Global numerical simulations of vortex-mediated pulsar glitches in full general relativity

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in collaboration with

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Sourie, Oertel & Novak, PRD, 2016
Sourie, Chamel, Novak & Oertel, submitted to MNRAS
1 Introduction
- Observations
- Vortex-mediated glitch theory

2 Simulations of pulsar glitches in GR
- Realistic equilibrium configurations
- Dynamics of giant glitches
- Astrophysical considerations

3 Conclusion
The glitch phenomenon

Observational features
Espinoza et al., MNRAS, 2011

- amplitude:
  \[ \frac{\Delta \Omega}{\Omega} \sim 10^{-11} - 10^{-5} \]

- short rise time:
  \[ \tau_r < 30 \text{ s} \]
  Vela

- exponential relaxation on several days or months.

→ glitch = manifestation of an internal process
(except possibly for highly magnetised neutron stars)
The glitch phenomenon

- **Observational features**
  - **amplitude:** \( \Delta \Omega/\Omega \sim 10^{-11} - 10^{-5} \)
  - **short rise time:** \( \tau_r < 30 \text{ s} \) ← Vela
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Espinoza et al., *MNRAS*, 2011
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Distinct glitching behaviors

Wang et al., Ap&SS, 2012

- The Crab pulsar
- The Vela pulsar

$\Delta \Omega / \Omega$ vs. Time Since Last Glitch (days)
Distinct glitching behaviors

Wang et al., *Ap&SS*, 2012

quasi-periodic giant glitches with a very narrow spread in size
Distinct glitching behaviors

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quasi-periodic giant glitches with a very narrow spread in size

glitches of various sizes at random intervals of time
Distinct glitching behaviors

  - quasi-periodic giant glitches with a very narrow spread in size

- glitches of various sizes at random intervals of time

Different models of glitches

- Haskell & Melatos, *IJMPD*, 2015
  - Rearrangement of the moment of inertia $\rightarrow$ crustquakes,
  - Angular momentum transfer between two fluids $\rightarrow$ superfluidity.
Superfluidity in neutron stars

Superfluid properties:

- zero viscosity,
- angular momentum quantized into vortex lines.

Theoretical predictions for NSs

\[ T \lesssim T_c \sim 10^9 - 10^{10} \, \text{K} \]

- superfluid neutrons in the core & in the inner crust of NSs.
- superconducting protons in the core.

Observational evidence

- Long relaxation time scales in pulsar glitches,
- Fast cooling in Cassiopeia A,
- QPOs from SGRs, ...
Vortex-mediated glitch theory


### Two-fluid model
Baym et al., *Nature*, 1969

- Charged particles:
  \[ \Omega_p = \Omega \leftrightarrow \text{pulsar} \]

---

**Graphical Representation**
- Angular velocities vs. time
  - \( \Omega_p \) decreases with time.
Vortex-mediated glitch theory

*Anderson & Itoh, Nature, 1975*

**Two-fluid model**

*Baym et al., Nature, 1969*

- Charged particles:
  \[ \Omega_p = \Omega \leftrightarrow \text{pulsar} \]

- Superfluid neutrons:
  \[ \Omega_n \gtrsim \Omega_p \]

**Key assumption:** the vortices can *pin* to the crust and/or to flux tubes.
Vortex-mediated glitch theory


**Two-fluid model**
Baym et al., *Nature*, 1969

- **Charged particles:**
  \[ \Omega_p = \Omega \leftrightarrow \text{pulsar} \]

- **Superfluid neutrons:**
  \[ \Omega_n \gtrsim \Omega_p \]

Once a critical lag \( \Omega_n - \Omega_p \) is reached:

- some vortices get **unpinned** and are allowed to move **radially**

\[ \rightarrow \] angular momentum **transfer** between the fluids
This work

**Question:**

What is the impact of **general relativity** on the global dynamics of superfluid neutron stars during a glitch spin-up?
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→ global simulations based on a smooth-averaged hydrodynamical approach (for Vela: $\sim 10^{17}$ vortices).
This work

**Question:**

What is the impact of **general relativity** on the global dynamics of superfluid neutron stars during a glitch spin-up?

→ global simulations based on a smooth-averaged *hydrodynamical* approach (for Vela: \( \sim 10^{17} \) vortices).

→ *fundamental hypothesis:*

*hydrodynamical time* \( \sim 0.1 \) ms \(<\) *rise time* (dissipation)

the glitch event can be well described by a sequence of quasi-stationary equilibrium configurations
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Assumptions & Ingredients


**Equilibrium configurations:**

- $T = 0$,
- no magnetic field,
- dissipative effects are **neglected**,
- **uniform** composition: $p$, $e^-$, $n$,
  - the crust is not considered,
- asymptotically flat, **stationary**, axisymmetric & **circular** metric,
- **rigid-body** rotation: $\Omega_n$, $\Omega_p$.

**Equations of state:**

- Polytropic EoSs,
- *Density-dependent RMF models* (DDH & DDH$\delta$).

<table>
<thead>
<tr>
<th>$M_G$ ($M_\odot$)</th>
<th>$R_p$ circ (km)</th>
</tr>
</thead>
<tbody>
<tr>
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</tr>
<tr>
<td>1</td>
<td>11.0</td>
</tr>
<tr>
<td>2</td>
<td>13.1</td>
</tr>
</tbody>
</table>

- PSR J0348+0432
- PSR J1614-2230

- DDH$\delta$ - 0 Hz
- DDH$\delta$ - 716 Hz
- DDH - 0 Hz
- DDH - 716 Hz

**Southampton - September 13, 2016**
Fluid couplings

Moments of inertia:

\[
dJ_X = I_{XX} \, d\Omega_X + I_{XY} \, d\Omega_Y \quad X, Y \in \{n, p\}
\]

\[
\hat{I}_X = I_{XX} + I_{XY}
\]

\[
\hat{I} = \hat{I}_n + \hat{I}_p
\]
Fluid couplings

Moments of inertia:

\[ \mathrm{d}J_X = l_{XX}\, \mathrm{d}\Omega_X + l_{XY}\, \mathrm{d}\Omega_Y \quad X, Y \in \{n, p\} \]

\[ \hat{l}_X = l_{XX} + l_{XY} \quad \hat{l} = \hat{l}_n + \hat{l}_p \]

In the slow-rotation approximation (\(\Omega_n, \Omega_p \ll \Omega_K\)), the fluids are mainly coupled through two non-dissipative mechanisms:

- **entrainment effect**
  - due to the strong interactions between nucleons *in the core*:
  
  \[ p_X^\alpha = K^{XX} n_X u_X^\alpha + K^{XY} n_Y u_Y^\alpha \]

  *Andreev & Bashkin, SJETP, 1976*

- **relativistic frame-dragging effect**
  - associated with the rotation of the two fluids, \(\Omega_n\) and \(\Omega_p\):
    
    \[ g_{t\varphi} \neq 0 \]

  *Carter, Annals of Physics, 1975*
Entrainment VS frame-dragging

Coupling coefficients:

\[ \hat{\varepsilon}_X = I_{XY} / \hat{I}_X \]

In the slow-rotation approximation:

\[
\hat{\varepsilon}_p = \frac{\tilde{\varepsilon}_p - \varepsilon_{LT}^{\rightarrow p}}{1 - \varepsilon_{p\rightarrow p}^{LT} - \varepsilon_{n\rightarrow p}^{LT}}
\]

Remarks:

- \( \tilde{\varepsilon}_X \) characterizes entrainment,

- in Newtonian gravity:

\[ \hat{\varepsilon}_X = \tilde{\varepsilon}_X \]

NB: \( \hat{\varepsilon}_n = \hat{I}_p / \hat{I}_n \times \hat{\varepsilon}_p \sim 0.05 \times \hat{\varepsilon}_p \)
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Angular momentum transfer

\[ \Omega_n - \Omega_p = \delta \Omega_0 \Rightarrow \text{the dynamics is governed by mutual friction forces} \]
Angular momentum transfer

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Assuming straight vortices, the mutual friction moment considered reads

\[ \Gamma_{\text{int}} = - \int \frac{R}{1 + R^2} R_n n_n \omega_n \chi^2_\perp d\Sigma \times (\Omega_n - \Omega_p) = -2 \vec{B} \hat{I}_n \Omega_n \zeta \times \delta \Omega \]
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superfluid vorticity
Angular momentum transfer

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- resistivity coefficient
- lag
- superfluid vorticity
- mean mutual friction parameter
Angular momentum transfer

\[ \Omega_n - \Omega_p = \delta \Omega_0 \implies \text{the dynamics is governed by mutual friction forces} \]

Assuming \textit{straight vortices}, the \textit{mutual friction moment} considered reads

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- \text{resistivity coefficient}
- \text{lag}
- \text{superfluid vorticity}
- \text{mean mutual friction parameter}

\[ \leadsto \text{the geometry of the vortex array and the interactions between superfluid vortices and superconducting flux tubes are \textbf{poorly known}.} \]
Spin-up time scale

**Evolution equations:**

\[
\begin{align*}
\dot{J}_n &= + \Gamma_{\text{int}}, \\
\dot{J}_p &= - \Gamma_{\text{int}}.
\end{align*}
\]

\[
\frac{\delta \Omega}{\delta \Omega} = - \frac{\hat{I}_n}{l_{nn}l_{pp} - l_{np}^2} \times 2\hat{B}\zeta\Omega_n
\]

**Theoretical rise time:**

\[
\delta\Omega(t) = \delta\Omega_0 \times e^{-\frac{t}{\tau_r}}
\]

**Numerical modelling:**

Computation of $\Omega_n(t)$ & $\Omega_p(t)$

profiles from $\Omega_{n,0} > \Omega_{p,0}$
\[ \frac{\Delta \Omega}{\Omega} = 10^{-6}, \quad \Omega_n^f = \Omega_p^f = 2\pi \times 11.19 \text{ Hz}, \]

\[ M_G = 1.4 \, M_\odot \quad & \quad \bar{B} = 10^{-4} \]

---

The spin-up time scale can be very precisely estimated from stationary configurations only.
Influence of general relativity on $\tau_r$

- polytropic EoSs
- compactness parameter:
  \[ \Xi = \frac{G M_G}{R_{c,eq} c^2} \]
  $NB$: for NSs, $\Xi \sim 0.2$
- these relative differences also depend on $\Omega$
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The Vela pulsar

\[ \Delta \Omega / \Omega = 10^{-6}, \ \Omega_n^f = \Omega_p^f = 2\pi \times 11.19 \text{ Hz} \]
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\[ \bar{B} \]

\[ \bar{B} \rightarrow \Rightarrow \tau_r \]

\[ \text{Constraint on } \bar{B}: \]

\[ \tau_r < 30 \text{ s} \Rightarrow \bar{B} > 10^{-5} \]
The Vela pulsar

$$\Delta \Omega / \Omega = 10^{-6}, \quad \Omega_n = \Omega_p^f = 2\pi \times 11.19 \text{ Hz}$$

<table>
<thead>
<tr>
<th>$\tau_r$ (s)</th>
<th>$10^{-5}$</th>
<th>$10^{-4}$</th>
<th>$10^{-3}$</th>
<th>$10^{-2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tau_r = 0.1$</td>
<td>DDH</td>
<td>DDH$\delta$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\tau_r = 1$</td>
<td>DDH</td>
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<td>$\tau_r = 10$</td>
<td>DDH</td>
<td>DDH$\delta$</td>
<td></td>
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<tr>
<td>$\tau_r = 30$</td>
<td>DDH</td>
<td>DDH$\delta$</td>
<td></td>
<td></td>
</tr>
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</table>

$\vec{B} > 0.5 \implies \tau_r > 0.6 \text{ ms}$

$\vec{B} > 10^{-5}$

The glitch event is a quasi-stationary process.
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Conclusion & perspectives

- **Additional coupling** through relativistic frame-dragging effects,
- **Relativistic corrections** on the spin-up time: $\sim 50\%$ (core),
  should be included in a quantitative model of glitches.

Future work:

- Improve our numerical models by including the crust and considering that only a small amount of vortices is involved in the glitch event,
- Compare with future accurate observations of glitches,
- Include interactions with flux tubes in a more realistic mutual friction moment.
Thank you!
$P - \dot{P}$ diagram

ATNF Pulsar Database; Manchester et al., Astron. Journal, 2005
Glitch activity

Observables

- Average glitch activity:
  \[ \bar{A} = \frac{1}{t_{\text{obs}}} \sum_i \Delta \Omega_i / \Omega \]

- Coupling parameter:
  \[ G = \frac{\Omega}{|\dot{\Omega}|} \times \bar{A} \]

\[ \rightarrow \text{Vela: } G \approx 1.62 \times 10^{-2} \]

\[ \rightarrow \text{Crab: } G \approx 1.45 \times 10^{-5} \]
Rotating neutron stars, at equilibrium, described by \((\mathcal{E}, g)\):

- **asymptotically flat**: \(g \to \eta\) at spatial infinity \((r \to +\infty)\),
- **stationary & axisymmetric**: \(\frac{\partial g_{\alpha\beta}}{\partial t} = \frac{\partial g_{\alpha\beta}}{\partial \varphi} = 0\),
- **circular**: perfect fluids \(\Rightarrow\) purely circular motion around the rotation axis with \(\Omega_n, \Omega_p\) (+ rigid rotation).

Spacetime metric in quasi-isotropc coordinates:

\[
g_{\alpha\beta} \, dx^\alpha \, dx^\beta = -N^2 \, dt^2 + A^2(dr^2 + r^2 \, d\theta^2) + B^2 r^2 \sin^2 \theta (d\varphi - \omega \, dt)^2
\]

**At spatial infinity**

\(N, A, B \to 1\) \& \(\omega \to 0\)
Metric potentials

\[ N(r, \theta) \]

\[ \frac{\omega}{\Omega_n}(r, \theta) \]

\[ A(r, \theta) \]

\[ B - A(r, \theta) \]
Relativistic two-fluid hydrodynamics


System = two perfect fluids:
- superfluid neutrons $\rightarrow \vec{n}_n = n_n \vec{u}_n$,
- protons & electrons $\rightarrow \vec{n}_p = n_p \vec{u}_p$.

Energy-momentum tensor

$$T_{\alpha\beta} = n_{n\alpha} p^n_{\beta} + n_{p\alpha} p^p_{\beta} + \Psi g_{\alpha\beta}$$

$\leftrightarrow$ conjugate momenta

Entrainment matrix:

$$\left\{ \begin{array}{ll} p^n_{\alpha} = \mathcal{K}^{nn} n^n_{\alpha} + \mathcal{K}^{np} n^p_{\alpha} \\ p^p_{\alpha} = \mathcal{K}^{pn} n^n_{\alpha} + \mathcal{K}^{pp} n^p_{\alpha} \end{array} \right.$$  

$\rightarrow$ entrainment effect

Equation of state

$$\mathcal{E}(n_n, n_p, \Delta^2)$$
Equations of state

Relativistic Mean-Field Theory:

strong interaction between nucleons ⇔ exchange of effective mesons

Gravitational mass:

\[ M_G = M^B + E_{\text{bind}}, \]

Circumferential radius:

\[ R_{\text{circ, eq}}^X = C_X / 2\pi. \]
**Entrainment effects**

**Dynamical effective mass:**

\[ 3\vec{p}_X = m^*_X \; 3\vec{u}_X \]

→ in the *rest frame* of the second fluid.

**Zero-velocity frame:**

\[ m^*_X = \mu^X \times (1 - \varepsilon^*_X) \]

special relativity

entrainment

\[ \varepsilon^*_X, \varepsilon^*_n, \varepsilon^*_p \]

DDH

DDH\(\delta\)

\[ n_b \text{ (fm}^{-3}) \]

\[ \varepsilon^*_X \]

\[ \varepsilon^*_n \]

\[ \varepsilon^*_p \]
3+1 formalism

Foliation of the spacetime \((\mathcal{E}, g)\) by \((\Sigma_t)_{t \in \mathbb{R}}\), with unit normal \(\vec{n}\)

**Eulerian** observer \(\mathcal{O}_n\): 4-velocity = \(\vec{n}\)

- **lapse function** \(N\): \(\vec{n} = -N \vec{\nabla} t\),
- **shift vector** \(\vec{\beta}\): \(\vec{\partial}_t = N \vec{n} + \vec{\beta}\).

3+1 metric:

\[
g_{\alpha\beta} \, dx^\alpha \, dx^\beta = -N^2 \, dt^2 + \gamma_{ij} \, (dx^i + \beta^i \, dt) \, (dx^j + \beta^j \, dt)\]
Numerical procedure

**Paramètres d’entrée :**
- une EOS
- $H_c^n$, $H_c^p$
- $\Omega_n$, $\Omega_p$

**Initialisation :**
- $N = A = B = 1$ et $\omega = 0, \forall (r, \theta)$
- $U_n = U_p = 0$
- $H^i_0(r, \theta) = H^i_c \left(1 - \frac{r^2}{R^2}\right)$

**Convergence threshold**

\[ |H^i_{k+1}(r, \theta) - H^i_k(r, \theta)| < \epsilon \]

**At each iteration**

For given values of $(\mu^n, \mu^p, \Delta^2)$, we compute:

1. $\Psi$, $n_n$, $n_p$ and $\alpha$ from the EoS
2. The source terms $E$, $p\psi$, $S^i_i$,
3. Einstein Equations are solved,
4. Kinetic terms $U_i$ et $\Gamma_i$,
5. Computation of $H^i_{k+1}$.
\[ M_G = 1.4 \, M_\odot, \, \Omega_n/2\pi = \Omega_p/2\pi = 716 \, \text{Hz} \]
Superfluid vorticity

\[ w_{\mu\nu} = \nabla_\mu p^\nu_n - \nabla_\nu p^\mu_n \quad \Rightarrow \quad \omega_n = \sqrt{\frac{w_{\mu\nu} w^{\mu\nu}}{2}} \]

\[ \Omega^n / 2\pi = \Omega^p / 2\pi = 716 \text{ Hz} \]
Angular momenta

Axisymmetry $\leftrightarrow \vec{\chi}$

Komar definition:

$$J_K = -\int_{\Sigma_t} T(\vec{n}, \vec{\chi}) \, d^3 V$$

Eulerian observer $\vec{n}$ (3+1)

Angular momentum of each fluid


$$p_\varphi = J_n n p_n + j^p_\varphi$$

$$J_X = \int_{\Sigma_t} j^X_\varphi A^2 Br^2 \sin \theta \, dr \, d\theta \, d\varphi$$
Fluid couplings

In the slow-rotation approximation and to first order in the lag \( \delta \Omega = \Omega_n - \Omega_p \), the **angular momentum of fluid** \( X \) reads

\[
J_X \approx \int \Sigma_t n_X \mu^X B \frac{B}{N} (\Omega_X - \omega) r^2 \sin^2 \theta \, d^3V \\
+ \int \Sigma_t n_X \mu^X \varepsilon_X B \frac{B}{N} (\Omega_Y - \Omega_X) r^2 \sin^2 \theta \, d^3V
\]

Introducing \( i_X \equiv n_X \mu^X B \frac{B}{N} r^2 \sin^2 \theta \), we characterize the couplings by

- **Entrainment:**

  \[
  \tilde{I}_X \tilde{\varepsilon}_X \equiv \int \Sigma_t i_X \varepsilon_X \, d^3V
  \]

- **Lense-Thirring:**

  \[
  \tilde{I}_X (\varepsilon^{LT}_X \Omega_X + \varepsilon^{LT}_Y \Omega_Y) \equiv \int \Sigma_t i_X \omega \, d^3V
  \]

where \( \tilde{I}_X \equiv \int \Sigma_t i_X \, d^3V \)

\[
J_X = \tilde{I}_X \left(1 - \varepsilon^{LT}_X\right) \Omega_X + \tilde{I}_X \left(\tilde{\varepsilon}_X - \varepsilon^{LT}_Y\right) \Omega_Y
\]
Fluid couplings

\[ \frac{c}{G M^2} J_n / \Omega_n / 2\pi = 0 \text{ Hz} \]

\[ \frac{c}{G M^2} J_p / \Omega_p / 2\pi = 0 \text{ Hz} \]

\[ \Omega_p / 2\pi = 0 \text{ Hz} \]

\[ \Omega_n / 2\pi \]
Influence of $\Omega$ on the couplings

Newtonian gravity

general relativity
Where does the vortex unpinning take place?

Glitches have been generally thought to originate from the crust, because:

- the core superfluid was expected to be strongly coupled to the crust [Alpar et al., *ApJ*, 1984]
- the analysis of glitch data suggested that the superfluid represents a few percent of the total angular momentum of the star [Link et al., *PRL*, 1999]

However, this scenario has been recently challenged:

- considering entrainment effects, the crust does not carry enough angular momentum [Andersson et al., *PRL*, 2012 & Chamel, *PRL*, 2013]
- a huge glitch has been observed in PSR 2334+61 [Alpar, *AIP Conf. Proc.*, 2011]
- the core superfluid could be decoupled from the rest of the star, if vortices are pinned to flux tubes [Gügercinoglu & Alpar, *ApJ*, 2014]

The core superfluid plays a more important role than previously thought.
Additional physical inputs

So far, we assumed that all the neutrons can decouple from the protons.

- Only a small fraction of the neutron fluid could be involved in the glitch:
  \[
  \frac{I_n}{I} > f \equiv I_n^{nc} / I \gtrsim G \times (1 - \varepsilon_n^{nc})
  \]

- We need to account for crustal entrainment (Bragg scattering):
  \[-14 \lesssim \varepsilon_n^{nc} \lesssim 0\]


See also: Chamel, *PRC*, 2012
Constraining the interior of NSs

\[ \tau_r = \frac{1 - f - \varepsilon_n^{nc}}{2\bar{\Omega}_n} \]

- **whole core:**
  \[ f = 0.94, \quad \varepsilon_n^{nc} = 0.03 \text{ (DDH)} \]
  \[ f = 0.96, \quad \varepsilon_n^{nc} = 0.02 \text{ (DDHδ)} \]

- **outer core:**
  \[ f = 0.016, \quad \varepsilon_n^{nc} = 0 \]

- **inner crust:**
  \[ f = 0.064, \quad \varepsilon_n^{nc} = -3 \]

\[ \tau_r^{min} \simeq 1 \text{ ms - 0.1 s} \quad \text{the glitch event is a quasi-stationary process} \]
Gravitational wave amplitude

\[ h_+(t) = -\frac{3}{2} \sin^2 i \frac{G}{Dc^4} \ddot{Q} = h_0 \sin^2 i e^{-\frac{t}{\tau_r}} \]

\[ h_0 \approx 1.0 \times 10^{-37} \left( \frac{D}{1 \text{ kpc}} \right)^{-1} \left( \frac{\bar{B}}{10^{-3}} \right)^2 \left( \frac{\Omega}{10^2 \text{ rad.s}^{-1}} \right)^4 \left( \frac{\Delta\Omega/\Omega}{10^{-6}} \right) \]

- \( D = 1 \text{ kpc}, \)
- \( \bar{B} = 10^{-3}, \)
- \( M_G = 1.4 M_\odot, \)
- DDH EoS.