A holographic description of hadronic structure

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Southampton U. 10/01/2019.

Based on **JHEP 1810 (2018) 084** and **JHEP 1804 (2018) 118**

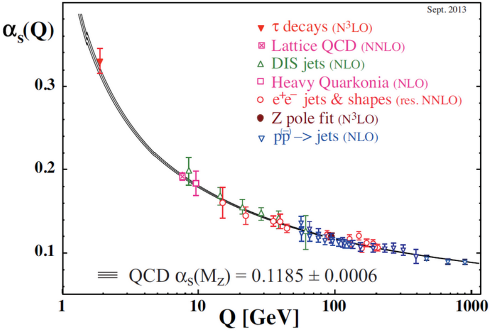
(with G. Michalski and M. Schvellinger)

Idea of the talk

Deep Inelastic Scattering in QCD

QCD: Asymptotic freedom and confinement

- Theory of gluons + quarks (strong interactions).
- $SU(3)$ gauge symmetry + fundamental matter.
- The coupling *constant* runs with momentum transfer (q):

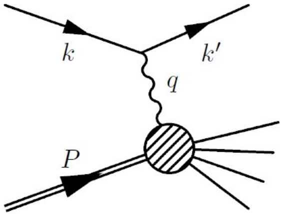


UV: Asymptotic freedom (partons)

IR: Confinement (hadrons)

Deep Inelastic Scattering

Goal: Exploring the internal structure of baryons and mesons.



- DIS regime: $q \gg P$.
- Inclusive process $\rightarrow \sum_{\text{final}}$
- **Bjorken**: $\boxed{x = \frac{-q^2}{2P\cdot q}}$.

Differential cross section ($y \equiv E^{-1}\Delta E$ of the lepton)

$$\frac{d\sigma}{dxdy} = \frac{e^4}{8\pi q^4}\, y\, l_{\mu\nu} W^{\mu\nu} \;,\; \text{(Physical range: } 0 < x, y < 1)$$

Leptonic tensor $l_{\mu\nu}$: computed easily in pQED. What about $W^{\mu\nu}$??

Hadronic tensor: symmetries and structure functions

$W^{\mu\nu}$ is defined from the EV of two em currents between hadronic states:

$$(W_{\mu\nu})_{\lambda\lambda'} = \frac{1}{4\pi}\int d^4x e^{iP\cdot x}\langle P,\lambda'|[J_\mu(x),J_\nu(0)]|P,\lambda\rangle$$

- **Difficult in pQCD.**
- Tensor structure fixed by symmetries:

Symmetries of W

$1)$**Parity** $2)$**Time reversal**
$3)$**Hermiticity** $4)$**Translations**

from $g_{\mu\nu}$, $\varepsilon_{\mu\nu\rho\sigma}$, P^μ, q^μ, $(S^\mu$, $\zeta^\mu)$.

Unknown scalar functions (x, q^2):

Spin 0

2 Functions: F_1, F_2.

Spin $1/2$

4 Functions: $F_{1,2}$ and $g_{1,2}$.

Spin 1

8 Functions: $F_{1,2}$, $g_{1,2}$ and $b_{1,2,...}$

Form of $W^{\mu\nu}$ for spin-0 and spin-1/2

A general $W^{\mu\nu}$ is decomposed as

$$W^{\mu\nu} = W^{\mu\nu}_{\rm sym} + iW^{\mu\nu}_{\rm asym}$$

where

$$\begin{aligned} W^{\mu\nu}_{\rm sym} &= F_1(x,q^2)\left(\eta^{\mu\nu} - \frac{q^\mu q^\nu}{q^2}\right) + \\ &\quad F_2(x,q^2)\,\frac{2x}{q^2}\left(P^\mu + \frac{q^\mu}{2x}\right)\left(P^\nu + \frac{q^\nu}{2x}\right) + [b_i\text{−terms}] \\ W^{\mu\nu}_{\rm asym} &= \epsilon^{\mu\nu\alpha\beta}\frac{q_\alpha}{P\cdot q}\left[S_\beta\, g_1(x,q^2) + \left(S_\beta - \frac{S\cdot q}{P\cdot q}P_\beta\right) g_2(x,q^2)\right] \end{aligned}$$

Interpretation - Parton model

$F_i(x) \leftrightarrow$ Distribution functions of partons with momentum xP

Why is the small-x region interesting?

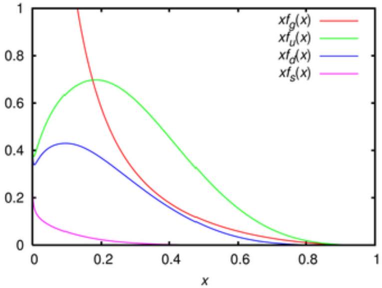
For *moderate values* of x:

- Large q DIS.
- Low parton densities.

$\Rightarrow$ weak coupling $\alpha_s(q)$

$\Rightarrow$ *See* asymptotic freedom

<u>Distribution Functions:</u>



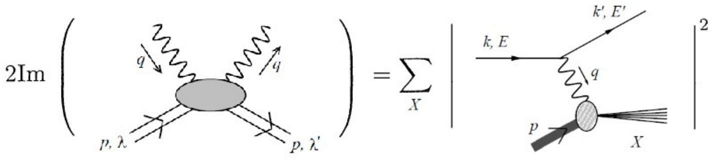
For small values of x:

- **Exp. constraint:** $\boxed{q^2_{\text{max}} \approx E_{\text{lep}}\, x\, (2\, M_{\text{had}})}$ $\Rightarrow$ only lower q.
- Higher parton densities (lots of gluons!).

$\Rightarrow$ probably strongly coupled physics involved!

Optical theorem and Forward Compton Scattering

Since $S = S^\dagger$, DIS is related to another process, the **FCS**:



The $W^{\mu\nu}(F_i)$ DIS and the $T^{\mu\nu}(\tilde{F}_i)$ of FCS are similar:

DIS *vs* FCS

$$W^{\mu\nu} \sim \text{Im}\,(T^{\mu\nu}) \Rightarrow F_i(x,q^2) = 2\pi\,\text{Im}\left(\tilde{F}_i(x,q^2)\right)$$

In practice, we work with the *holographic dual* of the **FCS** process.

Note that $s \equiv -(P+q)^2$ so

Small-x DIS $\Rightarrow$ high CM energy ($s \approx q^2/x$) for the FCS.

Regge theory

- One of the first attempts to describe **hadronic physics**.
- Based only on the **basics properties** of any S-**matrix**

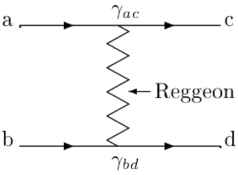
$$S_{ab} = \langle b_{\text{out}} | a_{\text{in}} \rangle .$$

Postulates for S:

Lorentz (4pt: s,t) Unitarity "Analyticity" (Analytic cont.)

At $s \gg t$: t-channel exchange and factorization

We get $\mathcal{A}(s \gg t) \approx \beta(t) s^{\alpha(t)} \rightarrow$ exchange of an effective spin $\alpha(t)$ mode.



$$\mathcal{A} \sim \gamma_{ac}(t)\gamma_{bd}(t)s^{\alpha_R(t)}$$

$t = 0$ case (DIS) from pQFT [1]

The amplitude can be written from a *Kernel* and two *impact factors*:

$$\mathcal{A} \sim \int \frac{dp_{\rm tr}}{p_{\rm tr}} \int \frac{dp'_{\rm tr}}{p'_{\rm tr}} \, \gamma_{ac}(p_{\rm tr}) \, \gamma_{bd}(p'_{\rm tr}) \, \mathcal{K}(s, p_{\rm tr}, p'_{\rm tr}),$$

where for large N and weak 't Hooft coupling $\lambda = g_{\rm YM}^2 N$

$$\mathcal{K}(s, p_{\rm tr}, p'_{\rm tr}) \approx s^{j_0} \times \frac{1}{\sqrt{4\pi D \log s}} \exp\left[\frac{-1}{4D \log s} (\log p_{\rm tr}/p'_{\rm tr})^2\right]$$

with

$$j_0 = 1 + \frac{\log 2}{\pi^2}\lambda \;\; \text{y} \;\; D = \frac{7\zeta(3)}{8\pi^2}\lambda.$$

Thus, we see a Regge-type factor s^{j_0} but also **diffusion** in the *transverse momentum* (with a characteristic scale D).

Comparison with experimental data for $F_2^p(x, q^2)$

The functional form fits very well, but with

$$j_0 \approx 1.25 \;\Rightarrow\; 1 < \lambda < 10 \;\Rightarrow\; ???$$

[1] [BFKL: Balitsky, Fadin, Lipatov, and Kuraev, 1976-78]

Our methods and goal

Holography[2] provides an analytic tool for studying strongly-coupled phenomena.

I want to...

- Recall how it can describe DIS in a strong coupling scenario.
- Describe how the full string theory (and not only supergravity) plays an important role in the most interesting parametric regime.
- Briefly review the Pomeron from the ST perspective.
- Give a precision test by fitting the recent data for the proton's structure function g_1 at small-x.

[2] [Maldacena, Witten, GKP, ...1997-]

Holography and DIS

The AdS/CFT conjecture

- Based on the large N expansion of gauge theories and on the holographic principle.
- Equates gravity (string theory) theories on AdS backgrounds and conformal gauge theories on their conformal boundary.

Most studied example:

$\mathcal{N}=4$ SYM in 4d with $U(N)$ gauge group	$\leftrightarrow$	Type IIB String Theory on an AdS$_5 \times S^5$ background

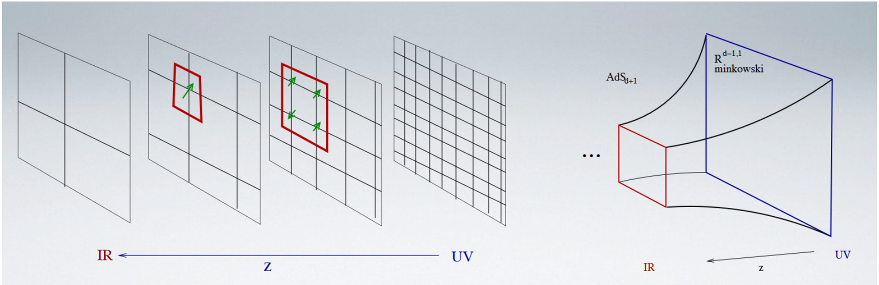
- Global symmetries of the QFT are associated with isometries of the gravity background.
- It is a **Weak/Strong duality:**

$$(\lambda,\, N) \quad \leftrightarrow \quad (\alpha' \sim R^2_{\rm AdS}/\sqrt{\lambda},\, g_s \sim g^2_{\rm YM})$$

Thus, for $N \gg \lambda \gg 1$ we have a classical gravity description.

The AdS/CFT conjecture

- The radial direction ($r \sim 1/z$) of AdS is identified with the RG scale.

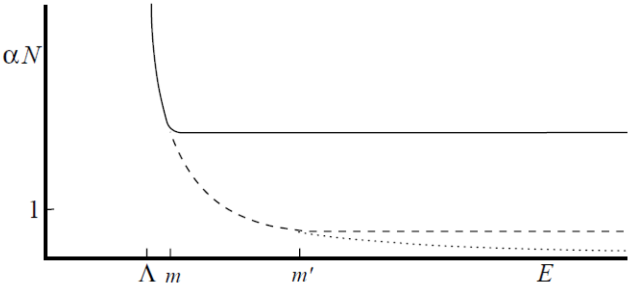


- Relevant deformations of the QFT will change the bulk interior (IR).
- The W-GKP *Ansatz* gives a prescription to compute CFT correlation functions from the bulk perspective:

$$Z_{\rm CFT}\left[\phi_0(x)\right] = \langle \exp \int \phi_0(x)\mathcal{O}(x)\rangle \equiv Z_{\rm grav}\left[\phi(x,z)|_{\rm bdy} = \phi_0(x)\right]$$

The RG flow of the theories we work with

$$\mathcal{N} = 4 \text{ SYM (UV, } \beta_g = 0) \ \to_m \ \mathcal{N} = 1 \text{ SYM (IR, confines at } \Lambda)$$



- $m \gg \Lambda \Rightarrow$ pQFT in the UV.
- $m \sim \Lambda \Rightarrow$ strong coupling in the UV, but perturbative dual description from ST. The geometry is $\sim$AdS only up to $r \sim \Lambda R^2$.

AdS/CFT dictionary for the dual FCS[3]

Confinement at characteristic scale Λ

Implemented through an IR deformation: **Hard-wall**, *soft-wall*, etc.

Conserved "em" current

Obtained by gauging a $U(1) \subset SU(4)_R$.

Boundary theory vs AdS modes

Virtual photon $\leftrightarrow \delta g^{MN} \propto A^m K^a$ (non-normalizable)

Hadronic states $\leftrightarrow$ on-shell modes

Finite λ corrections $\leftrightarrow$ Stringy contributions (α')

Non-planar corrections (1/N) $\leftrightarrow$ Quantum corrections ($\star$)

Mesonic targets $\leftrightarrow$ modes from flavor *probe* D-branes ($\star$).

[3][Polchinski & Strassler 2001-2002]

The idea of the holographic DIS (FCS) model

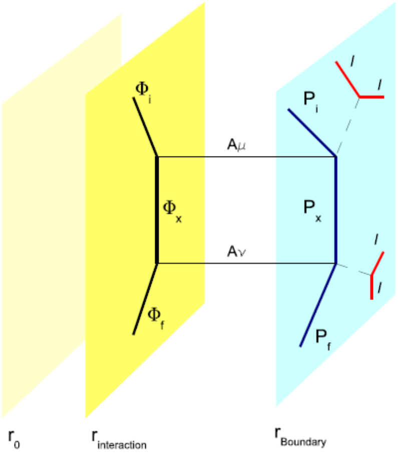
$$ds^2 \approx ds^2_{\mathrm{AdS}_5} + R^2 d\Omega_5^2$$

$$ds^2_{\mathrm{AdS}_5} = \frac{r^2}{R^2} dx^2 + \frac{R^2}{r^2} dr^2$$

$$R^2_{\mathrm{AdS}_5} \sim \alpha' \sqrt{\lambda}$$

$$r_0 \sim \Lambda R^2$$

$$r_{\mathrm{int}} \sim q R^2$$



3 distinct parametric regimes

Center-of-Mass energy in the 10d scattering process:

$$\tilde{s} = g^{MN}(P+q)_M(P+q)_N \sim \frac{1}{\sqrt{\lambda}}\left(\frac{1}{x}-1\right) \times \frac{1}{\alpha'}$$

Thus, we have to be careful with the actual physics involved:

Supergravity regime: $\tilde{s} \ll \alpha'$

- Valid for $1/\sqrt{\lambda} \ll x < 1$.

String theory regimes I and II

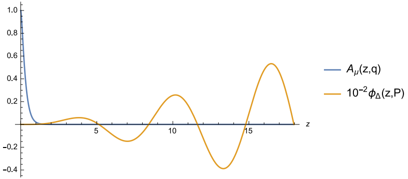
- Valid for $x \ll 1/\sqrt{\lambda}$.
- The subdivision is associated to whether or not the process is effectively local ($\sim$ 4pt-scattering) in the radial direction.

First regime: Supergravity.

Solutions, planar limit and SUGRA structure functions

$$W^{\mu\nu} \propto \mathrm{Im}\, T^{\mu\nu} \propto \sum_{X\ \mathrm{on-shell}} \langle h, P|\tilde{J}^{\mu}(q)|X, P+q\rangle\langle X, P+q|J^{\nu}(0)|h, P\rangle$$

We use AdS/CFT to compute the current 1pt-functions.



$$A_\mu(q,z) \sim n_\mu e^{iq\cdot x} qzK_1(qz)$$
$$\phi_\Delta(P,z) \sim e^{iP\cdot x} z^2 J_{\Delta-2}(Pz)$$

$$S_{A\phi\phi} \sim \frac{1}{N}\int d^{10}x\,\sqrt{-g}A_\mu J^\mu[\phi]$$

Computing $S_{A\phi\phi}$ *on-shell* y comparing with $W_{\mu\nu}$ we get

$$F_2 \sim \left(\frac{\Lambda^2}{q^2}\right)^{\Delta-1} x^{\Delta+1}(1-x)^{\Delta-2}$$

- Interaction: $z_{int} = 1/q$.
- Full Hadron.

Second regime: string theory.

Toy example: the scalar target case

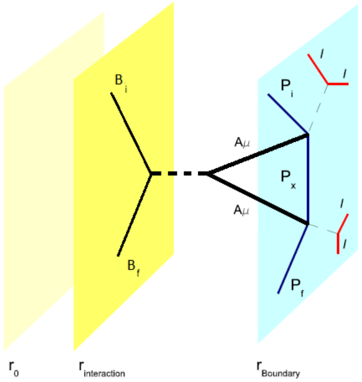
We need to include the exchange of stringy states since

$$x \ll \lambda^{-1/2} \Rightarrow \tilde{s} > 1/\alpha'$$

$$n_\mu n_\nu W^{\mu\nu} \sim \int_{\mathrm{AdS}} \mathrm{Im}\mathcal{A}^{\mathrm{cl.st.}}_{\mathrm{flat}}(2h, 2\phi)$$

Modus Operandi (t-channel)

- Compute $\mathcal{A}_{\mathrm{flat}} = \mathcal{G} \times \mathcal{K}$.
- Take $\tilde{s} \propto s$ and $\tilde{t} \to 0$.
- Take the Im. part.
- Use local approx. $S_{\mathrm{eff}}(4\mathrm{pt})$.
- Insert AdS sol's and integrate.



Details of the local approximation

Idea: superposition of local processes

- Separate: $X^M(\tau,\sigma) = x^M + \tilde{X}^M(\tau,\sigma)$.
- $\alpha'/R^2 \sim 1/\sqrt{\lambda}$ plays the role of $\hbar$

$\Rightarrow$ the path integral for $\tilde{X}^M$ is approximately gaussian.

$\Rightarrow S \approx i \int d^4x d^6y \sqrt{-G} \mathcal{A}^{4p}_{loc} \approx i(2\pi)^4 \delta(\sum p) \int d^6y \sqrt{-G} \mathcal{A}^{4p}_{loc}$

$$\mathcal{A} = \mathcal{G}(\alpha') \times \mathcal{K}\ ,\ \operatorname{Im} \mathcal{G} \sim \sum_{m=1}^{\infty} \delta\left(m - \frac{\alpha'\tilde{s}}{4}\right) \left(\frac{\alpha'\tilde{s}}{4}\right)^{\alpha'\tilde{t}/2} \sim \sum_{\text{exc}}$$

Mandelstam invariants in AdS

$$\alpha'\tilde{s} \approx \frac{\alpha' R^2}{r^2} s + \mathcal{O}(\lambda^{-1/2})\ ,\ \alpha'\tilde{t} \approx 0 + \mathcal{O}(\lambda^{-1/2})$$

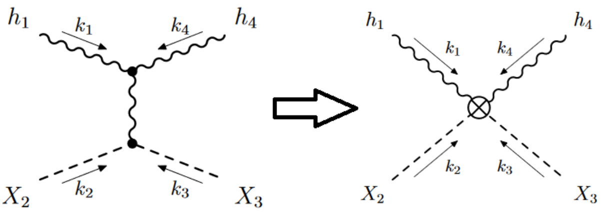
Heuristic interpretation of the effective action

- High energies $\Rightarrow \mathcal{A} \propto s^j$ (spin-j exchange).

Leading process at small-x

t-channel exchange of a (*reggeized*) graviton.

$$L_{eff} \sim T_A G_{\rm grav} T_\phi \sim F^{\mu m} F^{\nu}{}_{n} \partial_\mu \phi \partial_\nu \phi$$



- This can be seen from the field theory OPE $JJ \sim T$.
- The **Im** part comes from $\tilde{s}^2/\tilde{t} \to {\rm Im}\,\mathcal{G}(\alpha', \tilde{s} \gg \tilde{t}) \sim \frac{\pi\alpha'}{4} \sum_{\rm exc}$.

Small-x: results for scalar targets

Inserting the AdS solutions and integrating over the full 10d space:

$$F_1(x,q^2) \sim \frac{1}{\sqrt{\lambda}} \left(\frac{\Lambda^2}{q^2}\right)^{\Delta-1} \left(\frac{1}{x}\right)^2 , \; F_2(x,q^2) = \left(\frac{2\Delta+3}{\Delta+2}\right) 2xF_1(x,q^2)$$

- Power-like growth x^{-2} for $x \to 0$ (from $\mathcal{A} \sim s^2$).
- Still the same suppression factor from Λ^2/q^2.
- Callan-Gross type relations: $F_2 \propto 2xF_1$. ($\sim$ partons $s = 1/2$)

- What about spin-$1/2$ targets? ($\mathcal{A}(h,h,\psi,\psi)$?)

Where is g_1 at small-x? [5]

- The amplitude $\mathcal{A}(h, h, \psi, \psi)$ should include all possible processes. It gives similar results for $F_{1,2}$ but no contribution to $g_{1,2}$.
- The effect from the chirality of the ψ solution is sub-leading in this regime.

However, the explicit reduction of 10d type IIB SUGRA on S^5 shows that (at the linear level) A_m is a combination of modes from the graviton **and from the RR 4-form**[4]:

$$h_{ma} \sim A_{(m}K_{a)} \ , \ a_{mabc} \sim A_{[m}Z_{abc]} \, ,$$

with K^a the Killing vectors on S^5 and $Z_{abc} \propto \epsilon_{abcde}\nabla^d K^e$.

New amplitudes

We must consider amplitudes with in/out-going modes from the $\mathcal{F}_5$.

[4][Kim, Romans & van Nieuwenhuizen, 1985]

[5][Hatta, Hueda & Xiao, 2009]

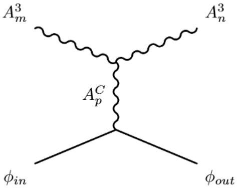
g_1: heuristic analysis

Is there some hint for extra contributions from the SUGRA point of view?

- $\mathcal{A} \sim s^j \Rightarrow$ after h $(j=2)$, look for $j=1$.

This process is possible due to the Chern-Simons term in 5d!

$$S_{CS} = \frac{i\kappa}{96\pi^2} \int d^5x \, d_{ABC} \, \varepsilon^{mnopq} A_m^A \partial_n A_o^B \partial_p A_q^C + \cdots$$



- $\boxed{L_{eff} \sim \mathcal{J}_A^m G_{mn}^{\text{gauge}} J_\phi^n}$.
- $\mathcal{J}_{\text{DIS}}^{m(A)} \sim d_{ABC} \varepsilon^{mnopq} \partial_n A_o^B \partial_p A_q^C$.
- Effective charge: $\mathcal{Q} \equiv d_{33C} \mathcal{Q}^C$.

This can be seen from the QFT OPE term $J^\mu(q) J^\nu(0) \sim \varepsilon^{\mu\nu\rho\sigma} q_\rho J_\sigma(0)$.

g_1: Computation from string theory

This same S_{eff} is obtained from first principles. **MO:**

1. Compute the 4pt amplitude $\mathcal{A}_{\text{flat}}(\mathcal{F}_5, \mathcal{F}_5, \psi, \psi)$.
2. Take $\tilde{s} \gg \tilde{t}$ and construct the effective action S_{eff} as before.
3. Reduce to 5d to compare with the previous expression for $S_{\text{eff}}[\text{CS}]$.

Results for ψ:

$$g_1(x, q^2) \sim \left(\frac{\Lambda^2}{q^2}\right)^{\Delta - 1} \frac{1}{x} \text{ and } g_2 = 0$$

Note that we find...

- The expected x^{-1} growth of g_1 as $x \to 0$.
- BUT the same suppression factor Λ^2/q^2.
- AdS/CFT Burkhardt-Cottingham sum rule for g_2.

Third regime: Pomeron physics.

Breakdown of the local approximation

As we saw, the relevant ST amplitudes factorize as $\mathcal{A} = \mathcal{G} \times \mathcal{K}$ with

$$\mathcal{G}(\alpha', \tilde{s}, \tilde{t}, \tilde{u}) = -\frac{\alpha'^3}{64} \frac{\Gamma\left(-\alpha'\tilde{s}/4\right)\Gamma\left(-\alpha'\tilde{t}/4\right)\Gamma\left(-\alpha'\tilde{u}/4\right)}{\Gamma\left(1+\alpha'\tilde{s}/4\right)\Gamma\left(1+\alpha'\tilde{t}/4\right)\Gamma\left(1+\alpha'\tilde{u}/4\right)}$$

Remember that

$\mathrm{Im}\,\mathcal{G} \propto \sum_{\mathrm{exc}} (\alpha'\tilde{s})^{\alpha'\tilde{t}}$ with $\tilde{s} \propto s \sim q^2/x$ and $\alpha'\tilde{t} \approx 0 + \mathcal{O}(\lambda^{-1/2})$

Thus, the local approximation breaks down at large but finite coupling,

$$\lambda \to \infty \;,\; s \to \infty \text{ with } \frac{\log s}{\sqrt{\lambda}} \text{ constant.}$$

Here, $\tilde{t}$ acts as a differential operator: the t-channel laplacian.

$$j \approx 2 \Rightarrow \alpha'\,\tilde{t} = \alpha'\,\nabla_t^2(h) \approx (\alpha'/R^2)\,\Delta_2$$

Pomeron exchange[6]

In this context, the amplitude can be expressed as $(u \equiv -\log z)$

$$\mathrm{Im}\mathcal{A}(s,t=0) \sim \int du \int du' P_A(u) P_\phi(u') \mathcal{K}(u,u',s) \;, \; \mathcal{K} \sim s^{2-2/\sqrt{\lambda}}$$

- The Kernel is $\mathcal{K}(u,u',s) \sim s^{2-2/\sqrt{\lambda}} \times \exp\left[-(u-u')^2/4\tau\right]$.
- x-dependence: $\boxed{F_1 \sim (1/x)^{2-2/\sqrt{\lambda}} \text{ y } F_2 \sim (1/x)^{1-2/\sqrt{\lambda}}}$.
- **Including confinement** modifies the form of $\mathcal{K}(u,u',s,z_{\text{hard-wall}})$.
- Multi-Pomeron exchange (loops) can be addressed through the *Eikonal* (important for unitarity/saturation effects).

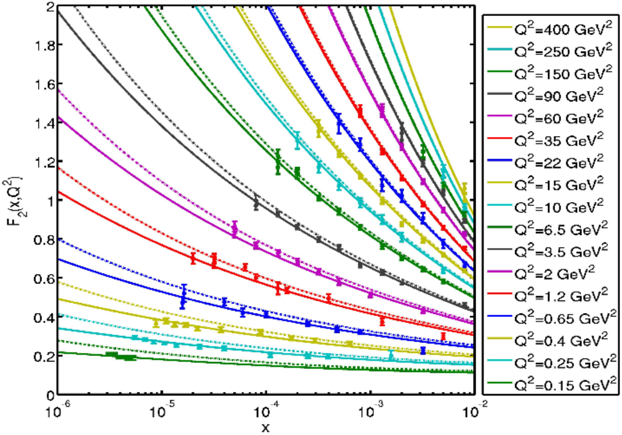
Another way of getting this:

Directly taking the OPE on the products of the vertex operators inserted on the worldsheet.

[6] [Polchinski, Brower, Strassler & Tan, 2006]

Phenomenology the proton F_2 (ZEUS-HERA) [7]

- Comparison with F_2 for protons.
- Approximation for the impact factors.
- Fitted with free λ, q' y Λ.
- They get fixed on reasonable values.



They also included loop corrections.

[7] [Brower, Durić, Sarcević and Tan, 2010]

New contribution to g_1: $j \approx 1$

In this case differential operator changes

$$\alpha' \tilde{t} = \alpha' \nabla_t^2(A) \approx (\alpha'/R^2)(\Delta_1 + 3)$$

and the Kernel becomes $(\rho \equiv -2 \log z)$

$$\mathcal{K}(\rho, \rho', t = 0, j = 1) = (\alpha' \tilde{s})^{1 - \frac{1}{2\sqrt{\lambda}}} \, e^{-\frac{1}{2}(\rho + \rho')} \sqrt{\frac{\lambda^{1/2}}{2\pi\tau}} e^{-\frac{\sqrt{\lambda}}{8\tau}(\rho - \rho')^2}$$

- Confinement effects (and loops?) can be included as before.

Comparison of the results

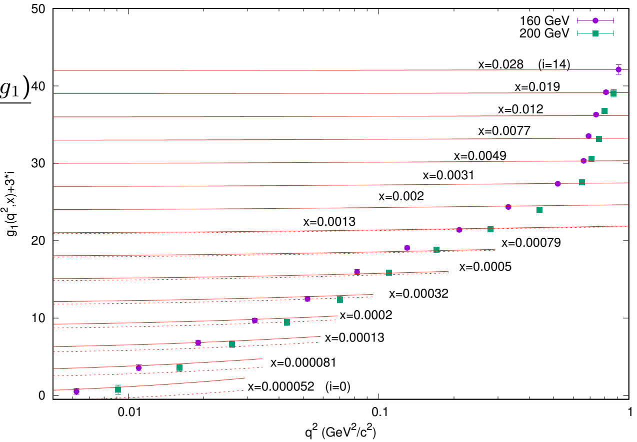
The exponent correction is different:

$$F_2 \sim \left(\frac{1}{x}\right)^{1 - \frac{2}{\sqrt{\lambda}}} \quad \text{while} \quad g_1 \sim \left(\frac{1}{x}\right)^{1 - \frac{1}{2\sqrt{\lambda}}} .$$

Phenomenology for g_1 (COMPASS LHC-CERN 2017)

Spin-$1/2$ vs Proton data (g_1)

- Best fit for the same values for λ, q' y Λ.
- Only one extra parameter (global constant)
- Less data available for g_1 at small-x.



Summary and outlook

Holography is very useful, but as a tool for describing **real world** experiments has pros and cons:

- It provides an analytic approach to strongly coupled physics (mainly at large N).
- The easiest cases we know have supersymmetry and conformal symmetry. They can be deformed, but the holographic dual to QCD is not known.

We can only describe real-world scenarios where the physics involved is fairly universal.

In this talk I have...

- Presented an example of such a scenario: **Deep Inelastic Scattering in the small-x regime.**
- Shown how to obtain both **qualitative insights** and **quantitative results**, focusing on the case of the spin-dependent structure function g_1 of spin-$1/2$ targets.

Summary and outlook

Future work:

- Analyze mesonic-DIS with it's polarized structure functions. (*)
- Study other "small-x" processes.
- Include non-planar corrections. (*)
- Consider more realistic targets (instantons).

Thank you for your time! Any questions?