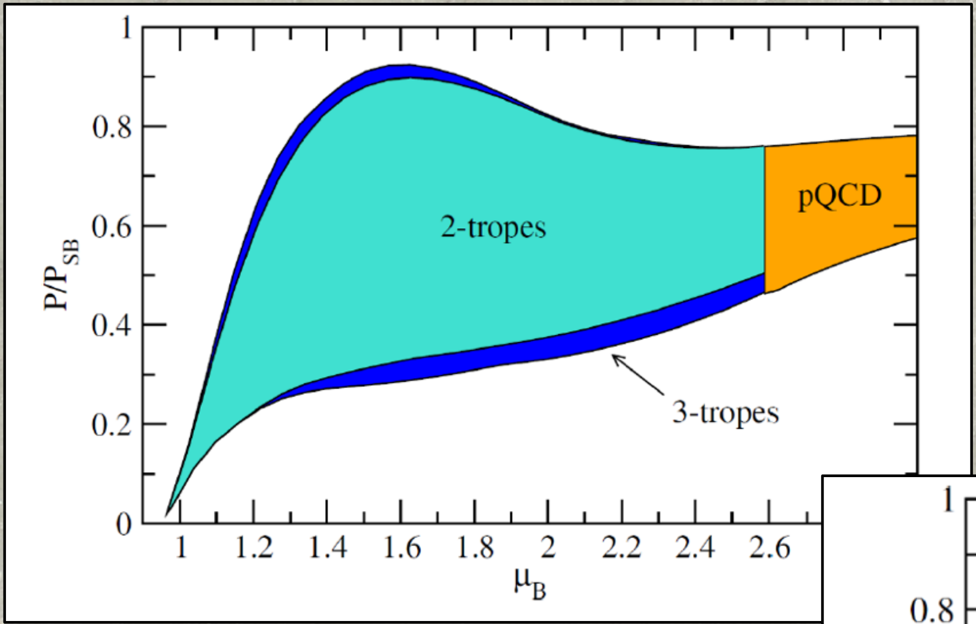
Larger masses imply very strict constraints on models!

Illustration - 2 tropes:

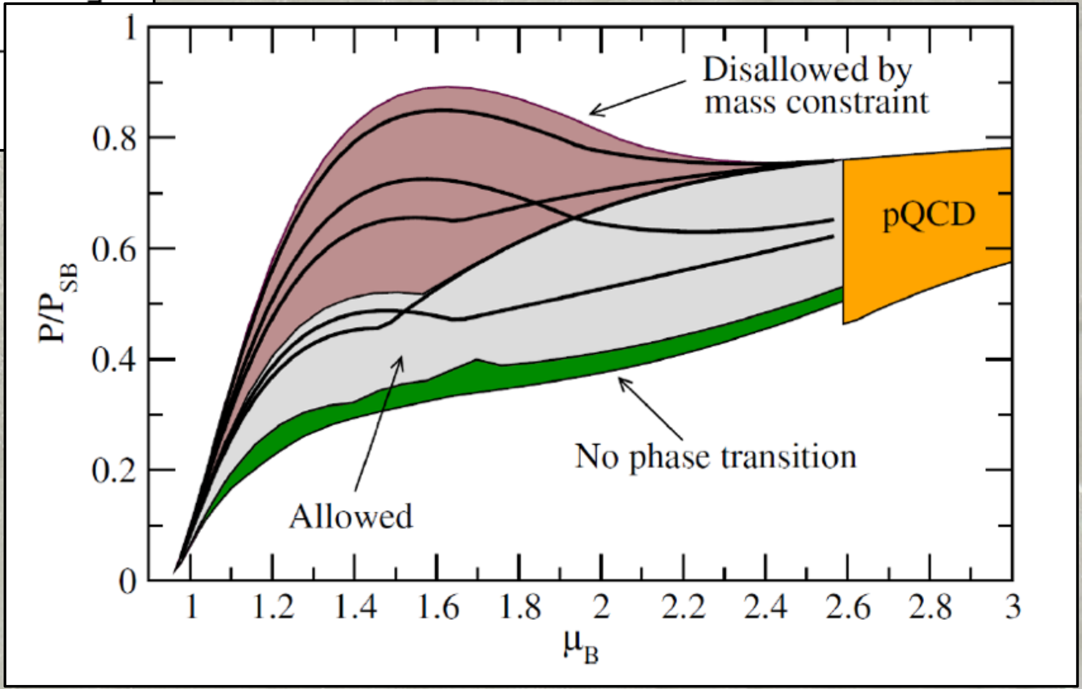


Solutions exist for $\mu_c \in [1.08, 2.05]$ GeV, $\gamma_1 \in [2.23, 9.2]$, $\gamma_2 \in [1.0, 1.5]$

Constraining the EoS



★ the 2-tropes case is the minimum necessary and also seems to be enough to do the job.

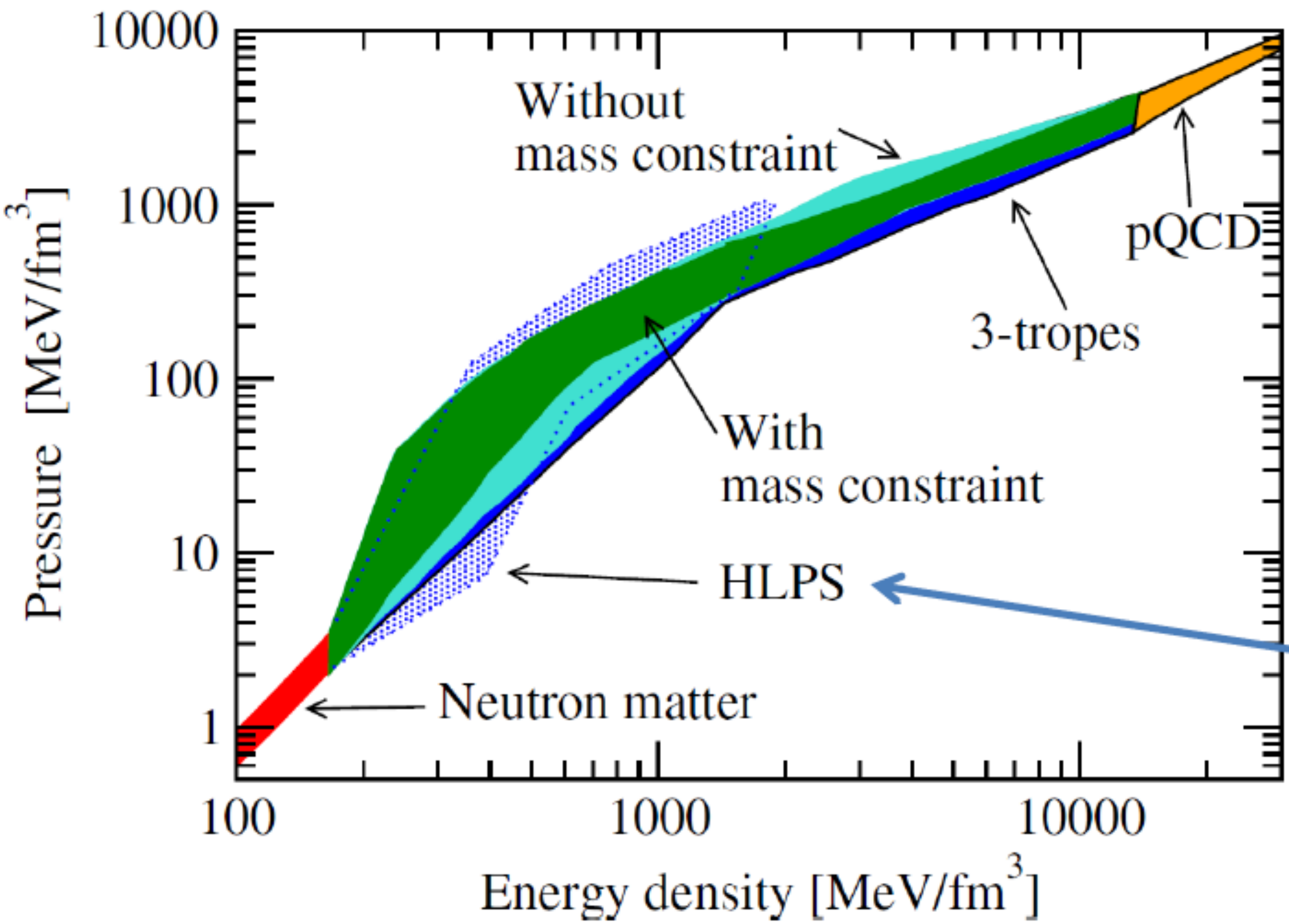


★ Allowing for $2M_{sun}$ stars constrains dramatically the band!

★ Implementing a 1st-order transition shrinks modestly the band.

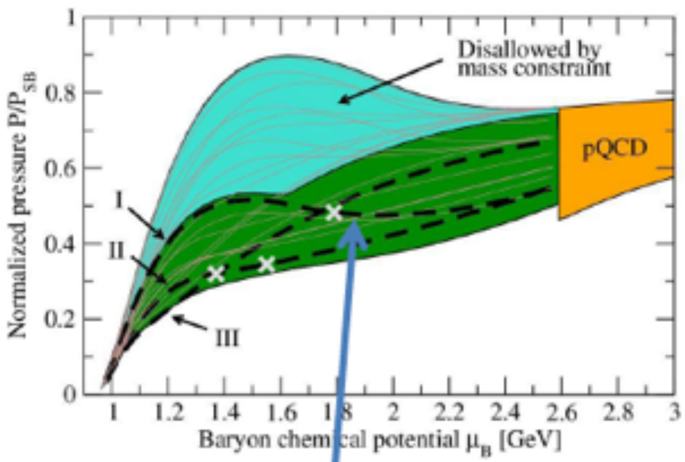
The band for pressure vs. energy density

[Kurkela, ESF, Schaffner-Bielich & Vuorinen (2014)]



HLPS [Hebeler, Lattimer, Pethick, Schwenk, APJ 773 (2013)] without high-density constraint

Implications for compact stars



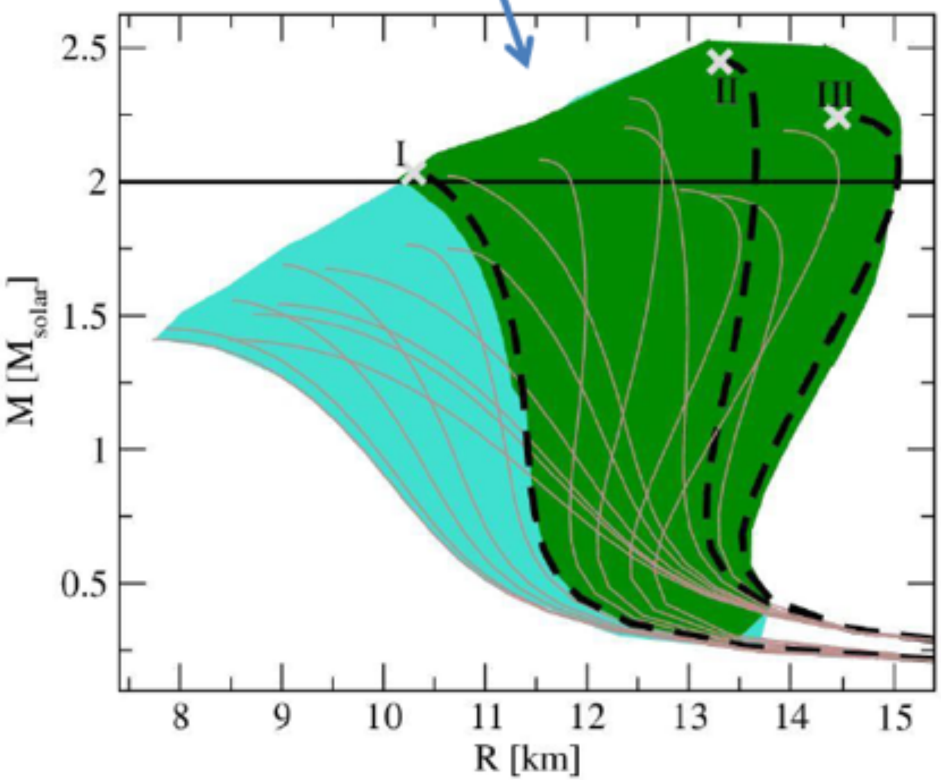
Maximal masses up to $2.5 M_{\rm sun}$!

Central densities

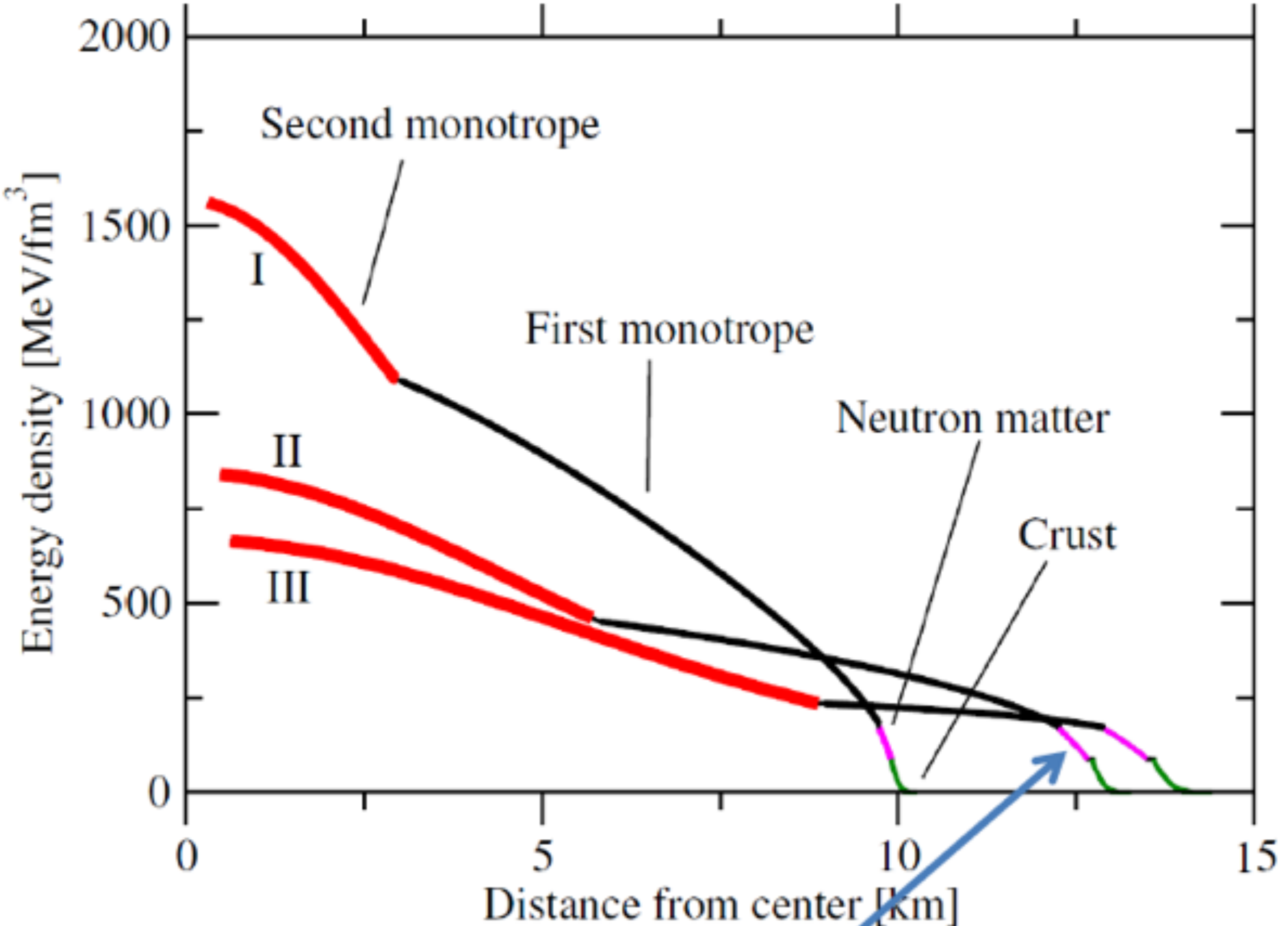
$n_{\rm central} \in [3.7, 14.3] n_{\rm s}$

Large reduction in EoS uncertainty due to tension from mass constraint: Large stellar masses require stiff EoS, matching to pQCD soft

$\Rightarrow$ EoS uncertainty down to 30% at all densities



Energy density profile



Nuclear matter EoS only used very close to star surface – yet important effects from matching

Final remarks

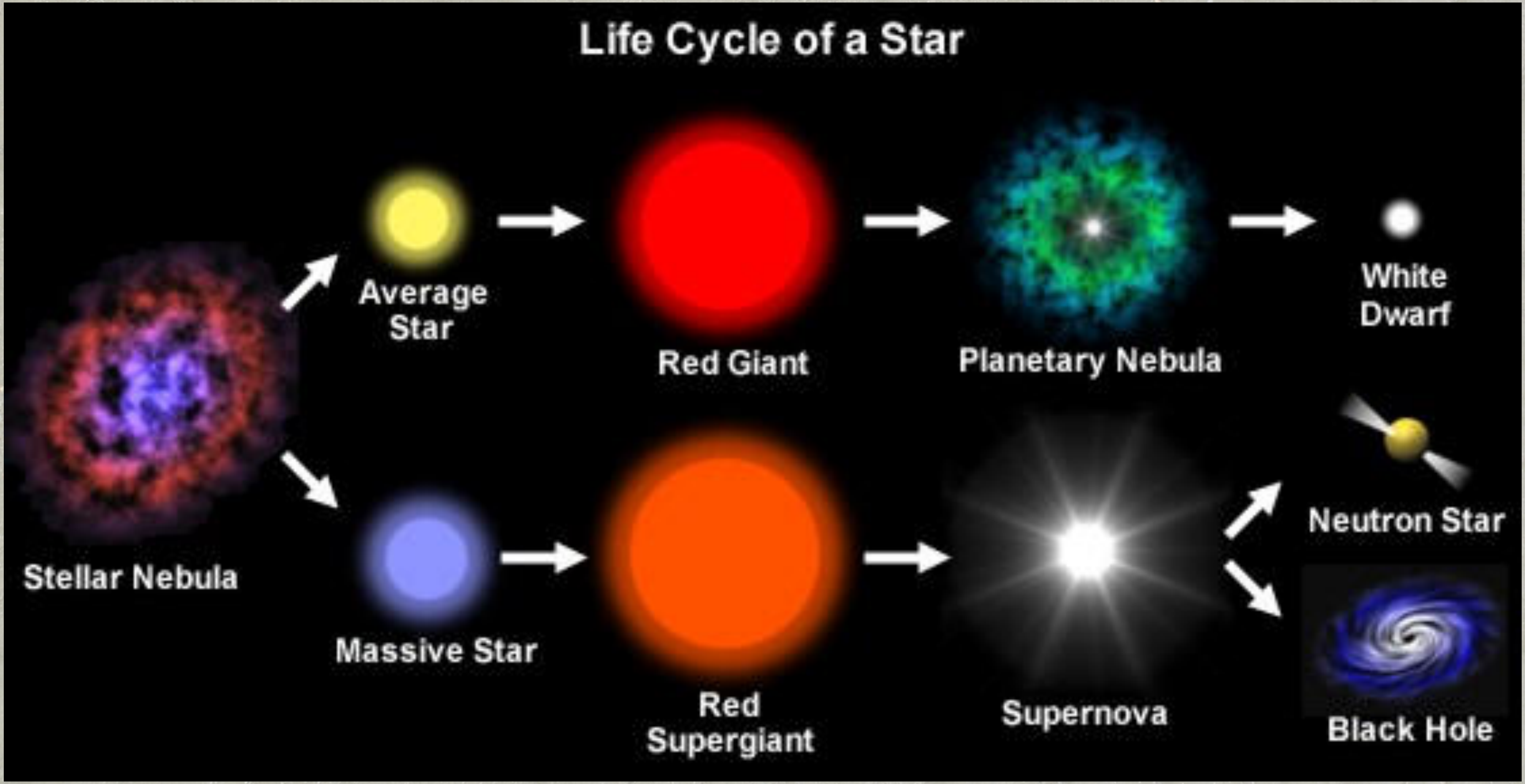
★ Message 1: Although strictly valid for very large densities, pQCD provides a controllable approximation for the EoS that can be systematically improved.

★ Message 2: even if there is no deconfined quark matter in the core of neutron stars, the form of the QCD EoS at very large densities affects dramatically the EoS for compact stars! A robust constraint!

★ The existence of 2 solar mass stars strongly constrains the EoS. Having a 1st-order transition or a crossover not so much.

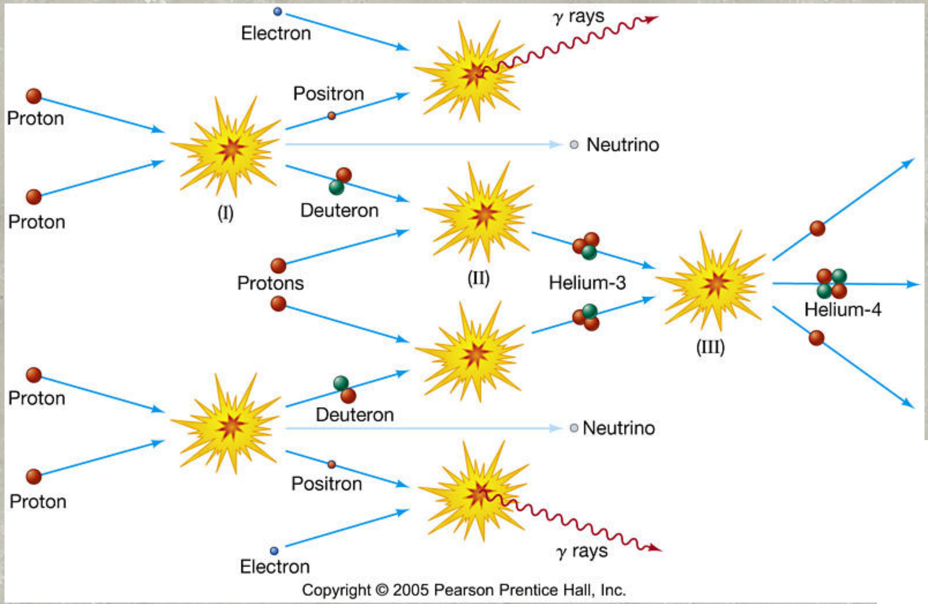
★ Fruitful interplay between particle/nuclear physics and neutron star physics!

First, a tiny bit on stellar formation and structure…



Inner pressure to equilibrate gravity...

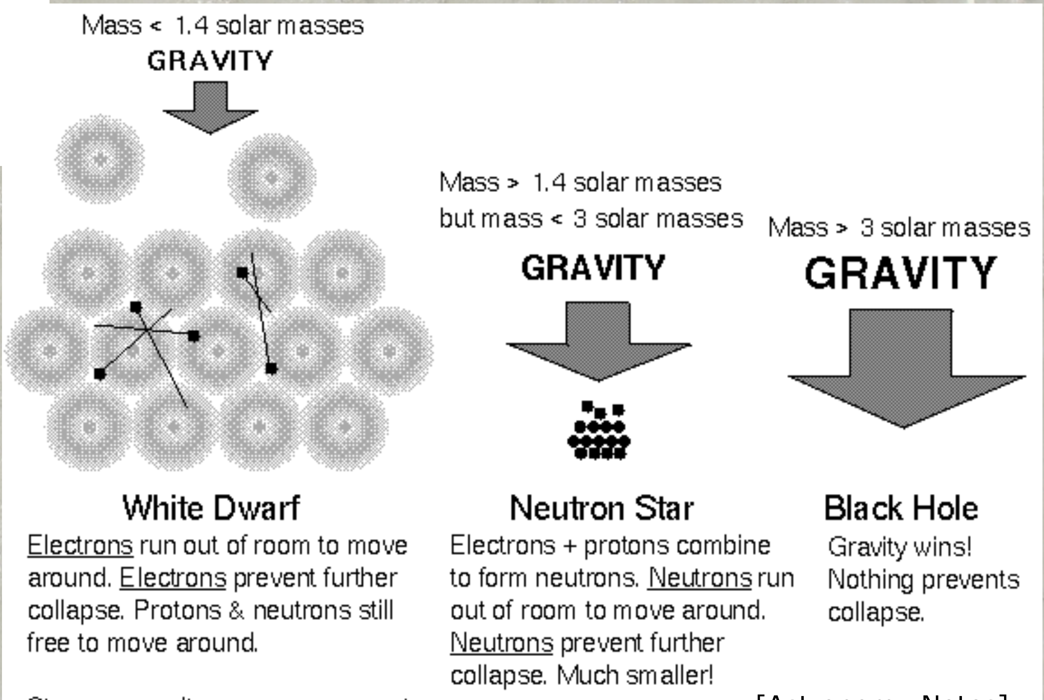
★ Average star (e.g., our Sun)



Inner pressure generated by thermonuclear reactions.

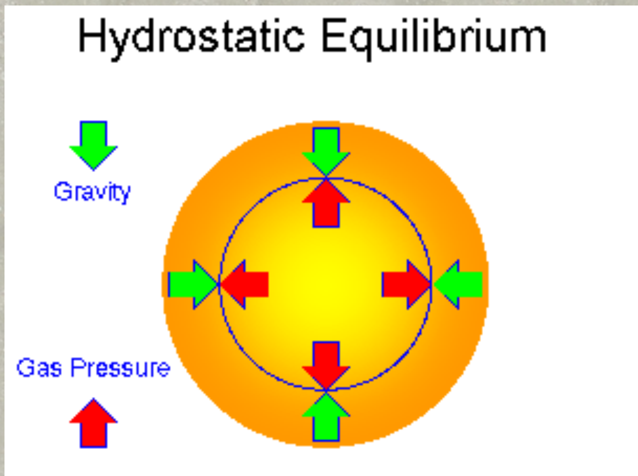
★ Compact star

Fermi pressure of degenerate fermion gas equilibrates gravity!



[Astronomy Notes]

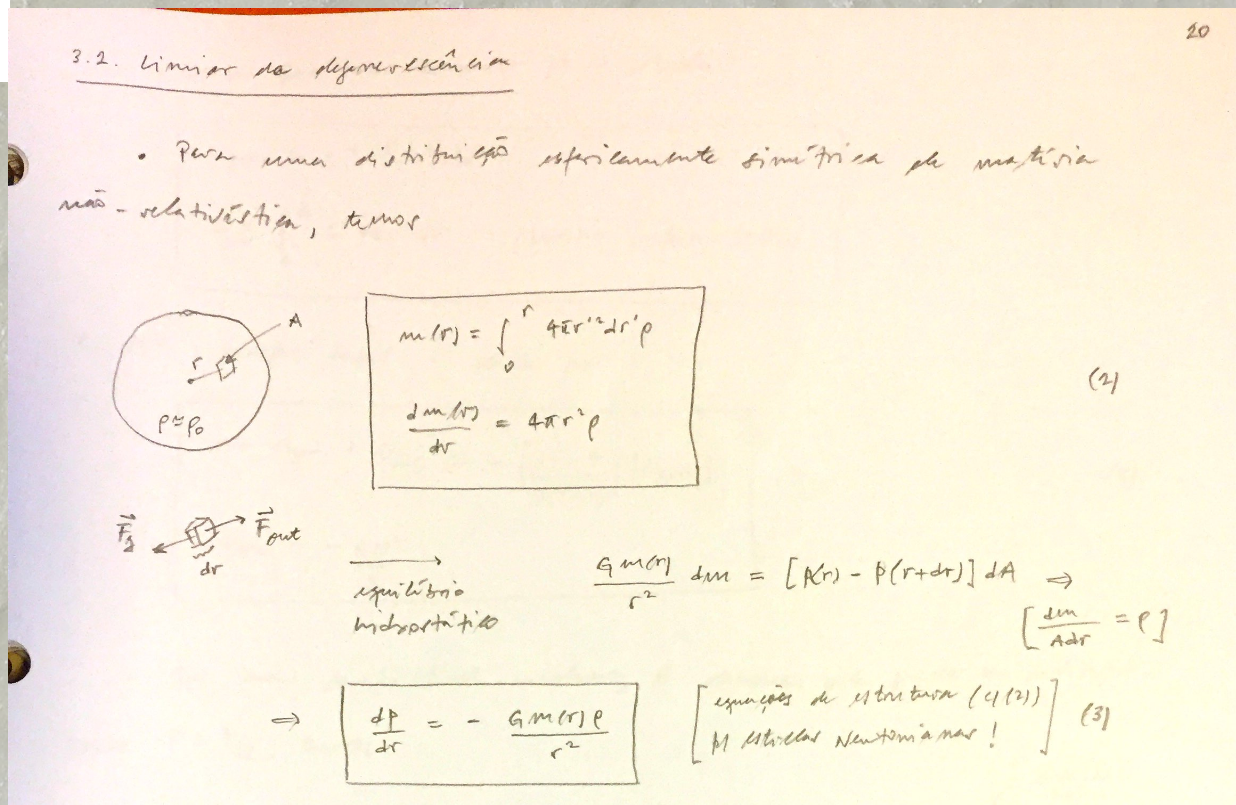
Hydrostatic equilibrium in stars – Newtonian stars:



[Astronomy, Ohio State]

★ Newton's law + spherical symmetry + hydrostatic equilibrium.

★ Inner pressure generated by thermonuclear reactions.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3.2. Limiar da degenerescência

• Para uma distribuição esfericamente simétrica de matéria não-relativística, temos

$\rho \simeq \rho_0$, r, A

$$\boxed{m(r) = \int_0^r 4\pi r'^2 dr' \rho \qquad \frac{dm(r)}{dr} = 4\pi r^2 \rho} \quad (2)$$

$\vec{F}_g$, $\vec{F}_{out}$, dr

$\xrightarrow[\text{hidrostático}]{\text{equilíbrio}}$ $\frac{G\,m(r)}{r^2}\,dm = [P(r) - P(r+dr)]\,dA \Rightarrow$

$\left[\frac{dm}{A\,dr} = \rho\right]$

$$\Rightarrow \boxed{\frac{dP}{dr} = -\frac{G\,m(r)\,\rho}{r^2}} \quad (3)$$

[equações de estrutura (c/(2)) p/ estrelas Newtonianas!]

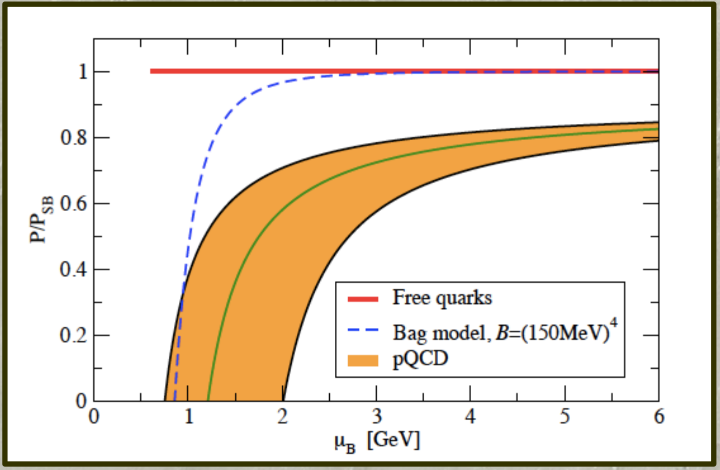
★ Application to "neutron" stars gained momentum with the idea of strange matter as the ground state of QCD and the possibility of "strange stars" [Witten (1984), Alcock, Farhi & Olinto (1985)].

★ Easy-to-use framework to describe the EoS in any situation -> became very popular in describing quark matter in hybrid stars.

★ Played a very important role in first studies and building a basic understanding of hybrid stars that contain a quark matter core.

★ Still relevant as a first approach to any given feature of NS, aiming at very qualitative hints (which might also prove wrong!)

★ However, statements e.g. on the existence or not of quark matter in neutron stars based on the bag model are very naive and dangerous!



★ E.g., it is well known that the QCD pressure goes very slowly (log) to the free gas result.

★ Feature totally missed by the bag model...

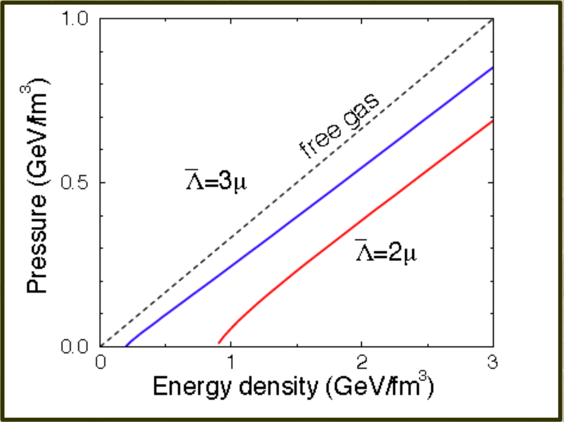
[ESF, Kurkela & Vuorinen (2013)]

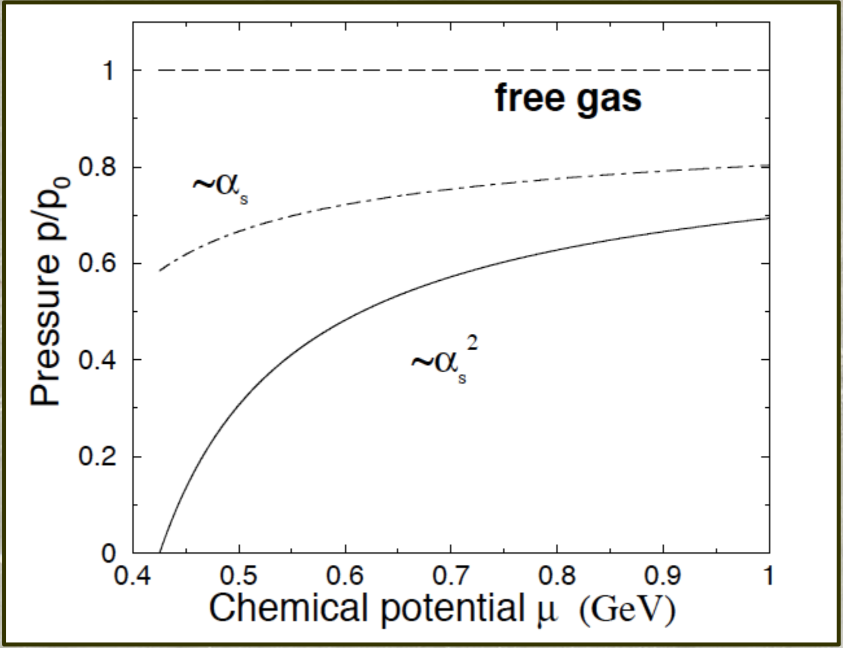
EoS and NS masses from cold (T=0) pQCD — 15 years ago:

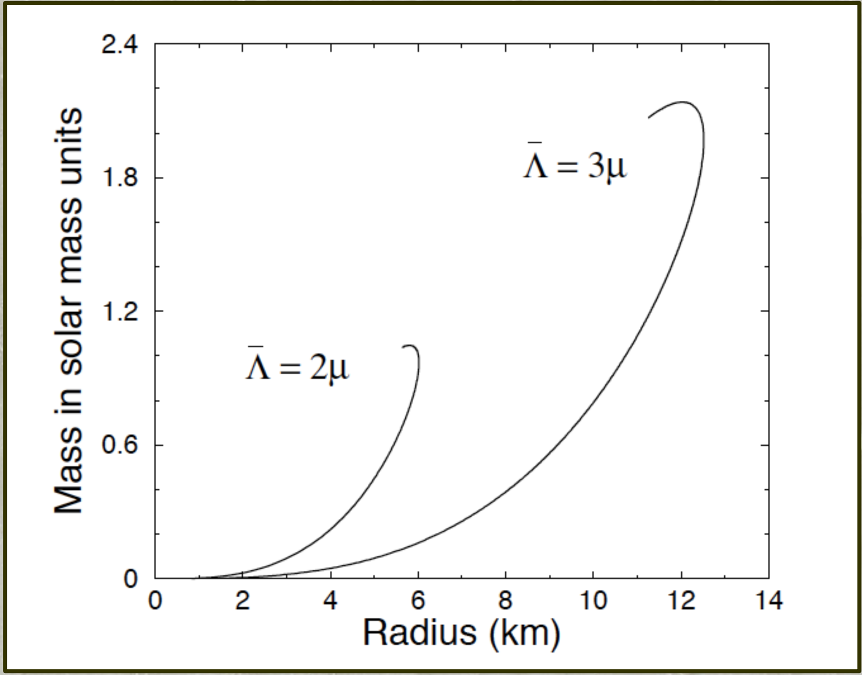
[ESF, Pisarski & Schaffner-Bielich (2001)]

• Gas of massless u, d, s quarks: charge neutrality and β equilibrium achieved for $\mu_s = \mu_d = \mu_u$ (no need for electrons).

• Interaction taken into account perturbatively up to (RG running) α_s^2 , no bag constant.



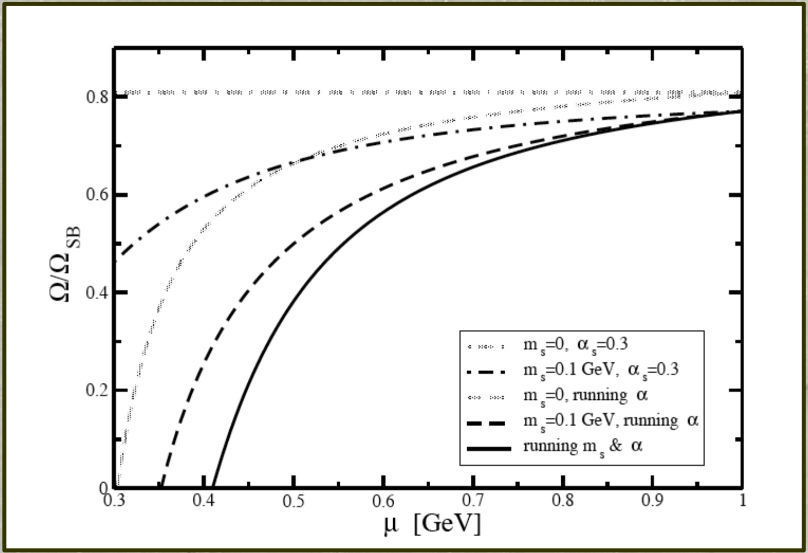




★ Large masses allowed for quark stars!

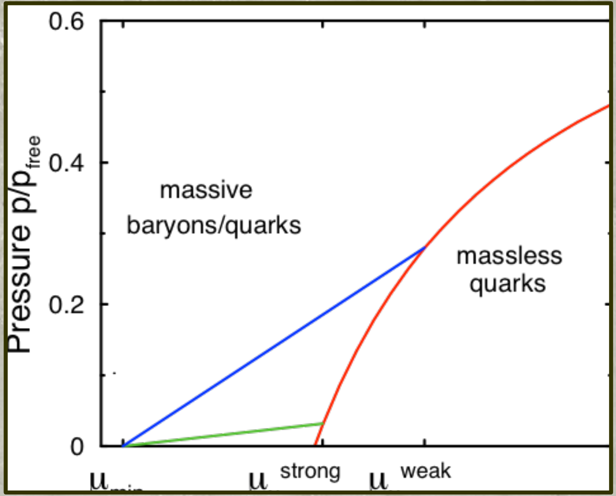
★ Strange quark mass correction:

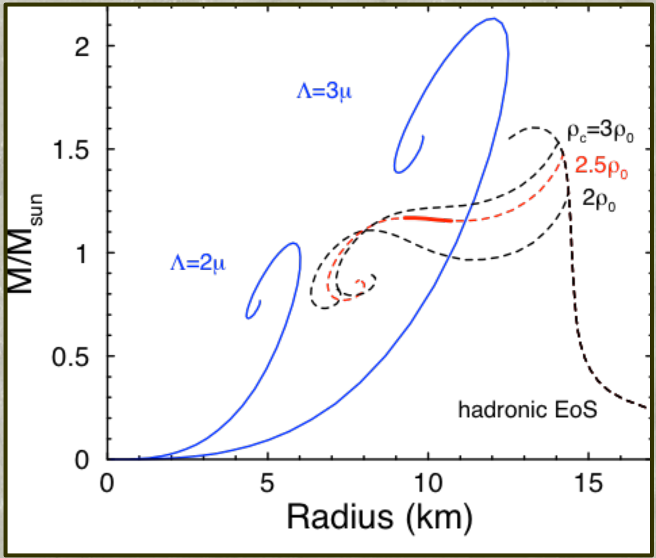
[ESF & Romatschke (2005)]



★ Matching and new class of stars (from pQCD + hadronic mantle):

[ESF, Pisarski & Schaffner-Bielich (2002)]





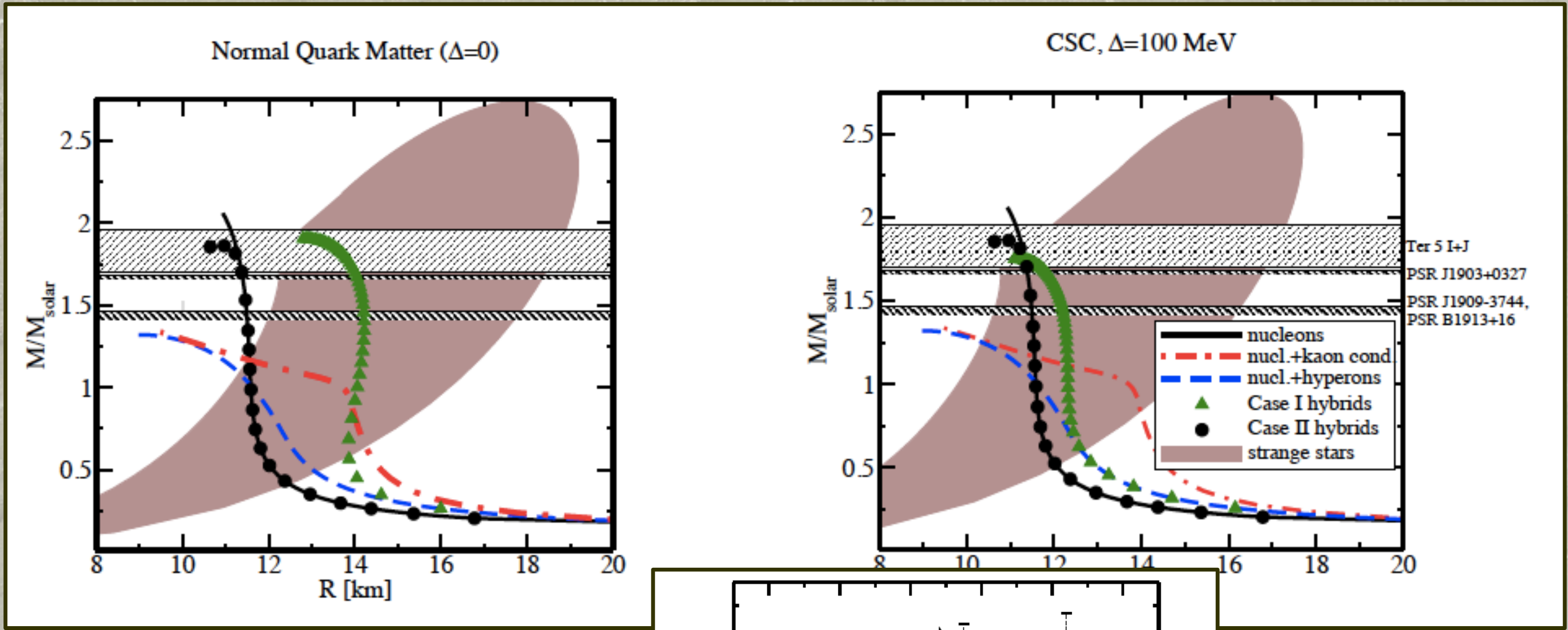
- Quark core with a hadronic mantle.
- Smaller, denser companion (twin) to a hybrid (more hadronic) star.
- Several results for twin stars in the literature.

EoS and NS masses from state-of-the-art cold pQCD:

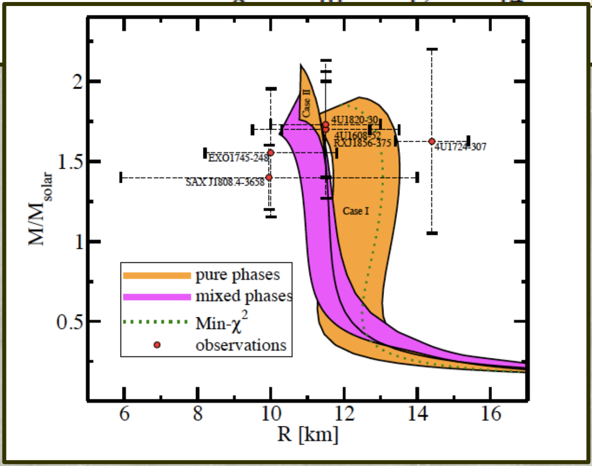
[Kurkela, Romatschke & Vuorinen (2010)]

★ NLO, NNLO: vacuum diagrams & ring resummation — cold QM.

★ large masses for hybrid (and even larger for strange stars) allowed!



Comparison to observations
[Kurkela, Romatschke, Vuorinen & Wu (2010)]



Sometimes, after a long and subtle calculation, one realizes that the complicated result can be very well adjusted (EoS, 1st & 2nd derivatives) by a simple and compact function:

Previously: Fraga, Pisarski & Schaffner-Bielich (2001)
Alford et al. (2005)

→ effective bag model

Using the complete results from Kurkela, Romatschke & Vuorinen (2010):

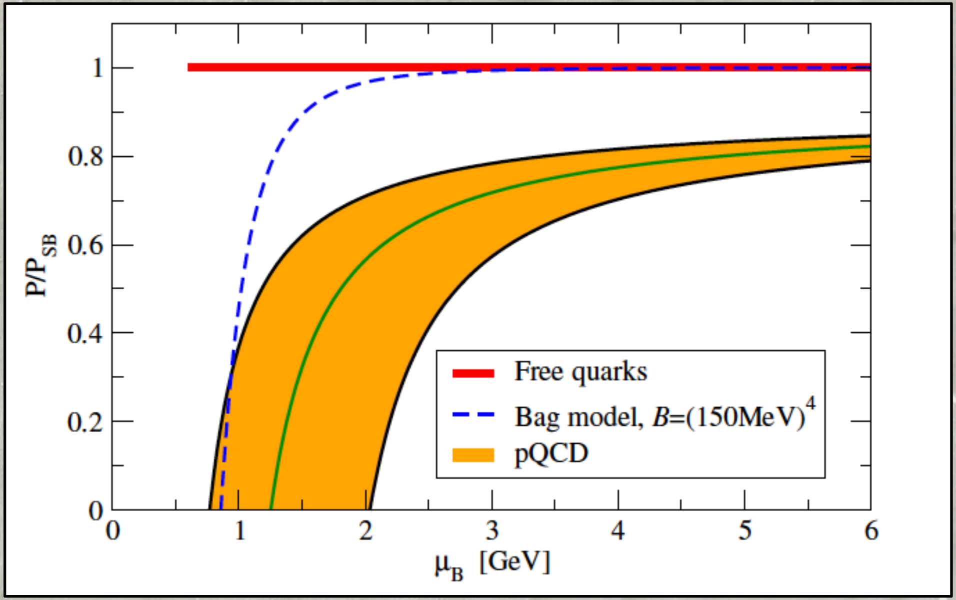
[ESF, Kurkela & Vuorinen (2013)]

$$\boxed{P_{\rm QCD}(\mu_B) = P_{\rm SB}(\mu_B)\left(c_1 - \frac{a(X)}{(\mu_B/\mathrm{GeV}) - b(X)}\right)}$$

$$\boxed{a(X) = d_1 X^{-\nu_1}, \quad b(X) = d_2 X^{-\nu_2}}$$

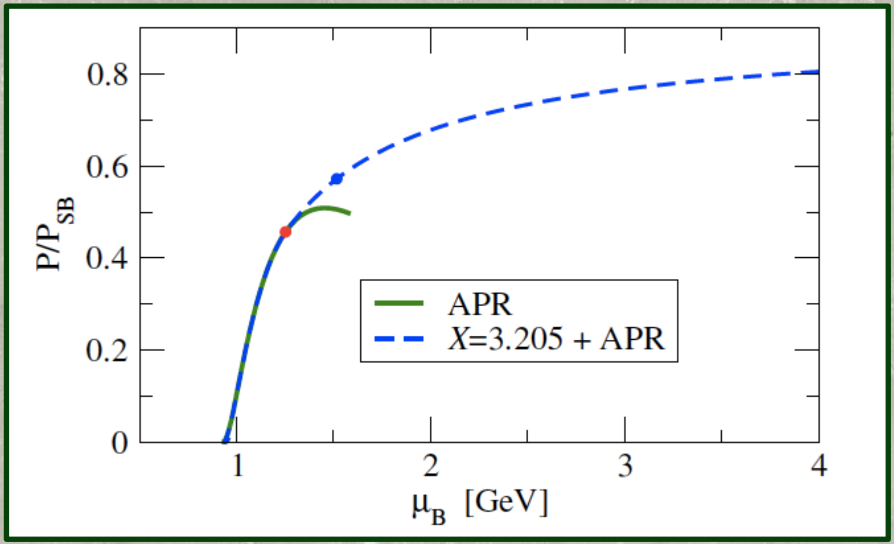
$$\boxed{P_{\rm SB} = \frac{3}{4\pi^2}(\mu_B/3)^4}$$

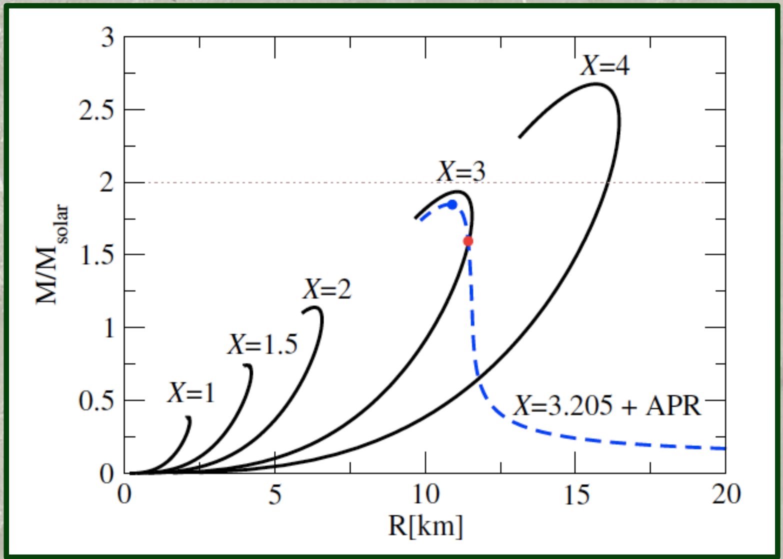
$$\boxed{X \equiv 3\bar{\Lambda}/\mu_B}$$



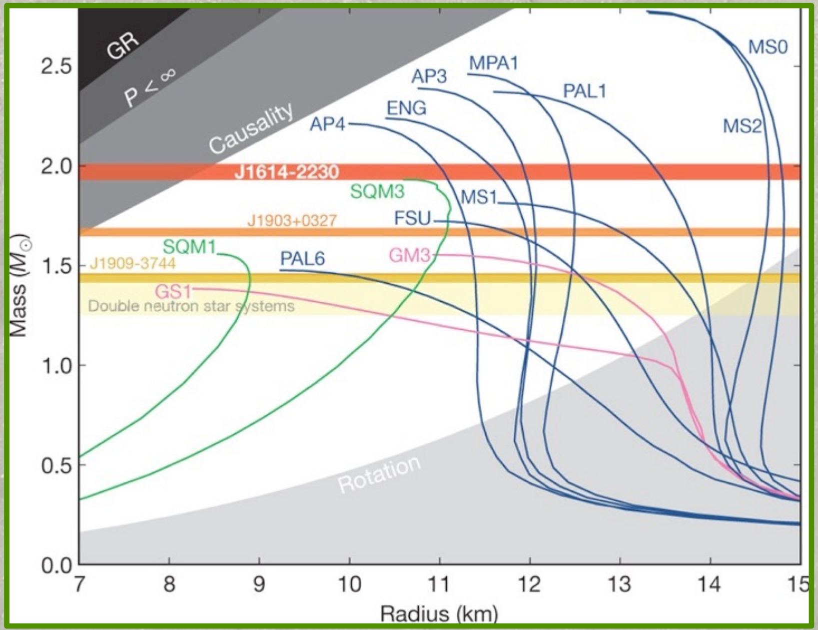
Plug-and-play cold QCD EoS!

★ From QCD plug-and-play and effective bag model:

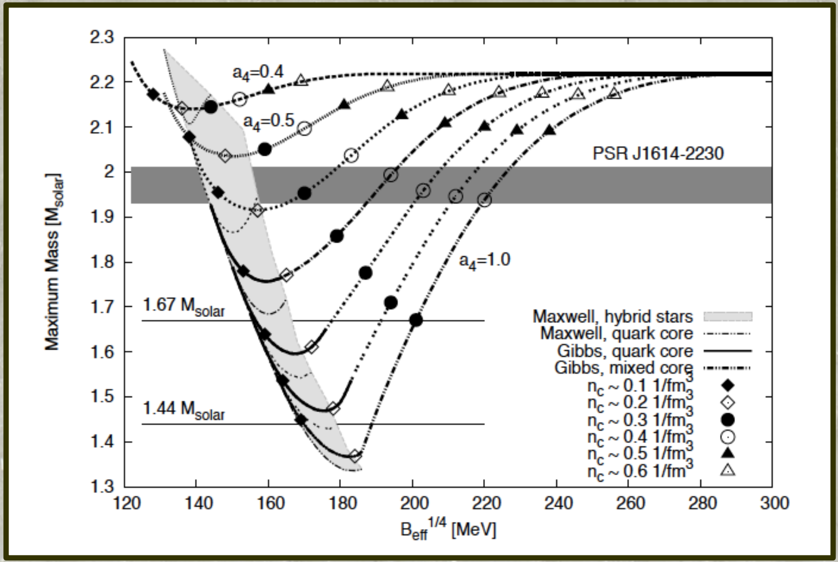




[ESF, Kurkela & Vuorinen (2013)]



[Demorest et al (2010)]



[Weissenborn, Sagert, Pagliara, Hempel, Schaffner-Bielich (2011)]

Some pros and cons for the pQCD EoS

⋆ Of course, its validity is assured only for very high densities. Strictly speaking, way beyond the relevant region for NS. However, this is the theory of strong interactions there. Nevertheless, one should be cautious when extrapolating down in density.

⋆ On the other hand, where can we rely on the bag model, or even on effective chiral models (NJL, etc)? Where are we sure it is safe?

⋆ There is always a dependence on the renormalisation scale. This is unfortunate, of course, but it also provides a measure of our uncertainty (which is a good thing!). And it can be improved, as was for the finite temperature case (which is probably worse to tame).

⋆ So, one should always take pQCD results for the EoS of neutron stars with a grain of salt. But trading this for the bag model (or even chiral models) does not seem to be a good idea to me...

⋆ Besides, pQCD gives us something that none of these models can deliver: the correct behavior at large enough densities and robust constraints. And so we can ask different questions...

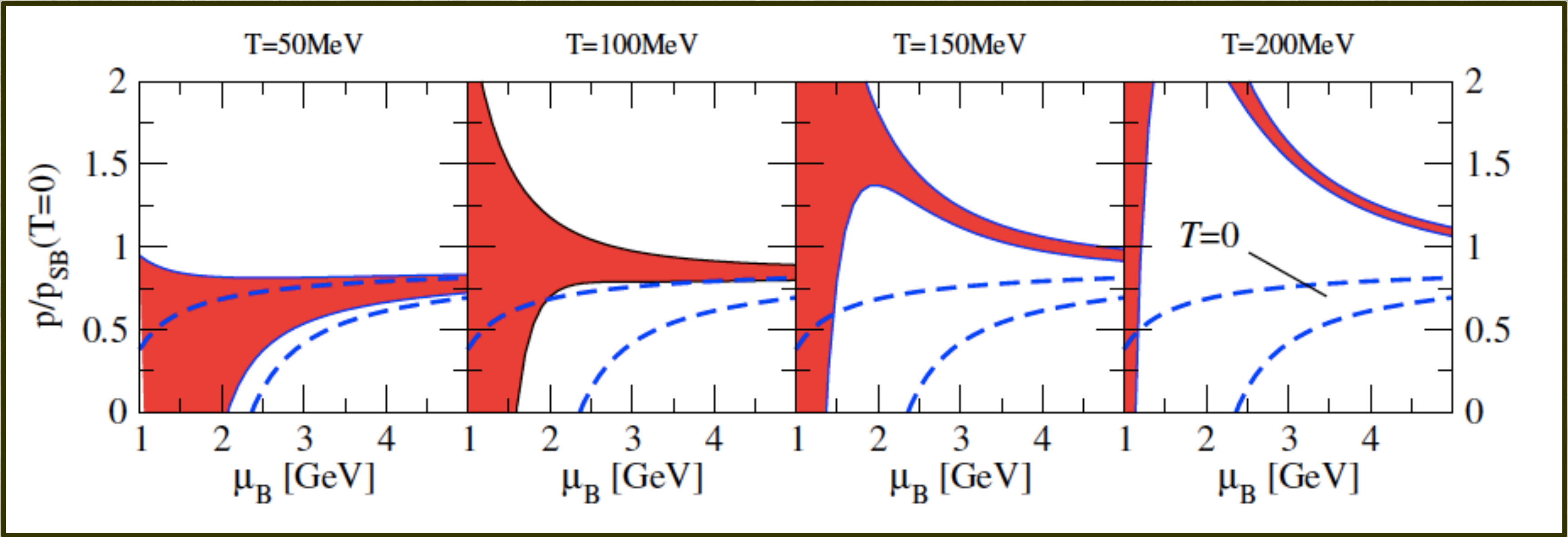
EoS from state-of-the-art cool pQCD:

[Kurkela & Vuorinen (2016)]

⋆ Generalization of the state-of-the-art pQCD EoS of cold quark matter to nonzero temperatures, in and out of beta equilibrium — cool QM.

⋆ Relevant in the description of neutron star mergers and core collapse processes.

⋆ Zero-Matsubara mode sector via dimensionally-reduced effective theory (EQCD), soft non-zero modes resummed using HTL.



➡ In the vacuum phase, charged pions decay weakly into leptons, with a characteristic lifetime of $\tau_{vac} \approx 10^{-8}$ s.

➡ In the condensed phase, the analogous weak process is quite different. Since the spontaneously broken symmetry group corresponds to the local gauge group of electromagnetism, the pion condensed phase is a superconductor, where the Goldstone mode is a linear combination of the electric charge eigenstates π^+ and π^-.

➡ In the presence of dynamical photons (in the unitary gauge) this mode disappears from the spectrum via the Higgs mechanism, at the cost of a nonzero photon mass $m_\gamma \propto e|\sigma\pi|$. In addition, the other linear combination of π^+ and π^- develops a mass above μ_I and is not excited if the temperature is sufficiently low.

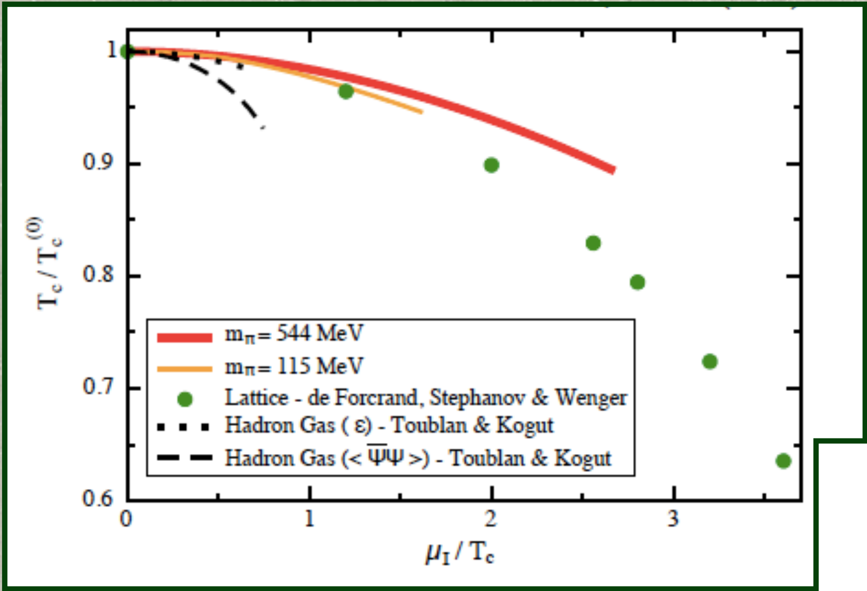
➡ Thus, there is no light, electrically charged excitation that would decay weakly.

➡ However, besides condensation in the pseu- doscalar channel, the ground state also exhibits an axial vector condensate that couples directly to the charged weak current.

➡ The coupling of σ_A to the weak current results in the depletion of the condensate and the production of charged anti-leptons and neutrinos. The characteristic lifetime τ of this process is

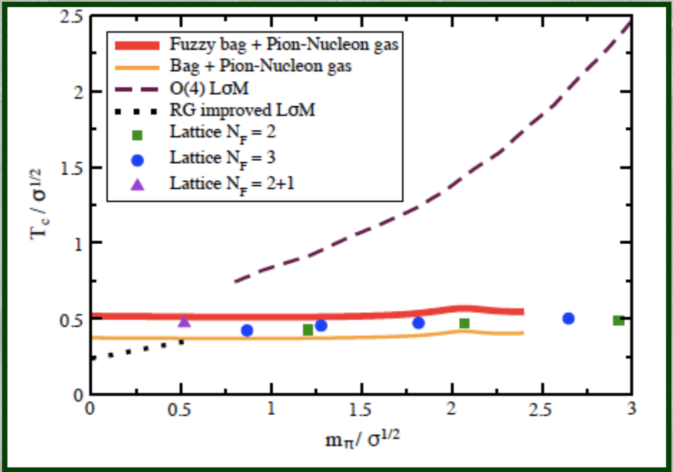
$$\boxed{\mu_I > m_\pi : \qquad \frac{\tau}{\tau_{\text{vac}}} = \frac{\mu_I^3}{m_\pi^3} \left[\frac{1 - m_\ell^2/m_\pi^2}{1 - m_\ell^2/\mu_I^2} \right]^2}$$

Remark: T_c vs. μ_I not so easy to get from models...

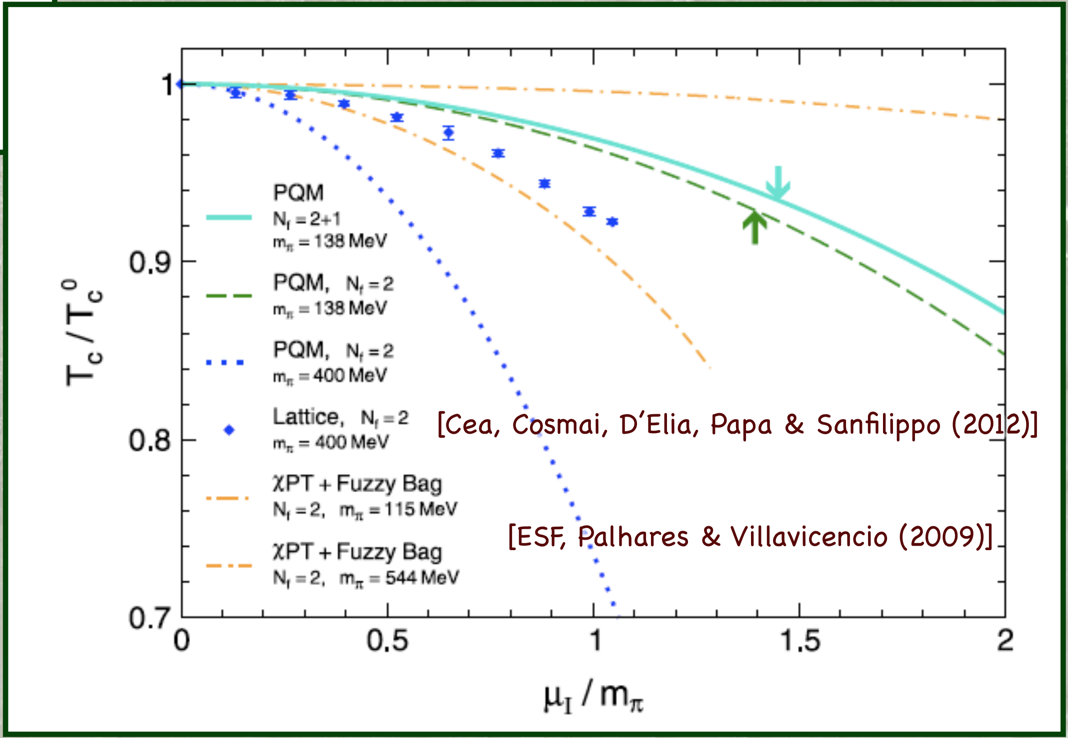


[ESF, Palhares & Villavicencio (2009)]

- Chiral models usually don't do well for μ_I & mass dependence of T_c.
- Region not accessible to pQCD.
- Best description (surprisingly!) given by bag & fuzzy bag... Need other approaches!



[ESF, Palhares & Villavicencio (2009)]



[Stiele, ESF & Schaffner-Bielich (2014)]

Low-energy description - effective chiral model

Ingredients:

⋆ Isospin rotations:

$$\begin{array}{lll} \Lambda_V : & \psi_L \to e^{-i\vec{\tau}.\vec{\theta}_V/2}\,\psi_L\,, & \psi_R \to e^{-i\vec{\tau}.\vec{\theta}_V/2}\,\psi_R \\ \Lambda_A : & \psi_L \to e^{-i\vec{\tau}.\vec{\theta}_A/2}\,\psi_L\,, & \psi_R \to e^{+i\vec{\tau}.\vec{\theta}_A/2}\,\psi_R \end{array}$$

⋆ For massless quarks, QCD is symmetric under Λ_V & Λ_A.

⋆ Λ_A spontaneously broken by the quark condensate.

⋆ Pions are the pseudo-Goldstone bosons, the lightest hadrons.

⋆ Λ_A rotates the π degrees of freedom into σ ones.

⋆ Symmetry breaking of Λ_A is associated to $\langle\sigma\rangle \neq 0$ and results in very different masses for π's and σ ⟶ pions dominate the low-energy regime!

LSM-iso & Lattice

[Brandt, Endrodi, ESF, Hippert, Schaffner-Bielich & Schmalzbauer (in prep.)]

Ingredients for the Linear Sigma Model with finite isospin (LSM-iso)

★ Lagrangian: [Gell-Mann & Levy (1960); Scavenius, Mócsy, Mishustin & Rischke (2001); ...]

$$\mathcal{L}_{LSM} = \frac{1}{2}\,\partial_\mu\sigma\,\partial^\mu\sigma + \frac{1}{2}\,\partial_\mu\vec{\pi}\cdot\partial^\mu\vec{\pi} - \frac{\lambda}{4}(\sigma^2+\vec{\pi}^2-v^2)^2 + h\,\sigma$$

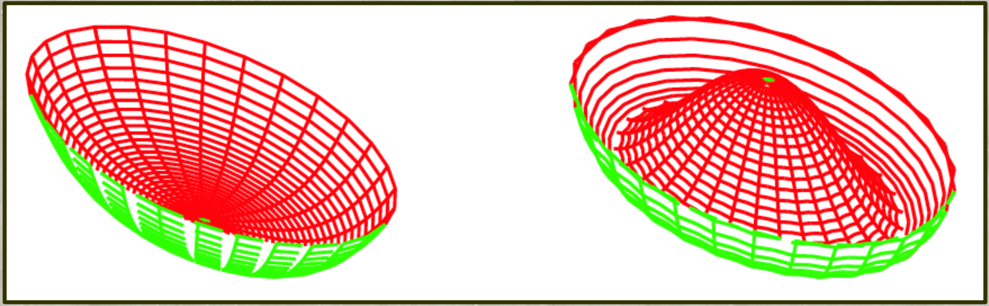
★ SU(2) x SU(2) spontaneously broken + explicit breaking by massive quarks.

★ All parameters chosen to reproduce the vacuum features of mesons.

★ Classical potential:

$$\begin{aligned} V_\phi(\sigma,\vec{\pi}) &= \frac{\lambda}{4}(\sigma^2+\vec{\pi}^2-v^2)^2 - h\sigma \\ &= \frac{1}{2}m_\sigma^2\sigma^2 + \frac{1}{2}m_\pi^2(\pi^0)^2 + m_\pi^2\pi^+\pi^- + \ldots \end{aligned}$$

T >> T_c: T << T_c:



Then, we have

★ Partition function:

$$\mathcal{Z} = \mathrm{Tr}\, \mathrm{e}^{-\beta \int \mathrm{d}^3 \mathrm{x}\, (\mathcal{H} - \mu_\mathrm{I}\, \rho_\mathrm{I})}$$

★ Isospin density (from Noether):

$$\rho_I = i\,(\pi_-\, \partial_0 \pi_+ - \pi_+\, \partial_0 \pi_-)$$

★ Effective Lagrangian:

$$\mathcal{L}_E = \frac{1}{2}(\partial_\tau \sigma)^2 + \frac{1}{2}(\partial_\tau \pi_3)^2 + \frac{1}{2}(\partial_\tau \pi_1 + i\,\mu_I\,\pi_2)^2 + \\ + \frac{1}{2}(\partial_\tau \pi_2 - i\,\mu_I\,\pi_1)^2 + \frac{1}{2}(\nabla \vec{\pi})^2 + \frac{1}{2}(\nabla \sigma)^2 + \mathcal{U}(\sigma, \vec{\pi})$$

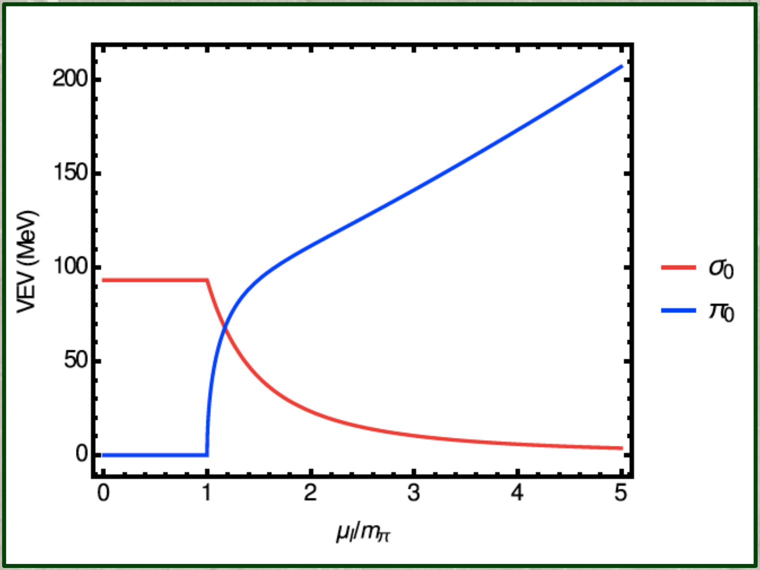
★ Shifting (from vacuum expectation values):

$$\vec{\pi} \to (\pi_0 + \pi_{\shortparallel}, \pi_\perp, \pi_3)\,, \qquad \sigma \to \sigma_0 + \sigma$$

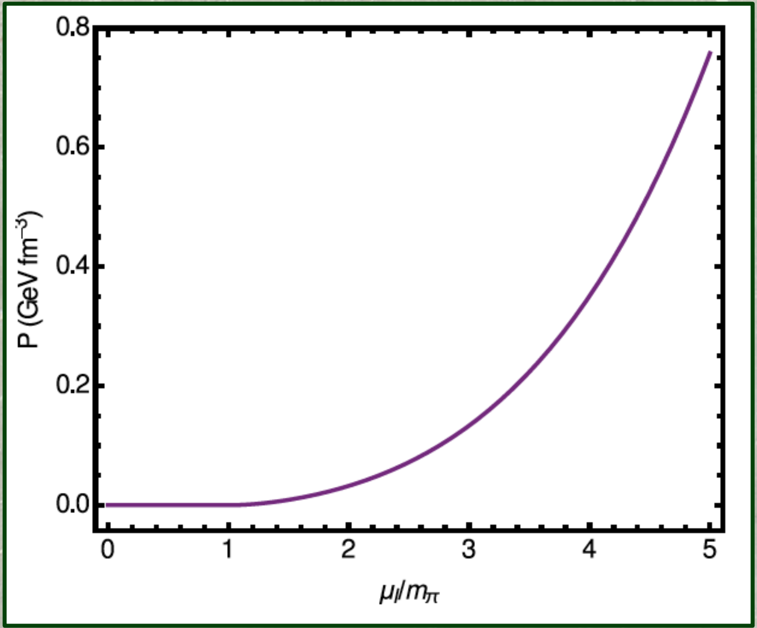
★ Classical approximation for the thermodynamic potential:

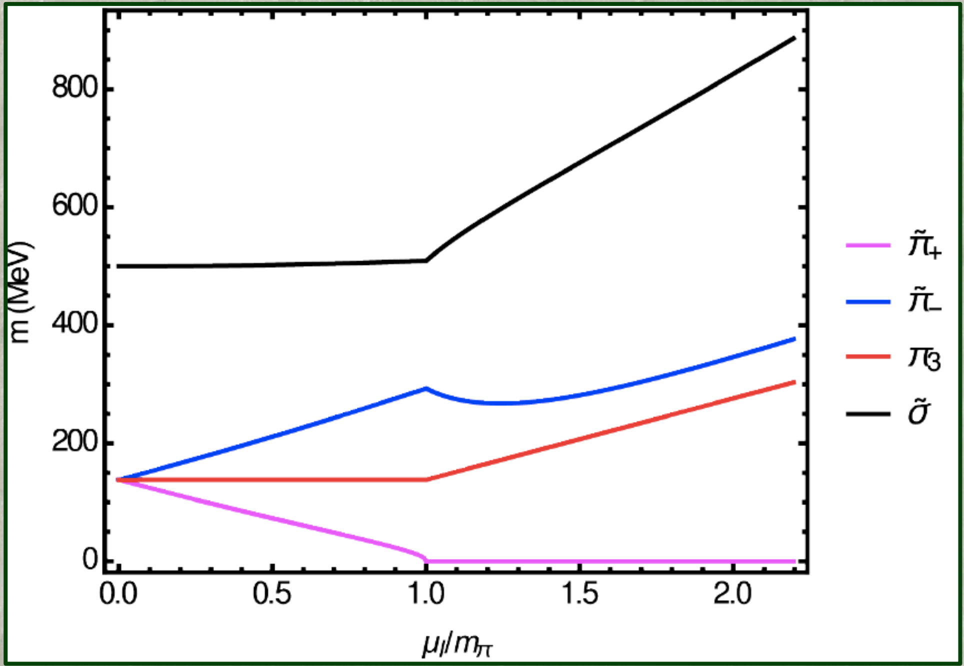
$$\Omega(\sigma_0, \vec{\pi}_0) \approx \Omega_C(\sigma_0, \pi_0) = -\frac{1}{2}\mu_I^2\,\pi_0^2 + \frac{\lambda}{4}(\sigma_0^2 + \pi_0^2 - v^2)^2 - h\,\sigma_0$$

Results - condensates, EoS & excitations



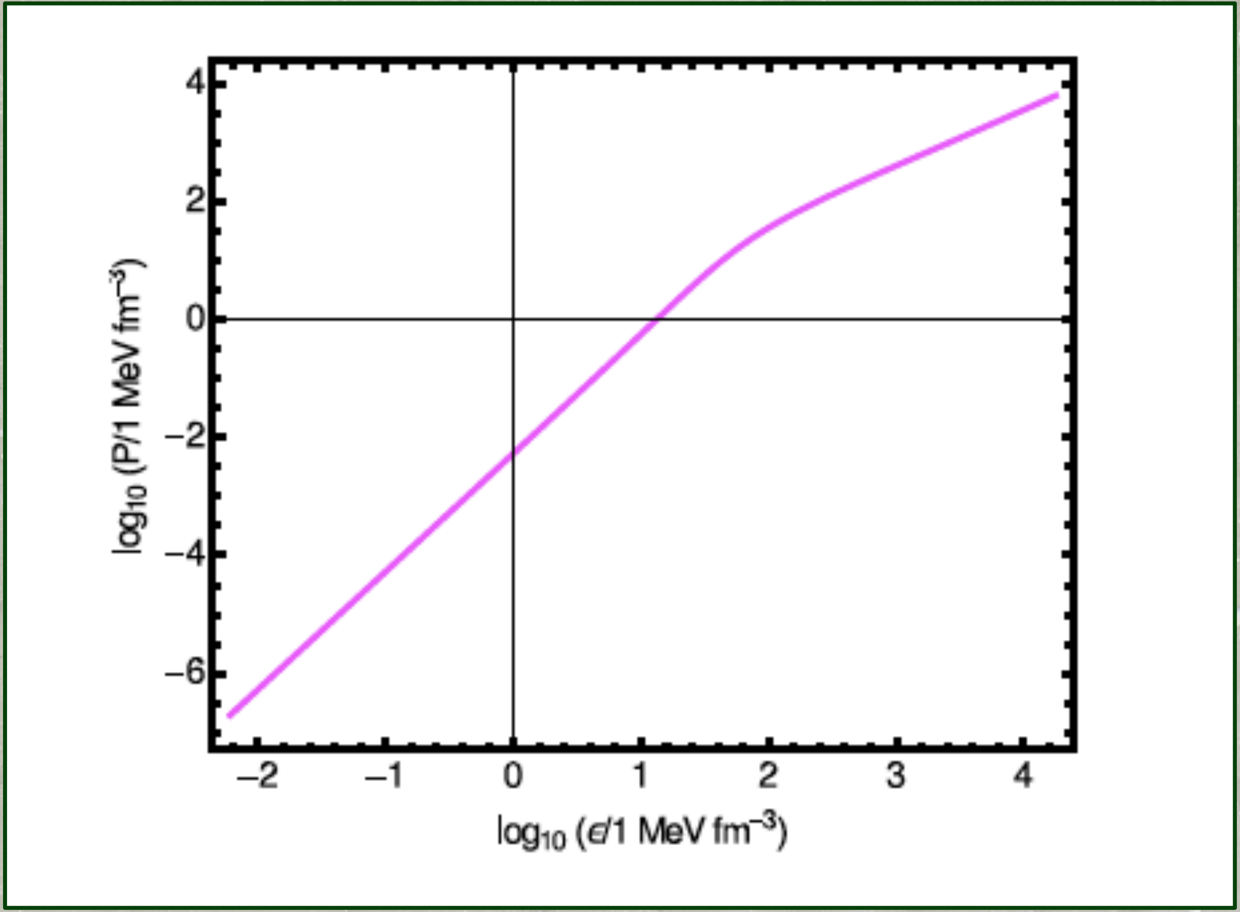
- Diagonalization of quadratic part of S_E $\rightarrow$ dispersion relations.
- Splitting of isospin triplet.
- masses: excitation energy for zero momentum.
- $|\mu_I| \geq m_\pi$: mixing between π and σ.



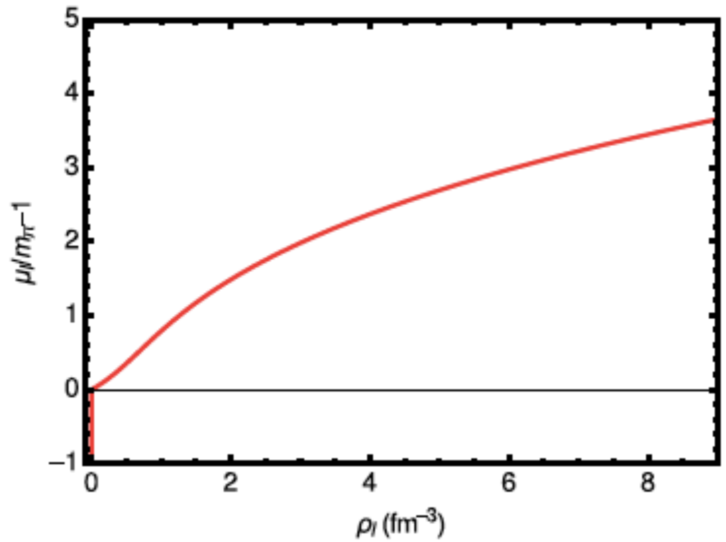


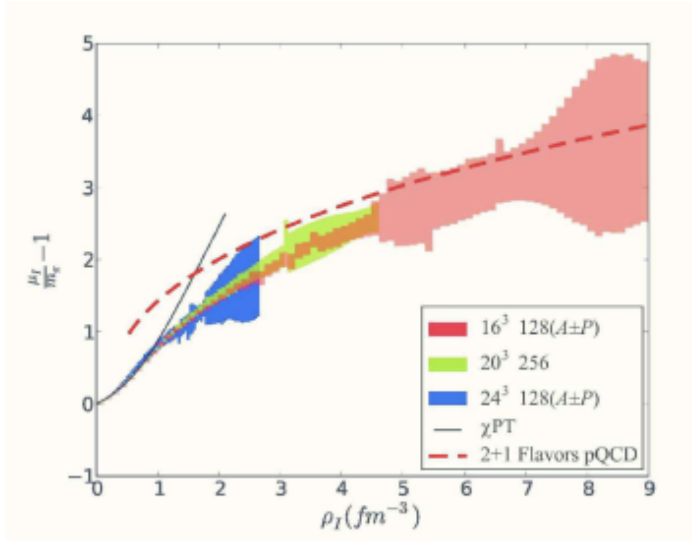
★ Power-law behavior in the EoS: $P \propto \varepsilon^{\Gamma}$

★ Transition from $\Gamma \approx 2$ to $\Gamma \approx 1$.



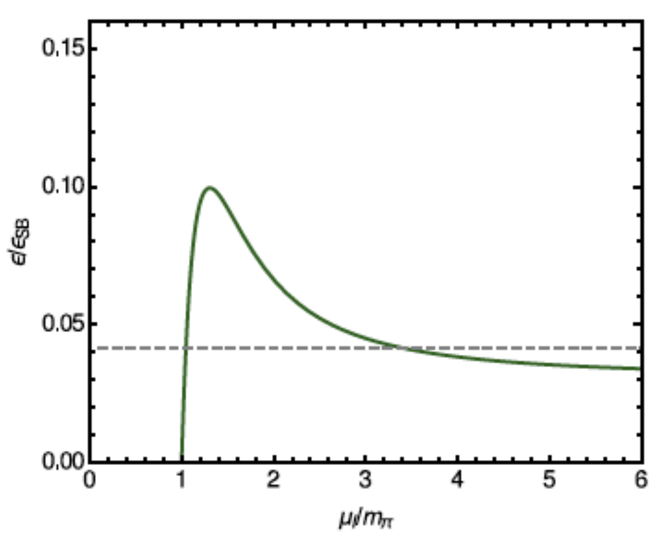
Results - comparing LSM-iso to Lattice QCD [m_π = 390 MeV]

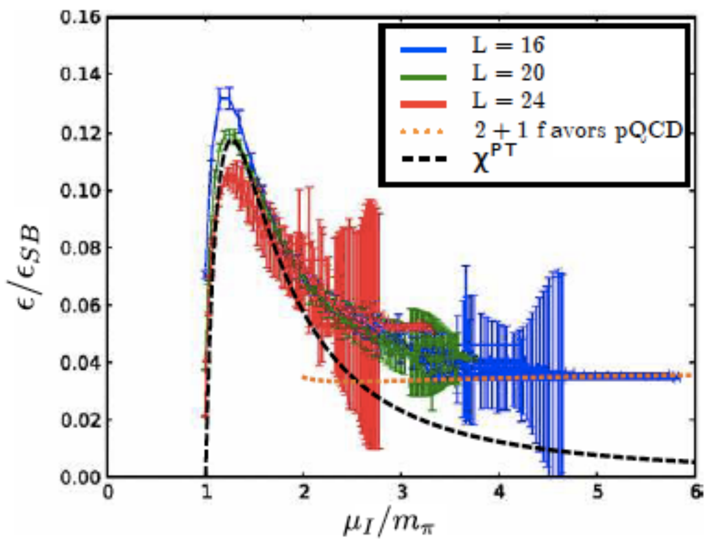




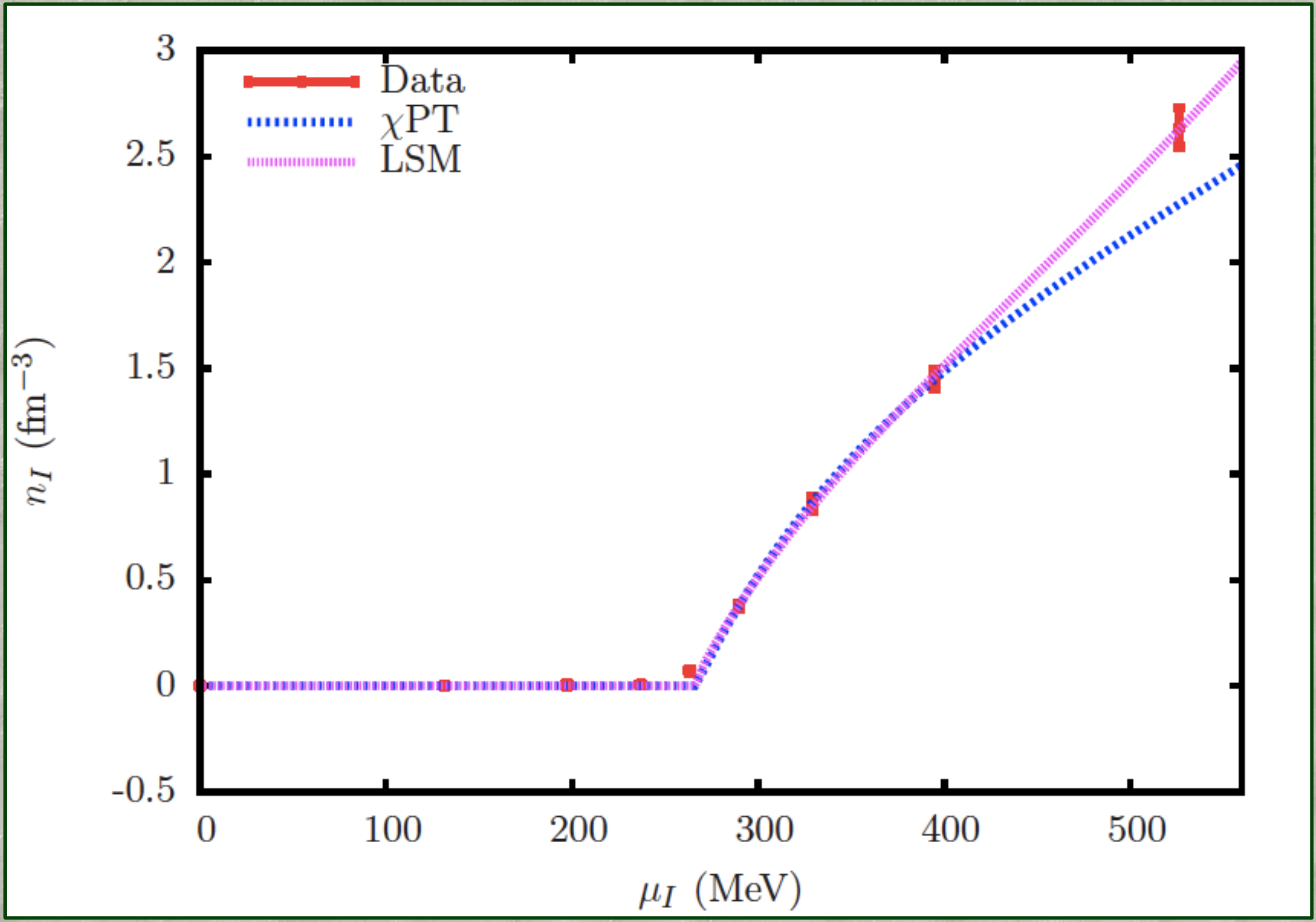
NOT fits!!

- Peak position independent of LSM-iso parameter choice!
- Depends only on m_π.





★ LSM-iso and Lattice for the EoS:



⋆ Message 3: Boson stars could provide an exciting possibility for a new class of compact stars and even contribute to the Dark Matter content of the universe.

➡ The issue of absence of repulsive interactions (which would lead to gravitational collapse) is not a problem for a pion star.

➡ Combining the (charged) pion condensate with leptons (electrons/muons), one can balance charge and find compact stars with $M \approx 250 M_\odot$ and $R \approx$ 30,000 km.

➡ The first issue that arises is related to stability (or metastability) against decay of the condensate into leptons via excitations. Probably the pion star will evaporate, but one still has to estimate its rate.

➡ The second is the mechanism of formation for pion stars, which is hard to guess. But, if it can be there, maybe Nature has found a way...

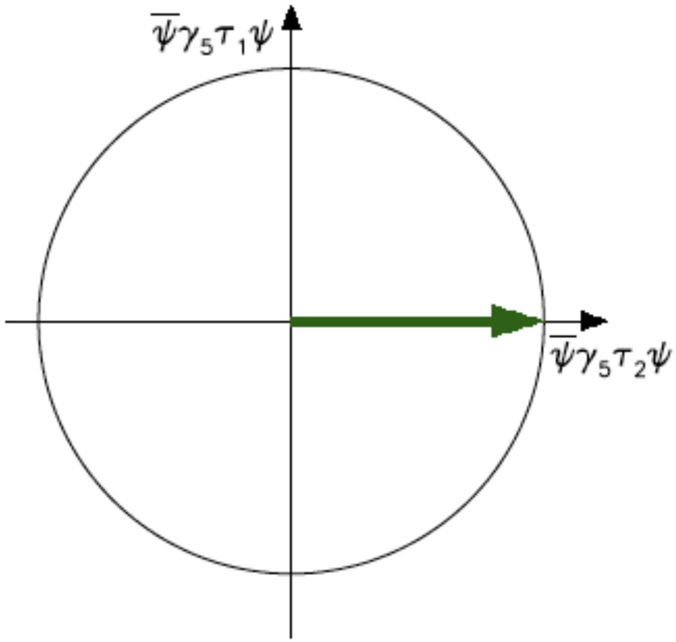
Symmetry breaking

- QCD with light quark matrix

$$M = \not{D} + m_{ud}\mathbb{1} + \mu_I\gamma_0\tau_3 + i\lambda\gamma_5\tau_2$$

- chiral symmetry (flavor-nontrivial)

$$\mathrm{SU}(2)_V \to \mathrm{U}(1)_{\tau_3} \to \varnothing$$



- spontaneously broken by a pion condensate

$$\left\langle \bar{\psi}\gamma_5\tau_{1,2}\psi \right\rangle$$

- a Goldstone mode appears
- add small explicit breaking

[Endrodi (2017)]