Charge Algebra in Al(A)dS Spacetimes

The Λ-BMS Group and the Flat Limit

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References

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  2. The Λ-BMS$_4$ charge algebra
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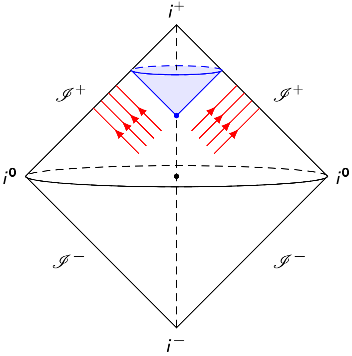
Plan

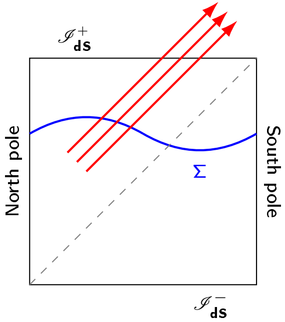
Asymptotically Flat Spacetimes

- Leaky boundary conditions = Boundary conditions that yield some flux through the conformal boundary
  $\implies$ The charges are not conserved
  $\implies$ The variational principle is not stationary on solutions
  $\implies$ This describes open gravitational systems
- Leaky boundary conditions are essential in asymptotically flat spacetimes at null infinity to consider radiative spacetimes.
  [Bondi-van der Burg-Metzner '62] [Sachs '62]
- Non-conservation of the charges : "Bondi mass loss formula".



Asymptotically de Sitter Spacetimes

- In asymptotically de Sitter (dS) spacetimes, essential to consider leaky boundary conditions
  $\Longrightarrow$ Otherwise, that would highly constrain the Cauchy problem
  [Anninos-Ng-Strominger '12] [Ashtekar-Bonga-Kesavan '15]



Asymptotically Anti-de Sitter Spacetimes

- In asymptotically anti-de Sitter (AdS) spacetimes : previous analyses considered "conservative" or "reflective" boundary conditions

  $\Longrightarrow$ Conserved charges, well-defined variational principle, closed system

  (see *e.g.* [Hawking '83] [Ashtekar-Magnon '84] [Henneaux-Teitelboim '85] [Papadimitriou-Skenderis '05])

- However, considering leaky boundary conditions in AdS is appealing :

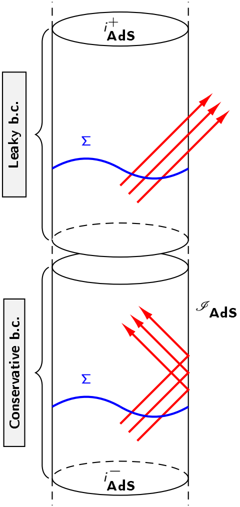
  $\Longrightarrow$ Quest for the "most general" boundary conditions

  (see *e.g.* [Grumiller-Riegler '16] [Grumiller-Sheikh-Jabbari-Zwikel '20] [Freidel-Geiller-Pranzetti '20])

  $\Longrightarrow$ BMS symmetries in AdS requires flux at infinity [Compère-Fiorucci-Ruzziconi '19]

  $\Longrightarrow$ Black hole evaporation requires external system [Almheiri-Mahajan-Maldacena '19]

  $\Longrightarrow$ Brane-world interacting with higher-dimensional spacetimes [Randall-Sundrum '99]



Asymptotically Locally (A)dS$_{d+1}$ Spacetimes

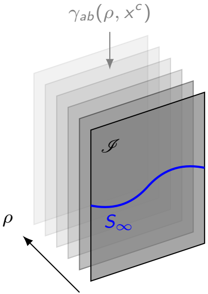
- Study of leaky boundary conditions in (A)dS$_{d+1}$ spacetimes
- Start from the most general Al(A)dS$_{d+1}$ spacetime ($d > 1$)
- Starobinsky/Fefferman-Graham gauge in $d+1$ dimensions

[Starobinsky '83] [Fefferman-Graham '85]

$$ds^2 = \eta \frac{\ell^2}{\rho^2} d\rho^2 + \gamma_{ab}(\rho, x^c) dx^a dx^b$$

with $\gamma_{ab} = \mathcal{O}(\rho^{-2})$ (conformal compactification)

- Coordinates : $x^\mu = (\rho, x^a)$, $a = 1, \ldots, d$
- Boundary at $\rho = 0$ and $\rho > 0$ into the bulk
- Valid for both $\Lambda > 0$ (dS), $\Lambda < 0$ (AdS) ($\Lambda = -\eta \frac{d(d-1)}{2\ell^2}$, $\eta = -\text{sgn}(\Lambda)$)



Solution Space

- Solutions of $G_{\mu\nu} + \Lambda g_{\mu\nu} = 0$:

$$\gamma_{ab} = \rho^{-2} g^{(0)}_{ab} + g^{(2)}_{ab} + \cdots + \rho^{d-2} g^{(d)}_{ab} + \rho^{d-2} \ln \rho^2 \tilde{g}^{[d]}_{ab} + \mathcal{O}(\rho^{d-1})$$

where the logarithmic term appears only for even d

- This expansion is completely determined by specifying $g^{(0)}_{ab}$ and $g^{(d)}_{ab}$
- Holographic stress energy tensor

[Balasubramanian-Kraus '99][de Haro-Skenderis-Solodukhin '00] :

$$T^{[d]}_{ab} = \frac{d}{16\pi G} \frac{\eta}{\ell} \left(g^{(d)}_{ab} + X^{[d]}_{ab}[g^{(0)}] \right)$$

- Einstein equations also imply

$$D^a T^{[d]}_{ab} = 0, \qquad g^{ab}_{(0)} T^{[2k+1]}_{ab} = 0$$

but $g^{ab}_{(0)} T^{[2k]}_{ab} \neq 0 \Rightarrow$ Weyl anomalies in the dual theory

[Henningson-Skenderis '98]

Residual Gauge Diffeomorphisms

- Diffeomorphisms preserving the Starobinsky/Fefferman-Graham gauge are generated by vector fields $\xi = \xi^\rho \partial_\rho + \xi^a \partial_a$ satisfying

$$\mathcal{L}_\xi g_{\rho\rho} = 0, \qquad \mathcal{L}_\xi g_{\rho a} = 0$$

- Solution :

$$\xi^\rho = \sigma(x^a)\rho, \qquad \xi^a = \bar{\xi}^a(x^b) - \eta\ell^2 \partial_b \sigma \int_0^\rho \frac{d\rho'}{\rho'} \gamma^{ab}(\rho', x^c)$$

where $\sigma(x^a)$ and $\bar{\xi}^a(x^b)$ are arbitrary functions

- Using modified Lie bracket that takes into account the field-dependence of the vector fields [Barnich-Troessaert '10]

$$[\xi_1, \xi_2]_\star = [\xi_1, \xi_2] - \delta_{\xi_1}\xi_2 + \delta_{\xi_2}\xi_1$$

we obtain

$$[\xi(\sigma_1, \bar{\xi}^a_1), \xi(\sigma_2, \bar{\xi}^a_2)]_\star = \xi(\hat{\sigma}, \hat{\bar{\xi}}^a),$$

$$\text{with} \begin{cases} \hat{\sigma} = \bar{\xi}^a_1 \partial_a \sigma_2 - \delta_{\xi_1}\sigma_2 - (1 \leftrightarrow 2), \\ \hat{\bar{\xi}}^a = \bar{\xi}^b_1 \partial_b \bar{\xi}^a_2 - \delta_{\xi_1}\bar{\xi}^a_2 - (1 \leftrightarrow 2). \end{cases}$$

$\Longrightarrow$ Field-dependent structure constants for generic cases

$\Longrightarrow$ For $\delta\sigma = 0 = \delta\bar{\xi}^a$, we have Diff($\mathscr{I}$)⋉Weyl

Variation of the Solution Space

- The solution space is parametrized by $(g^{(0)}_{ab}, T^{[d]}_{ab})$.
- Variation of the solution space under infinitesimal gauge diffeomorphisms :

$$\delta_\xi g^{(0)}_{ab} = \mathcal{L}_{\bar{\xi}} g^{(0)}_{ab} - 2\sigma g^{(0)}_{ab}$$
$$\delta_\xi T^{[d]}_{ab} = \mathcal{L}_{\bar{\xi}} T^{[d]}_{ab} + (d-2)\sigma T^{[d]}_{ab} + A^{[d]}_{ab}[\sigma]$$

  where $A^{[d]}_{ab}[\sigma]$ is the inhomogeneous part of the transformation related to Weyl anomalies, $A^{[2k+1]}_{ab}[\sigma] = 0$ but $A^{[2k]}_{ab}[\sigma] \neq 0$

- These variations satisfy

$$[\delta_{\xi_1}, \delta_{\xi_2}](g^{(0)}_{ab}, T^{[d]}_{ab}) = -\delta_{[\xi_1,\xi_2]_\star}(g^{(0)}_{ab}, T^{[d]}_{ab})$$

  where $[\delta_{\xi_1}, \delta_{\xi_2}] = \delta_{\xi_1}\delta_{\xi_2} - \delta_{\xi_2}\delta_{\xi_1}$

- Lie algebroid structure (Base space = solution space $(g^{(0)}_{ab}, T^{[d]}_{ab})$, algebra at each point = $\{\xi(\sigma, \bar{\xi}^a)\}$ with $[.,.]_\star$)

Phase Space

- Holographic renormalization in (A)dS [de Haro-Solodukhin-Skenderis '01] :

$$S_{ren} = \int_{\mathscr{M}} \boldsymbol{L}_{EH} + \int_{\mathscr{I}} \boldsymbol{L}_{GHY} + \int_{\mathscr{I}} \boldsymbol{L}_{ct} + \int_{\mathscr{I}} \boldsymbol{L}_{\circ}$$

$\Longrightarrow$ This action is finite on-shell, $S_{ren} = \mathcal{O}(\rho^0)$

$\Longrightarrow$ The term $\boldsymbol{L}_{\circ}$ is the freedom to add a finite term to the action

- This process removes the divergences from the sympectic structure [Papadimitriou-Skenderis '05] [Compère-Marolf '08] :

$$\begin{aligned} \boldsymbol{\Theta}_{ren}[g;\delta g]\Big|_{\mathscr{I}} &= \boldsymbol{\Theta}_{EH} - \delta \boldsymbol{L}_{GHY} - \delta \boldsymbol{L}_{ct} - \delta \boldsymbol{L}_{\circ} + d\boldsymbol{\Theta}_{ct} + d\boldsymbol{\Theta}_{\circ}\Big|_{\mathscr{I}} \\ &= -\frac{1}{2}\sqrt{|g^{(0)}|}\, T^{ab}_{[d]}\, \delta g^{(0)}_{ab}\, (d^d x) \end{aligned}$$

where $\boldsymbol{\Theta}_i$ is the presymplectic potential defined through

$$\delta \boldsymbol{L}_i = \frac{\delta \boldsymbol{L}_i}{\delta g}\delta g + d\boldsymbol{\Theta}_i[g;\delta g]$$

- Variational principle : $\delta S_{ren} = -\int_{\mathscr{I}} \boldsymbol{\Theta}_{ren}[g;\delta g]\Big|_{\mathscr{I}}$

$\Longrightarrow$ Well-defined for Dirichlet boundary conditions ($\delta g^{(0)}_{ab} = 0$)

Conservative *vs* Leaky Boundary Conditions

- The presymplectic current is obtained through $\boldsymbol{\omega}_{ren}[g;\delta g,\delta g] = \delta \boldsymbol{\Theta}_{ren}[g;\delta g]$. Explicitly,

$$\boldsymbol{\omega}_{ren}[g;\delta g,\delta g]\Big|_{\mathscr{I}} = -\frac{1}{2}\delta\left(\sqrt{|g^{(0)}|}\,T^{ab}_{[d]}\right)\wedge \delta g^{(0)}_{ab}\,(d^d x)$$

- Encodes the "flux of charges" going through the spacetime boundary
- Conservative boundary conditions would require $\boldsymbol{\omega}_{ren}|_{\mathscr{I}} = 0$

  $\Longrightarrow$ Conserved charges

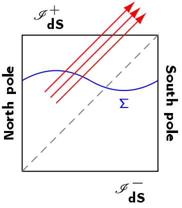
  $\Longrightarrow$ Action principle with S_{ren} can be made well-defined
- Here, we consider leaky boundary conditions : we allow $\boldsymbol{\omega}_{ren}|_{\mathscr{I}} \neq 0$

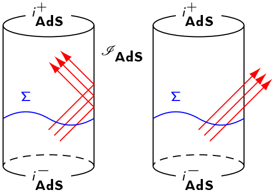
  $\Longrightarrow$ Non-conserved charges

  $\Longrightarrow$ S_{ren} is not stationary on solutions

  $\Longrightarrow$ Open system with external sources encoded in $\delta g^{(0)}_{ab}$

  $\Longrightarrow$ Natural in dS, non-standard in AdS (non-globally hyperbolic spacetime) [Ishibashi-Wald '04]





Infinitesimal Charges in Al(A)dS$_{d+1}$ Spacetimes

- The infinitesimal charges are obtained from the renormalized symplectic structure [Iyer-Wald '94] [Barnich-Brandt '02]

$$\delta\!\!\!/ H_\xi[g;\delta g] = \int_\Sigma \boldsymbol{\omega}_{ren}[g;\delta_\xi g,\delta g] = \int_{S_\infty} \boldsymbol{k}_{\xi,ren}[g;\delta g]$$

  where $S_\infty = \partial\Sigma$ and $d\boldsymbol{k}_{\xi,ren}[g;\delta g] = \boldsymbol{\omega}_{ren}[g;\delta_\xi g,\delta g]$

- The explicit expression is given by

$$\delta\!\!\!/ H_\xi[g;\delta g] = \int_{S_\infty} (d^{d-1}x)\Big[\underbrace{\delta\left(\sqrt{|g^{(0)}|}g^{tc}_{(0)}T^{[d]}_{bc}\right)\bar{\xi}^b - \frac{1}{2}\sqrt{|g^{(0)}|}\bar{\xi}^t T^{bc}_{[d]}\delta g^{(0)}_{bc}}_{\text{Boundary diffeomorphism charge}} + \underbrace{W^{[d]t}_{\sigma}[g;\delta g]}_{\text{Weyl charge}}\Big]$$

- Observations :
  1. The charges are not conserved, $d\boldsymbol{k}_{\xi,ren}[g;\delta g]|_{\mathscr{I}} = \boldsymbol{\omega}_{ren}[g;\delta_\xi g,\delta g]|_{\mathscr{I}} \neq 0$
  2. The charges are non-integrable, $\delta\!\!\!/ H_\xi[g] \neq \delta(\dots)$

  $\Longrightarrow$ Typical features of an open dissipative system

Weyl Charges

- Weyl charges : $W_\sigma^{[2k+1]t}[g;\delta g] = 0$, but $W_\sigma^{[2k]t}[g;\delta g] \neq 0$
- Explicit expressions :

$$W_\sigma^{[d=2]t}[g;\delta g] = -\frac{\ell}{16\pi G} D_b \sigma \left[\sqrt{|g^{(0)}|}\delta g^{tb}_{(0)} + 2\delta\sqrt{|g^{(0)}|} g^{tb}_{(0)} \right] - \ell\sigma\Theta^t_{EH}[g^{(0)};\delta g^{(0)}],$$

$$\begin{aligned} W_\sigma^{[d=4]t}[g;\delta g] = & \frac{\eta\,\ell^3}{16\pi G} \left[\frac{1}{6}\sqrt{|g^{(0)}|} R^{(0)} D_b\sigma\delta g^{tb}_{(0)} + \frac{1}{3} R^{(0)} D^t\sigma\delta\sqrt{|g^{(0)}|} \right. \\ & \left. -\frac{1}{2} R^{tc}_{(0)} D_c\sigma\delta\sqrt{|g^{(0)}|} + \frac{1}{4}\sqrt{|g^{(0)}|} R^{(0)}_{cb} D^t\sigma\delta g^{bc}_{(0)} - \frac{1}{2}\sqrt{|g^{(0)}|} R^{(0)t}_c D_b\sigma\delta g^{bc}_{(0)} \right] \\ & -\eta\frac{\ell^3}{4}\sigma\left[\Theta^t_{QCG(1)}[g^{(0)};\delta g^{(0)}] - \frac{1}{3}\Theta^t_{QCG(2)}[g^{(0)};\delta g^{(0)}] \right] \end{aligned}$$

  where Θ^t_{EH}, $\Theta^t_{QCG(1)}$ and $\Theta^t_{QCG(2)}$ are the presymplectic potentials of EH and quadratic curvature gravity
- Non-zero Weyl charges due to the presence of Weyl anomalies in the dual theory (not free to choose the conformal compactification factor)
- Weyl charges only visible if $\delta g^{(0)}_{ab} \neq 0$
- For more physics related to Weyl charges in $d = 2$, see [Alessio-Barnich-Ciambelli-Ruzziconi '20]

  $\Longrightarrow$ Non-conservation interpreted as an anomalous Ward–Takahashi identity of the boundary theory

Charge Algebra in Al(A)dS$_{d+1}$ Spacetimes

- When charges are integrable, *i.e.* $\delta\!\!\!/ H_\xi[g] = \delta H_\xi[g]$, then we have the representation theorem [Barnich-Compère '07]

$$\{H_{\xi_1}, H_{\xi_2}\} \equiv \delta_{\xi_2} H_{\xi_1}[g] \quad \Longrightarrow \quad \{H_{\xi_1}, H_{\xi_2}\} = H_{[\xi_1,\xi_2]_\star}[g] + K_{\xi_1,\xi_2}$$

  where $K_{\xi_1,\xi_2} = -K_{\xi_2,\xi_1}$ is a central extension satisfying the 2-cocycle condition

$$K_{[\xi_1,\xi_2]_\star,\xi_3} + \text{cyclic}(1,2,3) = 0$$

- What does this representation theorem become for non-integrable charges ?
  $\Longrightarrow$ Use the modified Barnich-Troessart bracket [Barnich-Troessaert '11]
  $\Longrightarrow$ Works in many different contexts, including asymptotically flat spacetimes (see *e.g.* [Barnich-Troessaert '11][Compère-Fiorucci-Ruzziconi '18]), or at the BH horizon (see *e.g.* [Donnay-Giribet-González, Pino '16])
  $\Longrightarrow$ We used it in the present context of Al(A)dS$_{d+1}$ spacetimes

- Total charge in Al(A)dS$_{d+1}$: $\delta\!\!\!/ H_\xi[g;\delta g] = \delta H_\xi[g] + \Xi_\xi[g;\delta g]$ where

$$H_\xi[g] = \int_{S_\infty} (d^{d-1}x) \left[\sqrt{|g^{(0)}|} g^{tc}_{(0)} T^{[d]}_{bc} \bar{\xi}^b \right]$$

$$\Xi_\xi[g;\delta g] = \int_{S_\infty} (d^{d-1}x) \left[-\frac{1}{2} \sqrt{|g^{(0)}|} \bar{\xi}^t T^{bc}_{[d]} \delta g^{(0)}_{bc} + W^{[d]t}_\sigma [g;\delta g] \right] - H_{\delta\xi}[g]$$

(the split between integrable and non-integrable parts is ambiguous)

- With the Barnich-Troessart bracket,

$$\{H_{\xi_1}, H_{\xi_2}\}_\star \equiv \delta_{\xi_2} H_{\xi_1}[g] + \Xi_{\xi_2}[g;\delta_{\xi_1} g] \quad \Longrightarrow \quad \{H_{\xi_1}, H_{\xi_2}\}_\star = H_{[\xi_1,\xi_2]_\star}[g] + K^{[d]}_{\xi_1,\xi_2}[g]$$

where $K^{[d]}_{\xi_1,\xi_2}[g] = -K^{[d]}_{\xi_2,\xi_1}[g]$ is a field-dependent 2-cocycle satisfying the generalized condition :

$$K^{[d]}_{[\xi_1,\xi_2]_\star,\xi_3}[g] + \delta_{\xi_3} K^{[d]}_{\xi_1,\xi_2}[g] + \text{cyclic}(1,2,3) = 0$$

(the form of the charge algebra is unambiguous)

- Physically, the algebra contains the information on the flux-balance laws at $\mathscr{I}$ ($\xi_2 \equiv \partial_t$, $\xi_1 \equiv \xi$) :

$$\frac{d}{dt} H_\xi[\phi] = -\Xi_{\partial_t}[\delta_\xi \phi;\phi] + K^{[d]}_{\xi,\partial_t}[g]$$

- $K^{[2k+1]}_{\xi_1,\xi_2}[g] = 0$ ($k \in \mathbb{N}_0$). For even d, we have explicitly

$$K^{[d=2]}_{\xi_1,\xi_2}[g] = \frac{\ell}{16\pi G}\int_{S_\infty}(d^{d-1}x)\sqrt{|g^{(0)}|}\Big[2\left(\sigma_1 D^t\sigma_2 - \sigma_2 D^t\sigma_1\right) + R^{(0)}\left(\sigma_1\bar{\xi}^t_2 - \sigma_2\bar{\xi}^t_1\right)\Big],$$

$$K^{[d=4]}_{\xi_1,\xi_2}[g] = \frac{\eta\,\ell^3}{16\pi G}\int_{S_\infty}(d^{d-1}x)\sqrt{|g^{(0)}|}\Bigg[\left(R^{tb}_{(0)} - \frac{1}{2}R^{(0)}g^{tb}_{(0)}\right)\left(\sigma_1 D_b\sigma_2 - \sigma_2 D_b\sigma_1\right), + \frac{1}{4}\left(R^{bc}_{(0)}R^{(0)}_{bc} - \frac{1}{3}R^2_{(0)}\right)\left(\sigma_1\bar{\xi}^t_2 - \sigma_2\bar{\xi}^t_1\right)\Bigg].$$

- We checked explicitly the generalized 2-cocycle condition
- For $d = 2$, if we impose Dirichlet boundary conditions ($\delta g^{(0)}_{ab} = 0$), the field-dependent 2-cocycle reduces to the Brown-Henneaux central extension [Brown-Henneaux '86]
$i\{\mathtt{L}^\pm_m, \mathtt{L}^\pm_n\} = (m-n)\mathtt{L}^\pm_{m+n} - \frac{c^\pm}{12}m(m^2-1)\delta^0_{m+n}$, $\{\mathtt{L}^\pm_m, \mathtt{L}^\mp_n\} = 0$ where $c^\pm = \frac{3\ell}{2G}$

BMS Group in 4d Asymptotically Flat Spacetimes

- Consider radiative 4d asymptotically flat spacetimes at null infinity
- What you may naively expect as asymptotic symmetry group :

$$\text{Poincaré} = SO(3,1) \ltimes \text{Translations}$$

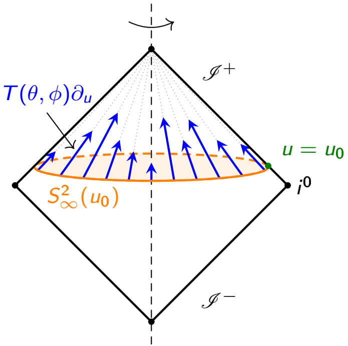
- What a careful analysis gives as asymptotic symmetry group

[Bondi-van der Burg-Metzner '62] [Sachs '62] :

$$\text{BMS} = SO(3,1) \ltimes \text{Supertranslations}$$

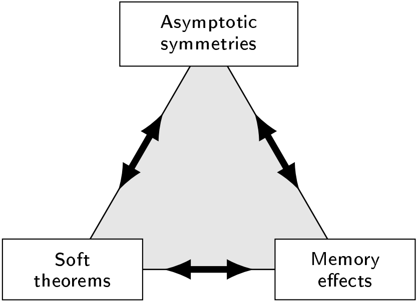
$\implies$ The supetranslations are necessary to include radiation

$\implies$ Boundary conditions yield some flux through the spacetime boundary



BMS and the Infrared Triangle

- Infrared sector of gauge theories described by a web of connections :
  [Strominger '17]



- Gravity :
  Supertranslations $\Leftrightarrow$ Displacement memory effect
  $\Leftrightarrow$ Soft graviton theorem

Extensions of BMS

- Recently, two extensions of the global BMS_4 have been proposed :
  1. Extended $\text{BMS}_4 = (\text{Diff}(S^1) \times \text{Diff}(S^1)) \ltimes \text{Supertranslations}^*$ [Barnich-Troessaert '10]
     $\Rightarrow$ Not globally well-defined on the celestial sphere (poles)
  2. Generalized $\text{BMS}_4 = \text{Diff}(S^2) \ltimes \text{Supertranslations}$ [Campiglia-Laddha '14]

- These extensions have important consequences :
  1. Physical processes (breaking of a cosmic string via black hole pair creation [Strominger-Zhiboedov '16])
  2. Superrotations $\Leftrightarrow$ Spin/refraction/velocity kick memory effects $\Leftrightarrow$ Subleading soft graviton theorem [Strominger '17] [Compère-Fiorucci-Ruzziconi '18]
  3. Celestial holography [Donnay-Puhm-Strominger '18]
  4. Edge mode symmetries [Donnelly-Freidel '16]
  5. ...

Questions

Natural questions arise :

- Is it possible to define the analogue of the BMS group in (A)dS ($\Lambda \neq 0$) ?
  $\Longrightarrow$ We call it the Λ-BMS group(oid)
- Is there a concept of flat limit ? ($\Lambda \to 0$ limit)
  $\Longrightarrow$ We want Λ-BMS $\to$ BMS in flat space when $\Lambda \to 0$

Leaky Boundary Conditions and Λ-BMS$_{d+1}$

- We consider partial Dirichlet boundary conditions in (A)dS :

$$g^{(0)}_{tt} = -\frac{\eta}{\ell^2}, \qquad g^{(0)}_{tA} = 0, \qquad \sqrt{|g^{(0)}|} = \frac{1}{\ell}\sqrt{\mathring{q}}$$

  where $x^a = (t/\ell, x^A)$, $A = 2, \dots, d$
- Fluctuations of $g^{(0)}_{AB}$ allowed ($\delta g^{(0)}_{AB} \neq 0$)
- Always reachable using the residual gauge diffeomorphisms ($d+1$ parameters $\bar{\xi}^a$ and σ for $d+1$ conditions)
  $\implies$ Does not constrain the Cauchy problem in dS
  (valid for both signs of Λ)
- Writing $\bar{\xi}^a \partial_a = \bar{\xi}^t \partial_t + \bar{\xi}^A \partial_A$, the residual gauge diffeomorphisms preserving the boundary conditions have to satisfy

$$\partial_t \bar{\xi}^t = \frac{1}{(d-1)} D_A \bar{\xi}^A, \qquad \partial_t \bar{\xi}^A = \frac{\eta}{\ell^2} g^{AB}_{(0)} D_B \bar{\xi}^t, \qquad \sigma = \frac{1}{(d-1)} D_A \bar{\xi}^A$$

- The generators satisfy the commutation relations

$$[\xi(\bar{\xi}_1^t, \bar{\xi}_1^A), \xi(\bar{\xi}_2^t, \bar{\xi}_2^A)]_\star = \xi(\hat{\bar{\xi}}_1^t, \hat{\bar{\xi}}_1^A)$$

where

$$\hat{\bar{\xi}}^t = \bar{\xi}_1^A D_A \bar{\xi}_2^t + \frac{1}{(d-1)} \bar{\xi}_1^t D_A \bar{\xi}_2^A - \delta_{\xi_1} \bar{\xi}_2^t - (1 \leftrightarrow 2),$$

$$\hat{\bar{\xi}}^A = \bar{\xi}_1^B D_B \bar{\xi}_2^A + \frac{\eta}{\ell^2} \bar{\xi}_1^t g_{(0)}^{AB} D_B \bar{\xi}_2^t - \delta_{\xi_1} \bar{\xi}_2^A - (1 \leftrightarrow 2)$$

$\Longrightarrow$ Field-dependent structure constants
$\Longrightarrow$ Λ-BMS$_{d+1}$ Lie algebroid

- In the flat limit $\ell \to \infty$, we obtain

$$\hat{\bar{\xi}}^t = \bar{\xi}_1^A D_A \bar{\xi}_2^t + \frac{1}{(d-1)} \bar{\xi}_1^t D_A \bar{\xi}_2^A - (1 \leftrightarrow 2),$$

$$\hat{\bar{\xi}}^A = \bar{\xi}_1^B D_B \bar{\xi}_2^A - (1 \leftrightarrow 2)$$

$\Longrightarrow$ This corresponds to the Generalized BMS$_{d+1}$ algebra (Diff(S^2) $\ltimes$ Supertranslations) of asymptotically flat spacetimes!

The Phase Space of Λ-BMS and its Flat Limit

- Symplectic structure :

$$\omega_{ren}[g;\delta g,\delta g]\Big|_{\mathscr{I}} = -\frac{\sqrt{\mathring{q}}}{\ell}\delta T^{AB}_{TF}\wedge\delta g^{(0)}_{AB}\,(d^d x)\neq 0$$

$\Longrightarrow$ Necessary to have some flux in dS
$\Longrightarrow$ Λ-BMS charges are not conserved, non-integrable

- The Fefferman-Graham gauge does not have a well-defined flat limit ($g_{\rho\rho}\to\infty$ when $\ell\to\infty$)
- Instead, one has to work in the Bondi gauge which admits a well-defined flat limit and exists for both $\Lambda\neq 0$ and $\Lambda = 0$
$\Longrightarrow$ Construct a diffeomorphisms from Fefferman-Graham to Bondi and translate all the results [Poole-Skenderis-Taylor '19] [Compère-Fiorucci-Ruzziconi '19]
$\Longrightarrow$ From now on, the discussion is valid only for $d = 3$
- When taking the flat limit of the solution space with our asymptotically (A)dS boundary conditions, one recovers the solution space of asymptotically flat spacetimes

- Flat limit works at the level of the symmetries and the solution space. What about the phase space ?
- When translated in Bondi gauge, one can show that

$$\boldsymbol{\omega}_{ren}[g;\delta g,\delta g]|_{\mathscr{I}} \sim \mathcal{O}(\Lambda^{-1})$$

$\Longrightarrow$ One cannot readily take $\Lambda \to 0$!

- The problem is solved by adding some corner terms in the holographically renormalized variational principle :

$$S_{ren} = \int_{\mathscr{M}} \boldsymbol{L}_{EH} + \int_{\mathscr{I}} \boldsymbol{L}_{GHY} + \int_{\mathscr{I}} \boldsymbol{L}_{ct} + \int_{\mathscr{I}} \boldsymbol{L}_{\circ}$$

with

$$\int_{\mathscr{I}} \boldsymbol{L}_{\circ} = \int_{(\partial\mathscr{I})_+} \boldsymbol{L}_C - \int_{(\partial\mathscr{I})_-} \boldsymbol{L}_C$$

- After this renormalization in Λ, one can safely take $\Lambda \to 0$
- We find the important result

$$\boldsymbol{\omega}_{ren(\rho,\Lambda)}[g;\delta g,\delta g]|_{\mathscr{I}} \to \boldsymbol{\omega}_{flat}[g;\delta g,\delta g]|_{\mathscr{I}} \quad \text{when} \quad \Lambda \to 0$$

where $\omega_{flat}[g;\delta g,\delta g]|_{\mathscr{I}}$ contains the Bondi mass loss in asymptotically flat spacetimes [Bondi-van der Burg-Metzner '62] [Sachs '62]

$$d\boldsymbol{k}_{\xi,flat}[g;\delta g]|_{\mathscr{I}} = \boldsymbol{\omega}_{flat}[g;\delta_\xi g,\delta g]|_{\mathscr{I}}$$

$\Longrightarrow$ Striking argument in favour of the existence of gravitational waves at the non-linear level of the theory

Summary

- Leaky boundary conditions in Al(A)dS$_{d+1}$ spacetimes
- Boundary diffeomorphism charges + Weyl charges
  $\Longrightarrow$ Weyl charges $\neq 0$ in even d
  $\Longrightarrow$ Sign of Weyl anomaly in the dual theory
- Charge algebra in Al(A)dS$_{d+1}$ spacetimes
  $\Longrightarrow$ Using the modified Barnich-Troessaert bracket
  $\Longrightarrow$ Exhibits a non-trivial field-dependent 2-cycle in even d
  $\Longrightarrow$ For $d = 2$, the latter reduces to the Brown-Henneaux central charge when imposing Dirichlet boundary conditions
- BMS-like symmetries in (A)dS
  $\Longrightarrow$ The Λ-BMS group(oid)
  $\Longrightarrow$ Flat limit to recover Generalized BMS

Perspectives

- Meaning of leaky boundary conditions in holography ?
  $\Longrightarrow$ Holography with "open" systems ?
  $\Longrightarrow$ Access to flat space holography through a flat limit process ?
  $\Longrightarrow$ Works for the Fluid/Gravity correspondence
  [Ciambelli-Marteau-Petropoulos-Ruzziconi '20]
- Implication of fluctuating boundary structure in (A)dS on the edge mode program ? [Donnelly-Freidel '16]
  $\Longrightarrow$ Interesting to have the maximum amount of symmetries
- Infrared triangle in (A)dS ?
  $\Longrightarrow$ Can we relate Λ-BMS with soft theorems and memory effects in (A)dS ? [Tolish-Wald '16] [Hinterbichler-Hui-Khoury '14]

Thank you !

Appendix : Non-Conservation and Variational Principle

- On-shell variational principle : $\delta S = \int_{\mathscr{I}} \boldsymbol{\Theta}[g;\delta g]|_{\mathscr{I}}$
- Presymplectic current : $\boldsymbol{\omega}[g;\delta g,\delta g] = \delta\boldsymbol{\Theta}[g;\delta g]$
- Flux-balance law controlling the non-conservation at infinity : $d\boldsymbol{k}_\xi[g;\delta g]|_{\mathscr{I}} = \boldsymbol{\omega}[g;\delta_\xi g,\delta g]|_{\mathscr{I}}$
- Conserved charges : $\boldsymbol{\omega}[g;\delta g,\delta g]|_{\mathscr{I}} = 0$
  $\Longrightarrow \boldsymbol{\Theta}[g;\delta g]|_{\mathscr{I}} = \delta\boldsymbol{B}[g]$
  $\Longrightarrow$ Add a boundary term to the action $S \to S' = S - \int_{\mathscr{I}} \boldsymbol{B}[g]$
  $\Longrightarrow$ Well-defined variational principle : $\delta S' = 0$
- Non-conserved charges : $\boldsymbol{\omega}[g;\delta g,\delta g]|_{\mathscr{I}} \neq 0$
  $\Longrightarrow \boldsymbol{\Theta}[g;\delta g]|_{\mathscr{I}} \neq \delta\boldsymbol{B}[g]$
  $\Longrightarrow$ Impossible to add a boundary term such that $\delta S = 0$.

Reduction to Dirichlet Boundary Conditions

- Dirichlet/Brown-Henneaux boundary conditions for AlAdS [Hawking '83] [Ashtekar-Magnon '84] [Brown-Henneaux '86] :

$$g^{(0)}_{ab} dx^a dx^b = -\frac{1}{\ell^2} dt^2 + \mathring{q}_{AB} dx^A dx^B$$

where $\mathring{q}_{AB}$ is the unit $(d-1)$-sphere metric and $x^a = (t/\ell, x^A)$, $A = 2, \dots, d$. For $d = 2$, the metric $\mathring{q}_{AB}$ has only one component that we take $\mathring{q}_{\phi\phi} = 1$.

- These boundary conditions are preserved under residual gauge diffeomorphisms $\xi(\bar{\xi}^a, \sigma)$ whose parameters satisfy

$$\mathcal{L}_{\bar{\xi}} g^{(0)}_{ab} = 2\sigma g^{(0)}_{ab}, \quad \sigma = \frac{1}{d} D_c \bar{\xi}^c$$

$\Longrightarrow$ Conformal algebra in d dimensions
(Witt$\oplus$Witt for $d = 2$ and $SO(d,2)$ for $d > 2$)

- Typical example of conservative boundary condition :

$$\boldsymbol{\omega}_{ren}[g; \delta g, \delta g]\Big|_{\mathscr{I}} = 0$$

- Charge algebra :
  1. $d > 2 \Longrightarrow$ No central extension [Henneaux '85]
  2. $d = 2$ [Brown-Henneaux '86] :
  $i\{\mathrm{L}^\pm_m, \mathrm{L}^\pm_n\} = (m-n)\mathrm{L}^\pm_{m+n} - \frac{c^\pm}{12} m(m^2-1)\delta^0_{m+n}$, $\{\mathrm{L}^\pm_m, \mathrm{L}^\mp_n\} = 0$ where $c^\pm = \frac{3\ell}{2G}$

Summary of Flat Limit