The classical evolution of binary black hole systems in scalar-tensor theories[1]

Gravity Seminar, University of Southampton

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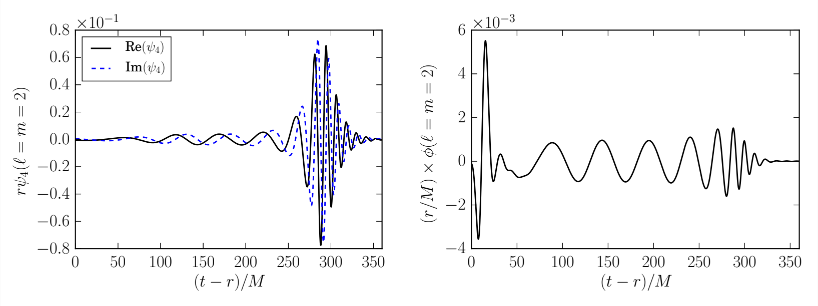
[1] Mostly based on arXiv:2011.03547

Outline and Summary

$$S = \frac{c^4}{16\pi G} \int d^4x \sqrt{-g} \left(R + X - V(\phi) + \alpha(\phi) X^2 + \beta(\phi) \mathcal{G} \right),$$

$$X \equiv -\frac{1}{2} g^{\mu\nu} \nabla_\mu \phi \nabla_\nu \phi, \qquad \mathcal{G} \equiv R^2 - 4 R_{\mu\nu} R^{\mu\nu} + R_{\mu\alpha\nu\beta} R^{\mu\alpha\nu\beta}$$



Goals: understand why we choose to study the above theory, and understand how we made these plots!

Outline and Summary

- Why study scalar-tensor gravity theories?
- Generating gravitational waveforms for scalar-tensor gravity theories
- Technical/mathematical advances that made this possible (if there is time/interest)

Planck units

- We will use *(reduced) Planck units*: $8\pi G = c = \hbar = k_B = 1$
- Everything can be phrased in terms of the *geometrized dimension L*
- Energy scale, etc. are multiples of:
  - Planck energy: $E_p = l_p c^4/G \sim 10^{16} ergs \sim 10^{19} GeV$
  - Planck length: $l_p = (G\hbar/c^3)^{1/2} \sim 10^{-33} cm$
  - Planck time: $t_p = l_p/c \sim 10^{-44} s$
  - Planck mass: $m_p = l_p c^2/G \sim 10^{-5} g$
  - Planck temperature $E_p/k_B \sim 10^{32} K$

Outline

Review: scalar-tensor gravity theories

Candidate theory: sEFT gravity

Shift symmetric

Conclusion

Scalar-tensor (Horndeski) gravity

Theories that have a tensor ($g_{\mu\nu}$) field and scalar (ϕ) field, and have second order equations of motion

$$
\begin{aligned}
S &= \int d^4x\sqrt{-g}\left(\mathcal{L}_1 + \mathcal{L}_2 + \mathcal{L}_3 + \mathcal{L}_4 + \mathcal{L}_5\right),\\
\mathcal{L}_1 &\equiv \frac{1}{2}R + X - V(\phi),\\
\mathcal{L}_2 &\equiv G_2\left(\phi, X\right),\\
\mathcal{L}_3 &\equiv G_3\left(\phi, X\right)\Box\phi,\\
\mathcal{L}_4 &\equiv G_4\left(\phi, X\right)R + \partial_X G_4\left(\phi, X\right)\delta^{\mu\nu}_{\alpha\beta}\nabla^\alpha\nabla_\mu\phi\nabla^\beta\nabla_\nu\phi,\\
\mathcal{L}_5 &\equiv G_5\left(\phi, X\right)G_{\mu\nu}\nabla^\mu\nabla^\nu\phi - \frac{1}{6}\partial_X G_5\left(\phi, X\right)\delta^{\mu\nu\rho}_{\alpha\beta\gamma}\nabla_\mu\nabla^\alpha\phi\nabla_\nu\nabla^\beta\phi\nabla_\rho\nabla^\gamma\phi,\\
X &\equiv -\frac{1}{2}\left(\nabla\phi\right)^2,
\end{aligned}
$$

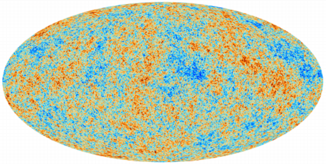
Why study scalar-tensor gravity?

- Find a complete theory of quantum gravity
- Model the dynamics of the very early universe
- Model the dynamics of the late universe
- Test GR for sake of basic science

Find a complete theory of quantum gravity

- GR is *nonrenormalizable*: the gravitational coupling constant, G, has units of $(M_P)^2$ (M_P is the Planck mass.)
- Nonrenormalizability hints that GR could/'should' be modified at energies around the Planck scale $l_p \sim 10^{-33} cm$

Cosmology and GR



- At the largest scales the universe is approximately:
  1. homogeneous
  2. isotropic
  3. expanding
  4. Spatial sections are geometrically flat (${}^{(3)}R_{ijkl} = 0$)
- Friedman-Lemaitre-Robertson-Walker (FLRW) solutions to the Einstein Equations
- With suitable matter contributions and a cosmological constant, the FLRW solutions match observational cosmological data extremely well

Late universe and GR

- To model the recent/late time expansion of the universe, need to add a *cosmological constant* Λ to the Einstein equations

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R + g_{\mu\nu}\Lambda = T_{\mu\nu}.$$

- Is there a physical mechanism that sets the value of the cosmological constant, or is it a new fundamental constant of nature?

Late universe and GR

- If you want to have "super-accelerated" expansion, where expansion happens *faster* than is possible with a cosmological constant (i.e. when the effective equation of state $w < -1$), then typically you need to modify gravity with higher derivative terms[2]

THE GALILEON AS A LOCAL MODIFICATION OF GRAVITY

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[2] e.g. Phys.Rev.D 79 (2009) 064036 arXiv:0811.2197 [hep-th]

Early universe cosmology and GR: basic questions

- What mechanism set the initial conditions for the universe?[3]
- FLRW cosmologies are *geodesically incomplete*: what preceded the 'big bang'?

Generalized G-inflation: Inflation with the most general second-order field equations

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Galilean Genesis: an alternative to inflation

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Fully stable cosmological solutions with a non-singular classical bounce

CrossMark

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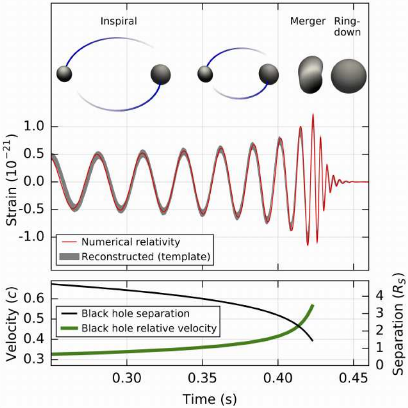
ARTICLE INFO ABSTRACT

[3] references to above papers: Prog.Theor.Phys. 126 (2011) 511-529, arXiv:1105.5723; JCAP 11 (2010) 021, arXiv:1107.0027; Phys.Lett.B 764 (2017) 289-294, arXiv:1609.01253

Test GR for the sake of basic science: gravitational waves



- Gravitational potential of earth $\sim 10^{-9}$
- Employ *matched filtering* to extract gravitational wave signals: need to accurately model the physics!

Test GR with gravitational waves: the need for accurate source modeling

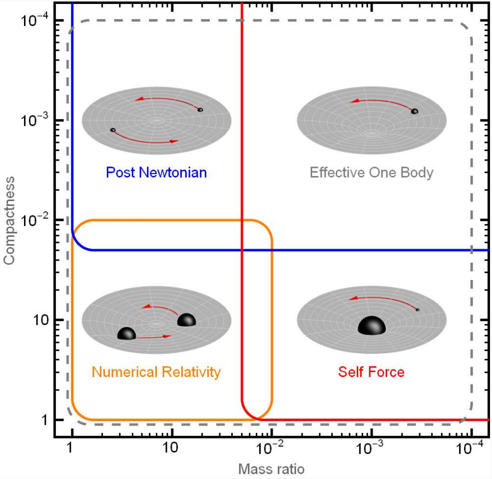


Figure: https://en.wikipedia.org/wiki/Two-body_problem_in_general_relativity

Guiding principles

Can we find a classical field theory that

1. Has a mathematically sensible interpretation?
2. Matches all current observations?
3. Addresses a current problem in physics?
   3.1 Renormalizable (or leading order interactions of a sensible quantum theory of gravity)?
   3.2 Incompleteness of early universe or black holes (and so admits NCC violating solutions)?
4. Can be tested/constrained with new observations?

Outline

Review: scalar-tensor gravity theories

Candidate theory: sEFT gravity

Shift symmetric

Conclusion

sEFT gravity

$$S = \frac{c^4}{16\pi G} \int d^4x \sqrt{-g} \left(R + X - V(\phi) + \alpha(\phi) X^2 + \beta(\phi) \mathcal{G} \right),$$

where

$$X \equiv -\frac{1}{2} g^{\mu\nu} \nabla_\mu \phi \nabla_\nu \phi,$$

$\mathcal{G}$: the *Gauss-Bonnet scalar*

$$\mathcal{G} \equiv R^2 - 4 R_{\mu\nu} R^{\mu\nu} + R_{\mu\alpha\nu\beta} R^{\mu\alpha\nu\beta}.$$

Why sEFT gravity?

1. Has a mathematically sensible interpretation?
   - Yes, provided the modified gravity corrections are "small"[4]
2. Matches all current observations?
   - Yes, provided we do not use this theory to model the late universe ESGB gravity not highly constrained by, e.g. binary pulsar tests[5]

[4] e.g. JLR & Pretorius, Class.Quant.Grav. 36 (2019) 13, 134001, Kovacs et. al. Phys.Rev.D 101 (2020) 12, 1240030

[5] e.g. Baker et. al. Phys.Rev.Lett. 119 (2017) 25, 251301, Yagi et. al. Phys.Rev. D93 (2016) no.2, 024010

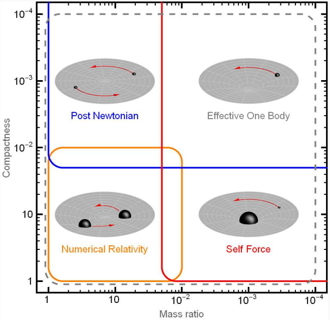
Why sEFT gravity?

1. Addresses a current problem in physics?
   - Theory captures leading order scalar-tensor parity invariant interactions, so captures the leading order corrections from many UV complete theories of gravity[6]
2. Can be tested/constrained with new observations?
   - Many versions of the theory have 'scalarized' black hole solutions, so will be strongly constrained by gravitational wave observations[7]

[6] e.g. Weinberg, Phys.Rev.D 77 (2008) 123541
[7] e.g. Kanti et. al. Phys.Rev.D 54 (1996) 5049-5058

Approaches to studying modified gravity theories[9]

- Order reduction approach to solve the equations of motion of a modified gravity theory [8]
- **Study exact (nonperturbative) solutions to particular modified gravity theories: useful for understanding physics in strong field, dynamical regime**



[8] e.g. Okounkova etl al., Class.Quant.Grav. 36 (2019) 5, 054001; Okounkova et. al., Phys.Rev.D 99 (2019) 4, 044019

[9] e.g. Cayuso, Ortiz, Lehner, Phys.Rev. D96 (2017) no.8, 084043; Allwright, Lehner, Class.Quant.Grav. 36 (2019) no.8, 084001

Outline

Shift symmetric ESGB gravity

- "Shift symmetric Einstein scalar Gauss-Bonnet" (ESGB) gravity

$$S_{ESGB} = \frac{1}{2}\int d^4x\sqrt{-g}\left(R - g^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi + 2\lambda\phi\mathcal{G}\right),$$

Shift symmetric ESGB gravity

- "Shift symmetric Einstein scalar Gauss-Bonnet" (ESGB) gravity

$$S_{ESGB} = \frac{1}{2}\int d^4x\sqrt{-g}\left(R - g^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi + 2\lambda\phi\mathcal{G}\right),$$

- This theory does not admit **stationary** Schwarzschild black hole solutions[10]; instead "hairy" scalar black holes should be end states in this theory

$$\Box\phi + \lambda\mathcal{G} = 0$$

,

[10] Sotiriou and Zhou, Phys.Rev. D90 (2014) 124063

Shift symmetric ESGB in a modified harmonic formulation[11]

- Collaboration with Will East
- Reformulate the equations of motion in *modified generalized harmonic* formulation
- Consider spinning black hole evolution (axisymmetric spacetime)
- Consider head on black hole collisions (axisymmetric spacetime)
- Consider binary black hole merger (no symmetry assumptions)

[11] arXiv:2011.03547

Modified generalized harmonic (MGH) formulation[12]

- Specify two auxiliary Lorentzian metrics $\hat{g}^{\mu\nu}$ and $\tilde{g}^{\mu\nu}$ in addition to the spacetime metric $g^{\mu\nu}$
- Specify the gauge/coordinate condition with:

$$\tilde{g}^{\mu\nu}\nabla_\mu\nabla_\nu x^\gamma = H^\gamma, \tag{1}$$

  where H^γ is source function
- Free parameters: $\hat{g}^{\mu\nu}$, $\tilde{g}^{\mu\nu}$, H^γ (more details given at end of talk)
- Besides using the MGH formulation, we begin with GR initial data, and use standard techniques from numerical relativity

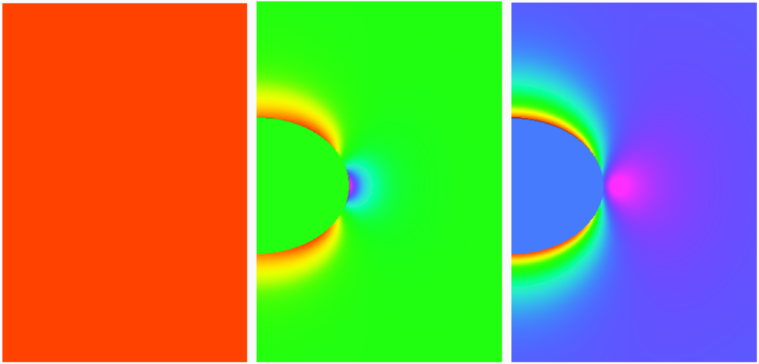
[12] Kovacs and Reall, Phys.Rev.D 101 (2020) 12, 124003, arXiv:2003.08398

Initial conditions

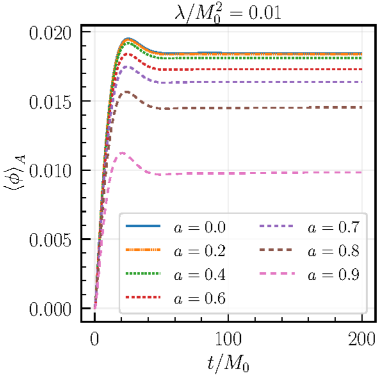
- For technical reasons, we always start with a GR solution (e.g. one spinning black hole, two boosted black holes), and then let the black holes grow scalar hair as we evolve in time
- After a finite amount of evolution, the black holes stop growing scalar hair (growth saturates)

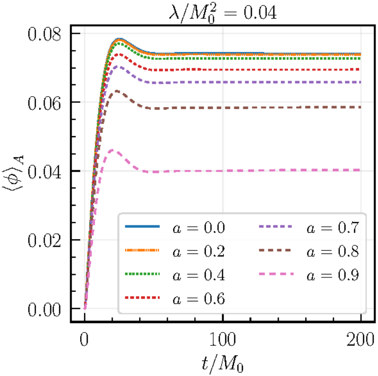
$$S_{ESGB} = \frac{1}{2}\int d^4x\sqrt{-g}\left(R - g^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi - 2\lambda\phi\mathcal{G}\right),$$

Scalar hair growth around spinning black holes



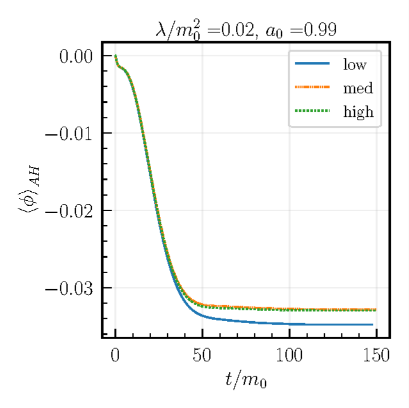
Scalar hair growth around spinning black holes





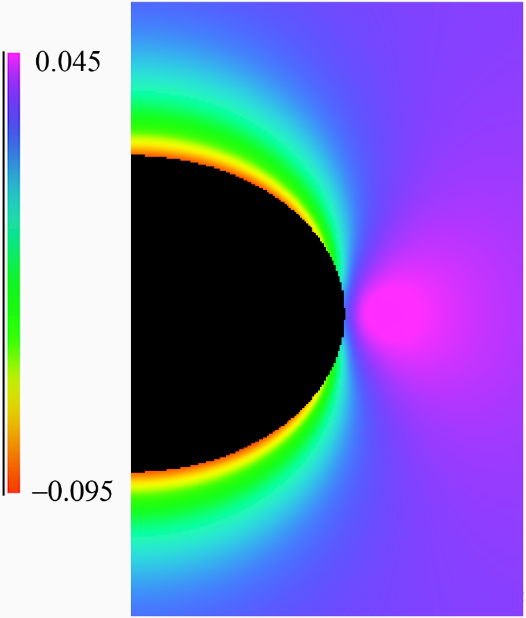
- $\langle\phi\rangle_A$: average scalar field value on black hole horizon
- a: initial dimensionless black hole spin

Scalar hair growth around spinning black holes

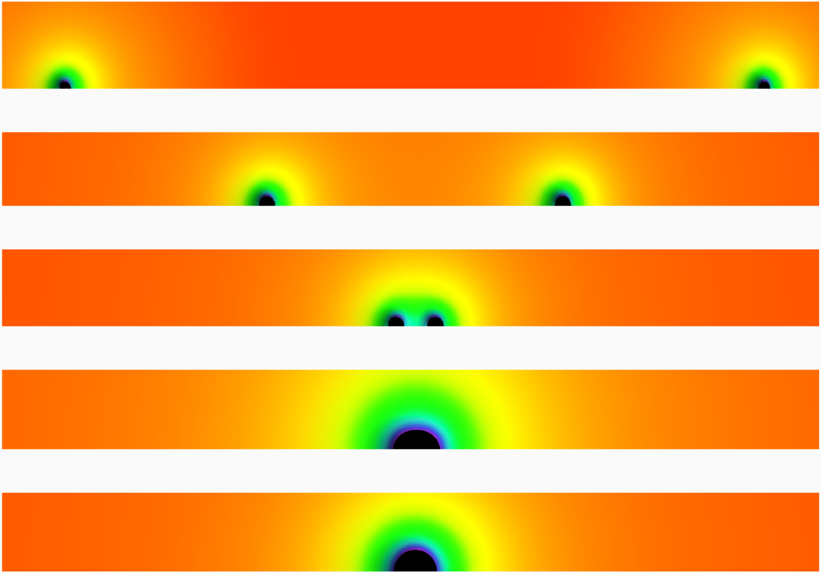


- $\langle\phi\rangle_A$: average scalar field value on black hole horizon, at three different resolutions (convergence study)

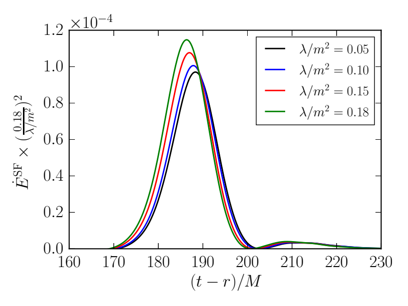
Scalar field density around a spinning black hole

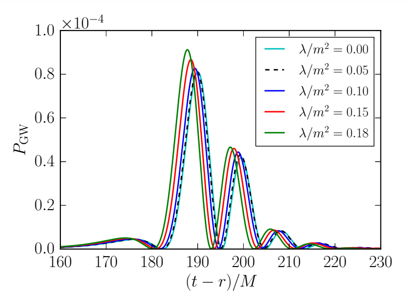


Head on black hole collisions



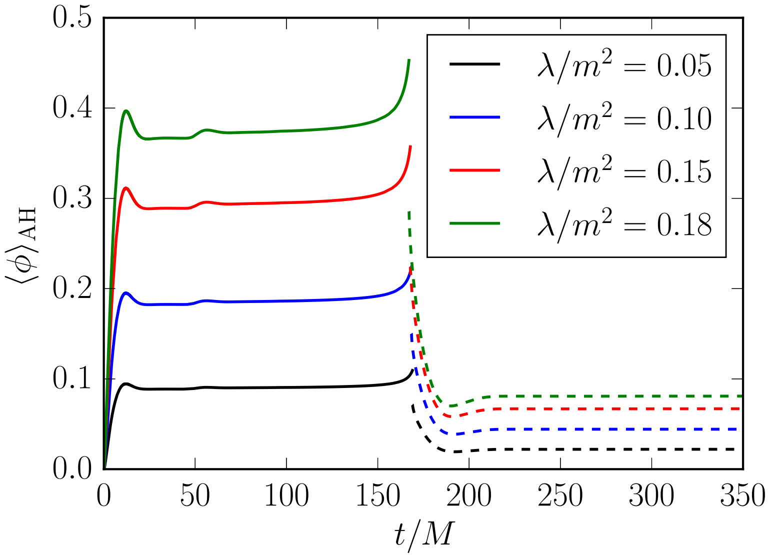
Head on black hole collisions: gravitational and scalar radiation



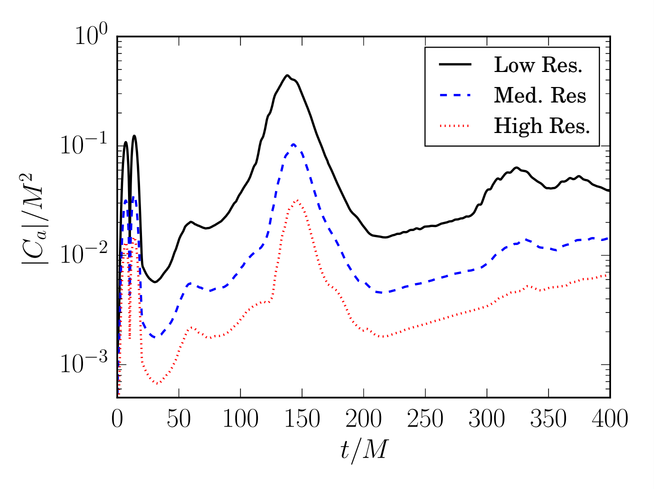


Flux of scalar field vs flux of gravitational waves

Head on black hole collisions: scalar field on horizon



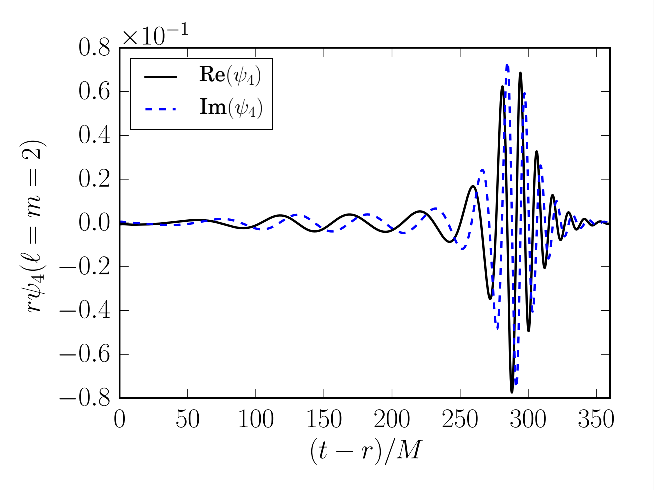
Head on black hole collisions: convergence



Convergence of “constraint violation”:

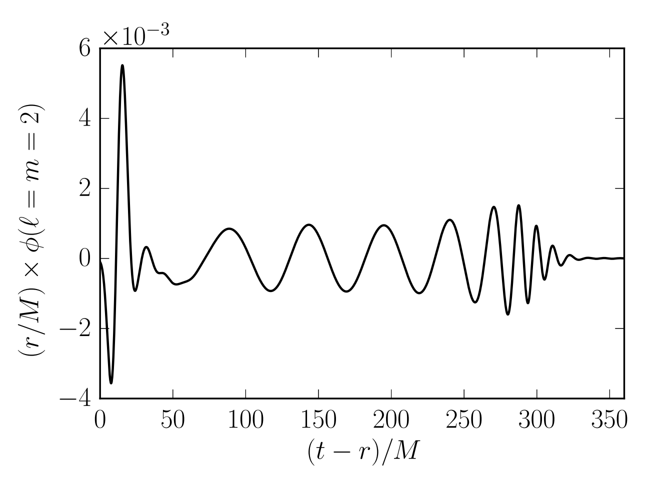
$$C^\alpha \equiv H^\alpha + \tilde{g}^{\mu\nu}\Gamma^\alpha_{\mu\nu} \tag{2}$$

Binary black hole collisions



Gravitational wave strain from two ESGB binary black holes merging

Binary black hole collisions



Radiated scalar waves

What was the main challenge? Finding a well-posed initial value formulation for the theory

- sEFT gravity has a well-posed initial value problem in generic spacetimes, provided the modified gravity corrections are "small", when one specifies their coordinate according to a *modified generalized harmonic* (MGH) condition[13]:

$$H^{\gamma} + \Gamma^{\gamma}_{\alpha\beta}\tilde{g}^{\alpha\beta} = 0. \tag{3}$$

- H^{γ}: free function one can choose
- $\tilde{g}^{\alpha\beta}$: "auxiliary" metric one can choose (*not* the "physical" metric $g^{\alpha\beta}$)
- In contrast to "generalized harmonic" formulation[14]: $H^{\gamma} + \Gamma^{\gamma}_{\alpha\beta}g^{\alpha\beta} = 0$

[13] Kovacs and Reall, Phys. Rev. D 101, 124003 (2020), Phys. Rev. Lett. 124, 221101 (2020)

[14] e.g. Pretorius, Class.Quant.Grav. 22 (2005) 425-452

Outline

Conclusion

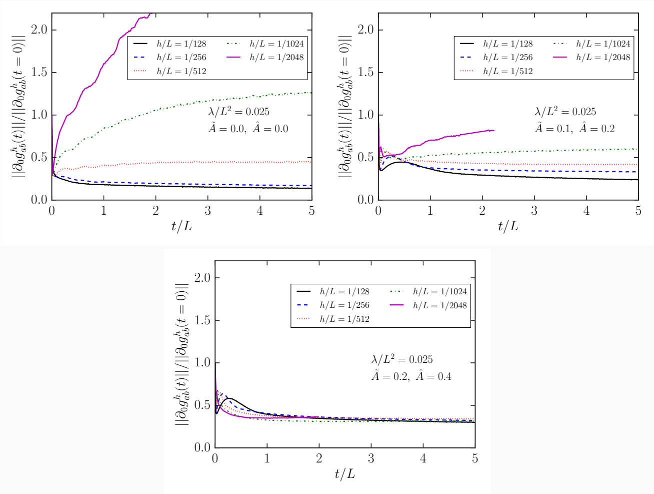
- GR is an extremely successful theory of gravity, but there are still reasons to study modified gravity theories
  - early universe: inflation, genesis, bouncing, ...
  - late universe: dark energy, ...
- Can test GR with gravitational waves
  - for that you need gravitational waveform templates to compare to data
- **Claim:** We now have the tools to produce gravitational waveforms produced during the merger of two black holes for a whole class of scalar-tensor gravity theories

Future directions

- Further develop the MGH formulation of general relativity and scalar-tensor gravity theories
  - What are "good" choices for the auxiliary metrics?
- Binary black hole waveform catalogues for other kinds of scalar-tensor gravity theories
- Consider early universe cosmological simulations in these theories

Backup slides

Hyperbolicity test: Self-convergence in harmonic vs modified harmonic gauge



Additional work on well-posedness of modified gravity theories

- Bernard, Lehner, and Luna[15] consider spherically symmetric dynamics of

$$\mathcal{L} = (1 + G_4(\phi))\, R + (\nabla\phi)^2 - V(\phi) + G_2\left(\phi, (\nabla\phi)^2\right).$$

see also Papallo and Reall, who study Horndeski theories in less symmetric spacetimes[16].

[15] Phys.Rev. D100 (2019) no.2, 024011

[16] Papallo, Reall, Phys.Rev. D96 (2017) no.4, 044019

Shift symmetric ESGB: equations of motion

$$S_{ESGB} = \frac{1}{2}\int d^4x\sqrt{-g}\left(R - g^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi - 2\lambda\phi\mathcal{G}\right),$$

$$\begin{aligned} R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R + 2\lambda\delta^{\gamma\delta\kappa\lambda}_{\alpha\beta\rho\sigma}R^{\rho\sigma}{}_{\kappa\lambda}\left(\nabla^\alpha\nabla_\gamma\phi\right)\delta^\beta{}_{(\mu}g_{\nu)\delta} & \\ -\nabla_\mu\phi\nabla_\nu\phi + \frac{1}{2}g_{\mu\nu}\left(\nabla\phi\right)^2 &= 0, \\ \Box\phi + \lambda\mathcal{G} &= 0. \end{aligned}$$

Order reduction approach for ESGB gravity[17]

Assume $\epsilon \sim \lambda$ and $|\epsilon| \ll 1$

$$\begin{aligned} g_{\mu\nu} =& g^{(0)}_{\mu\nu} + \epsilon g^{(1)}_{\mu\nu} + \epsilon^2 g^{(2)}_{\mu\nu} + \cdots \\ \phi =& \phi^{(0)} + \epsilon \phi^{(1)} + \epsilon^2 \phi^{(2)} + \cdots \end{aligned} \tag{4a}$$

$$\phi^{(0)} = 0, \tag{5a}$$

$$R_{\mu\nu}[g^{(0)}_{\alpha\beta}] - \frac{1}{2} g_{\mu\nu} R[g^{(0)}_{\alpha\beta}] = 0 \tag{5b}$$

$$\Box \phi^{(1)} = \lambda \mathcal{G}\left[g^{(0)}_{\alpha\beta}\right], \tag{6a}$$

$$R_{\mu\nu}[g^{(0)}_{\alpha\beta}] - \frac{1}{2} g_{\mu\nu} R[g^{(0)}_{\alpha\beta}] = 0 \tag{6b}$$

$$R_{\mu\nu}[g^{(2)}_{\alpha\beta}] - \frac{1}{2} g_{\mu\nu} R[g^{(2)}_{\alpha\beta}] = \lambda \times F\left[\phi^{(1)}\right] \tag{7}$$

[17] Okounkova, *Phys. Rev. D* 100 (2019)