A Three-Dimensional Model of the Penis for Analysis of Tissue Stresses during Normal and Abnormal Erection

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ABSTRACT: Approximately half of the males between the ages of 40 and 70 suffer erectile dysfunction. Because adequate mechanical interactions in the penis are necessary for functional erection, it is important to analyze stresses in the erect penis. Previous penis models were limited to simplified or two-dimensional geometry. Here we developed a three-dimensional model for structural analysis of normal erection as well as erections of a penis with substantial asymmetry of the corporal bodies, and Peyronie’s disease. The model was constructed based on anatomical images and included skin, tunica albuginea, corpus cavernosa, and spongiosum. The mechanical behavior of the tunica and skin were assumed to be three-dimensional-orthotropic, and other tissues as well as Peyronie’s plaque was taken as linear elastic. Stresses and deformations during erection were analyzed using a commercial finite elements (FE) solver. Erection was simulated by raising blood pressure in the corporal bodies to 100 mmHg. The tunica was found to be the most highly loaded tissue in the erect penis. Peak von Mises stresses in the healthy tunica, tunica of the asymmetric corpora model, and tunica with Peyronie’s disease were 114 kPa, 167 kPa, and 830 kPa, respectively. The angles of distortion of the penis with respect to the vertical axis were ∼4.5° and ∼2°, for the asymmetric and Peyronie’s cases, respectively. The model’s ability to determine internal stresses in the erect penis offers a new point of view on the mechanical factors involved with erection, and enables us to relate these data with different penile pathologies.

KEYWORDS: erectile dysfunction; impotence; Peyronie’s disease; tissue mechanics; finite element method

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INTRODUCTION

The penis, a vital structure allowing, in its erect state, vaginal penetration during intercourse, plays a critical role in human sexual activity. Knowledge of its mechanical behavior during erection, including the stress distribution developed within its structural components, is a key for understanding not only the normal sexual function but also allows better comprehension of common penile pathologies. Current technology is limiting direct measurements of the mechanical stress distribution within the living human penis during erection. Hence, analysis of computational simulations, conducted using a realistic three-dimensional biomechanical penis model, is the only feasible alternative to carry out such an investigation, which may open new approaches for treating many erectile dysfunction conditions.

Potential of Computational Modeling in Clinical Evaluation and Treatment of Erectile Dysfunction Conditions

Impotence may occur in some individuals solely due to unusual tissue geometry and/or mechanical factors, despite development of sufficient erectile pressure and existence of adequate hemodynamic function. For example, patients who are treated by a microvascular arterial bypass surgery may not reach sufficient penile rigidity during erection despite presenting improved hemodynamic findings due to concomitant abnormal geometric or material tissue properties. Development of a computational model capable of predicting the mechanical behavior of the three-dimensional penis during erection will provide the means for analysis of the effect of specific abnormal structural variations on the developed stresses and deformations. These variations could be further related with functional alterations in basic characteristics of the penis, such as its overall compliance, the amount of compression applied to the vascular bed, and the expandability of the cavernosal spaces.

In its erect state, the penis is vulnerable to blunt injuries (i.e., penile fracture), occurring during intercourse when the penis slips out of the vagina and is thrust against the partner’s perineum or pubic bone, or when the erect penis is subjected to accidental abnormal bending. While long-term consequences of blunt injury to the erect penis, including erectile dysfunction, are documented, only little is known about the damage mechanism. A biomechanical model indicating sites of elevated penile stresses during erection can identify the most vulnerable tissue components and explore the role of penile geometry and erectile pressure in injuries caused by abrupt loading.

The penis during actual erection is rarely straight, and frequently, some level of penile curvature is observed even in normal individuals. It can be hypothesized that a curvature of the erect penis is related with asymmetry of its corporal bodies, because if one corporal cavernosum is smaller than the other
it may restrict inflation of the other cavernosum, even though both cavernosa are subjected to the same erectile pressure. A computational model of the penis is ideal for investigating such interactions, because it allows isolation of the effect of corporal geometry, while keeping other factors (e.g., hemodynamics, tissue mechanical properties) constant. Ultimately, a computational model can be used for determining a quantitative relation between corporal geometry asymmetry and the level of penile curvature during erection, which will allow to predict if a certain level of corporal asymmetry allows vaginal penetration or not.

An accepted treatment of Peyronie’s disease, a connective tissue disorder of the penis resulting in fibrotic plaque formation, consists of plaque excision and patching with one of many potential patch materials. While several different biological and artificial patch materials are currently being used (superficial dorsal penile vein tissue, silicone fabric, dermabraded preputial flaps, etc.), the optimal patch material for covering the resultant defect has not yet been determined. Inadequate mechanical interaction between the patch and the surrounding penile tissues may induce sites of localized, elevated stresses, which may irritate the dense and delicate network of nerves or obstruct some of the penile blood vessels. The “biomechanical compatibility” of any given patch with the surrounding penile tissues can be characterized by incorporating the patch into the biomechanical penis model. The stresses developing around the patch can thus be analyzed, allowing a more optimal selection of transplant/implant materials as well as geometry. However, a first, basic step needed to be taken prior to such analyses is to determine the interference to the normal stress state in the erect penis, which is caused by a Peyronie’s plaque.

Past Modeling Work and Current Objectives

Despite the clinical importance in understanding the biomechanical perspectives of erection, as discussed above, quantitative structural analysis of the penis is still at its beginning. A basic, simplified model of the penis as a homogenous shaft having a circular cross-section was suggested by Udelson et al. to estimate the force required to cause penile buckling during intercourse. Later, Chen et al. developed a biomechanical model of the penis as a blood-filled cylindrical tube, and applied it to predict penile elongation during erection. Missing the different penile components and the development of erectile pressure, these models are not applicable for evaluating local tissue loading during erection.

A first two-dimensional model that quantitatively analyzed stresses in the natural anatomical structure of the human penis during erection was introduced by Gefen et al. This model was successfully applied to investigate the development process of Peyronie’s disease and optimize the engineering design of penile prostheses. Nevertheless, more realistic modeling of the penis during
erection necessarily confronts a three-dimensional problem. Stresses within the penis structure may be well affected by the three-dimensional geometry of its soft tissue components, including the anatomical areas through which erectile pressure is transferred and the physical constraints constituting its deformation during erection. Therefore, in the present study, a more complex three-dimensional finite element (FE) model of a normal penis structure was developed to characterize the mechanical stress state occurring during erection and to identify the most highly loaded tissue regions. The model was modified to simulate erection where substantial asymmetry exists between the sizes of the two corpora cavernosa, and the effect on penile curvature during erection was determined. Last, we included a Peyronie’s plaque in the tunica albuginea of the model to simulate the stress state in and around the plaque, and the resulted distortion of the penile erect shape. This is the first biomechanical model of the penis, which considers the three-dimensional penile geometry and three-dimensional tissue constitutive behavior.

METHODS

The Visible Human Male digital database (Fig. 1A) was used to determine the gross dimensions of a symmetrical three-dimensional model of the penis in its flaccid state, by segmenting tissue types in a commercial solid modeling software (SolidWorks 2006, SolidWorks Co., MA, USA, Fig. 1). The model included the skin, tunica albuginea, corpus cavernosa, corpus spongiosum, and 

FIGURE 1. Computational modeling of the penis: (A) cross-sectional image of the penis from the Visible Human Male database, upon which the modeling was based, (B) cross-section through the three-dimensional penis model, (C) a view of the three-dimensional solid model of the penis with gross dimensions.
glans (Figs. 1B and 1C). The initial length and diameter of the penis were taken as 8 and 4 cm, respectively. The model was transferred to a commercial nonlinear FE solver (MARC 2006, MSC Software Co., CA, USA) for strain/stress analyses based on the large deformation theory. For this purpose, the model was meshed into $\sim 15,000$ tetrahedron elements and $\sim 25,000$ respective nodes (Fig. 2A). An equivalent erectile pressure of 100 mmHg ($\sim 13.3$ kPa) was applied to the internal boundaries of the corpus cavernosa and spongiosum, and nodes on the penile base were fixed for radial movement (Figs. 2B and 2C). The skin and tunica albuginea were assumed to be transverse-orthotropic materials, which obey the following constitutive law:

$$
\begin{bmatrix}
\sigma_{xx} \\
\sigma_{yy} \\
\sigma_{zz} \\
\sigma_{xz} \\
\sigma_{xy}
\end{bmatrix} =
\begin{bmatrix}
\frac{1-v_{yz}v_{zy}}{E_xE_y\Delta} & \frac{v_{yx}+v_{zx}v_{yz}}{E_yE_z\Delta} & \frac{v_{zx}+v_{yx}v_{zy}}{E_xE_z\Delta} & 0 & 0 & 0 \\
\frac{v_{xy}+v_{zx}v_{yz}}{E_xE_z\Delta} & \frac{1-v_{yz}v_{zy}}{E_xE_y\Delta} & \frac{v_{xy}+v_{zx}v_{yz}}{E_xE_y\Delta} & 0 & 0 & 0 \\
\frac{v_{xz}}{E_xE_y\Delta} & \frac{v_{yz}}{E_xE_z\Delta} & \frac{1-v_{yx}v_{xy}}{E_xE_y\Delta} & 0 & 0 & 0 \\
0 & 0 & 0 & 2G_{yz} & 0 & 0 \\
0 & 0 & 0 & 0 & 2G_{zx} & 0 \\
0 & 0 & 0 & 0 & 0 & 2G_{xy}
\end{bmatrix}
\begin{bmatrix}
\varepsilon_{xx} \\
\varepsilon_{yy} \\
\varepsilon_{zz} \\
\varepsilon_{xz} \\
\varepsilon_{xy}
\end{bmatrix},
$$

(1)
TABLE 1. Mechanical properties assigned to the penile tissues for finite element modeling

<table>
<thead>
<tr>
<th>Tissue</th>
<th>E</th>
<th>v</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Glans</td>
<td>80  kPa</td>
<td>0.4</td>
<td>12</td>
</tr>
<tr>
<td>Peyronie’s Plaque</td>
<td>320 MPa</td>
<td>0.3</td>
<td>8</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Tissue</th>
<th>$E_x$</th>
<th>$E_y$</th>
<th>$E_z$</th>
<th>$G_{xy}$</th>
<th>$G_{yz}$</th>
<th>$G_{zx}$</th>
<th>v*</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tunica Albuginea</td>
<td>12 MPa</td>
<td>12 MPa</td>
<td>30 kPa</td>
<td>10 kPa</td>
<td>4 MPa</td>
<td>4 MPa</td>
<td>0.4</td>
<td>8,9</td>
</tr>
<tr>
<td>Skin</td>
<td>0.5 MPa</td>
<td>0.5 MPa</td>
<td>12.5 kPa</td>
<td>4.25 kPa</td>
<td>170 kPa</td>
<td>170 kPa</td>
<td>0.4</td>
<td>8,9</td>
</tr>
</tbody>
</table>

*Values for three-dimensional-orthotropic material coefficients were based on elastic and shear moduli that were published at the listed references. Under the three-dimensional orthotropic material model assumption, strains in the radial direction during erection were 1–4.5%.

**Poisson’s ratio was taken as 0.4 in all material directions.

where $\sigma_{ij}$ are tensorial tissue stresses, $\varepsilon_{ij}$ are tensorial tissue strains, $E_i$ are the elastic moduli, $G_{ij}$ are the shear moduli, $v_i$ are Poisson’s ratios and

$$
\Delta = \frac{1 - v_{xy}v_{yx} - v_{yz}v_{zy} - v_{zx}v_{xz} - 2v_{xy}v_{yz}v_{zx}}{E_xE_yE_z}.
$$

Poisson’s ratios $v_{ij}$ of the skin and tunica albuginea were all taken as 0.4 following Gefen et al.\(^7\). The other orthotropic material constants for the skin and tunica are provided in Table 1. The glans was assumed to be made of an incompressible, homogeneous, and linear elastic material with the elastic modulus similar to that of fat tissue, that is, 80 kPa, and Poisson’s ratio of 0.4.\(^{12}\)

First, we simulated normal erection, from flaccid, through tumescence, to rigidity. Then, we simulated erection with the same boundary conditions but with an asymmetrical penile geometry. Specifically, we set one of the corpus cavernosa to be larger than the other by $\sim 20\%$ in cross-sectional area (Fig. 3A).

To simulate the mechanical conditions in the penis during erection in a patient with Peyronie’s disease, we increased the elastic modulus and decreased the Poisson’s ratio of a proximal dorsal segment of the tunica albuginea,\(^{13}\) as specified in Table 1 and shown in Figure 3B. In each simulation, deformation, strain, and stress distributions were calculated along the penile body.

RESULTS

The simulations resulted in the stress/strain states in the normal penis model, the penis model with cavernosal asymmetry, and the model with Peyronie’s plaque. We specifically calculated the von Mises stresses, principal
FIGURE 3. Simulation cases: (A) asymmetrical penile geometry where one of the corpus cavernosa was set to be larger than the other by ~20% in cross-sectional area, and (B) Peyronie’s disease with the plaque shown in the tunica albuginea.

Compression stresses, principal tension stresses, and respective strains for each simulation case. For the abnormal simulation cases, we also determined the angle of maximal distortion of the erect penis with respect to the vertical axis. Overall, we found that the tunica albuginea was the most highly loaded tissue layer in the penis during erection.

Peak von Mises, principal compression, and principal tension stresses in the normal healthy tunica albuginea during erection were found to be 114 kPa, 32 kPa, and 122 kPa, respectively (FIG. 4A). For all types of stresses, the maximal value was located laterally on the corpus cavernosal cavity walls (FIG. 4A). Peak von Mises, principal compression, and principal tension strains in the tunica were found to be 29%, 12%, and 57%, respectively. These maximal strains were found to occur distally on the upper third of the corpus cavernosal walls, where maximal stresses occurred.

In the asymmetrical model configuration, peak von Mises (167 kPa), principal compression (102 kPa), and principal tension (181 kPa) stresses were found to be ~1.5-, ~3.2-, and ~1.5-fold higher than the corresponding stresses in the normal state, respectively. Maximal von Mises and principal tension stresses were located laterally on the larger corpus cavernosal wall (FIG. 4B). The maximal principal compression stress was located distally, however, on the tunical wall region between the corpus cavernosa and corpus spongiosum. Peak von Mises, principal compression, and principal tension strains during erection of the penis with asymmetric corpora were 31%, 12%, and 47%, respectively. The
von Mises maximal strain was mildly higher (1.07-fold) than in the normal model configuration, and the maximal principal tension strain was $\sim$0.8-fold lower than the respective strain in the normal condition. For all strain types, peak strain was located distally, on the larger corpus cavernosal wall.

Von Mises stresses during erection in the tunica albuginea affected by Peyronie’s disease are shown in FIGURE 4C. Maximal von Mises, principal compression, and principal tension stresses were found to be 830 kPa, 596 kPa, and 905 kPa, respectively, and were located within the plaque and around it (FIG. 4C). These stresses were $\sim$7.3-, $\sim$18.6-, and $\sim$7.4-fold greater than stresses in the normal tunica albuginea. Peak von Mises, principal compression, and principal tension strains in the penis model with Peyronie’s disease were found to be 35%, 12%, and 67%, respectively. The von Mises and principal tension strains were $\sim$1.2 higher than strains in normal condition. For all types of strains, peak values were located on the side opposite to the plaque, which overall induced a distorted, curved elongation of the erect penis.

The von Mises stress distribution along two major paths ($M$ and $N$) for each simulation case: normal, cavernosal asymmetry, and Peyronie’s disease is depicted in FIGURE 5. This analysis reveals that across all cases, peak stresses along path $M$ are located within the tunica albuginea’s lateral walls but in the asymmetry and Peyronie’s cases, peak stresses occur at the larger cavernosa or the plaque side, respectively. In all cases, peak stresses along path $N$ were located on the dorsal side of the tunica (i.e., the side opposite to the corpus spongiosum).

**FIGURE 4.** Distribution of von Mises stresses in (A) the normal penis model, (B) penis with asymmetrical geometry, and (C) Peyronie’s disease. A central cross-section is magnified on each stress diagram to show locations where maximal internal stresses occur.
FIGURE 5. Von Mises stress distributions along paths M (A) and N (B) for each simulation case.

The deformed geometry of the erect penis for the cases of asymmetric corpora and Peyronie’s disease are shown in Figure 6. The maximal angles of distortion of the penis with respect to the vertical axis were $\sim 4.5^\circ$ and $\sim 2^\circ$, for the asymmetric and Peyronie’s cases, respectively. Moreover, while in the asymmetrical case the distortion is rather homogenous toward one direction (Fig. 6A), in the case of a Peyronie’s plaque the distortion is clearly nonhomogenous, as evident by the deformed penile axis, which crosses both sides of the vertical reference axis (Fig. 6B).

DISCUSSION

In this study we used three-dimensional FE analyses to simulate the mechanical conditions (stresses, strains, deformations) during normal erection, erection of a penis with asymmetric corpora cavernosa, and erection of a penis with Peyronie’s disease. We found that the tunica albuginea was the most highly loaded tissue layer in the penis during erection. Peak von Mises stresses in the healthy tunica albuginea, tunica of the asymmetric corpora model, and tunica with Peyronie’s disease were 114 kPa, 167 kPa, and 830 kPa, respectively. The angles of distortion of the penis with respect to the vertical axis were $\sim 4.5^\circ$ and $\sim 2^\circ$, for the asymmetric and Peyronie’s cases, respectively. These results reveal that (i) asymmetrical corpora cavernosa sizes increase tissue local loads at the constrained side (i.e., near the smaller corpora), (ii) substantial asymmetry of corpora cavernosa (here 20% difference in corporal cross-sectional...
(A) The asymmetrical model and (B) the model with Peyronie’s plaque. The dashed contours show the geometry of the normal penis model in its erect state.

FIGURE 6. Distortion of the penile geometry during simulated erection of (A) the asymmetrical model and (B) the model with Peyronie’s plaque. The dashed contours show the geometry of the normal penis model in its erect state.

area) causes a visible penile curvature during erection (Fig. 6A), which, interestingly, was found to be more substantial than that predicted in a Peyronie’s disease model. (iii) A Peyronie’s plaque induces highly elevated stresses in tunical tissue around it (i.e., more than sevenfold stress increase with respect to normal), which is likely to influence the quality of erection as such focal stresses may irritate penile nerves and/or obstruct blood vessels.

The biomechanical model of the three-dimensional penile structure presented in this study is capable of predicting the distribution of stresses within the different components of the penis. The ability to acquire data characterizing the internal stress state in the penis during erection makes this model a basic clinical tool, as it offers a new point of view on the mechanical factors that are active during erection, and enables us to relate these data with different penile pathologies. For example, penile fractures in which injuries of the tunica albuginea occur due to abrupt bending of the erect penis (e.g., during vigorous coitus) are mainly reported to appear in the lateral-ventral parts of the tunica. This could be associated with the present findings, identifying the...
lateral walls of the tunica as a highly loaded structural segment of the normal erect penis. The compound loading of elevated internal stresses, particularly at the base of the penis, which is also subjected to bending moments owing to pelvis thrusting during coitus (not considered in the present simulations) highly loads the proximal-lateral aspects of the tunica. These mechanical conditions are very likely to make the proximal-lateral parts of the penis most vulnerable to penile fractures. Hence, being able to identify highly loaded soft tissue regions of the penis, the model can be used for understanding the development mechanisms of some common erectile disorders (like Peyronie’s disease considered herein). Moreover, the model can be potentially applied for development of novel clinical decision-making and penile treatment approaches, as suggested below.

Urologist surgeons frequently need objective information about the likelihood of success of a planned surgical intervention. The present penis model is able to provide such preoperative evaluation by simulating the biomechanical effects of the intended surgical intervention. For example, to enhance surgical correction in a penis with Peyronie’s plaque, the present model could be modified to simulate local tissue stiffening due to fibrosis. Virtual removals of some plaque elements could then be carried out until a more optimal structure is obtained, in terms of functional characteristics (e.g., penile alignment) and the resulted stress distribution during erection. Routine management of computational simulation procedures prior to reconstructive penile surgeries may reduce local stresses, and, thereby, may minimize fistulas, tissue disintegration, and other postsurgery complications. The consequences of replacement of tissue components with biological or artificial implants in these procedures may also be examined, and a more adequate penis-implant interaction could be obtained. Similarly, the biomechanical effects of penile prostheses for restoration of erectile function can be studied from the structural stress perspective, to analyze possible postoperative complications, such as severe pain during operation of the prosthesis, buckling of the prosthetic cylinders, and more.

Over the last decade, surgical applications of computational three-dimensional organ geometry reconstruction and biomechanical modeling are rapidly growing due to development of sophisticated, user-friendly systems that allow the clinician to obtain digital imaging data more easily, and use it for surgical planning. Currently, there are great opportunities to make use of this advanced technology in the field of urology, by employing it to select the most effective surgical intervention to restore erectile function. Further development of the present methodology, toward adaptation of a biomechanical model of the penis to anatomical characteristics of specific patients, is a promising way to accomplish the above aims. Indeed, in current clinical practice, reconstruction of computational penis models specifically made for presurgical assessment of individual patients is not practically feasible, mainly due to the complexity and time consumption of the development and simulation process. A possible approach to overcome these difficulties involves the use of parametric
solid modeling. Applying this approach, a limited set of anthropometric parameters (e.g., penile length, circumference, cavernosal cross-sectional area, tunical thickness, etc.) will be acquired through medical imaging techniques and consequently used to generate a custom-made solid model (based on a predefined parametric general-purpose model). Subsequently, hemodynamic measurements will be used to adjust the loading system of the model, that is, the characteristic erectile pressure.

Successful application of the present methodology to support the above-mentioned and other penile treatments is highly dependent on acquisition of experimental data characterizing the nonlinear and viscoelastic biomechanical properties of the penile tissues. Based on the present simulation results, particular attention should be given to characterization of the mechanical properties of the tunica albuginea and erectile tissue. After these data become available, a quasilinear viscoelastic approach can be useful to obtain an even more accurate representation of the structural behavior of the penis. As computer power increases and computational modeling advances, we are approaching a time when patient-specific modeling of the penis will be a standard routine in the clinical setting, as an integral part of patient evaluation and of planning interventions.

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Linder-Ganz and Gefen contributed equally to this work.

REFERENCES