Non-equilibrium layers in rough-wall channel flow

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From bent layers to broken waves – studies in engineering and environmental fluid dynamics
Workshop in honour of Ian Castro’s 65th birthday
Synopsis

• Fully-developed rough channel flow: use of large relative roughness to examine limits to classical scaling
• Self-similarity (log law) of mean velocity demonstrated \( k/h = 4\% \)
• Townsend outer similarity also demonstrated for higher-order moments
• However, log-law offset \( (d_+ = 130) \) considerably smaller than zero-plane adjustment available from momentum balance \( d_{m+} = 237 \)
• How does this arise? Is the log-law offset necessarily the same as the momentum balance zero-plane adjustment?
• What are the implications for the energy balance and scale separation?
$h = 50.8\ \text{mm}$

$x/h \sim 132$

$U_o \sim 30\ \text{m/s}$

$Re_\tau = 5,700 - 7,700$

$W/h = 15$

Birch & Morrison (2011)
Surface topology

“grit”

Heavy industrial abrasive

\( k/h \sim 4\% \)
Basic properties

• Larger roughness elements exert a disproportionately large effect - pressure drop across a particular element goes as $k^2$, that is, when roughness is large, form drag dominates and wall shear-stress variation with velocity is quadratic

• “Shielding” of smaller roughness elements by larger ones (Colebrook & White 1937)

• Hence, for the sake of argument, identify $k$ as the maximum roughness height in a distribution

• Grit peak-to-peak $k_{max} \approx 1.8$ mm, $k/h = 3.5$
  • Isotropic
  • Non-Gaussian, positively skewed

\[
\bar{U} = \frac{1}{h} \int_0^h U(y) dy
\]

<table>
<thead>
<tr>
<th>Surface</th>
<th>$U_{el}$</th>
<th>$\bar{U}$</th>
<th>$Re_x$</th>
<th>$Re_h \times 10^{-4}$</th>
<th>$ku_\tau/\nu$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Grit</td>
<td>24.7</td>
<td>21.0</td>
<td>4780</td>
<td>7.28</td>
<td>186</td>
</tr>
<tr>
<td>Grit</td>
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<td>22.8</td>
<td>5130</td>
<td>7.78</td>
<td>200</td>
</tr>
<tr>
<td>Grit</td>
<td>28.6</td>
<td>24.4</td>
<td>5540</td>
<td>8.43</td>
<td>216</td>
</tr>
</tbody>
</table>
Self-similarity

• Self-similarity implies that the constant in the log argument may be freely chosen - a consequence of simultaneous overlap

• Therefore as long as there is a sufficient separation of scales, the log law may be written with either inner or outer variables

\[ U^+ = \frac{1}{\kappa} \ln \left( \frac{y - d}{k} \right) + B - \Delta U^+ \]

\[ U^+_c - U^+ = -\frac{1}{\kappa} \ln \left( \frac{y - d}{h} \right) + B^* \]

• By extension, it may also be written in terms of roughness height, \( k \) :

\[ U^+ = \frac{1}{\kappa} \ln \left( \frac{y - d}{k} \right) + B_2 \quad \frac{\nu}{u_r} \ll k \ll h \]

• A sufficient separation of scales also implies that \( B, B^* \) and \( B_2 \rightarrow \) constants: “fully rough”

• Here \( k^+ \approx 200 \); self-similarity for mean velocity demonstrated:

\[ U^+ = \frac{1}{\kappa} \ln \left( \frac{y - d}{y_0} \right) \quad 205 < (y - d)^+ < 308, \quad d^+ = 130 \]
Log law – determination of $d$ and $y_0$

- Take origin at bottom of roughness elements
- Minimise value of objective function $f(y-d)$ with $\kappa = 0.41$
- Assume lower limit for two-parameter fit is smallest value of $(y-d)/h$ for which mean velocity is spanwise homogeneous
- Independent check by plotting both mean velocity deficit and second-order moment vs. $(y-d)/h$
- “Zero-plane displacement” is large relative to thickness of supposed logarithmic region: $d$ usually taken as mean height at which momentum is extracted (Jackson 1981)

$$U^+ = \frac{1}{\kappa} \ln \left( \frac{y-d}{y_0} \right) \quad 205 < (y-d)^+ < 308 \quad d^+ = 130, y_0^+ = 13$$

- A change in $\kappa$ of ±0.01 gives variation in $d$ and $y_0$ which is within experimental error
Mean velocity: inner and outer scaling

\[ U^+ = \frac{1}{0.41} \ln \left( \frac{y-d}{y_0} \right) \]

Simultaneous overlap:

\[ 0.04 < \frac{y-d}{h} < 0.06 \]

\[ \frac{U_{cl}-U}{u_\tau} = -\frac{1}{0.41} \ln \left( \frac{y-d}{h} \right) + B^* \]

\[ 0.038 < \frac{y-d}{h} < 0.076 \]

\[ \frac{y-d}{y_0} \]

- \[ Re_\tau = 4780 \]
- \[ Re_\tau = 5190 \]
- \[ Re_\tau = 5530 \]
Townsend outer scaling
Second- & third-order moments

Jiménez (2004) suggests that \( k/h < 2.5\% \) “before similarity laws can be expected”..
Zero-plane displacement – “offset”

- **Smooth surface**
  - Squire (1948) used a zero-plane displacement for the log law – a measure of the effect of viscous sublayer
  - George & Castillo (1997), Wosnik et al. (2000): “near-asymptotics” indicate log-law offset
  - Lindgren et al. (2004): Lie Group symmetry methods show appearance of offset – log argument must be independent of the origin

- **“Mesolayer”** (Long & Chen 1981): a region in which, while the mean velocity exhibits self-similarity, the separation of scales is insufficient for direct effects of viscosity to be insignificant. No offset proposed: $y_p^+ \propto \left(Re_r\right)^{\frac{1}{2}}$
  - “Viscous/advection balance mesolayer” (Wei et al. 2005)
  - Mesolayer as a location at which momentum balance undergoes “balance breaking & exchange” (Klewicki et al. 2009): analysis leads to an offset

- **By extension**, similar situation can occur on rough surface – even with large relative roughness Monin & Yaglom (1973), Jackson (1981)…
Momentum balance

\[ \text{Measured} \quad \overline{uv}^+ : \]
\[ = 1 - \frac{(y - d_m)^+}{h^+} - \frac{1}{\kappa(y - d_m)^+} \]
(assuming the log law)

\[ \overline{uv}^+ = (1 - (y - d_m)/h) \]

\[ d_m^+ = 237 \]

\[ \frac{1}{\kappa y^+} \times 10^2 \]
Log law with $d_m$ offset

\[ U^+ = \frac{1}{\kappa} \ln \left( \frac{y-d}{y_0} \right) \]

\[ \approx \frac{1}{\kappa} \ln \frac{y}{y_0} - \frac{1}{\kappa} \left( \frac{d^+}{y^+} \right) - \frac{1}{2\kappa} \left( \frac{d^+}{y^+} \right)^2 + \ldots \]

\[ d_m = 237 \]

\[ \kappa = 0.41 \]

\[ O(10) \quad O(1) \quad O(0.1) \]
Local-equilibrium approximation

- Expect local-equilibrium region to be coincident with one for a log law as deduced by a dimensional analysis that excludes both wall and outer influence other than through $u_\tau$

- Inertial subrange is self-similar spectral transfer, $T(k)$, as demonstrated by simultaneous overlap with inner and outer scaling

- Wavelet decomposition of DNS data (smooth channel) shows $T(k)$ much more spatially intermittent than equivalent terms for either $P$ or $\varepsilon$ (Dunn & M 2003)

- Therefore $T(k)$ is unlikely to scale simply

- Even then, energy balance at any point in space is an integration over all $k$ – so $P = \varepsilon$ will only ever be an approximation

- Brouwers (2007): turbulent transport is $O(\nu/h)$ error on local equilibrium, but is $Re$–dependent

- Bradshaw/Lumley: “first-order” subrange: -5/3 slope retained but conservative spectral flux relaxed. Typically $Re_\lambda > 100$
Superpipe scale separation at $y/R = 0.12$

\[ A = \left( \frac{u_\tau / v_\varepsilon}{R/\eta} \right)^3 \]
\[ B = \left( \frac{(U_{cl} - \bar{U})/v_\varepsilon}{R/\eta} \right)^3 \]

- No ambiguity from hot-wire resolution
- Kolmogorov variables from local equilibrium approximation
- Scale separation appears for $R^+ > 5000$
- This is a prerequisite for log law and -5/3 scaling (does not imply local isotropy)

McKeon & Morrison 2007
Similarity: scale separation

- Sufficient scale separation should lead to

\[ \varepsilon \equiv \frac{v_{\tau}^3}{\eta} \equiv \frac{u_{\tau}^3}{Ah} \]

A \approx \text{constant}

- In ‘centre’ of log region:

\[ \varepsilon \approx \frac{u_{\tau}^3}{\kappa(y-d)} \left( 1 - \frac{y-d}{h} - \frac{1}{\kappa(y-d)^+} \right) \]

\[ d^+ = 0 \]
\[ d^+ = 130 \]
Conclusions

- Simultaneous overlap (log law) demonstrated for mean velocity in channel $4780 < \text{Re}_\tau < 5540$ with large relative roughness $(k/h \approx 4\%)$
- Townsend outer similarity for higher-order moments $\bar{u}^n$ clearly defined
- However, offset from supposed log law $d^+ = 130$ is larger than thickness of log law itself $205 < (y - d)^+ < 308$
- Offset from momentum equation $d_m^+ = 237$ considerably larger than that from log law $d^+ = 130$: former does not lead to a log law
- Either offset definition is consistent with wall-normal position of ‘mesolayer’, $\frac{\partial \bar{u} \bar{v}}{\partial y^+} \rightarrow 0$, being below log region:
  $$ (y_p - d)^+ \approx 45 \quad (y_p - d_m)^+ \approx 75 $$
- Preliminary analysis suggests significant spatial transport $\frac{\partial (u^2 v + v^3)}{\partial y} \neq 0$ – local equilibrium approximation does not apply
- Hence, is simultaneous overlap a sufficient condition for a log law in a strongly perturbed wall layer?