Waves in Loudspeaker Cones

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Introduction

The development of numerical modelling methods, such as the finite element method or the dynamic stiffness matrix method, allows the efficient and flexible calculation of the dynamic behaviour of loudspeaker cones. Physical insight into the dynamic behaviour can, however, best be gained from a consideration of the types of wave that can propagate along the cone. The wave finite element method is a numerical method of investigating wave motion in waveguides and expressing the dynamics of the waveguide in terms of wavenumbers and wave modes[1]. The wave finite element analysis is used here to predict the wavenumbers and wave modes as a function of position, for a cone typical of those used in loudspeakers, and the overall results from the finite element analysis is decomposed and interpreted in terms of wave components [2].

Loudspeaker Cone Dynamics

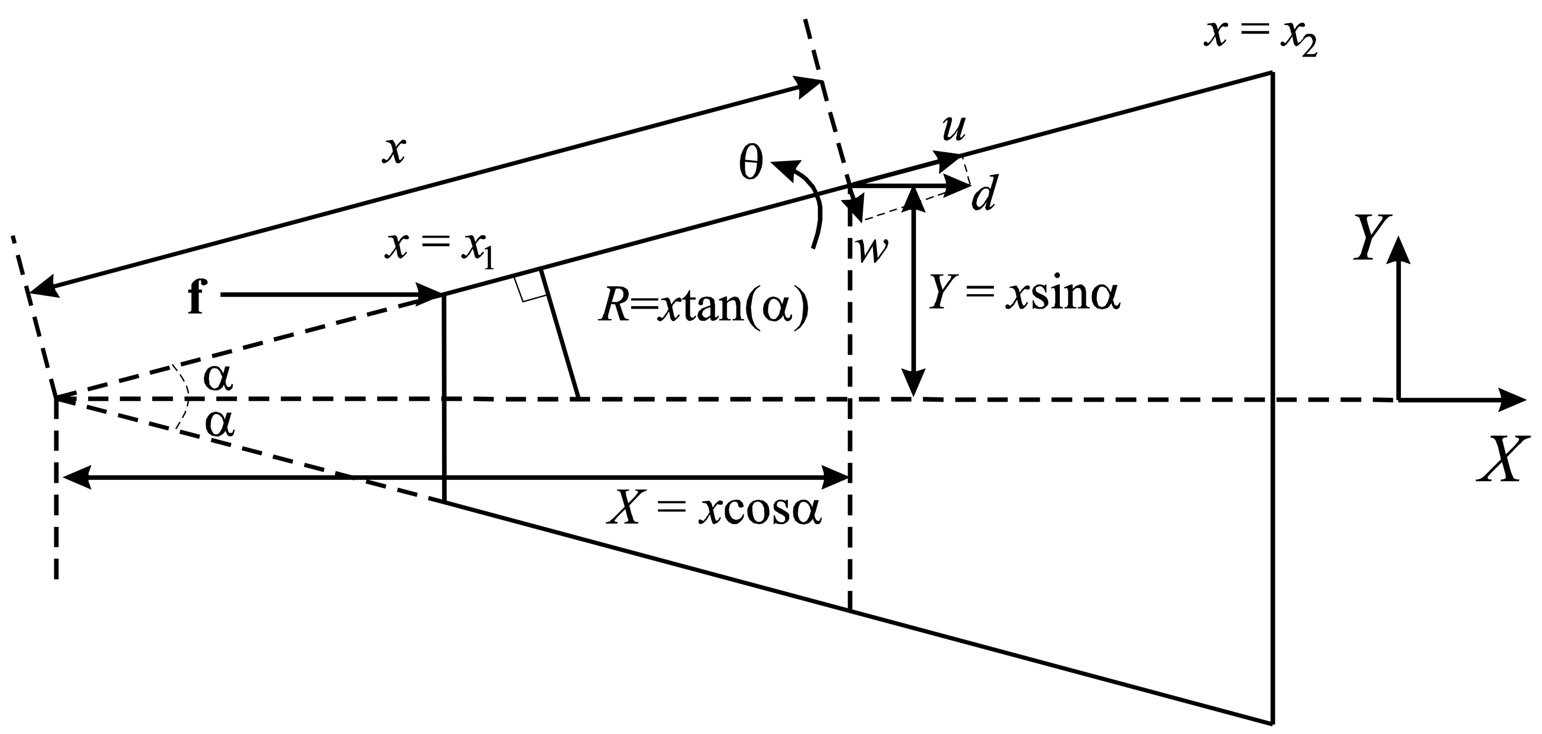


Fig.1. Side view of the conical cone in global coordinates.

The dynamic behaviour at a given position along the cone depends on whether the excitation is above or below the ring frequency at this location, which is given by [3] $f_R = c/2\pi R$, and can be seen in Fig. 2, where c is the speed of longitudinal waves and R is the distance between the cone and cone axis measured perpendicular to the cone meridian. Below the ring frequency the dynamics are dominated by the membrane stiffness, resulting in mostly in-plane motion. Above the ring frequency the dynamics are dominated by the bending stiffness, resulting in flexural motion.

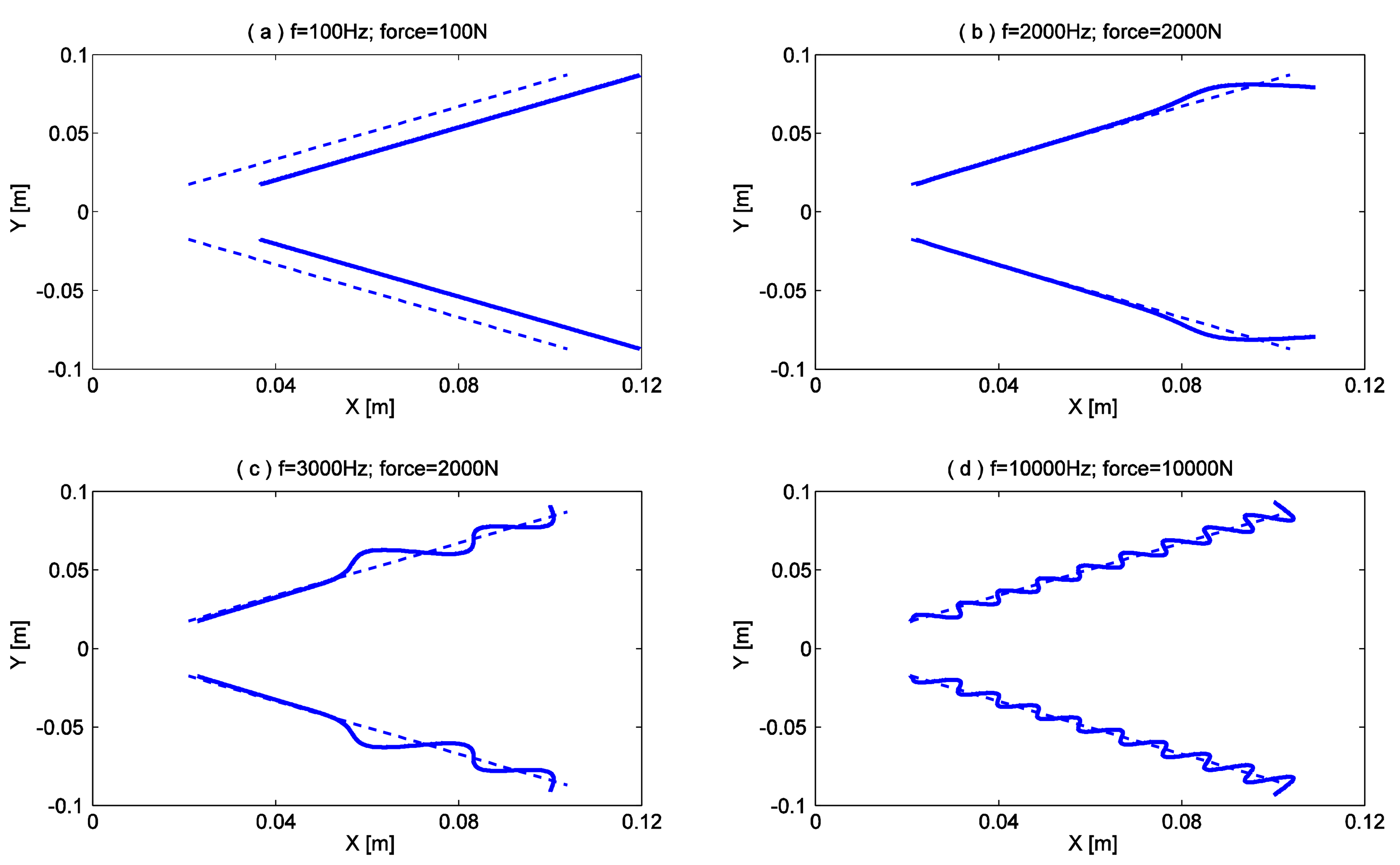


Fig.3. Forced responses of the cone at (a) 100 Hz, (b) 2000 Hz, (c) 3000 Hz and (d) 10000 Hz, the displacements are plotted in the global coordinates denoted by solid lines and the un-deformed middle surface of the cone is represented by dashed lines.

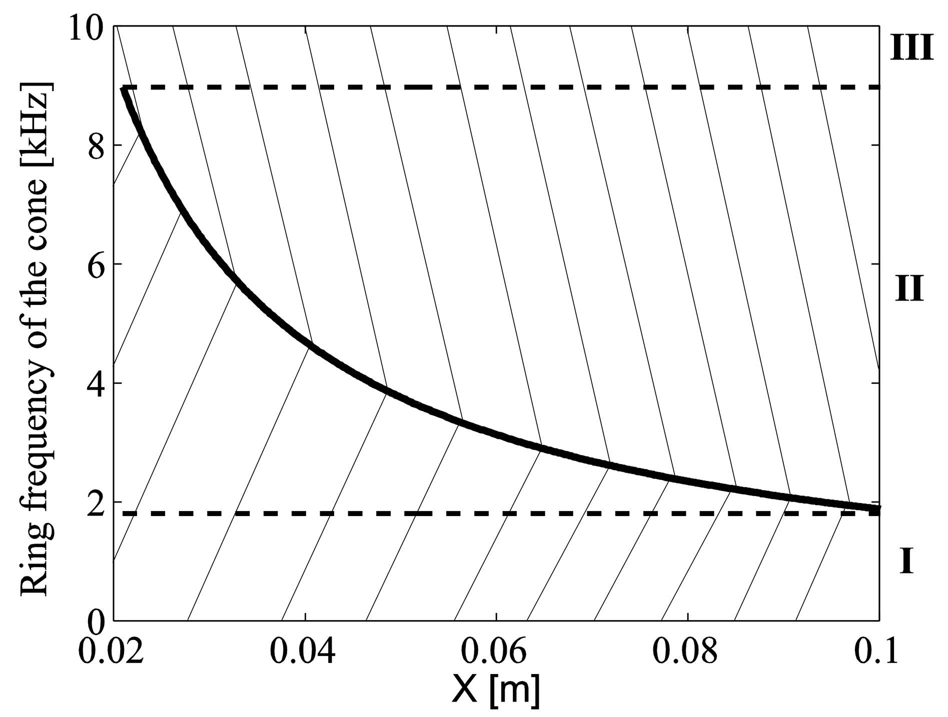


Fig.2. The ring frequency as a function of position along the cone axis indicating the transition between mostly in-plane (///) and mostly bending (\\\) behaviour.

In Fig.3, the cone moves almost as a rigid body at low frequency, 100 Hz for example, since the frequency is in region I. At higher frequencies, 2 kHz and 3 kHz when the excitation frequency is in region II, cone "break-up" occurs, in which the cone no longer vibrates as a rigid body, but some sections of the cone still move in phase. In this frequency range, a bending wave cannot propagate in the region close to the apical edge of the cone, due to the high stiffness, but can propagate on the outer part of the cone and then build up into a standing wave. The position, the "transition point", where bending wave starts to cut-on, moves towards the apical edge as driving frequency increases. At 10 kHz the excitation frequency is in region III and the whole cone moves with a bending motion.

Waves in the Loudspeaker Cone

$$\mathbf{Dq} = \mathbf{f} \qquad \begin{bmatrix} \mathbf{q}_R(n) \\ -\mathbf{f}_R(n) \end{bmatrix} = \mathbf{T}(n) \begin{bmatrix} \mathbf{q}_L(n) \\ \mathbf{f}_L(n) \end{bmatrix}$$

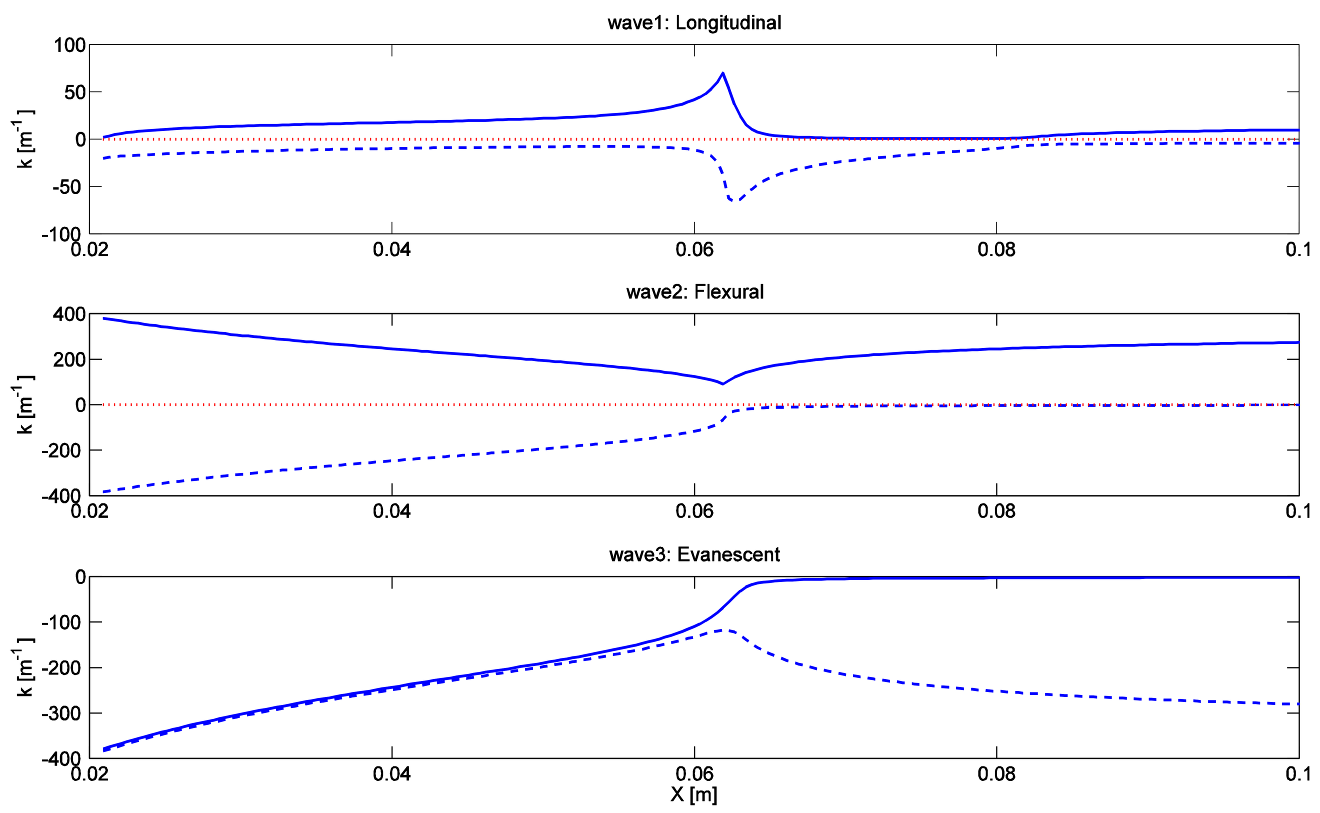
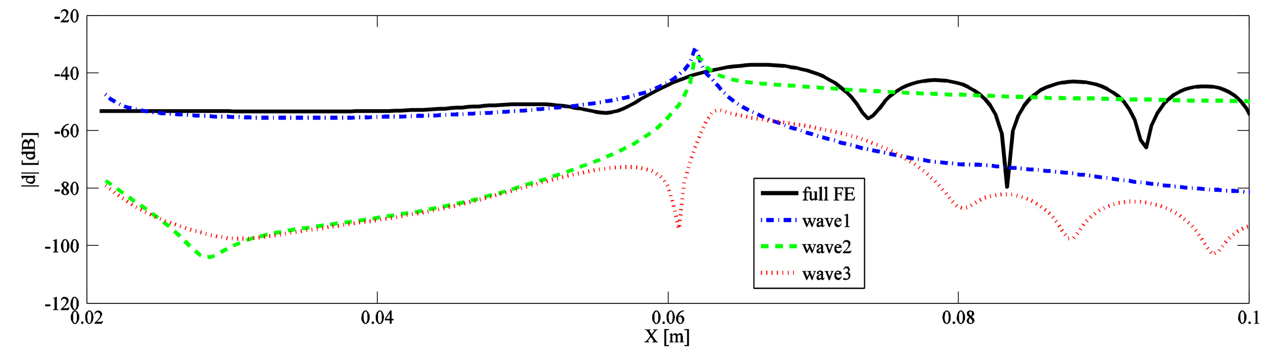


Fig.4. Wavenumber distribution along the cone axis at 3000 Hz using the WFE, with only wavenumbers corresponding to forward going waves plotted, and solid lines being the real component of the wavenumber k, and dashed lines being the imaginary component.

The wavenumber can be obtained directly from solving the eigenvalue problem for the transfer matrix $\mathbf{T}(n)$ [1]. Figure 4 shows the distribution of the wavenumber along the cone axis at 3000 Hz calculated using the wave finite element method. The cone can be split into 2 regions along the cone axis, region A includes apex to about 0.06 m corresponding to the transition point at 3000 Hz shown in Figure 2 and region B corresponds to from 0.06 m to the base of the cone. Wave 1 propagates with a gradually decreasing speed and decays less than the other two waves, since wave 1 has a small non-zero imaginary part of wavenumber. This shows that in the region A, the longitudinal motion dominates the vibration pattern of the cone. Beyond the position 0.06 m, wave 2 starts to propagate towards the base. The wave 1 becomes an evanescent wave beyond the position 0.06 m. Wave 3 has a large non-zero imaginary part of wavenumber along the whole range of the cone, which indicates that this wave does not play a significant role in cone vibration.



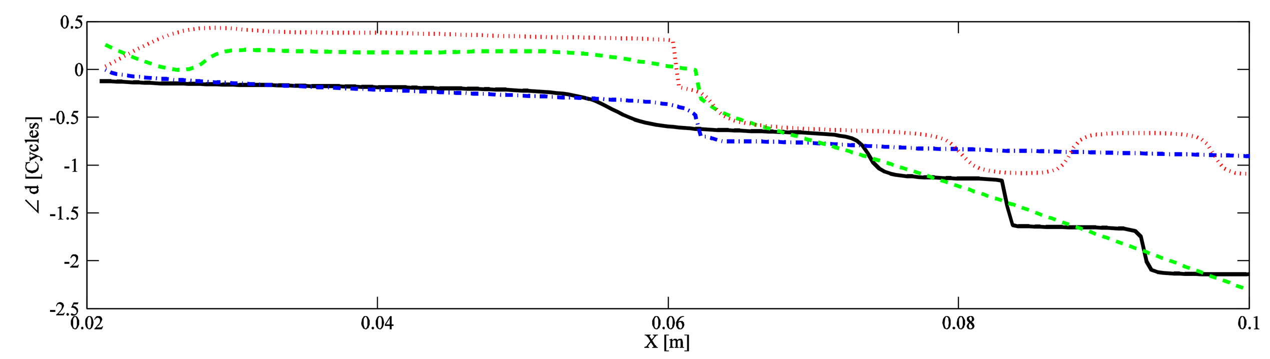


Fig.5. Decomposition of the overall axial displacement calculated from the full finite element model into components due to forward going waves in Figure 4, calculated from the wave finite element model.

In Fig. 5, the contribution of wave 1 is seen to be in reasonable agreement with the overall result from the full finite element method for positions apical to the peak response at this frequency, at about 0.06 m along the cone axis. The contribution of wave 1 is significantly less than the overall result of the full finite element for positions beyond the peak response, region B, however, where the contribution of wave 2 dominates the overall response. There is also a negative going component of wave 2 in this region, due to the reflection from the free basal end of the cone and the interference between this and the positive going wave 2 gives rise to the interference pattern seen in the full finite element results.
The contribution of wave 3 decays away on either side of this peak, and the amplitude is too small to significantly affect the overall response. It can be concluded that, inside region A the wavelength is long due to the predominance of longitudinal motion, and inside region B the wavelength is much shorter due to the predominance of bending motion. The energy is converted from longitudinal motion to bending motion at the transition point.

Conclusions

The wave finite element method provides a method of studying the waves that travel in the loudspeaker cone and, more importantly, decomposing the response of the full finite element model into the components due to each of these waves, in order to explore how they interact. In this way the insight provided by the wave approach can be brought to bear on the numerical results from more detailed finite element models.

References

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