Compact binaries and the gravitational self-force

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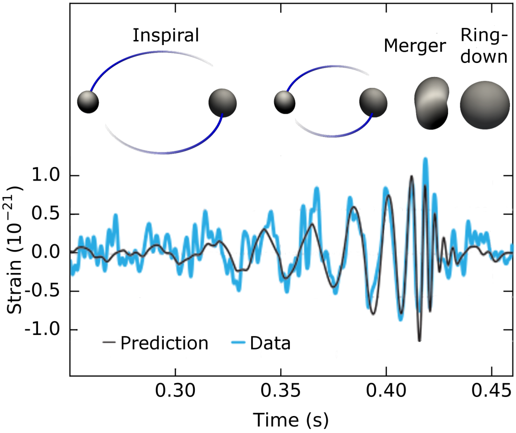
The first detections

Three years ago, LIGO detected the gravitational waves from a black hole binary merger



Several more binaries have since been detected by LIGO and Virgo

Gravitational waves and binary systems

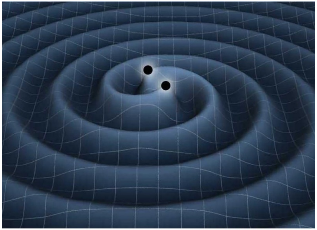


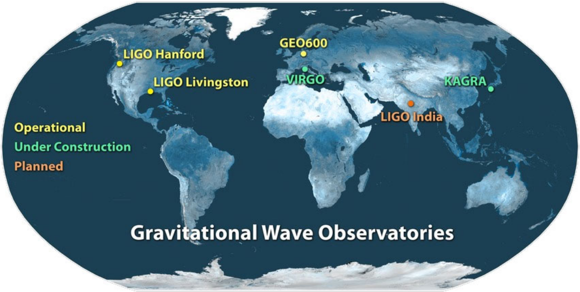
Image: Nasa/ GSFC/ The Washington Post

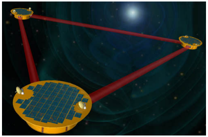
- compact objects (black holes or neutron stars) strongly curve the spacetime around them
- their motion in a binary generates gravitational waves, small ripples in spacetime

- waves propagate to detector
- to extract meaningful information from a signal, we require models that relate the waveform to the source

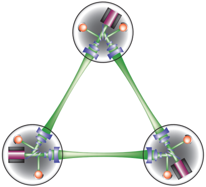
Many detectors

A multitude of detectors are in various stages of development on the ground and in space





LISA

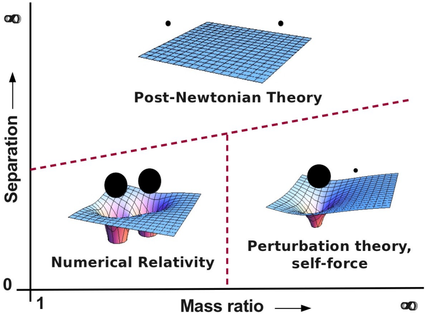


DECIGO

Many types of binaries

- different classes of binaries will be observed by different detectors and tell us different things
- share a common structure: emission of energy in gravitational waves drives inspiral and eventual collision
- but they require different modeling methods

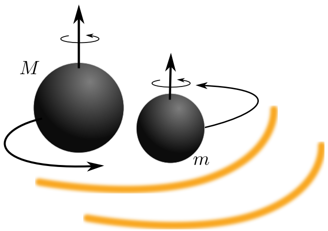
Binary parameter space



[Image credit: Leor Barack]

- models can be combined in phenomenological effective-one-body (EOB) theory

Comparable-mass inspirals



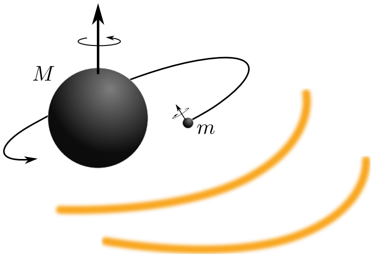
Science

- the common type of binary observed by ground-based detectors LIGO/Virgo
- observations will
  - constrain populations of stellar- and intermediate-mass BHs
  - constrain NS equation of state
  - test alternative theories of gravity

Modeling

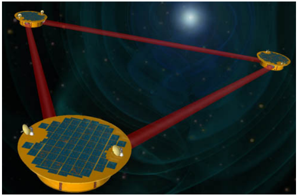
- early stages modeled by post-Newtonian (PN) theory: expansion in limit of $v/c \to 0$
- late stages modeled by numerical relativity (NR): numerical solution to nonlinear Einstein equation
- full evolution modeled by EOB

Extreme-mass-ratio inspirals (EMRIs)



Science

- space-based detector LISA will observe extreme-mass ratio inspirals of stellar-mass BHs or neutron stars into massive BHs
- small object spends $\sim M/m \sim 10^5$ orbits near BH $\Rightarrow$ unparalleled probe of strong-field region around BH



Modeling

- PN and NR don't work
- use black hole perturbation theory/self-force theory

More on EMRI science

Fundamental physics

- measure central BH parameters: mass and spin to $\sim .01\%$ error, quadrupole moment to $\sim .1\%$
  $\Rightarrow$ measure deviations from the Kerr relationship $M_l + iS_l = M(ia)^l$
  $\Rightarrow$ test no-hair theorem
- measure deviations from Kerr QNMs, presence or absence of event horizon, additional wave polarizations, changes to power spectrum
- constraints on modified gravity will be one or more orders of magnitude better than any other planned experiment

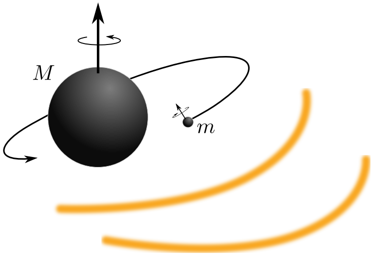
Astrophysics

- constrain mass function $n(M)$ (number of black holes with given mass)
- provide information about stellar environment around massive BHs

Cosmology

- measure Hubble constant to $\sim 1\%$

More on EMRI modeling: why self-force?



- highly relativistic, strong fields
- disparate lengthscales
- long timescale: inspiral is slow, produces $\sim \frac{M}{m} \sim 10^5$ wave cycles

- treat m as source of perturbation of M's metric $g_{\mu\nu}$:

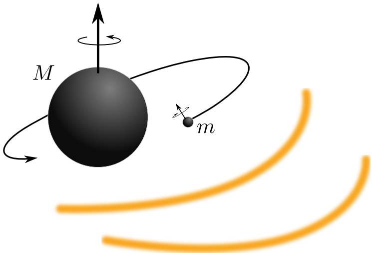
$$\mathsf{g}_{\mu\nu} = g_{\mu\nu} + \epsilon h^1_{\mu\nu} + \epsilon^2 h^2_{\mu\nu} + \dots$$

  where $\epsilon \sim m/M$

- represent motion of m via worldline z^μ satisfying

$$\frac{D^2 z^\mu}{d\tau^2} = \epsilon F^\mu_1 + \epsilon^2 F^\mu_2 + \dots$$

More on EMRI modeling: why self-force?



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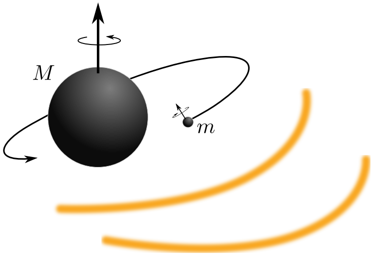

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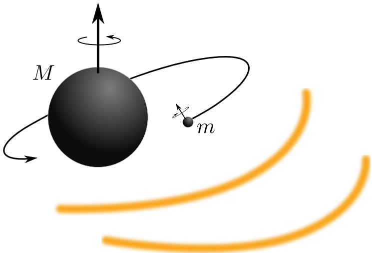

- highly relativistic, strong fields
  ⇒ *can't use post-Newtonian theory*
- disparate lengthscales
  ⇒ *can't use numerical relativity*
- long timescale: inspiral is slow, produces $\sim \frac{M}{m} \sim 10^5$ wave cycles
  ⇒ *need a model that is accurate to* $\ll 1$ *radian over those* $\sim 10^5$ *cycles*

- treat m as source of perturbation of M's metric $g_{\mu\nu}$:

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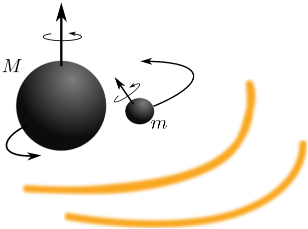
One more: intermediate-mass-ratio inspirals (IMRIs)

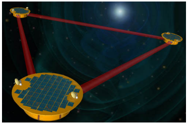


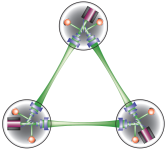
Science

- observable by ground-based (LIGO/Virgo) and space-based (LISA/DECIGO) detectors

Modeling

- pushes the limits of NR
- pushes the limits of self-force





Outline

1. Binaries and gravitational waves
2. EMRI model requirements
3. Self-force theory: the local problem
4. Self-force theory: the global problem
    - First order
    - Second order

Outline

How high order?

$$\frac{D^2 z^\mu}{d\tau^2} = \epsilon F_1^\mu + \epsilon^2 F_2^\mu + \dots$$

- force is small; inspiral occurs very slowly, on time scale $\tau \sim 1/\epsilon$
- suppose we neglect F_2^μ; leads to error $\delta\left(\frac{D^2 z^\mu}{d\tau^2}\right) \sim \epsilon^2$

  $\Rightarrow$ error in position $\delta z^\mu \sim \epsilon^2 \tau^2$

  $\Rightarrow$ after time $\tau \sim 1/\epsilon$, error $\delta z^\mu \sim 1$

$\therefore$ accurately describing orbital evolution requires second order

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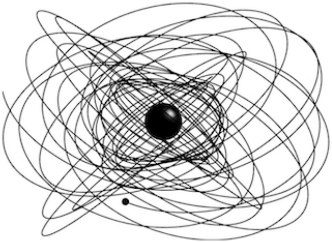
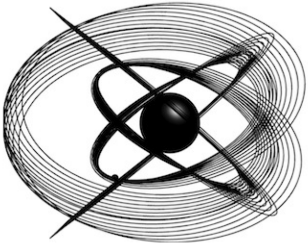
Zeroth-order approximation: geodesics in Kerr



[image courtesy of Steve Drasco]

- geodesic characterized by three constants of motion:
  1. energy E
  2. angular momentum L_z
  3. Carter constant Q, related to orbital inclination

- E, L_z, Q related to frequencies of r, ϕ, and θ motion
- *resonances* occur when two frequencies have a rational ratio



[image courtesy of Steve Drasco]

Hierarchy of self-force models [Hinderer and Flanagan]

- when self-force is accounted for, E, L_z, and Q evolve with time
- on an inspiral timescale $t \sim 1/\epsilon$, the phase of the gravitational wave has an expansion (excluding resonances)

$$\phi = \frac{1}{\epsilon}\left[\phi_0 + \epsilon\phi_1 + O\big(\epsilon^2\big)\right]$$

- a model that gets ϕ_0 right should be enough for signal detection
- a model that gets both ϕ_0 and ϕ_1 should be enough for parameter extraction

Hierarchy of self-force models [Hinderer and Flanagan]

Adiabatic order

determined by

- averaged dissipative piece of F_1^{μ}

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Hierarchy of self-force models [Hinderer and Flanagan]

Adiabatic order

determined by

- averaged dissipative piece of F_1^μ

Post-adiabatic order

determined by

- averaged dissipative piece of F_2^μ
- conservative piece of F_1^μ
- oscillatory dissipative piece of F_1^μ

- when self-force is accounted for, E, L, and Q evolve with time
- on an inspiral timescale $t \sim 1/\epsilon$, the phase of the gravitational wave has an expansion (excluding resonances)

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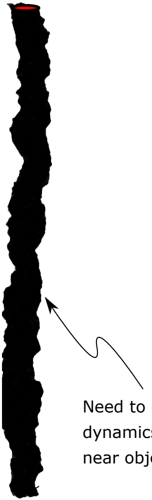
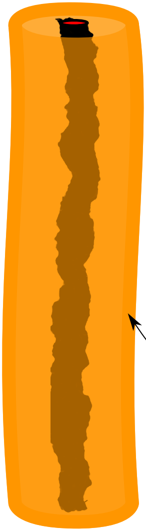
What is the problem we want to solve?



A small, compact object of mass and size $m \sim l \sim \epsilon$ moves through (and influences) spacetime

- Option 1: tackle the problem directly, treat the body as finite sized, deal with its internal composition

What is the problem we want to solve?



A small, compact object of mass and size $m \sim l \sim \epsilon$ moves through (and influences) spacetime

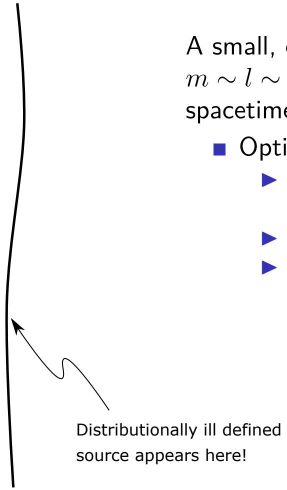
- Option 2: restrict the problem to distances $s \gg m$ from the object, treat m as source of perturbation of external background $g_{\mu\nu}$:

$$\mathsf{g}_{\mu\nu} = g_{\mu\nu} + \epsilon h^1_{\mu\nu} + \epsilon^2 h^2_{\mu\nu} + \dots$$

- This is a free boundary value problem

Metric here must agree with metric outside a small compact object; and "here" moves in response to field

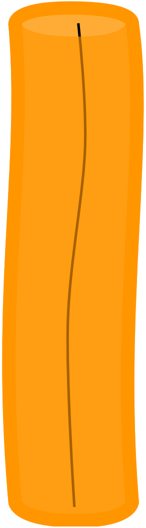
What is the problem we want to solve?



A small, compact object of mass and size $m \sim l \sim \epsilon$ moves through (and influences) spacetime

- Option 3: treat the body as a point particle
  - takes behavior of fields outside object and extends it down to a fictitious worldline
  - so $h^1_{\mu\nu} \sim 1/s$ (s =distance from object)
  - second-order field equation $\delta G[h^2] \sim -\delta^2 G[h^1] \sim (\partial h^1)^2 \sim 1/s^4$ —no solution unless we restrict it to points off worldline, which is equivalent to FBVP

What is the problem we want to solve?

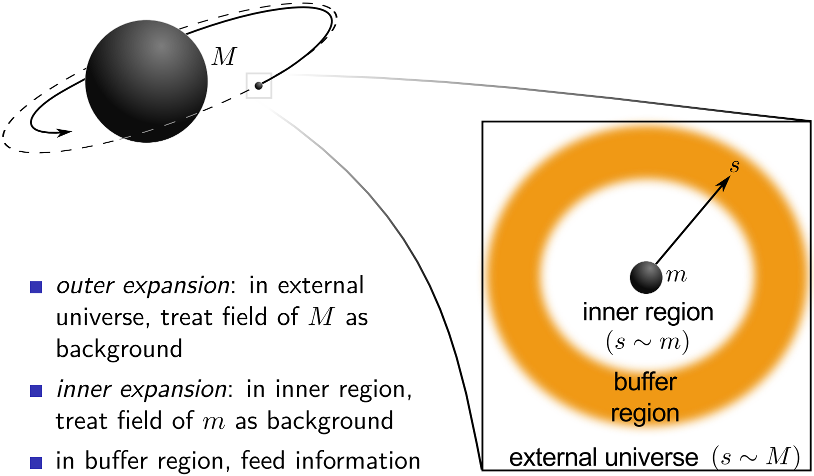


A small, compact object of mass and size $m \sim l \sim \epsilon$ moves through (and influences) spacetime

- Option 4: transform the FBVP into an *effective* problem using a *puncture*, a local approximation to the field outside the object
- This will be the method emphasized here

[Mino, Sasaki, Tanaka 1996; Quinn & Wald 1996; Detweiler & Whiting 2002-03; Gralla & Wald 2008-2012; Pound 2009-2017; Harte 2012]

Matched asymptotic expansions

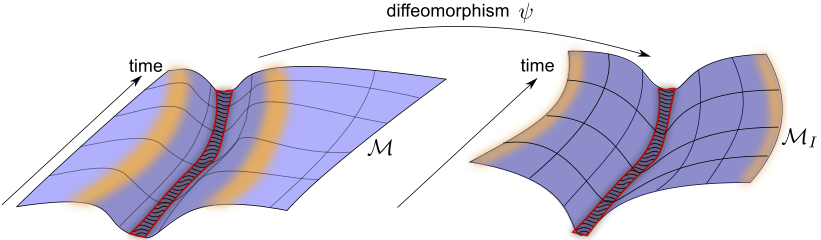


- *outer expansion*: in external universe, treat field of M as background
- *inner expansion*: in inner region, treat field of m as background
- in buffer region, feed information from inner expansion into outer expansion

The *inner expansion*

Zoom in on object

- unperturbed object defines background spacetime $g^{\rm obj}_{\mu\nu}$ in inner expansion
- buffer region at asymptotic infinity $s \gg m$
  $\Rightarrow$ can define object's multipole moments as those of $g^{\rm obj}_{\mu\nu}$



General solution in buffer region

General solution compatible with existence of inner expansion [Pound 2009, 2012]:

First order

- $h^{(1)}_{\mu\nu} = h^{S(1)}_{\mu\nu} + h^{R(1)}_{\mu\nu}$
- $h^{S(1)}_{\mu\nu} \sim 1/s + O(s^0)$ defined by mass monopole m
- $h^{R(1)}_{\mu\nu}$ is undetermined homogenous solution regular at $s = 0$
- evolution equations: $\dot{m} = 0$ and $a^\mu_{(0)} = 0$

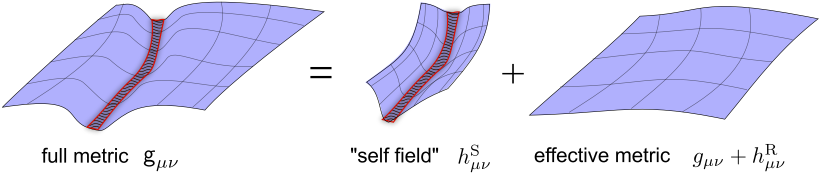
  (where $\frac{D^2 z^\mu}{d\tau^2} = a^\mu_{(0)} + \epsilon a^\mu_{(1)} + \ldots$)

Second order

- $h^{(2)}_{\mu\nu} = h^{S(2)}_{\mu\nu} + h^{R(2)}_{\mu\nu}$
- $h^{S(2)}_{\mu\nu} \sim 1/s^2 + O(1/s)$ defined by
  1. monopole correction δm
  2. mass dipole M^μ (set to zero)
  3. spin dipole S^μ
- evolution equations: $\dot{S}^\mu = 0$, $\dot{\delta m} = \ldots$, and $a^\mu_{(1)} = \ldots$

Self-field and effective field

- we've locally split metric into a "self-field" and an effective metric

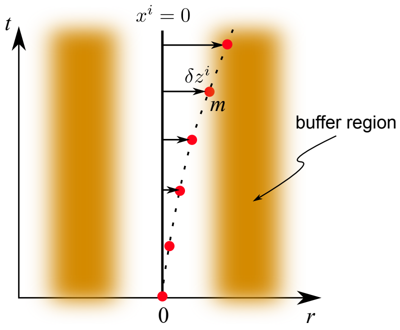


- $h^{S}_{\mu\nu}$ directly determined by object's multipole moments
- $g_{\mu\nu} + h^{R}_{\mu\nu}$ is a *smooth vacuum metric* determined by global boundary conditions

Defining object's position [Mino et al, Gralla-Wald, Pound]

Reminder: mass dipole moment M^i:

- small displacement of center of mass from origin of coordinates
- e.g., Newtonian field $\dfrac{m}{|x^i - \delta z^i|} \approx \dfrac{m}{|x^i|} + \dfrac{m\delta z^j n_j}{|x^i|^2} \ \Rightarrow\ M^i = m\delta z^i$



Definition of object's worldline:

- work in coordinates (t, x^i) centered on a curve γ
- equation of motion of z^μ: whatever ensures $M^\mu \equiv 0$

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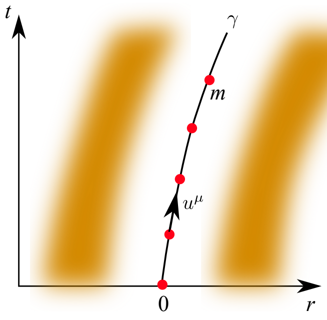

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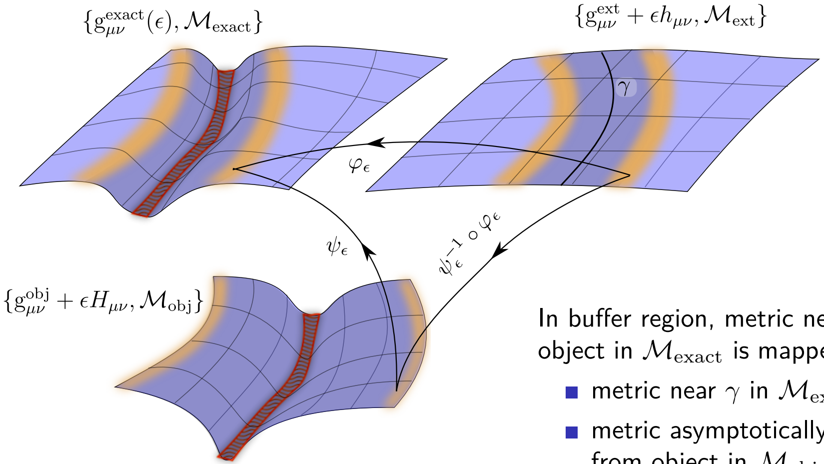
Where is the worldline?



In buffer region, metric near object in $\mathcal{M}_{\rm exact}$ is mapped to

- metric near γ in $\mathcal{M}_{\rm ext}$
- metric asymptotically far from object in $\mathcal{M}_{\rm obj}$

0th-, 1st-, and 2nd-order equations of motion

0th order, arbitrary object: $\frac{D^2 z^\mu}{d\tau^2} = O(m)$ (geodesic motion in $g_{\mu\nu}$)

1st order, arbitrary compact object [MiSaTaQuWa 1996]:

$$\frac{D^2 z^\mu}{d\tau^2} = -\tfrac{1}{2}\left(g^{\alpha\delta} + u^\alpha u^\delta\right)\left(2h^{\rm R1}_{\delta\beta;\gamma} - h^{\rm R1}_{\beta\gamma;\delta}\right)u^\beta u^\gamma + \tfrac{1}{2m}R^\alpha{}_{\beta\gamma\delta}u^\beta S^{\gamma\delta} + O(m^2)$$

(motion of spinning test body in $g_{\mu\nu} + h^{\rm R1}_{\mu\nu}$)

2nd-order, nonspinning, spherical compact object [Pound 2012]:

$$\frac{D^2 z^\mu}{d\tau^2} = -\tfrac{1}{2}\left(g^{\mu\nu} + u^\mu u^\nu\right)\left(g_\nu{}^\rho - h^{\rm R}_\nu{}^\rho\right)\left(2h^{\rm R}_{\rho\sigma;\lambda} - h^{\rm R}_{\sigma\lambda;\rho}\right)u^\sigma u^\lambda + O(m^3)$$

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- all these results are derived *directly from EFE outside the object*; there's no regularization of infinities, and no assumptions about $h^{\rm R}_{\mu\nu}$

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$$\frac{D^2 z^\mu}{d\tau^2} = -\tfrac{1}{2}\left(g^{\mu\nu} + u^\mu u^\nu\right)\left(g_\nu{}^\rho - h^{\rm R}_\nu{}^\rho\right)\left(2h^{\rm R}_{\rho\sigma;\lambda} - h^{\rm R}_{\sigma\lambda;\rho}\right) u^\sigma u^\lambda + O(m^3)$$

(geodesic motion in $g_{\mu\nu} + h^{\rm R}_{\mu\nu}$)

- all these results are derived *directly from EFE outside the object*; there's no regularization of infinities, and no assumptions about $h^{\rm R}_{\mu\nu}$

0th-, 1st-, and 2nd-order equations of motion

0th order, arbitrary object: $\frac{D^2 z^\mu}{d\tau^2} = O(m)$ (geodesic motion in $g_{\mu\nu}$)

1st order, arbitrary compact object [MiSaTaQuWa 1996]:

$$\frac{D^2 z^\mu}{d\tau^2} = -\tfrac{1}{2}\left(g^{\alpha\delta} + u^\alpha u^\delta\right)\left(2h^{\rm R1}_{\delta\beta;\gamma} - h^{\rm R1}_{\beta\gamma;\delta}\right) u^\beta u^\gamma + \tfrac{1}{2m} R^\alpha{}_{\beta\gamma\delta} u^\beta S^{\gamma\delta} + O(m^2)$$

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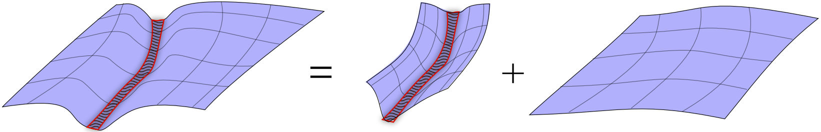
Point particles and punctures [Barack et al, Detweiler, Gralla-Wald, Pound]

- replace "self-field" with "singular field"



- at 1st order, can use this to *replace object with a point particle*

$$T^1_{\mu\nu} := \frac{1}{8\pi}\delta G_{\mu\nu}[h^1] \sim m\delta(x-z)$$

- beyond 1st order, point particles not well defined—but can replace object with a *puncture*, a local singularity in the field, moving on γ, equipped with the object's multipole moments

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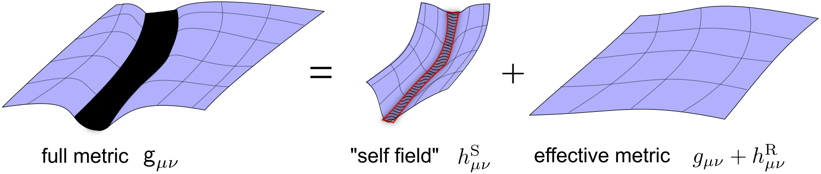

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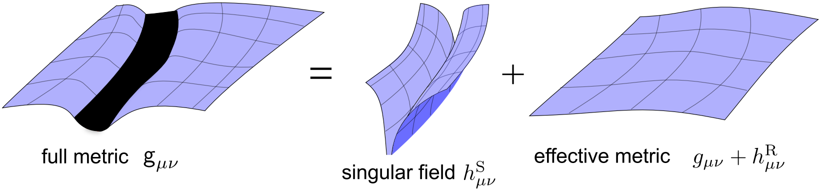

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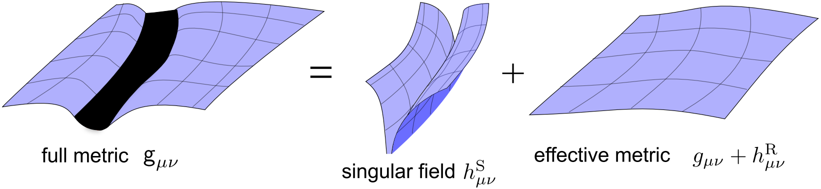

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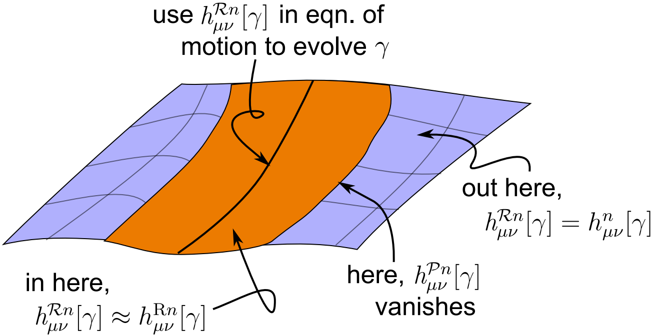
How you replace an object with a puncture

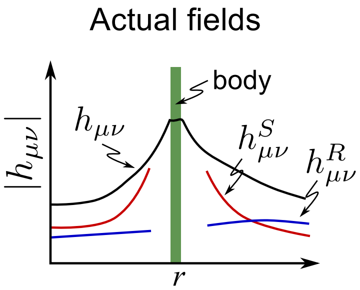
- use a local expansion of $h^{\mathrm{S}n}_{\mu\nu}$ as a puncture $h^{\mathcal{P}n}_{\mu\nu}$ that moves on γ
- transition $h^{\mathcal{P}n}_{\mu\nu}$ to zero at some distance from γ, solve field equations for the *residual field*

$$h^{\mathcal{R}n}_{\mu\nu} := h^{n}_{\mu\nu} - h^{\mathcal{P}n}_{\mu\nu}$$

- move the puncture with eqn of motion (using $\partial h^{\mathcal{R}n}_{\mu\nu}|_\gamma = \partial h^{\mathrm{R}n}_{\mu\nu}|_\gamma$)



More on puncturing



- Note: self-force literature often speaks of "regularizing" singular fields and forces
- but we *introduce* the singular field as a tool to compute a specific regular field
- self-force theory *does not involve regularizing divergent quantities*

Outline

Solving the Einstein equations globally

- solving the local problem told us how to replace the small object with a moving puncture in the field equations:

$$\delta G_{\mu\nu}[h^{\mathcal{R}1}] = -\delta G_{\mu\nu}[h^{\mathcal{P}1}]$$
$$\delta G_{\mu\nu}[h^{\mathcal{R}2}] = -\delta^2 G_{\mu\nu}[h^1, h^1] - \delta G_{\mu\nu}[h^{\mathcal{P}2}]$$
$$\frac{D^2 z^\mu}{d\tau^2} = -\frac{1}{2}(g^{\mu\nu} + u^\mu u^\nu)(g_\nu{}^\delta - h_\nu^{\mathcal{R}\delta})(2h^{\mathcal{R}}_{\delta\beta;\gamma} - h^{\mathcal{R}}_{\beta\gamma;\delta})u^\beta u^\gamma$$

where $\delta G_{\mu\nu}[h] \sim \Box h_{\mu\nu}$, $\delta^2 G_{\mu\nu}[h,h] \sim \partial h \partial h + h \partial^2 h$

- the global problem: how do we solve these equations in practice?

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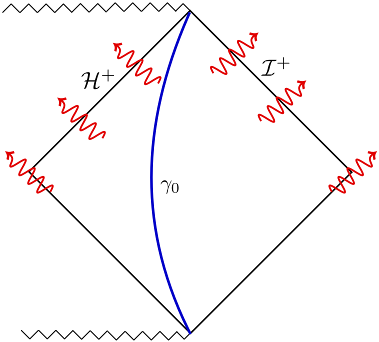
Typical calculation at first order [Barack et al, van de Meent, many others]



- approximate the source orbit as a bound geodesic
- impose outgoing-wave BCs at $\mathcal{I}^+$ and $\mathcal{H}^+$
- solve field equation numerically, compute self-force from solution
- system radiates forever; at any given time, BH has already absorbed infinite energy
- but on short sections of time the approximation is accurate
- breaks down on *dephasing time* $\sim 1/\sqrt{\epsilon}$, when $|z^\mu - z_0^\mu| \sim M$

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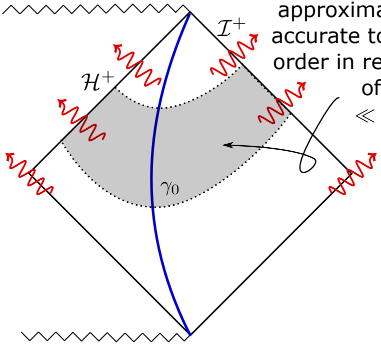

Geodesic approximation accurate to 1st order in region of size $\ll 1/\sqrt{\epsilon}$

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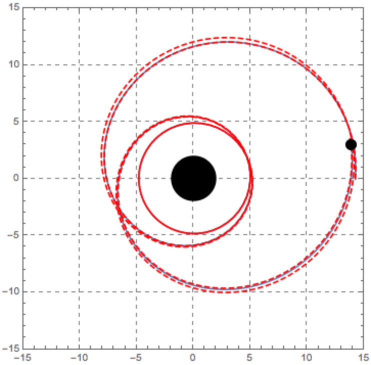
First-order results: orbital evolution

- adiabatic evolution schemes in Kerr already devised and implemented (modulo resonances) [Mino, Drasco et al, Sago et al]



[image courtesy of Warburton]

- complete inspirals also simulated in Schwarzschild using full F_1^μ [Warburton et al]
- and F_1^μ has been computed on generic orbits in Kerr [van de Meent]
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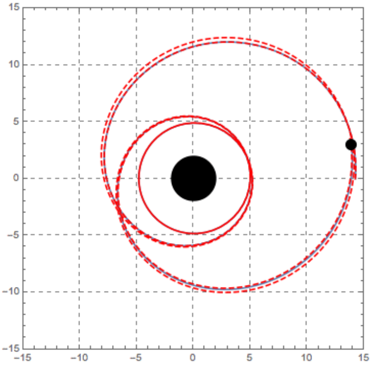

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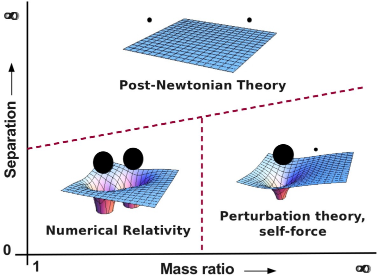
First-order results: improving other binary models

PN and EOB models have been improved using data for *conservative* effects of the self-force (computed by "turning off" dissipation)

- orbital precession [Barack et al., van de Meent]
- ISCO shift [Barack and Sago, Isoyama et al.]
- Detweiler's redshift invariant $\frac{dt}{d\tau^R}$ on circular orbits [Detweiler, Shah et al., Dolan and Barack]
- averaged redshift $\left\langle \frac{dt}{d\tau^R} \right\rangle$ on eccentric orbits [Barack et al., van de Meent & Shah]
- spin precession [Dolan et al, Bini et al]
- quadrupolar and octupolar self-tides [Dolan et al, Damour and Bini]

Binary parameter space

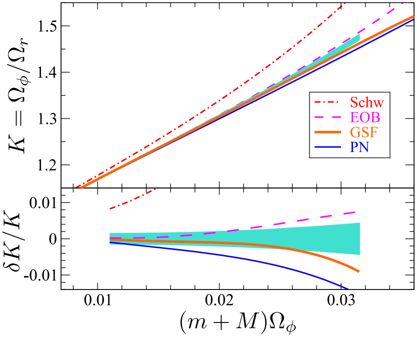


[image courtesy of Leor Barack]

First-order results: using SF to *directly* model IMRIs

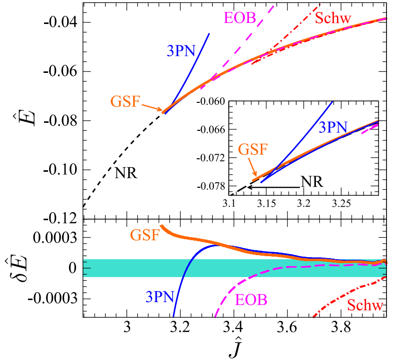
Comparisons for **equal-mass** binaries

Orbital precession



[Le Tiec et al]

Gravitational binding energy



- SF results use “mass symmetrized” model: $\frac{m}{M} \to \frac{mM}{(m+M)^2}$
- with mass-symmetrization, second-order self-force might be able to directly model even comparable-mass binaries

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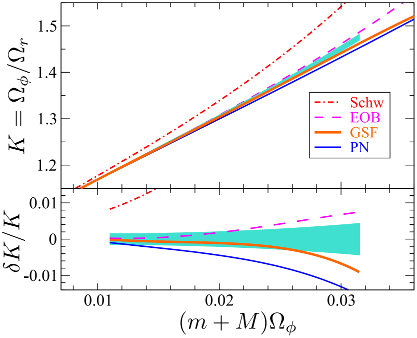

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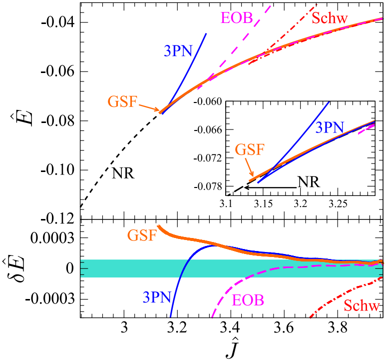

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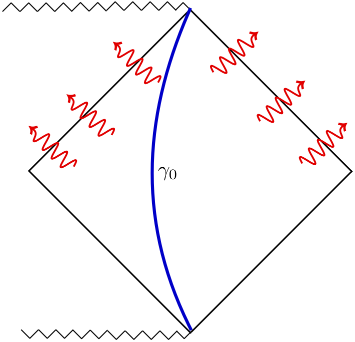
Outline

1. Binaries and gravitational waves
2. EMRI model requirements
3. Self-force theory: the local problem
4. Self-force theory: the global problem
   - First order
   - Second order

Infrared problems at second order [Pound 2015]



- suppose we try to use "typical" $h^1_{\mu\nu}$ to construct source for $h^2_{\mu\nu}$
- because $|z^\mu - z^\mu_0|$ blows up with time, $h^2_{\mu\nu}$ does likewise
- because $h^1_{\mu\nu}$ contains outgoing waves at all past times, the source $\delta^2 R_{\mu\nu}[h^1]$ decays too slowly, and *its retarded integral does not exist*
- instead, we must construct a uniform approximation
  - $h^1_{\mu\nu}$ must include evolution of orbit
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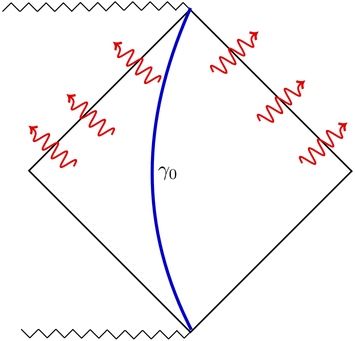

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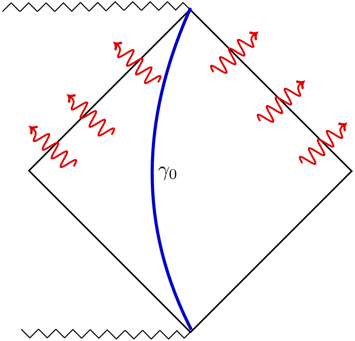

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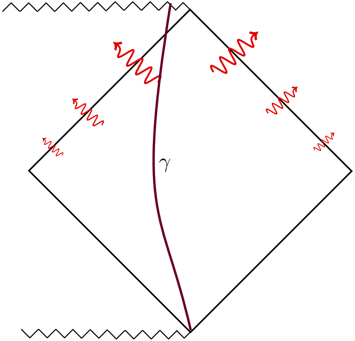

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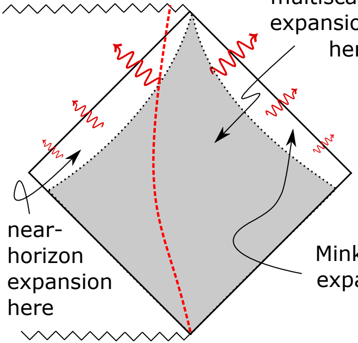
Matched expansions [Pound, Moxon, Flanagan, Hinderer, Yamada, Isoyama, Tanaka]



Multiscale expansion

- multiscale expansion: expand orbital parameters and fields as

$$J^\alpha = J_0^\alpha(\tilde{t}) + \epsilon J_1^\alpha(\tilde{t}) + \ldots$$

$$h^n_{\mu\nu} \sim \sum_{k^\alpha} h^n_{k_\alpha}(\tilde{t}) e^{-ik^\alpha q_\alpha(\tilde{t})}$$

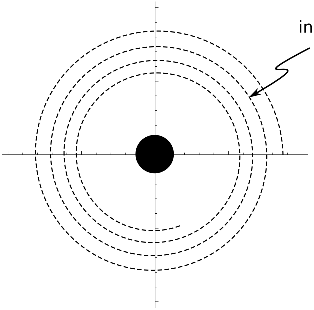
where (J^α, q_α) are action-angle variables for z^μ, and $\tilde{t} \sim \epsilon t$ is a “slow time”

- solve for $h^n_{k^\alpha}$ at fixed $\tilde{t}$ with standard frequency-domain techniques

Get boundary conditions from

- post-Minkowski expansion: expand $h^n_{\mu\nu}$ in powers of M
- near-horizon expansion: expand $h^n_{\mu\nu}$ in powers of gravitational potential near horizon

Quasicircular orbits in Schwarzschild [Pound, Wardell, Warburton, Miller]



Multiscale expansion

- expand orbital radius as $r_p = r_0(\tilde{t}) + \epsilon r_1(\tilde{t}) + \ldots$
- expand field as

$$h^n_{\mu\nu} = \sum_{ilm} h^n_{ilm}(\tilde{t}, r) e^{-im\phi_p(\tilde{t})} Y^{ilm}_{\mu\nu}$$

- use post-Minkowski and near-horizon expansions to obtain punctures at $r \gg M$ and $r \sim 2M$

- solve numerically for h^n_{ilm} at fixed $\tilde{t}$ using same frequency-domain methods as at 1st order
- evolve $\tilde{t}$ dependence using equation of motion

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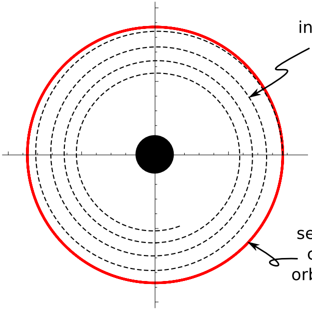

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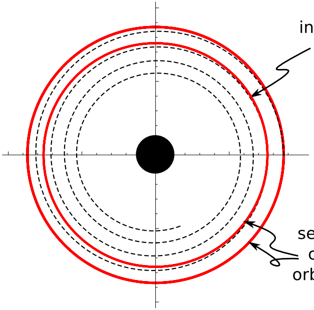

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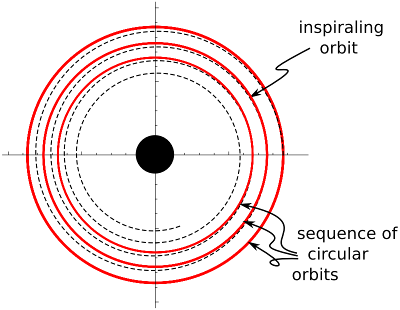

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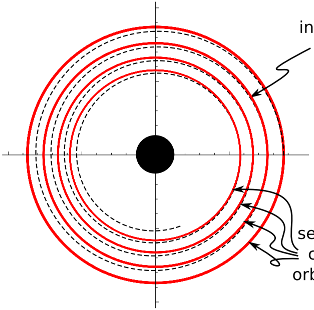

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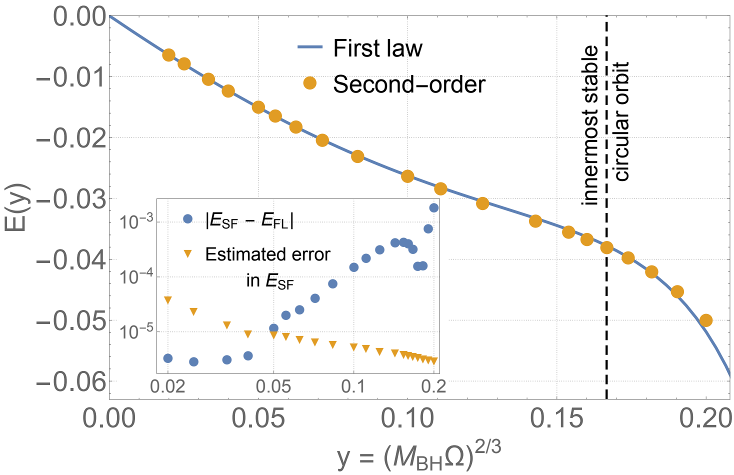
Binding energy for quasicircular orbits

Second-order piece of $E_{\rm bind} = M_{\rm Bondi} - m - M_{BH}$



Calculations performed as of 2005

		Adiabatic	1st order	2nd order
Schwarz.	circular	✓		
	generic	✓		
Kerr	circular	✓		
	generic (w/o resonances)			
	generic (w/ resonances)			**holy grail**

Calculations performed as of 2017

		Adiabatic	1st order	2nd order
Schwarz.	circular	✓	✓	
	generic	✓	✓	
Kerr	circular	✓	✓	
	generic (w/o resonances)	✓	✓	
	generic (w/ resonances)	*underway*	*underway*	**holy grail**

Calculations performed as of 2018

		Adiabatic	1st order	2nd order
Schwarz.	circular	✓	✓	✓
	generic	✓	✓	
Kerr	circular	✓	✓	
	generic (w/o resonances)	✓	✓	
	generic (w/ resonances)	*underway*	*underway*	**holy grail**

Conclusion

Modeling binaries

- self-force theory required for modeling of EMRIs; could be best model for IMRIs; improves models of comparable-mass binaries
- need second-order accuracy for modeling

Status of formalism and computations

- "local problem" solved, but still missing higher-moment effects at second order
- "global problem" solved in some cases
- wealth of numerical results at first order, computations at second order are underway

For more information, see recent review by Barack and Pound in Reports on Progress in Physics