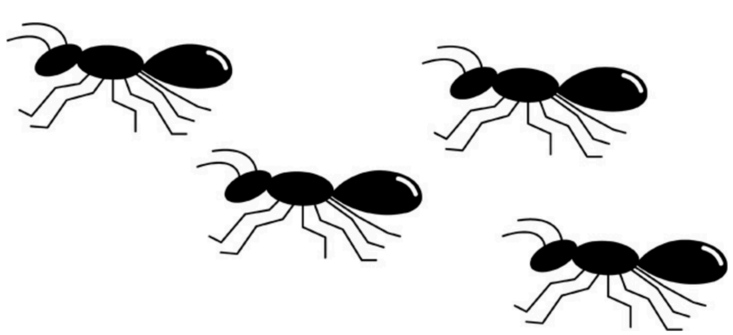
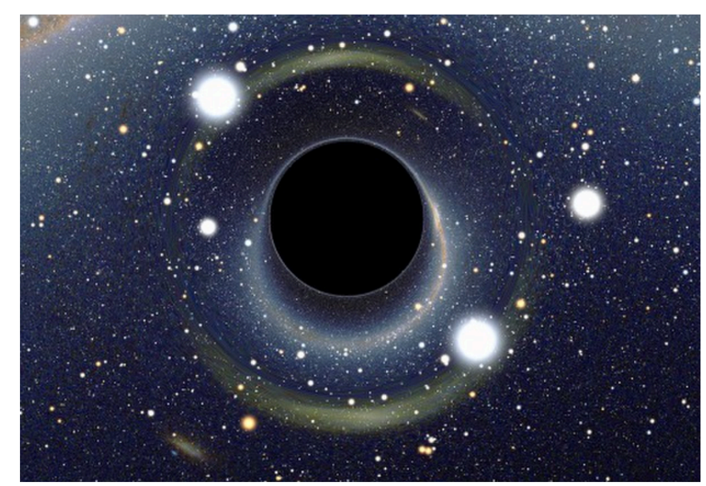
Black Hole/Ant Trail Correspondence and Partial Deconfinement

Masanori Hanada
University of Southampton

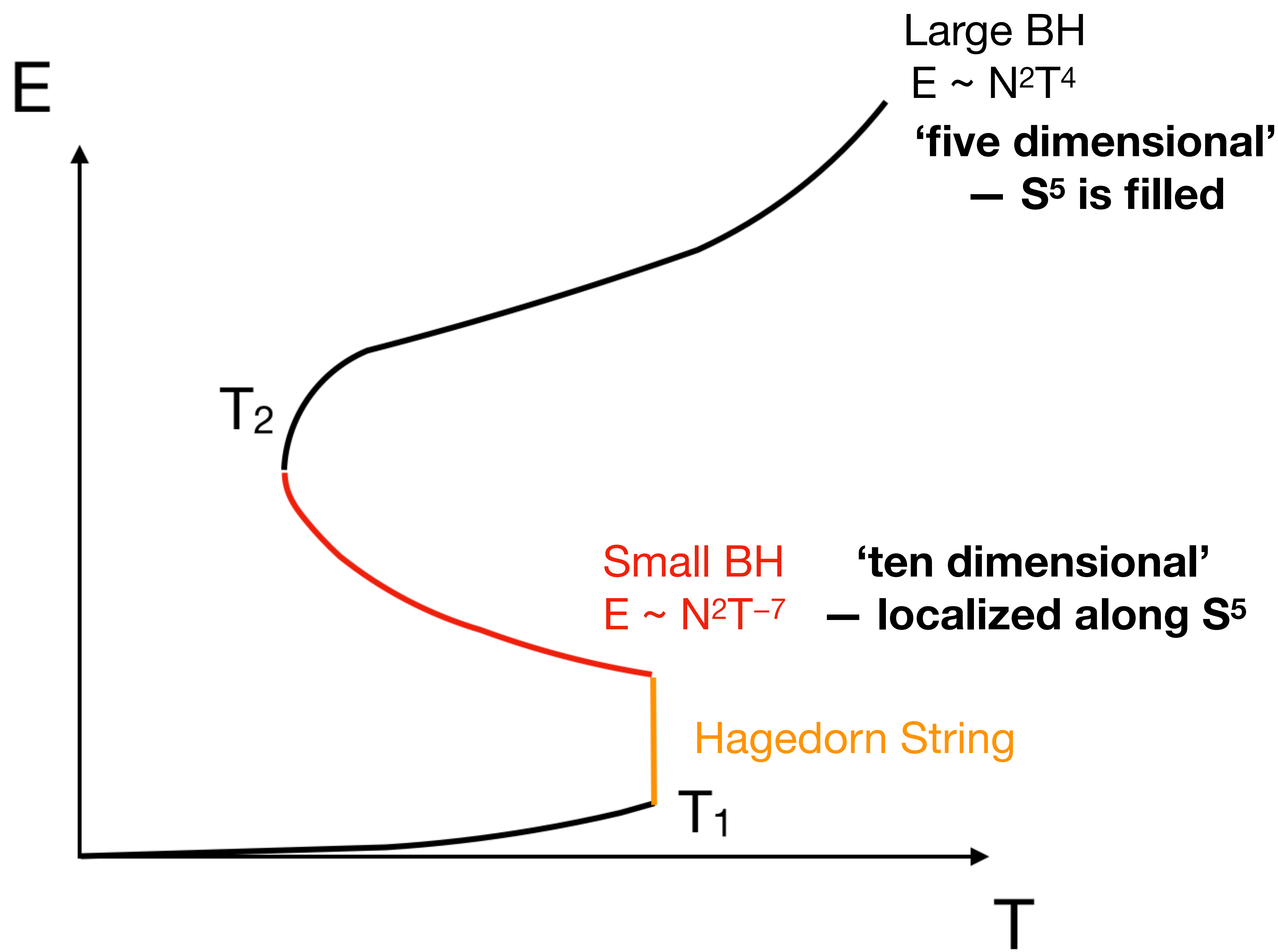




Cotler-MH-Ishiki-Watanabe, in preparation
(+ MH-Maltz, 2016)

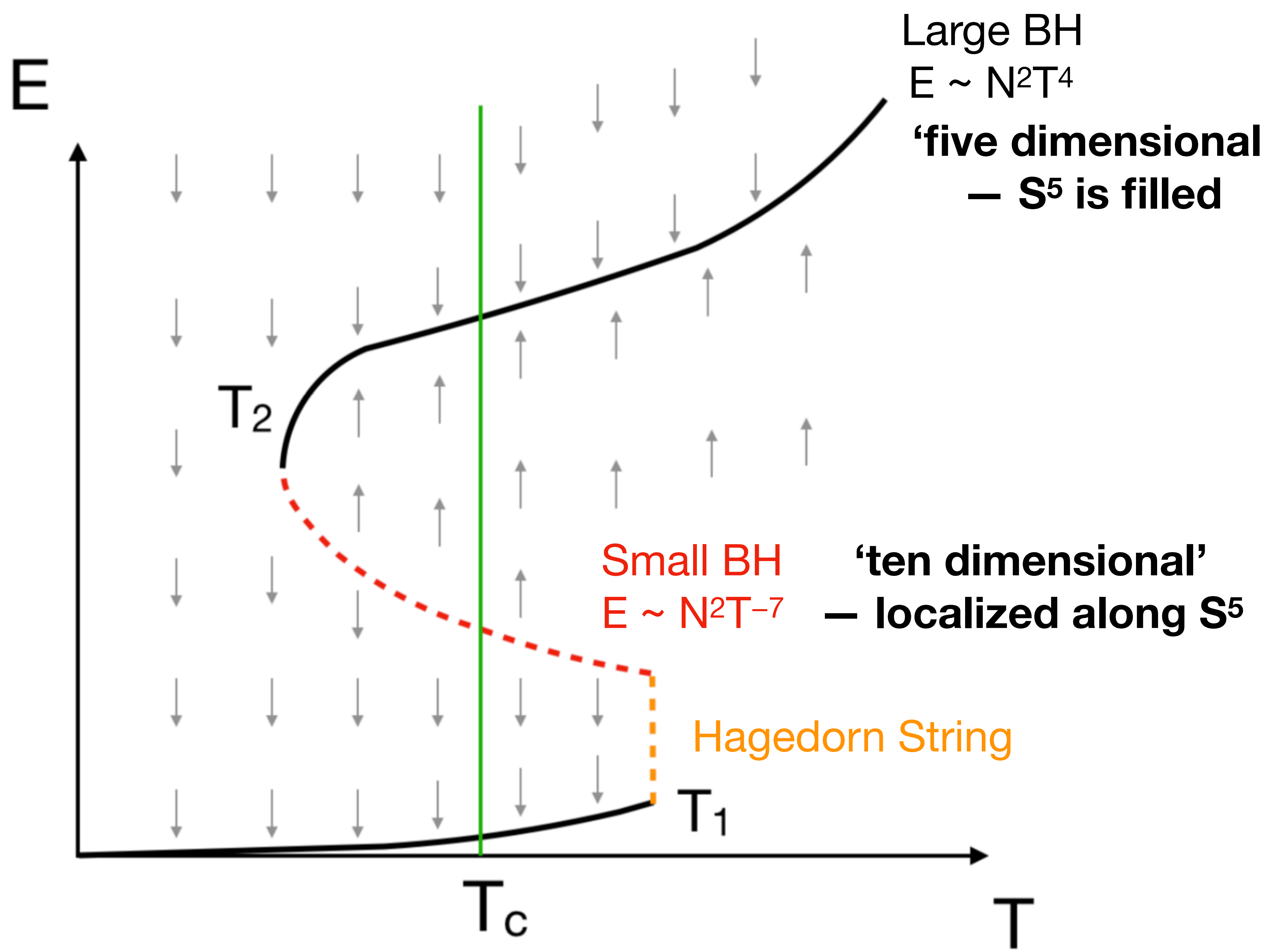
Black Hole in $AdS_5 \times S^5$ = 4d N=4 SYM on S^3

microcanonical ensemble (E fix)

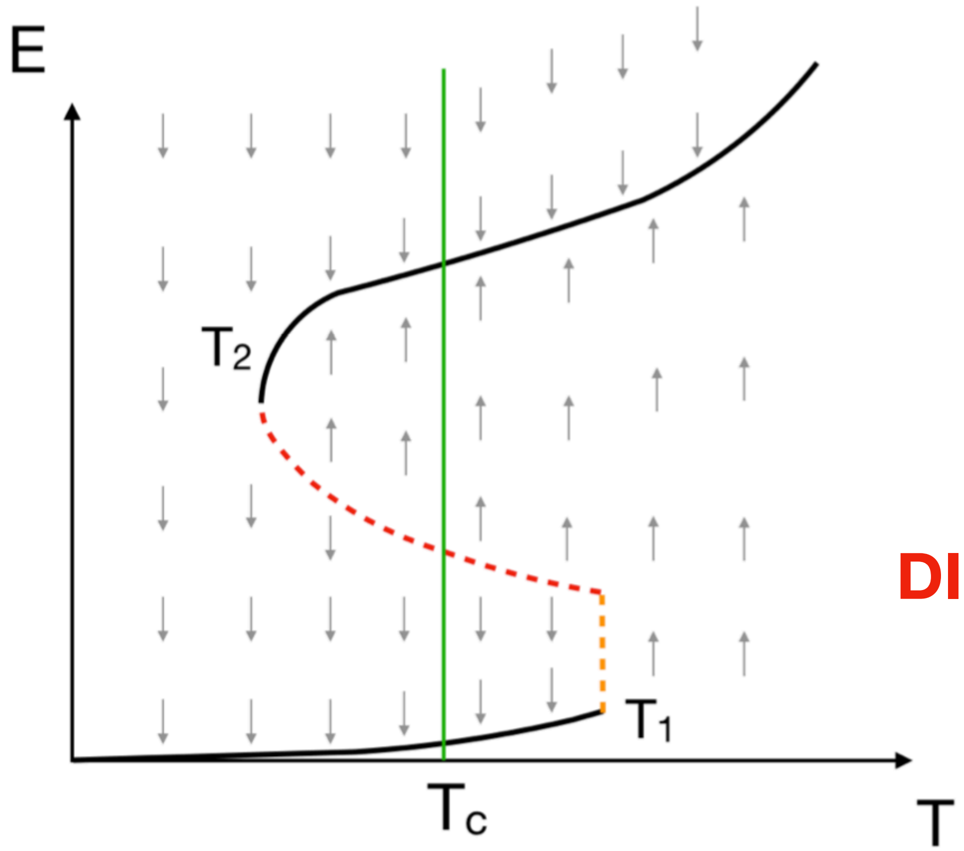


Black Hole in $AdS_5 \times S^5$ = 4d N=4 SYM on S^3

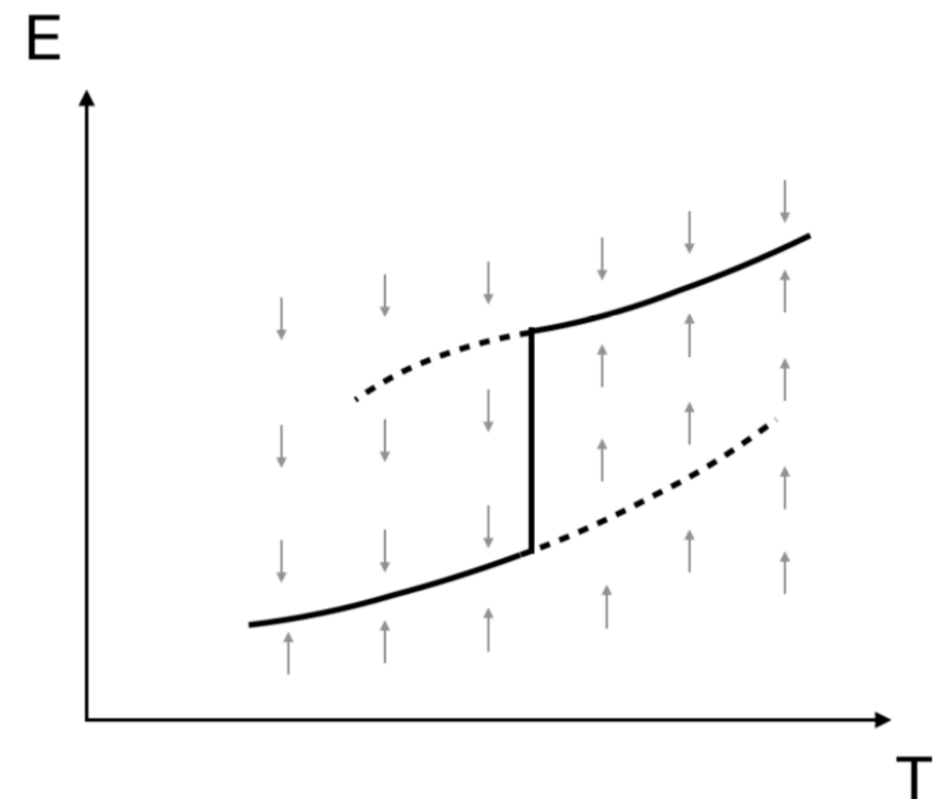
canonical ensemble (T fix)



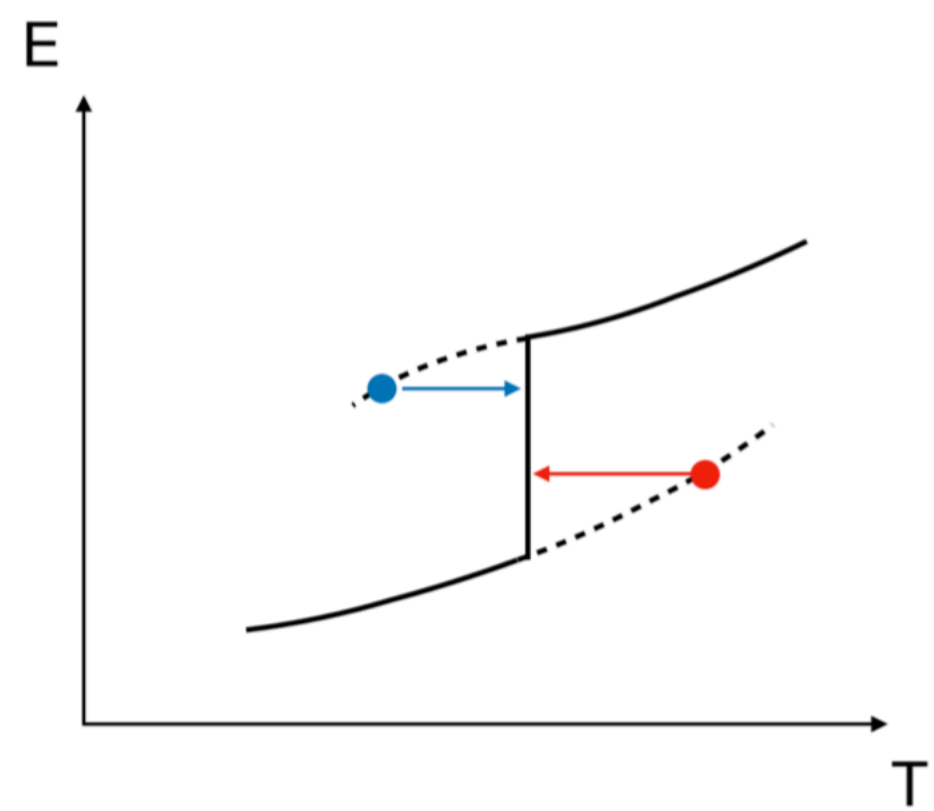
strongly coupled
4d SYM



water/ice



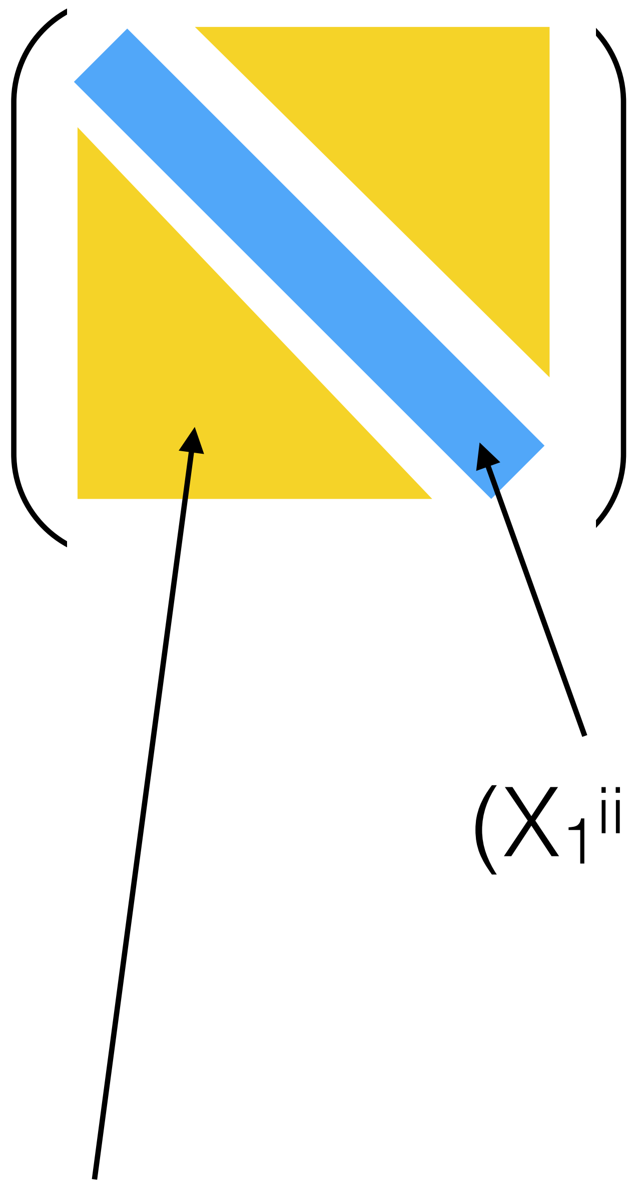
VERY DIFFERENT!

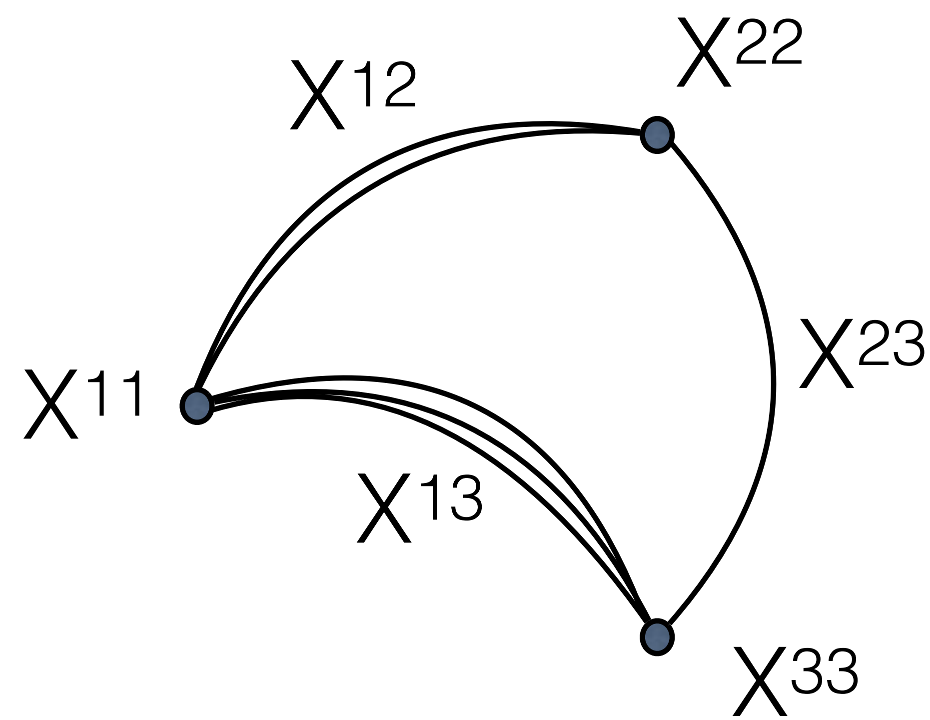


How can we explain such difference?

D-brane bound state and Gauge Theory

$X_M =$





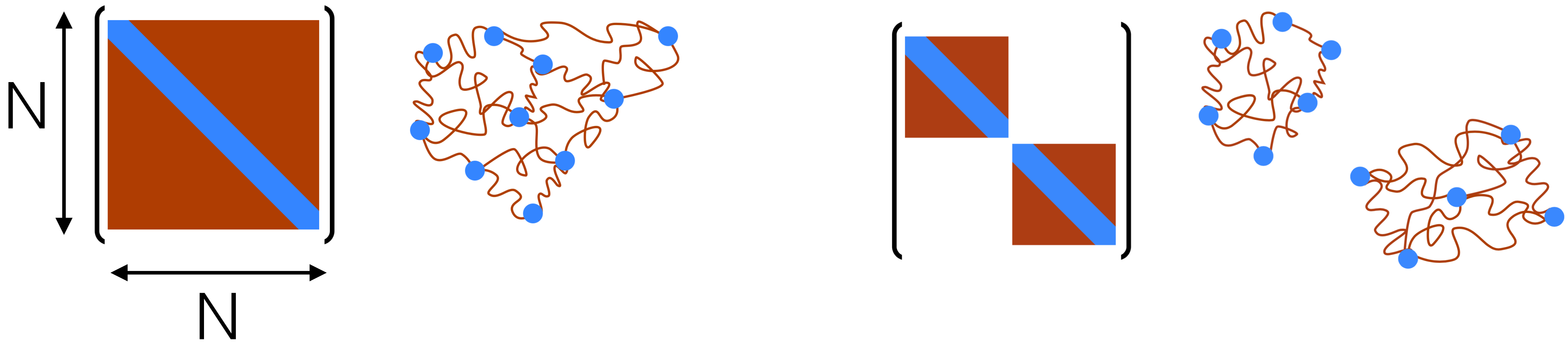
$(X_1^{ii}, X_2^{ii}, \ldots, X_6^{ii})$

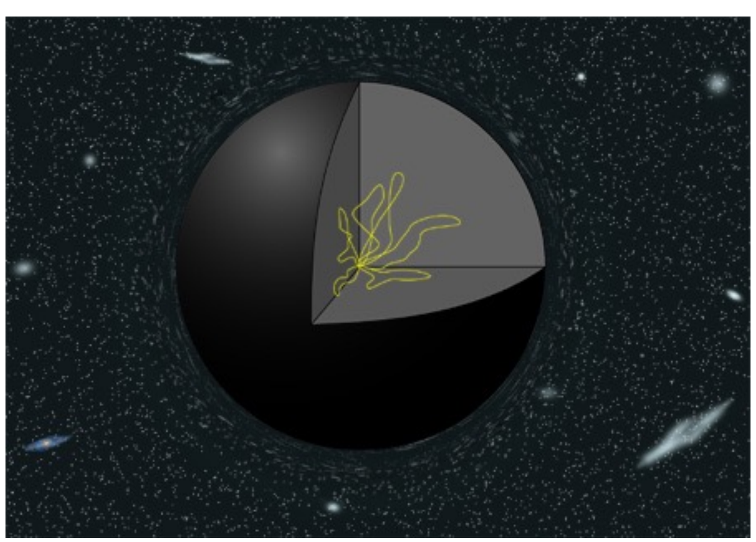
location of i-th D-brane

X_M^{ij} : open strings connecting i-th and j-th D-branes.

large value → a lot of strings are excited

(Witten, 1994)



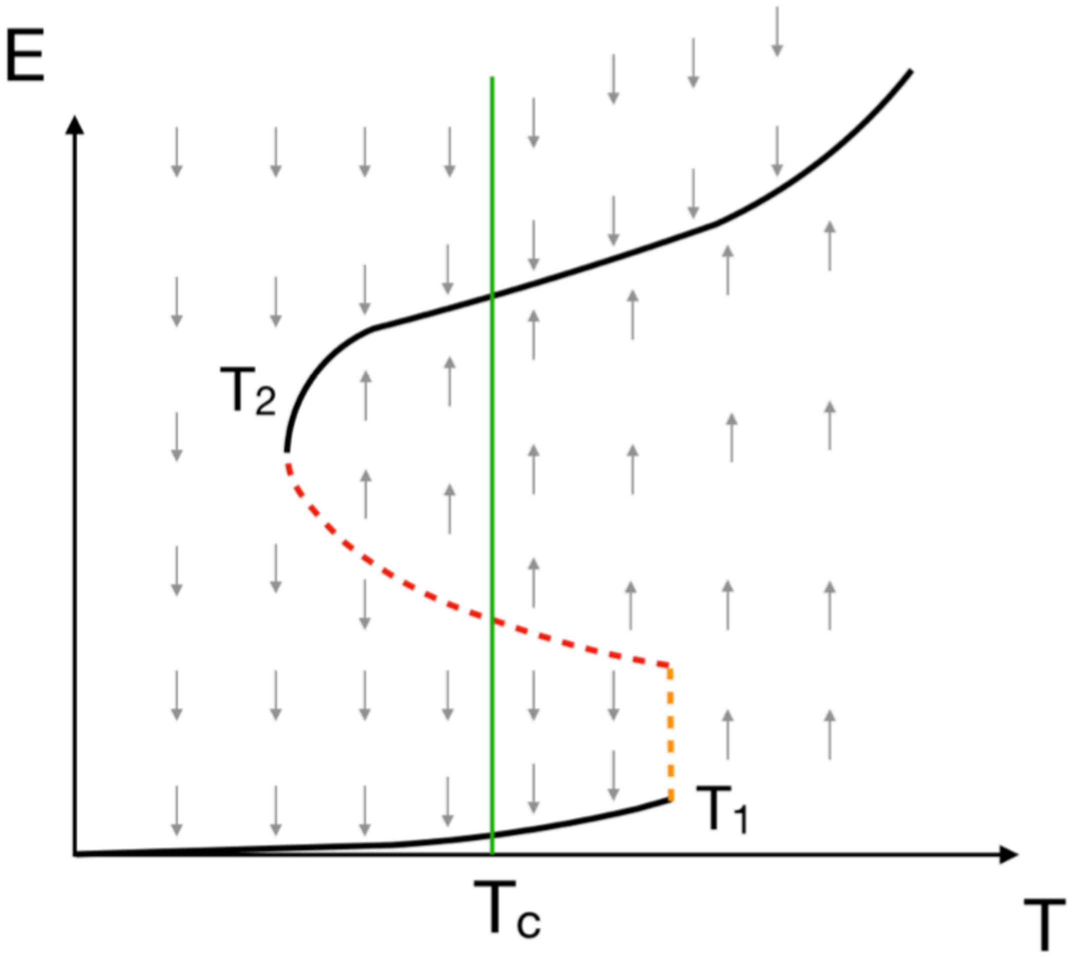


diagonal elements = particles (D-branes)
off-diagonal elements = open strings

(Witten, 1994)

black hole = bound state of D-branes and strings

strongly coupled 4d SYM



VERY DIFFERENT!

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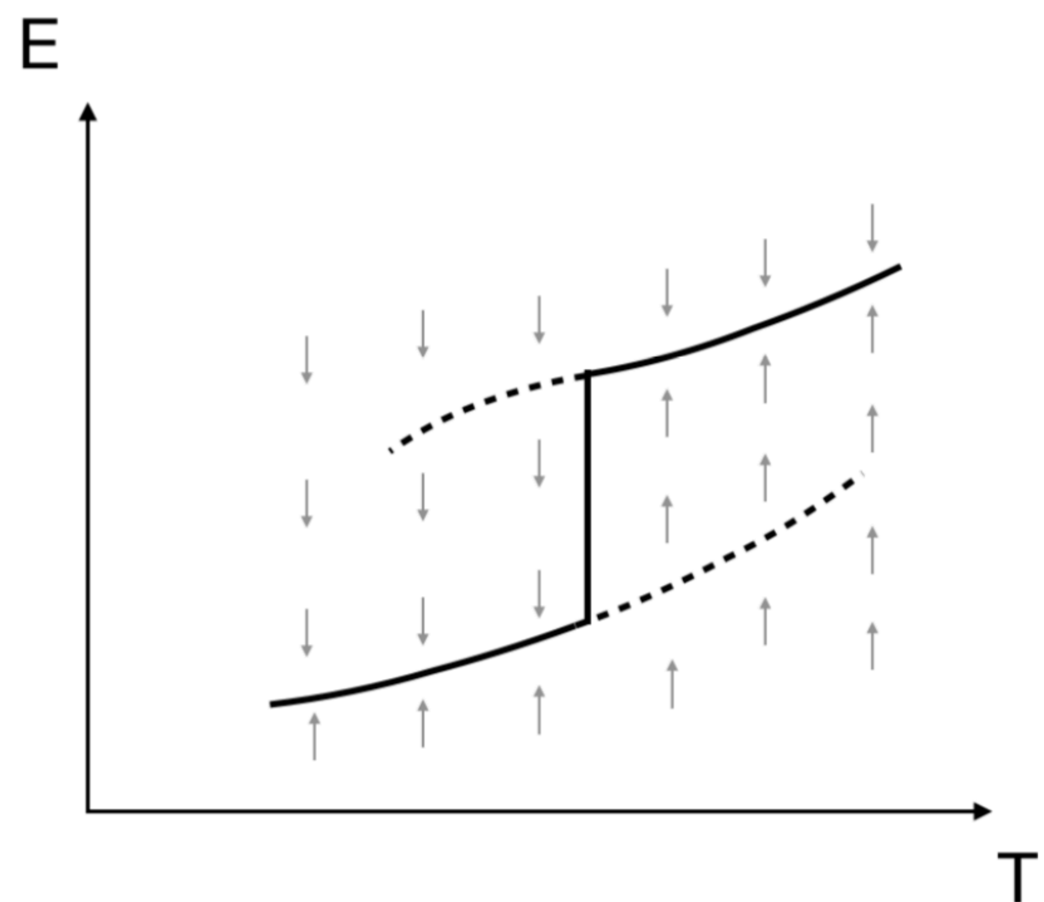
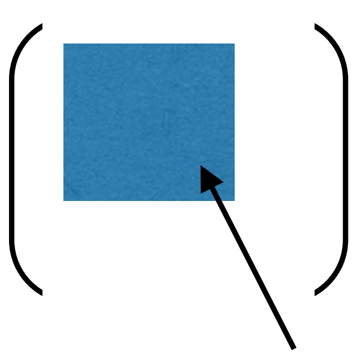
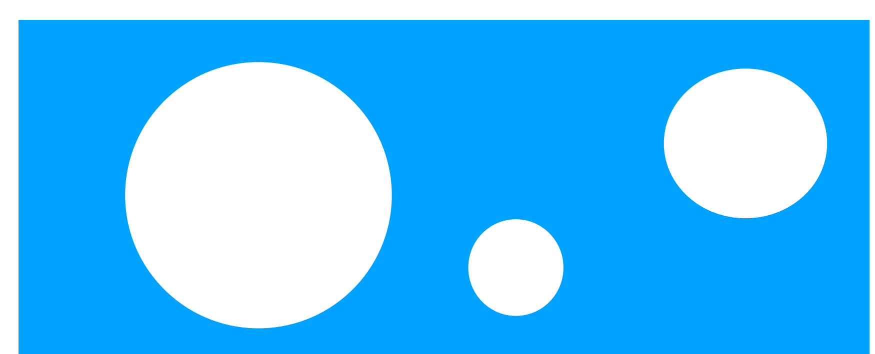
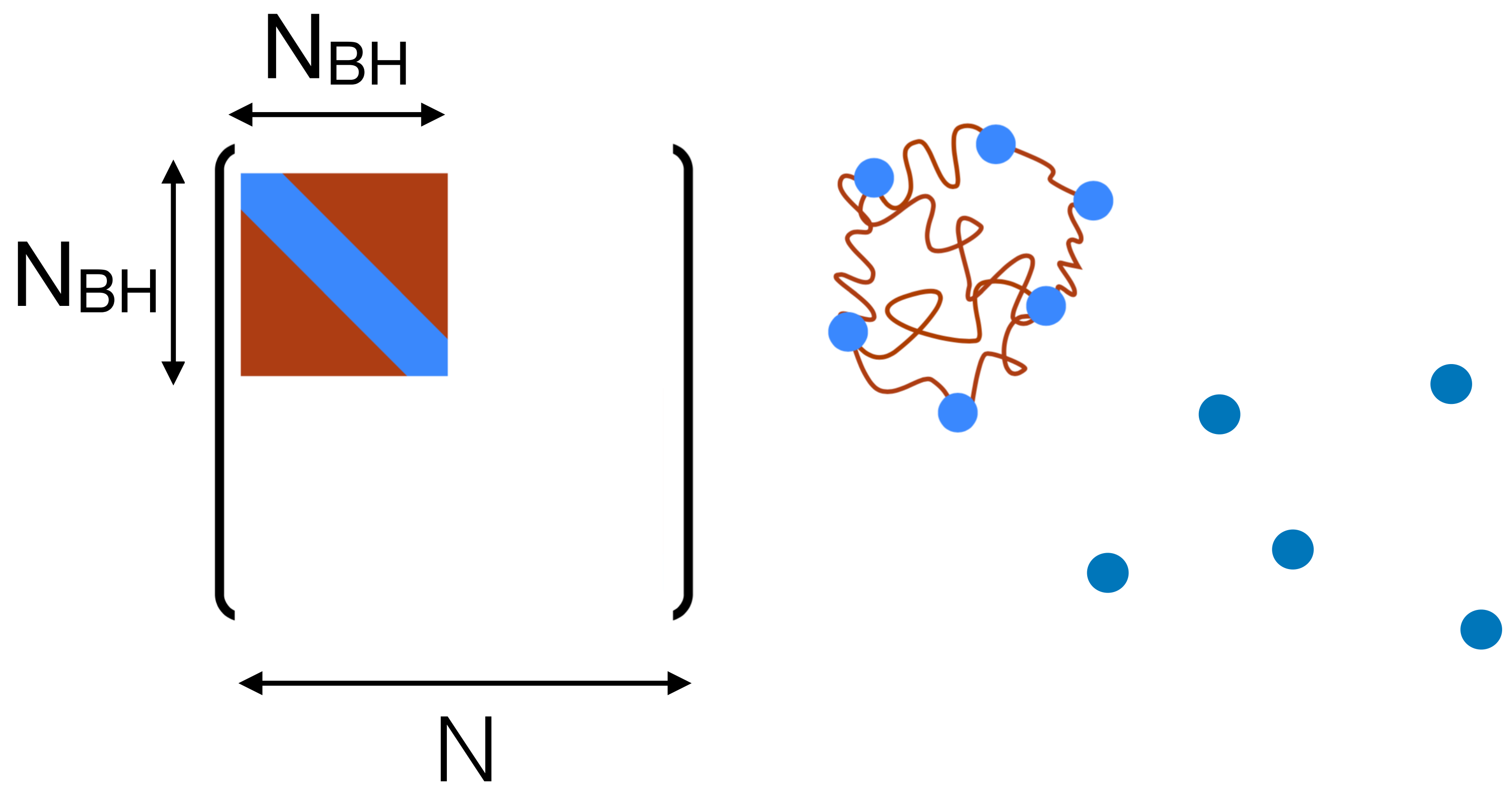


separation in color d.o.f



separation in space





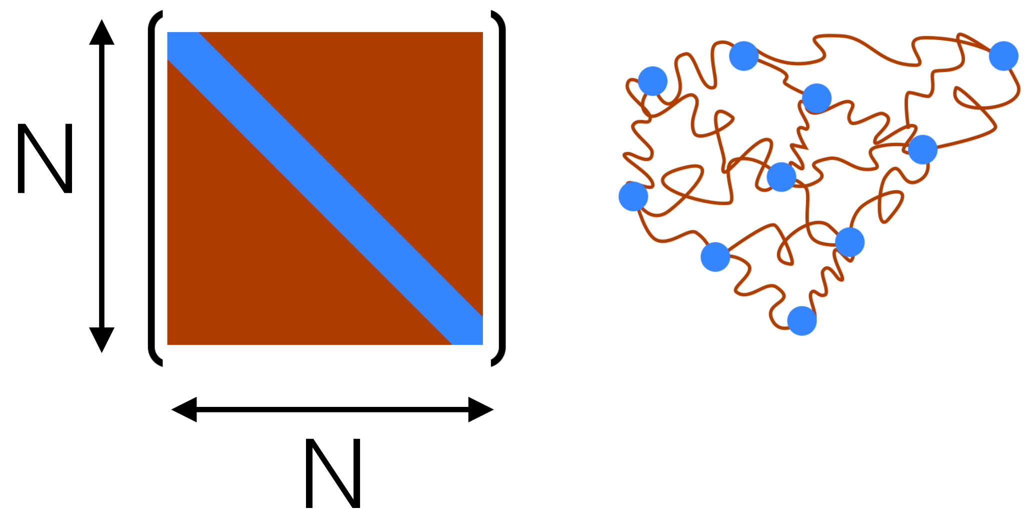
N_{BH} D-branes form the bound state

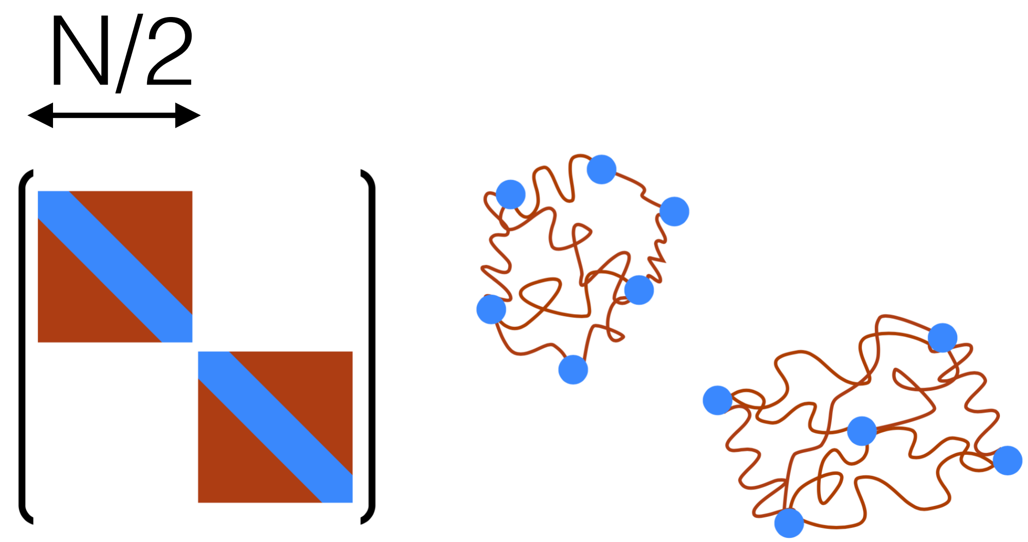
$U(N_{BH})$ is deconfined — ‘partial deconfinement’

Can explain $E \sim N^2T^{-7}$ for 4d SYM, N^2T^{-8} for ABJM

(String Theory → 10d) (M-Theory → 11d)

Why can negative specific heat appear?





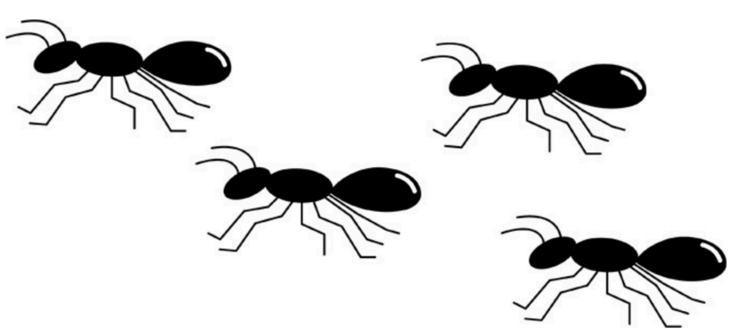
$T \sim E/N^2$

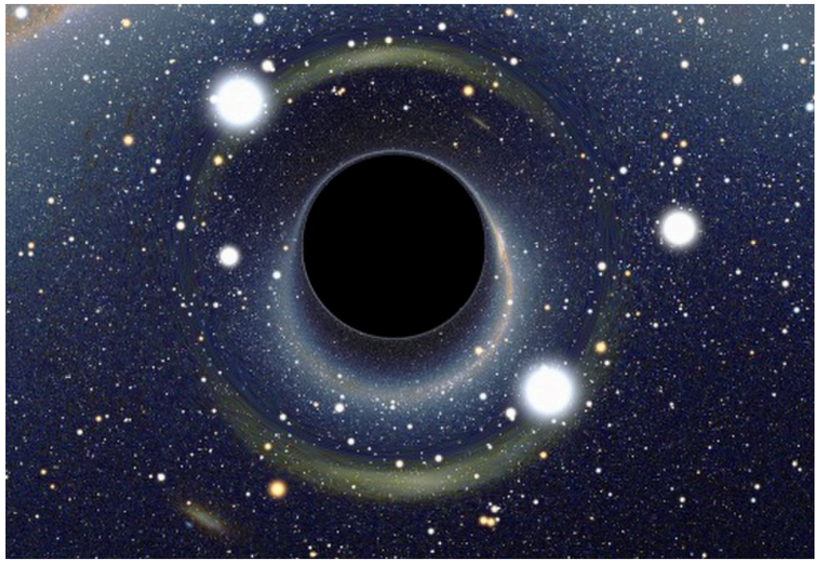
$T' \sim E'/[2\times(N/2)^2]$

$T' > T$ if $E' > E/2$

(more analyses later)

Ant trail/black hole correspondence





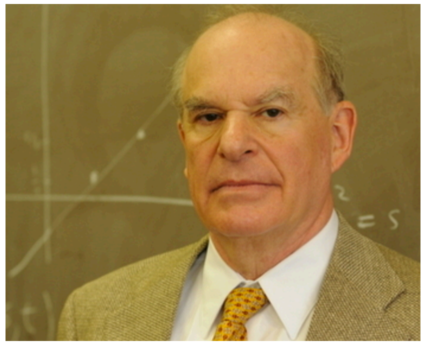
Cotler-MH-Ishiki-Watanabe, in preparation

50th Anniversary

John H. Schwarz

California Institute of Technology

Strings 2018 June 29, 2018

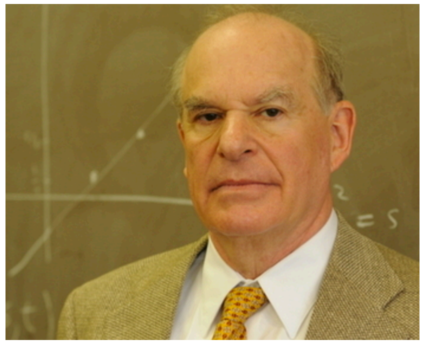


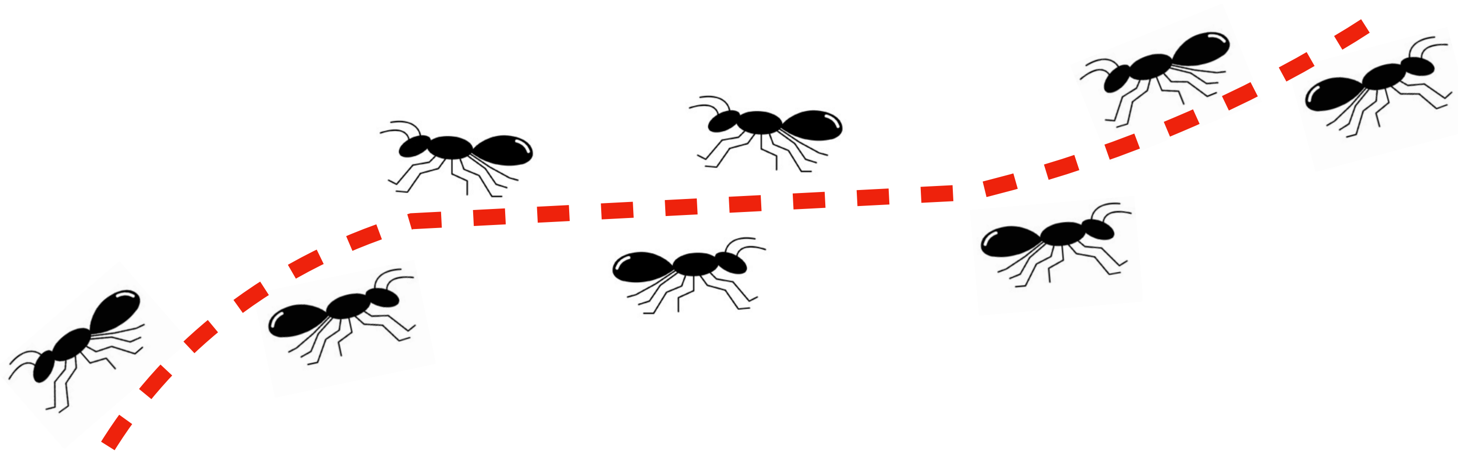
Lesson #2: Take "coincidences" seriously.

Example 1: The massless states of type IIA superstring theory correspond to the massless states of 11d supergravity on a circle. This was known for more than a decade before it was taken seriously.

Example 2: It was well known that the Lorentzian conformal group in d dimensions is the same as the Anti de Sitter isometry group in $d+1$ dimensions many years before AdS/CFT duality was proposed.

Lesson #2: Take “coincidences” seriously.



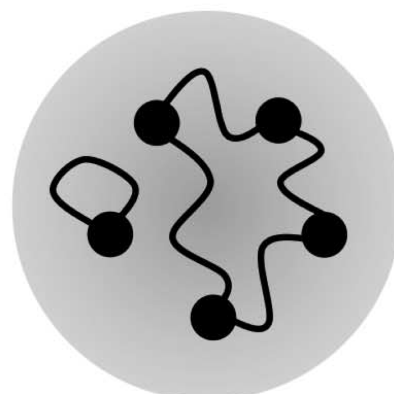


- Ant ‘trail’ is called 行列 in Japanese.
- ‘Matrix’ is called 行列 in Japanese.
- Gauge/gravity duality says BH is matrix.

black hole = ant trail?

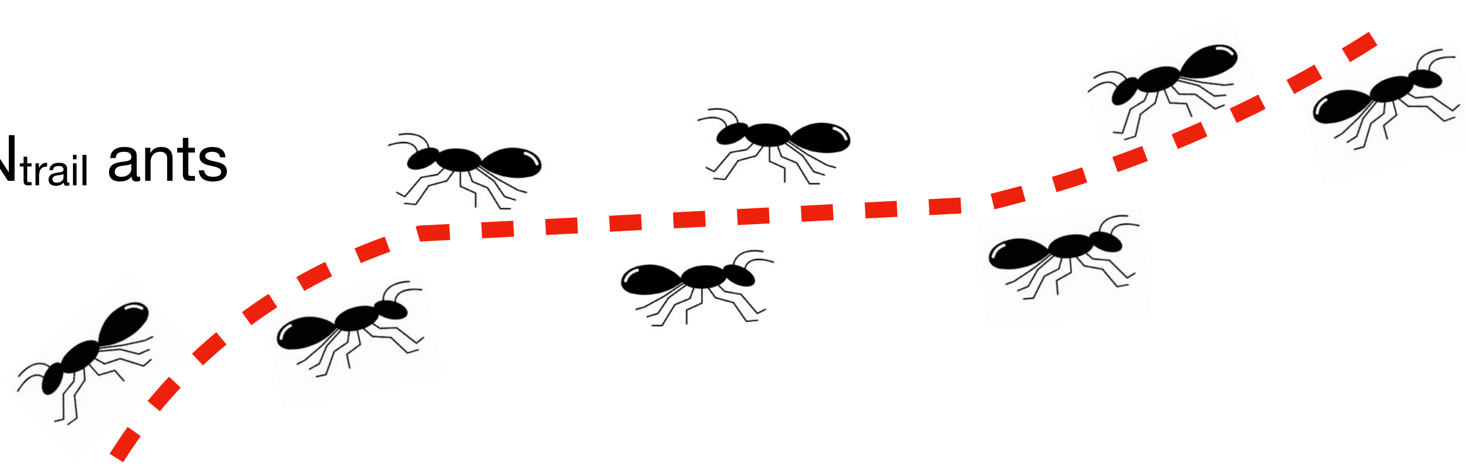
Black hole = D-brane bound by open strings

N_{BH} D-branes

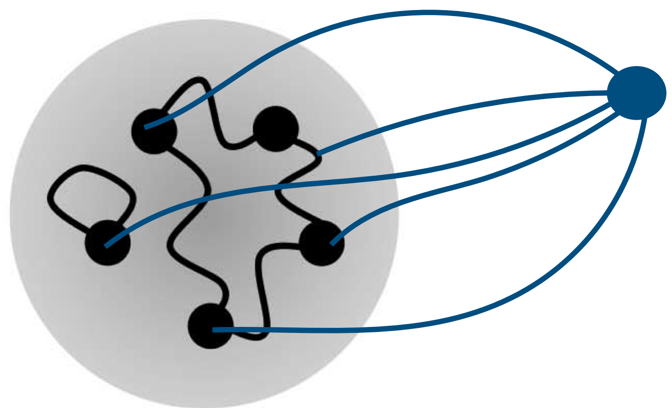


Ant trail = ants bound by pheromone

N_{trail} ants

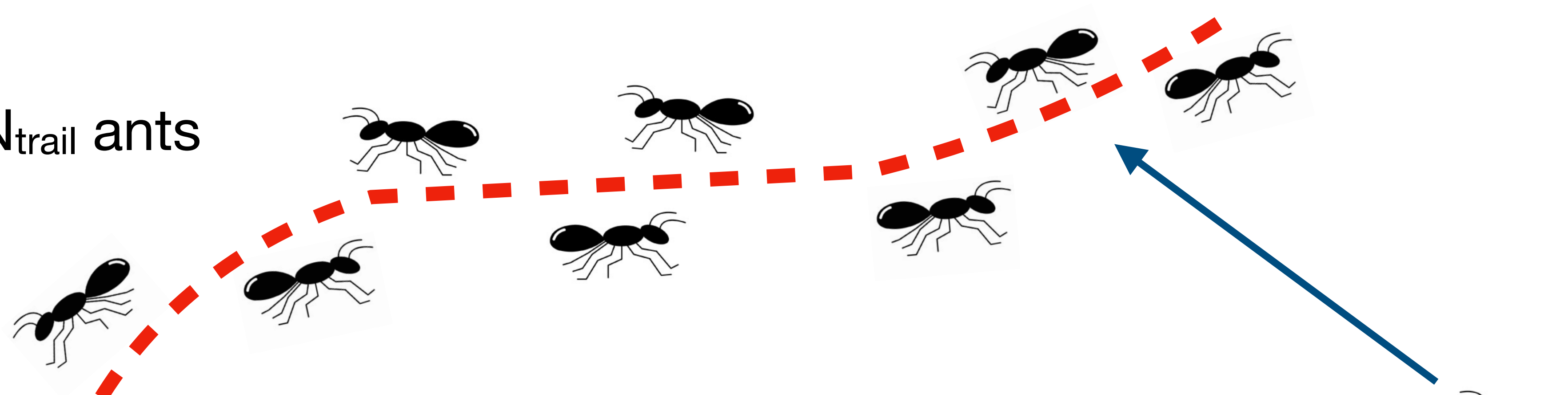


Black hole = D-brane bound by open strings



$$N_{\rm trail} \longleftrightarrow N_{\rm BH}$$

Ant trail = ants bound by pheromone



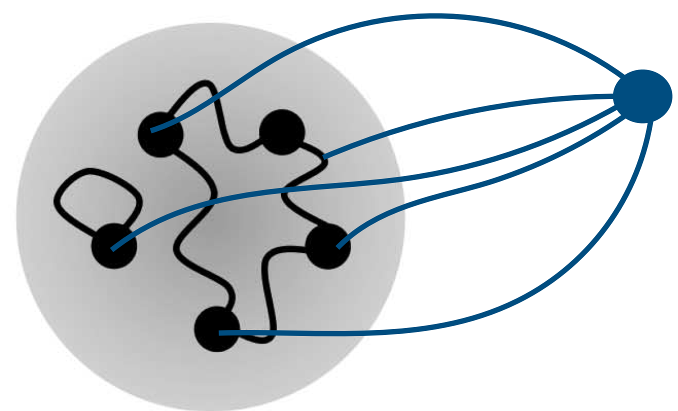
hot ∼ strong pheromone

$T \sim p$

Lesson #2: Take "coincidences" seriously.

Black hole = D-brane bound by open strings

N_{BH} D-branes



N_{BH} open strings
try to capture
the other D-brane

$$N_{\mathrm{trail}} \longleftrightarrow N_{\mathrm{BH}},$$
$$p \longleftrightarrow T.$$

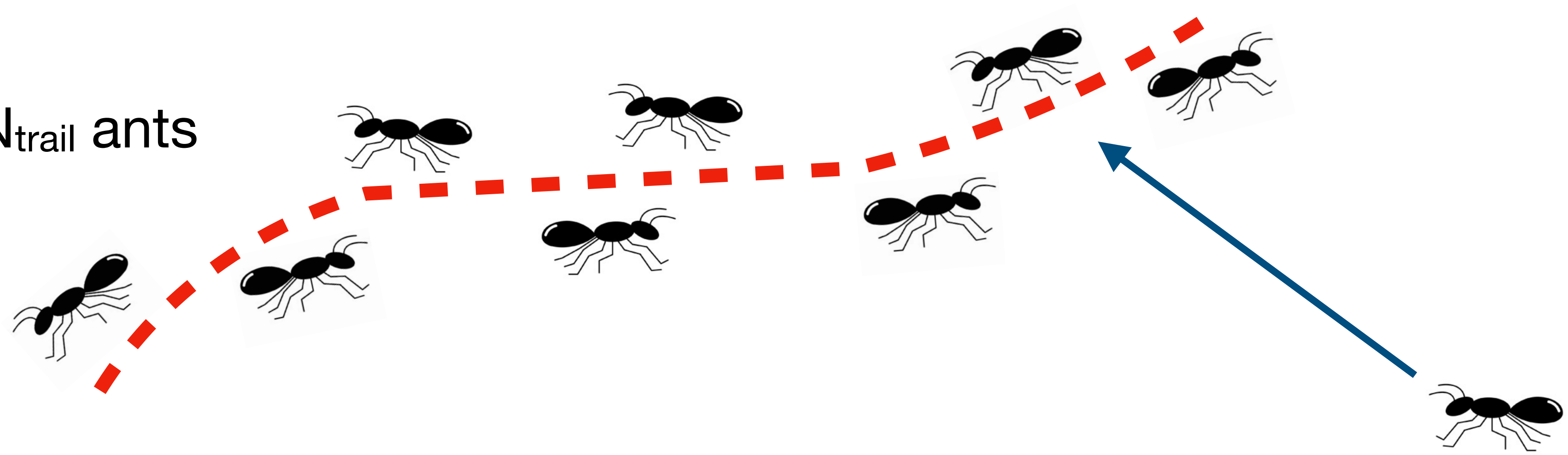
high T ~ each mode is excited more
~ stronger pheromone from each ant

Ant trail = ants bound by pheromone

pheromone strength = $p \times N_{trail}$

p: pheromone from each ant

N_{trail} ants



The ant equation

Phase transition between disordered and ordered foraging in Pharaoh's ants

Madeleine Beekman[*,†], David J. T. Sumpter[‡], and Francis L. W. Ratnieks[*]

[*]Laboratory of Apiculture and Social Insects, Department of Animal and Plant Sciences, Sheffield University, Sheffield S10 2TN, United Kingdom; and [‡]Centre for Mathematical Biology, Mathematical Institute, Oxford University, 24-29 St. Giles, Oxford OX1 3LB, United Kingdom

Communicated by I. Prigogine, Free University of Brussels, Brussels, Belgium, June 7, 2001 (received for review August 12, 2000)

PNAS Proceedings of the National Academy of Sciences of the United States of America

$$\frac{dN_{\text{trail}}}{dt} = (\text{ants beginning to forage at feeder}) - (\text{ants losing pheromone trail})$$

$$= (\alpha + \underline{pN_{\text{trail}}})(N - N_{\text{trail}}) - \frac{sN_{\text{trail}}}{s + N_{\text{trail}}}.$$

stringy term

Natural large-N limit: $\alpha \sim N^0, p \sim N^0, s \sim N^1$

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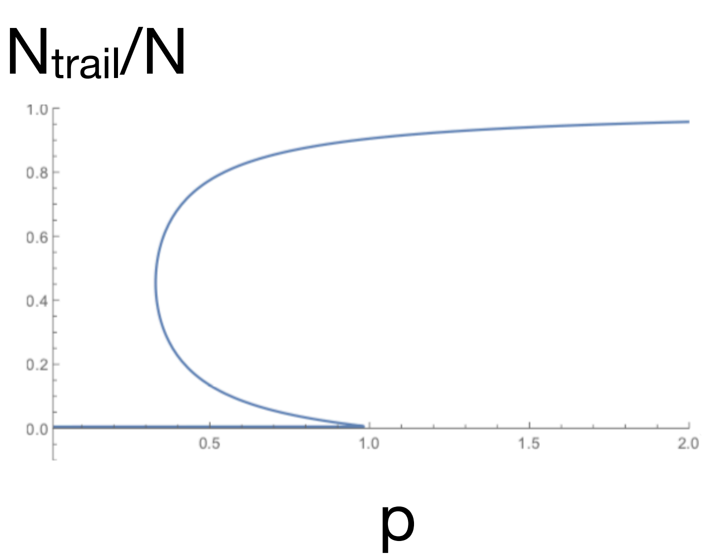

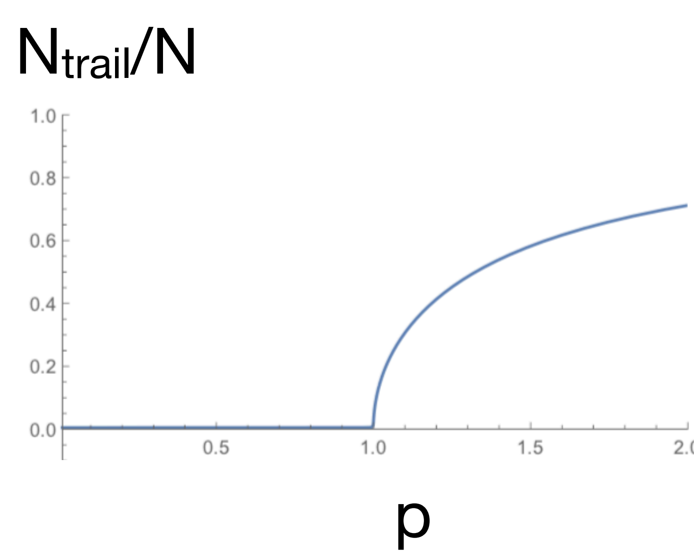

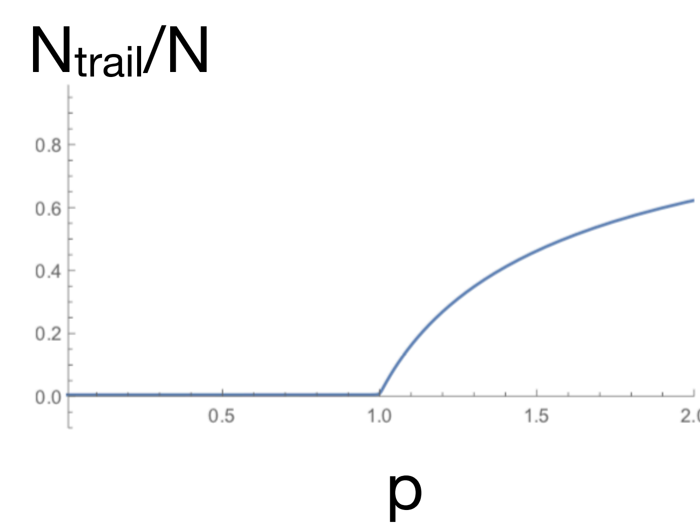

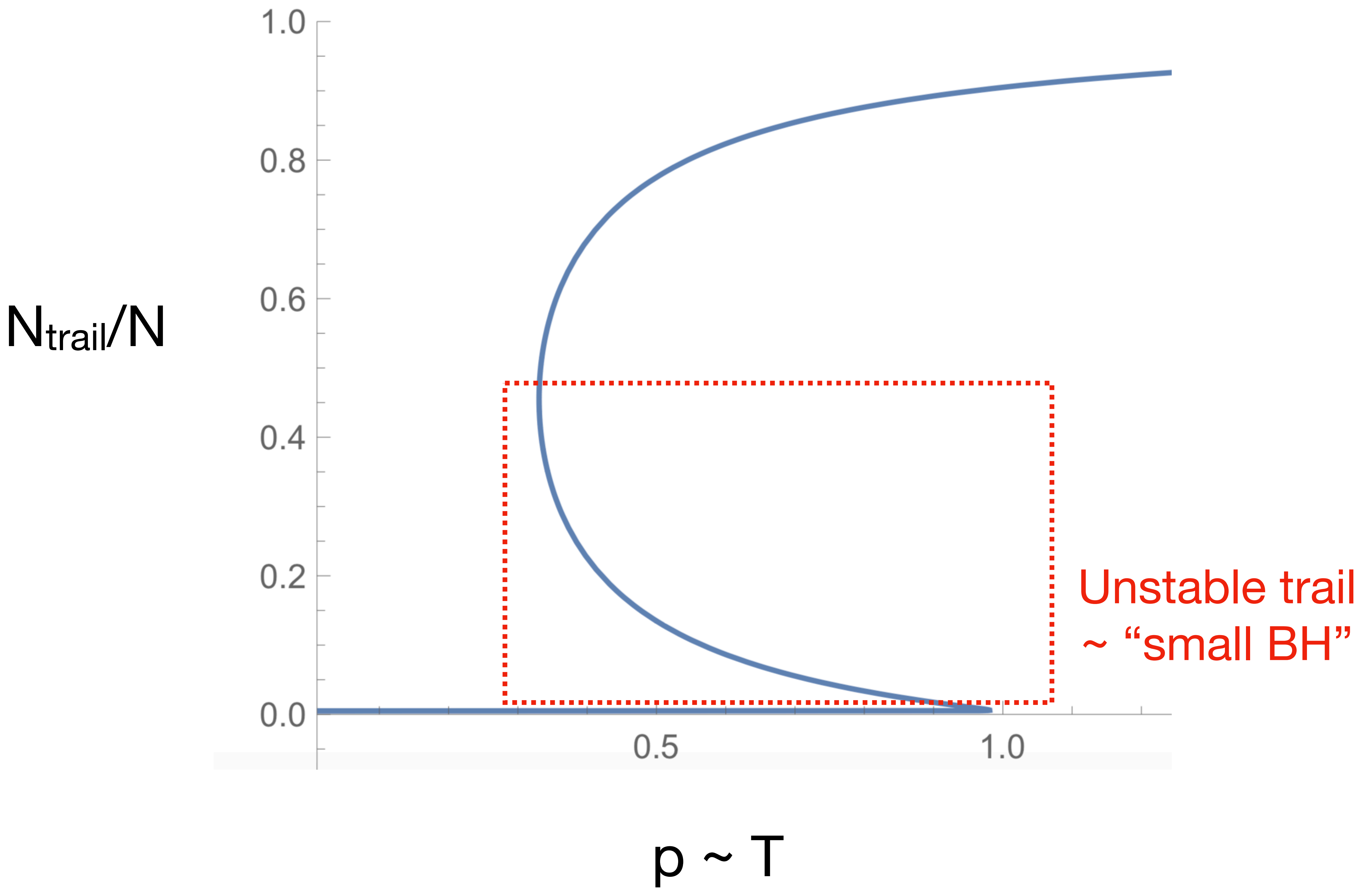

1.0
0.8
0.6
0.4
0.2
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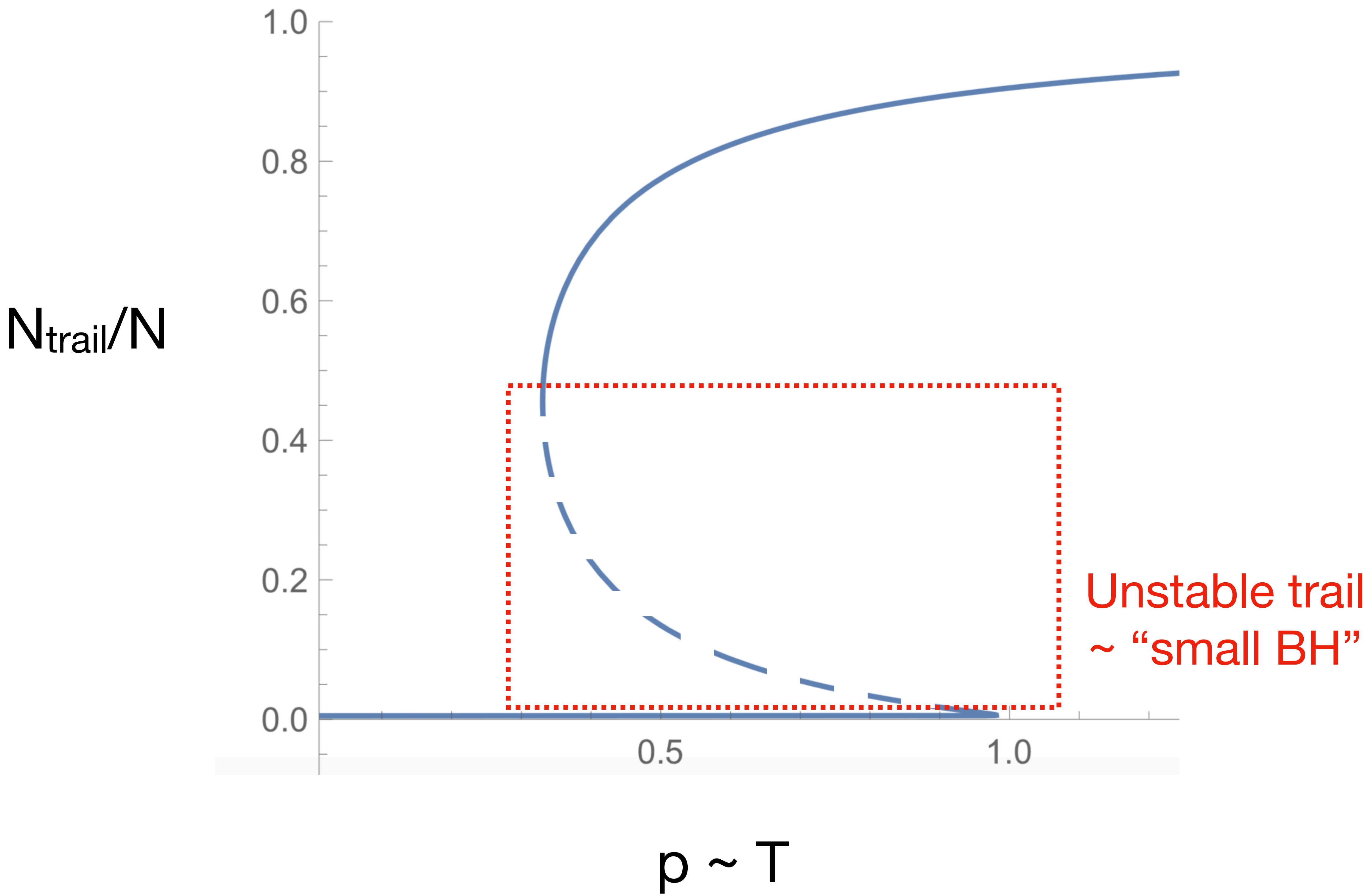
0.5
1.0
Ntrail/N
p ~ T
Unstable trail
~ “small BH”



1.0
0.8
0.6
0.4
0.2
0.0
N_{trail}/N
0.5
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Unstable trail
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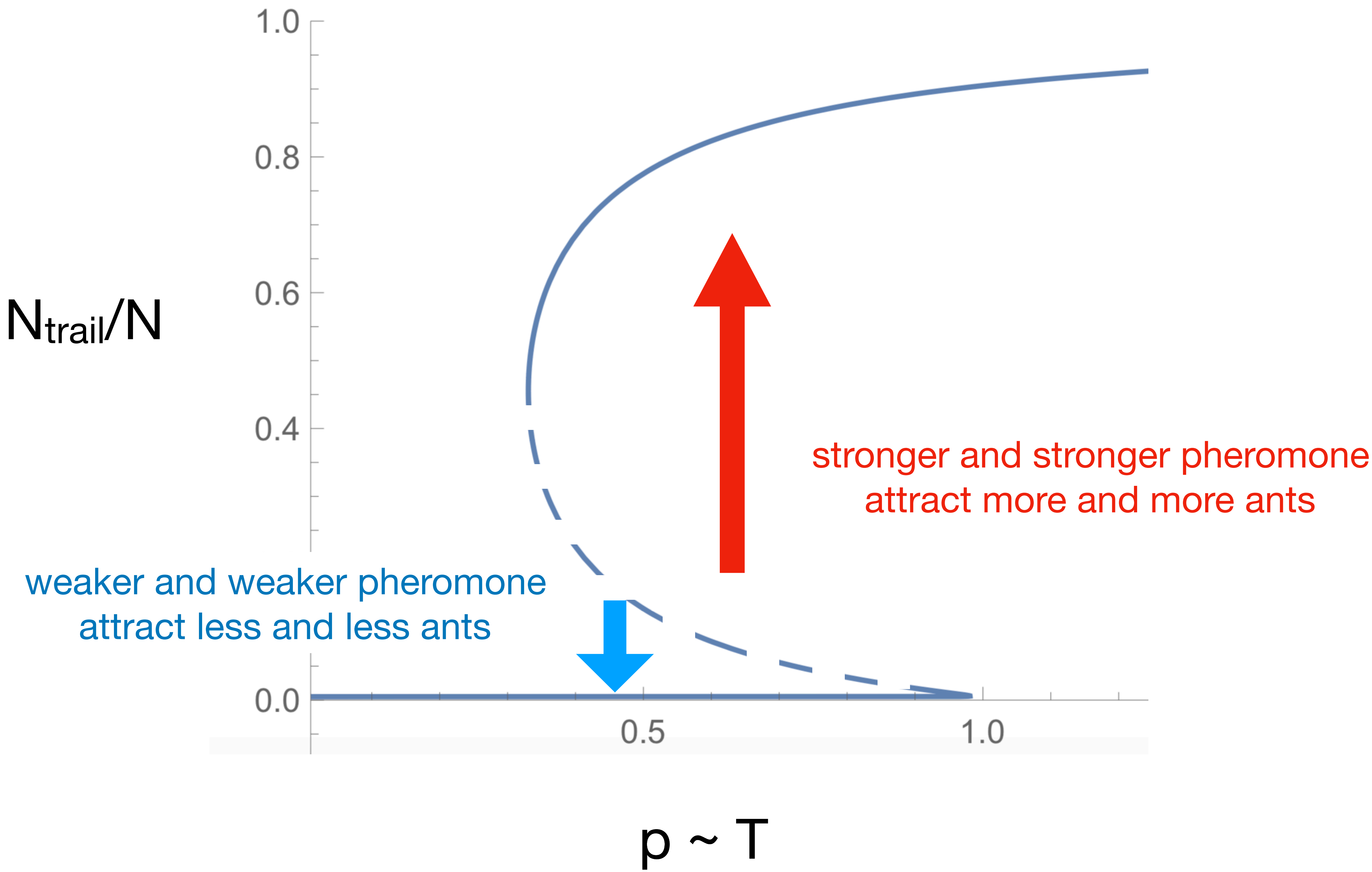
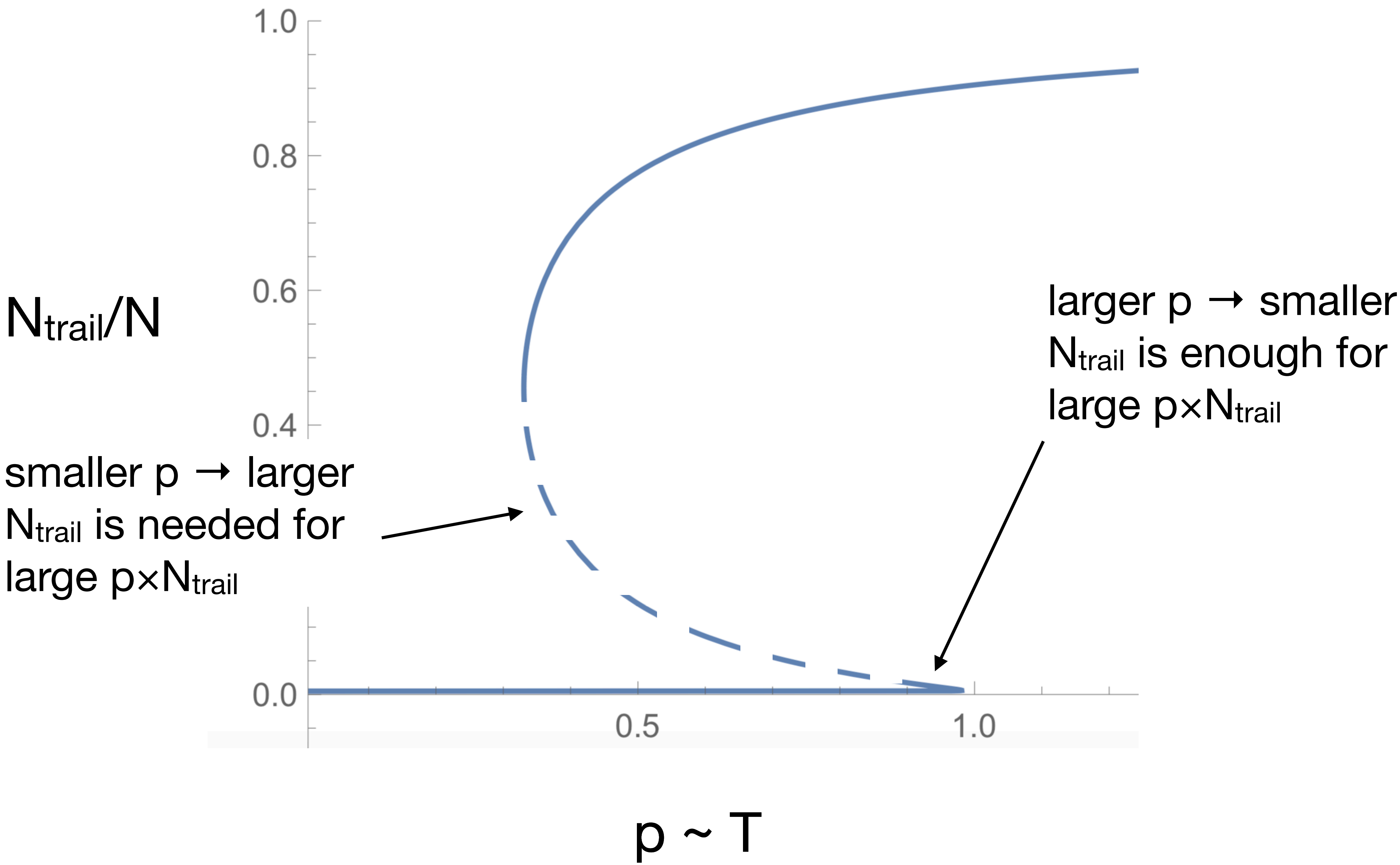
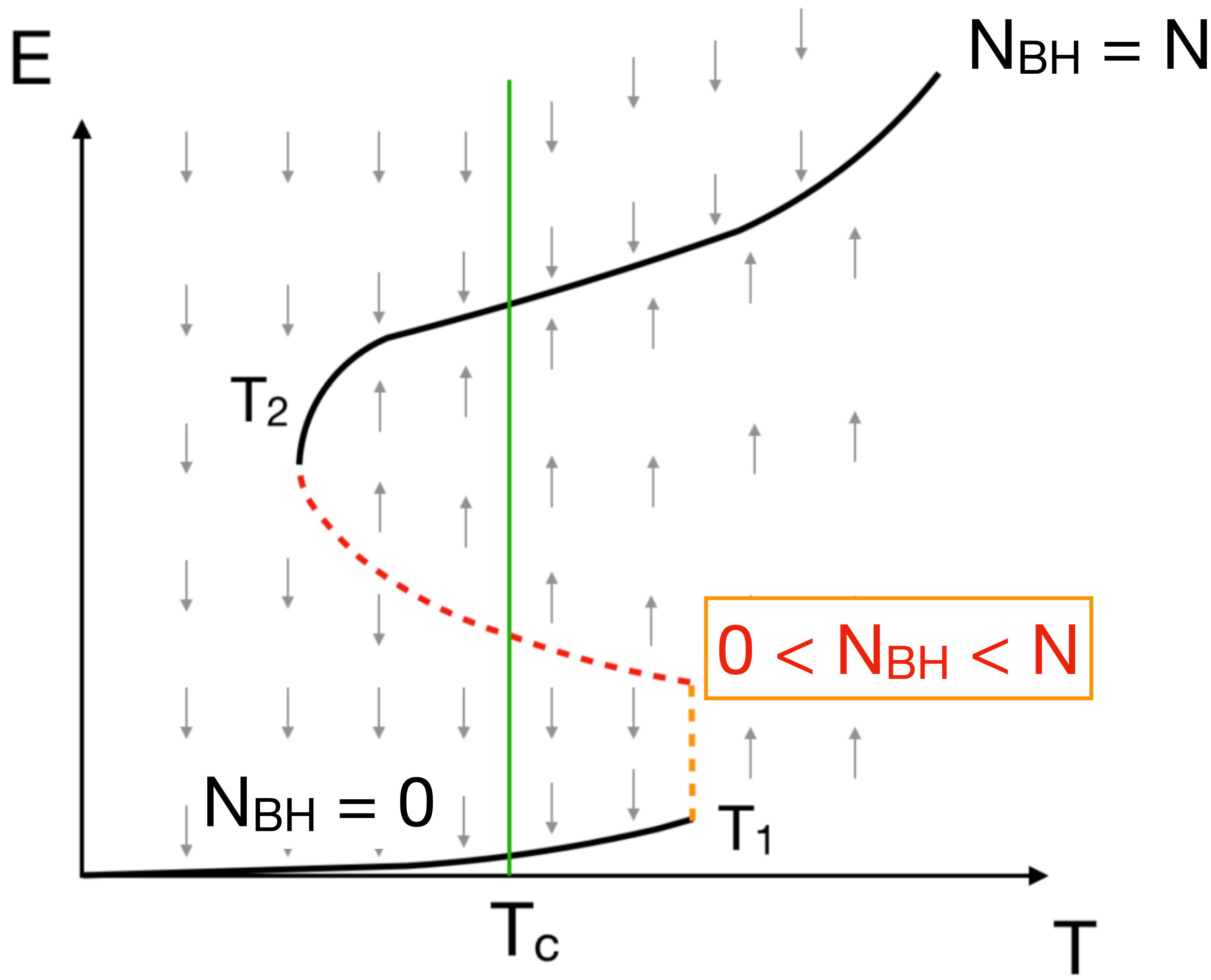


1.0
0.8
0.6
0.4
0.0
0.5
1.0
N_{trail}/N
stronger and stronger pheromone
attract more and more ants
weaker and weaker pheromone
attract less and less ants
p ~ T



1.0
0.8
0.6
0.4
0.0
0.5
1.0
Ntrail/N
larger p → smaller
Ntrail is enough for
large p×Ntrail
smaller p → larger
Ntrail is needed for
large p×Ntrail
p ~ T



N_{BH} D-branes form the bound state

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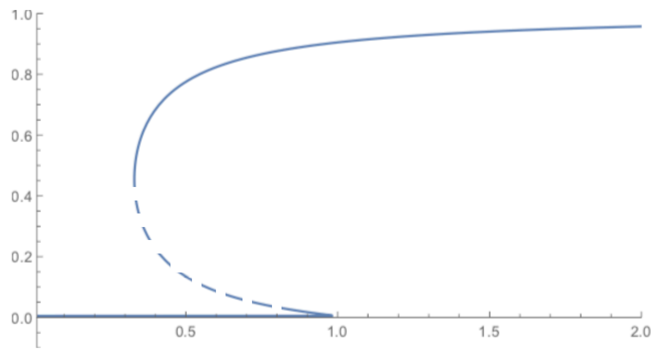
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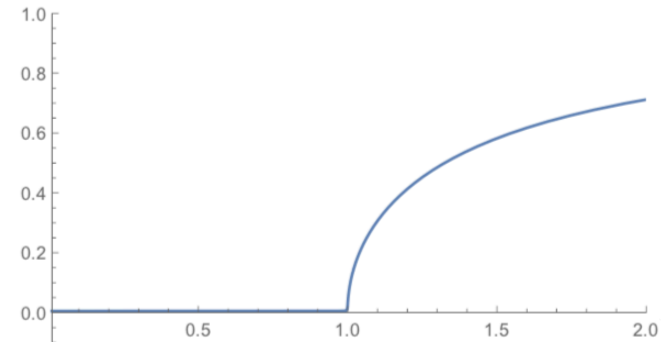
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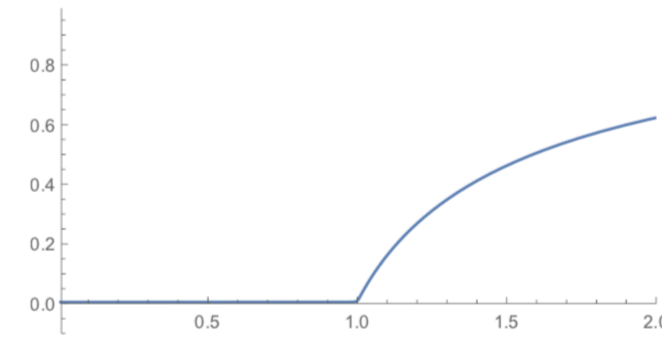
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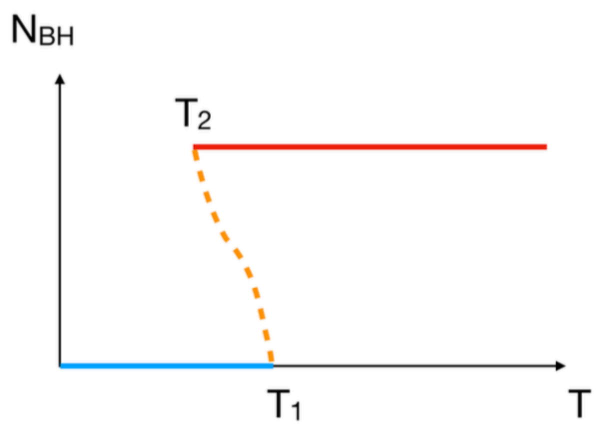
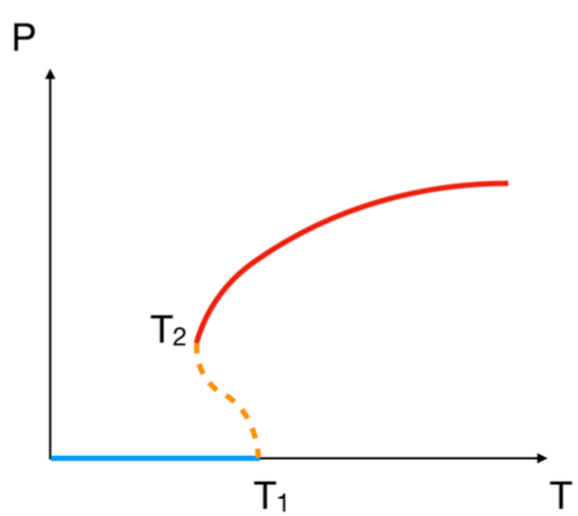
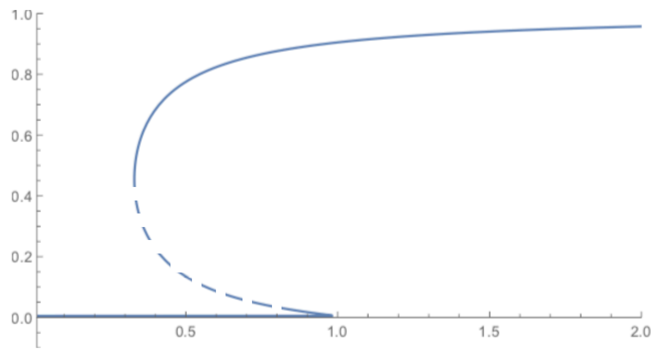

N_{BH}
T_2
T_1
T


P
T_2
T_1
T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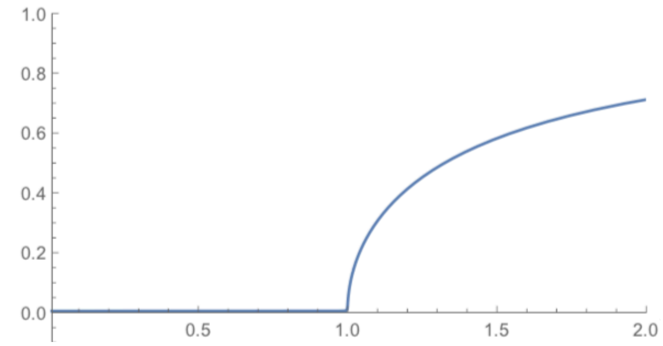
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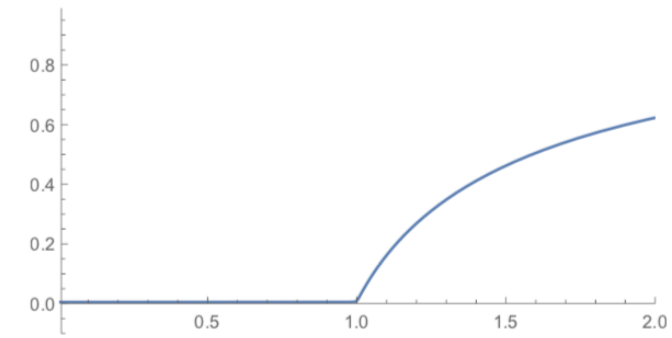
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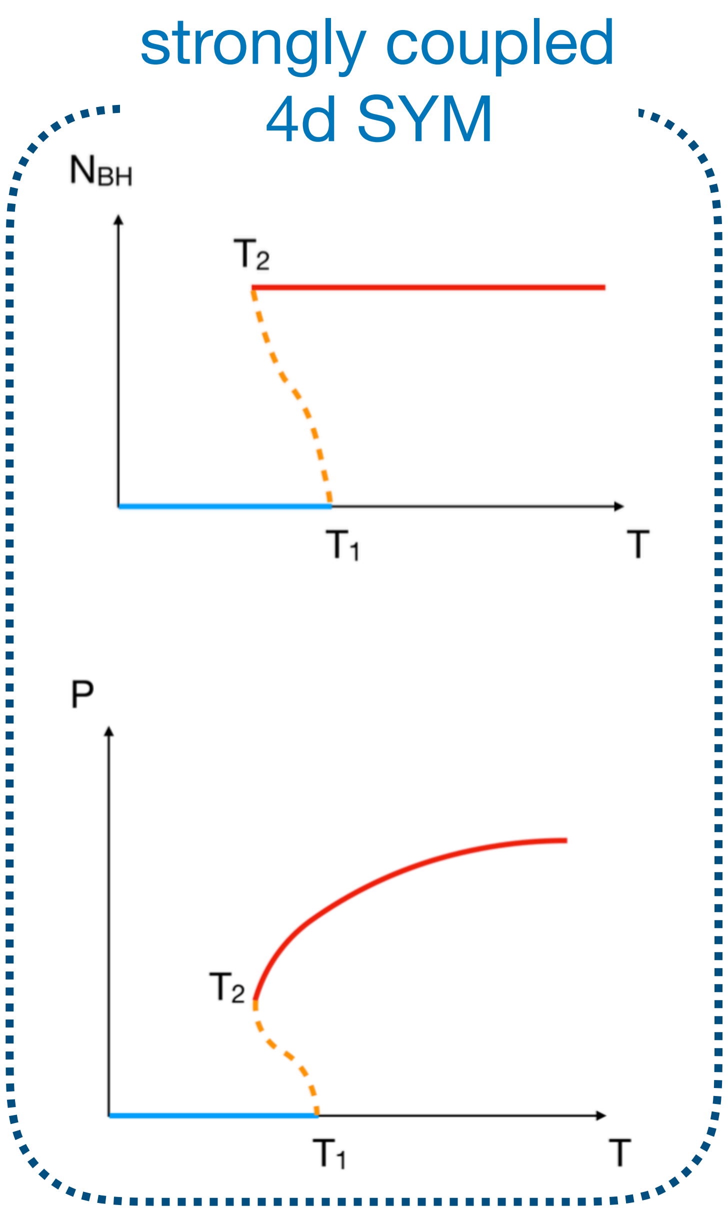
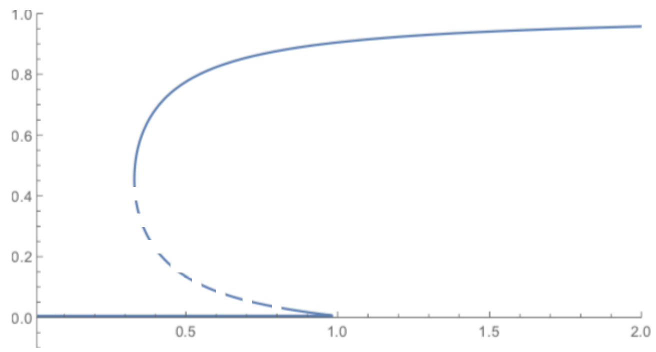

strongly coupled
4d SYM
N_{BH}
T_2
T_1
T
P
T_2
T_1
T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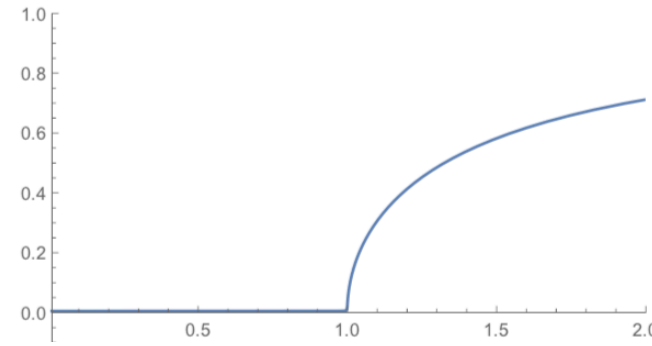
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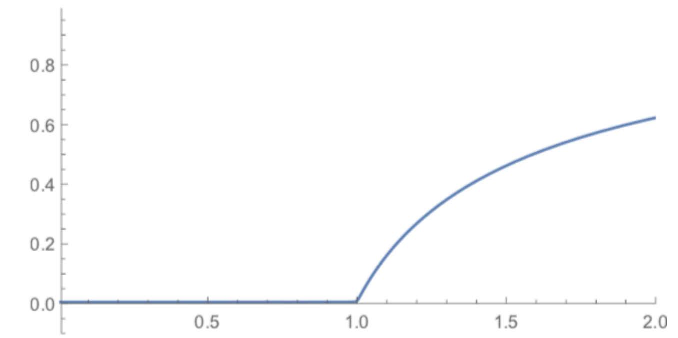
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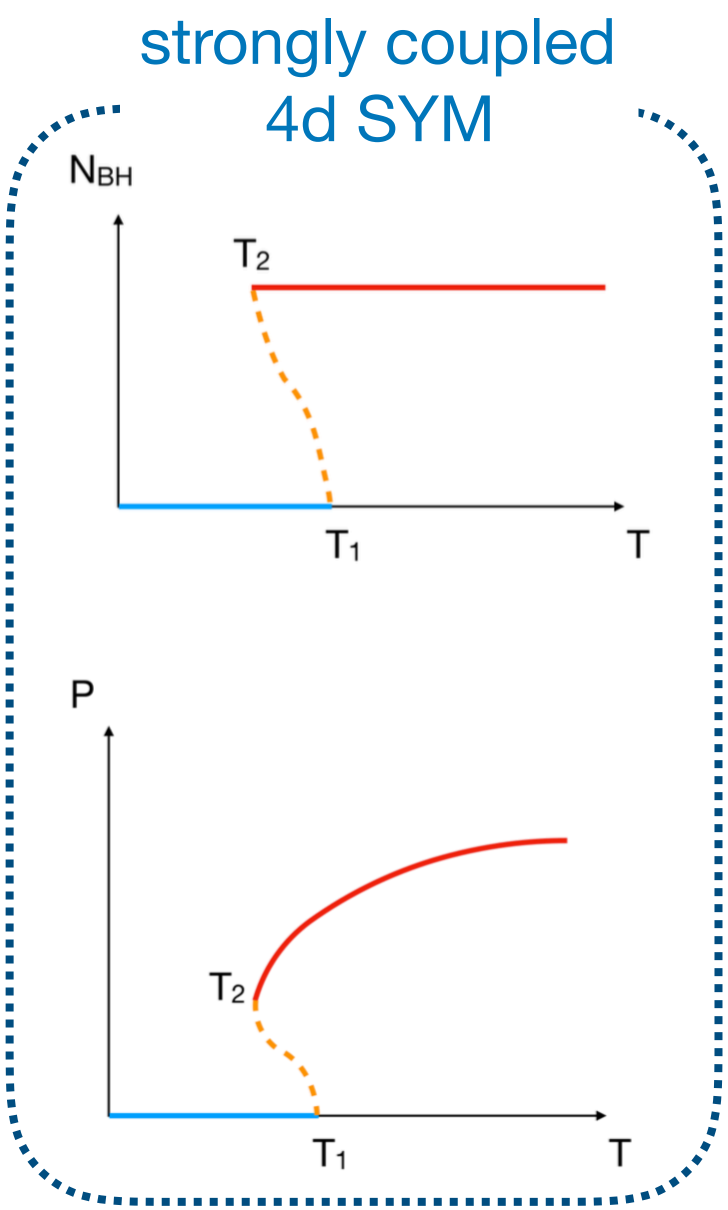
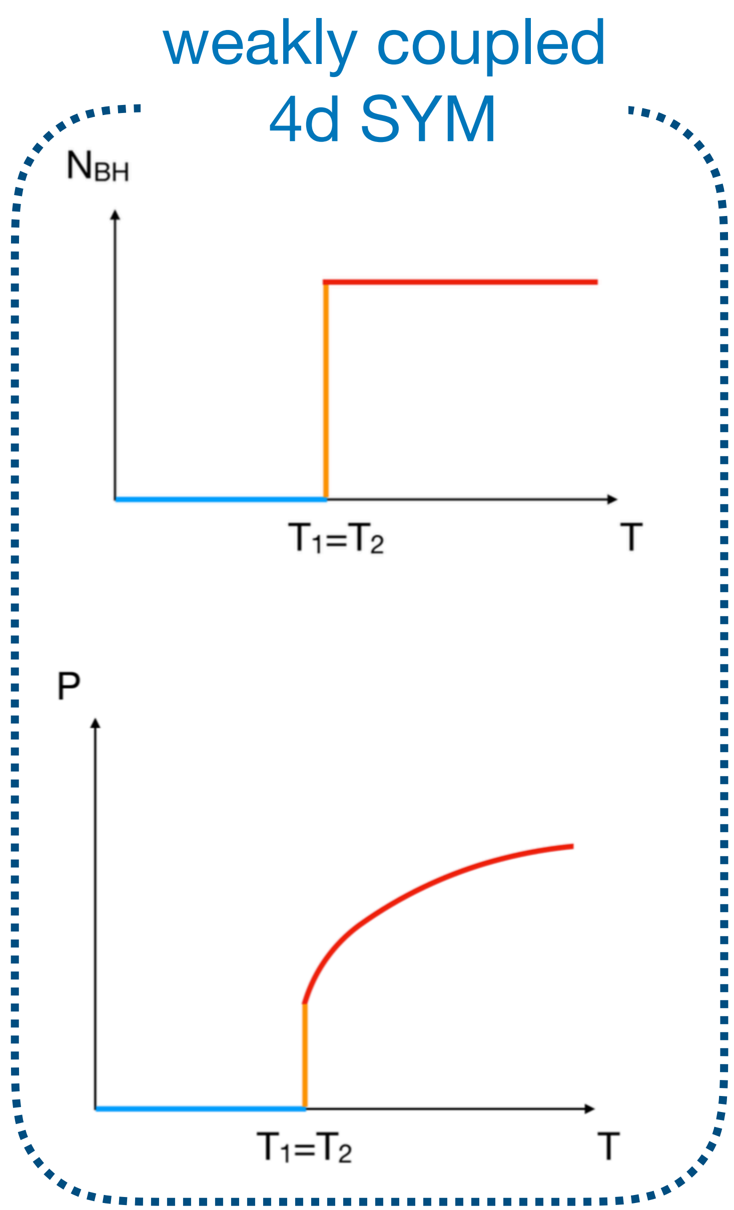
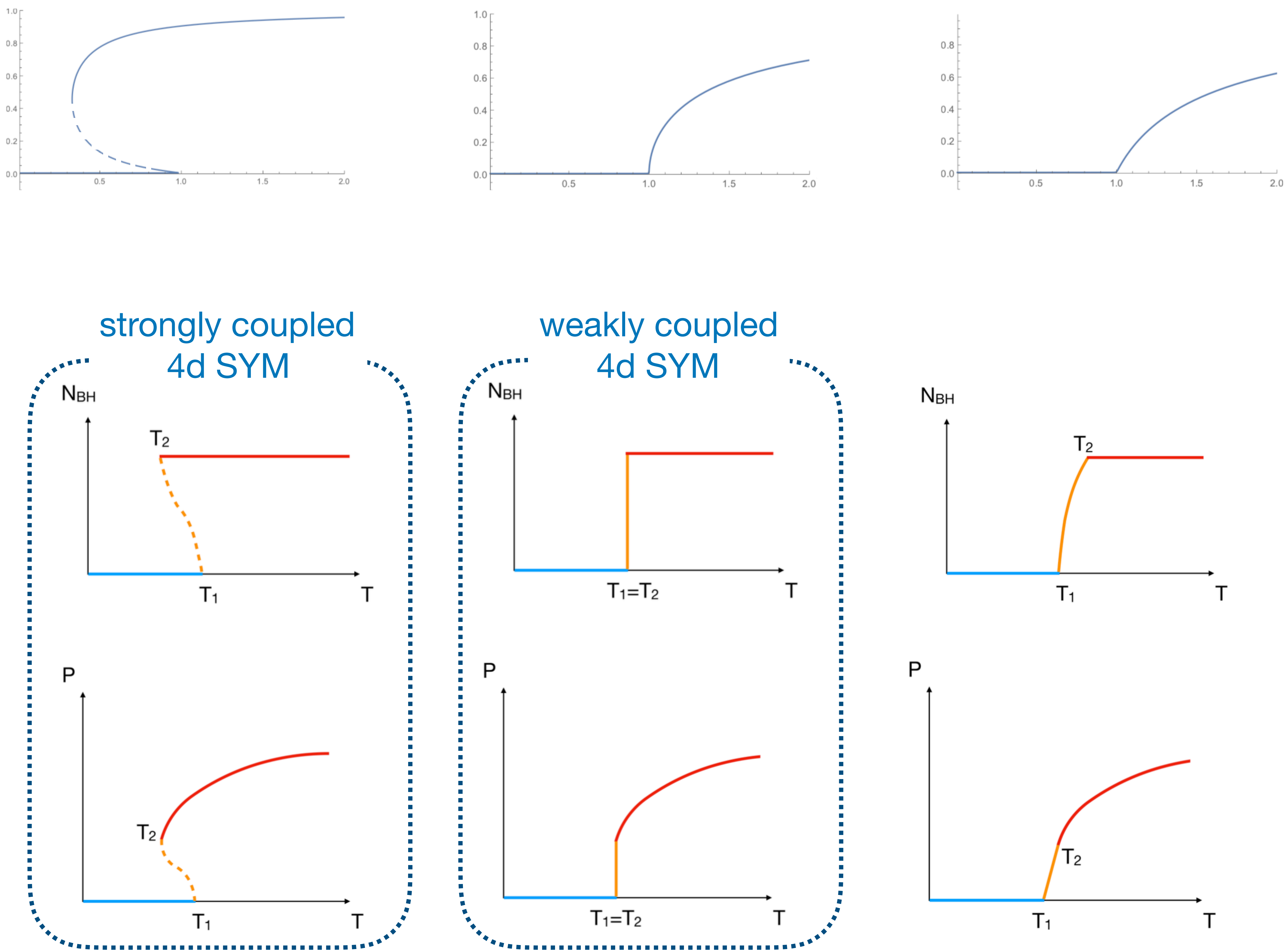
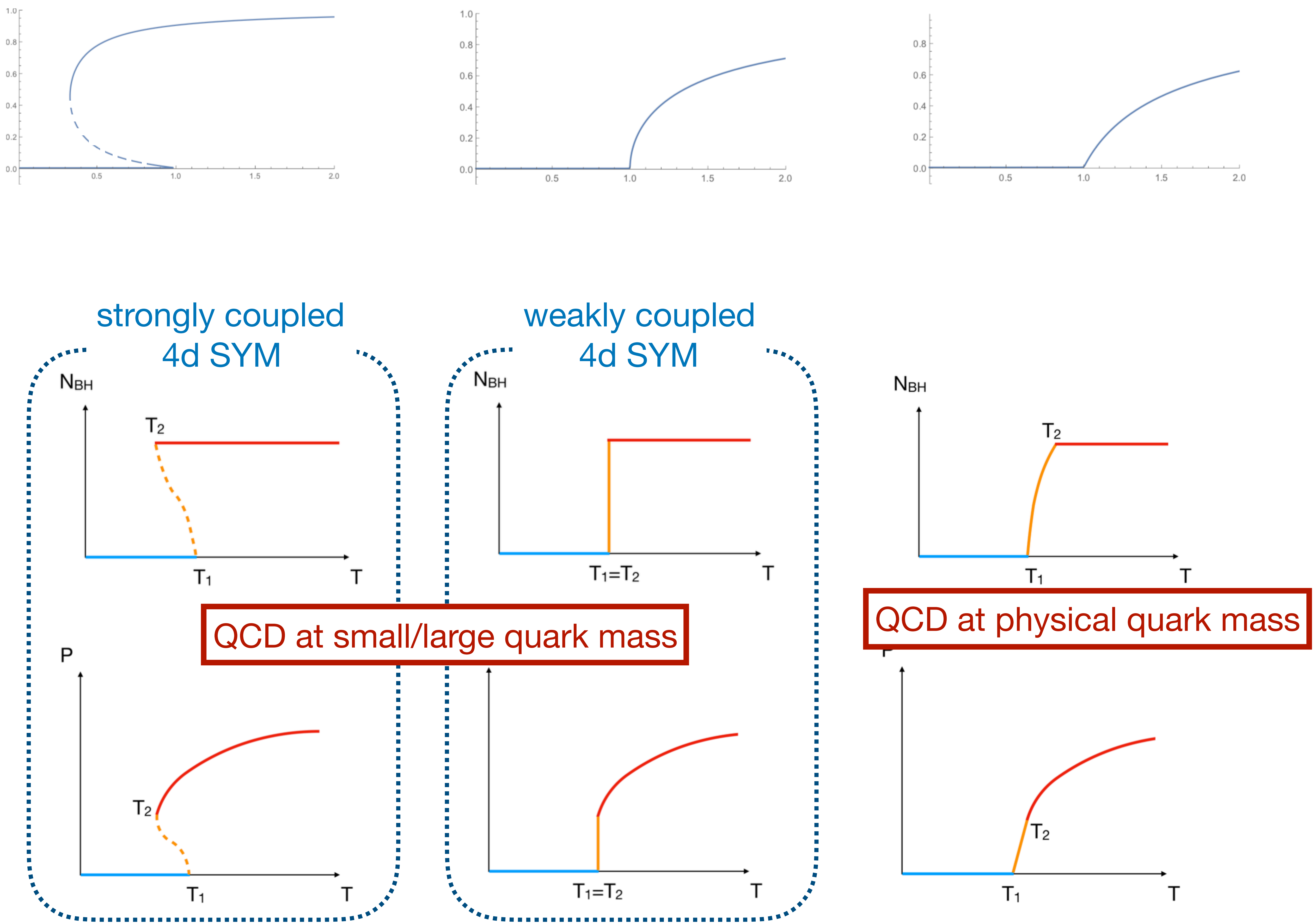

strongly coupled
4d SYM
N_BH
T2
T1
T
P
T2
T1
T

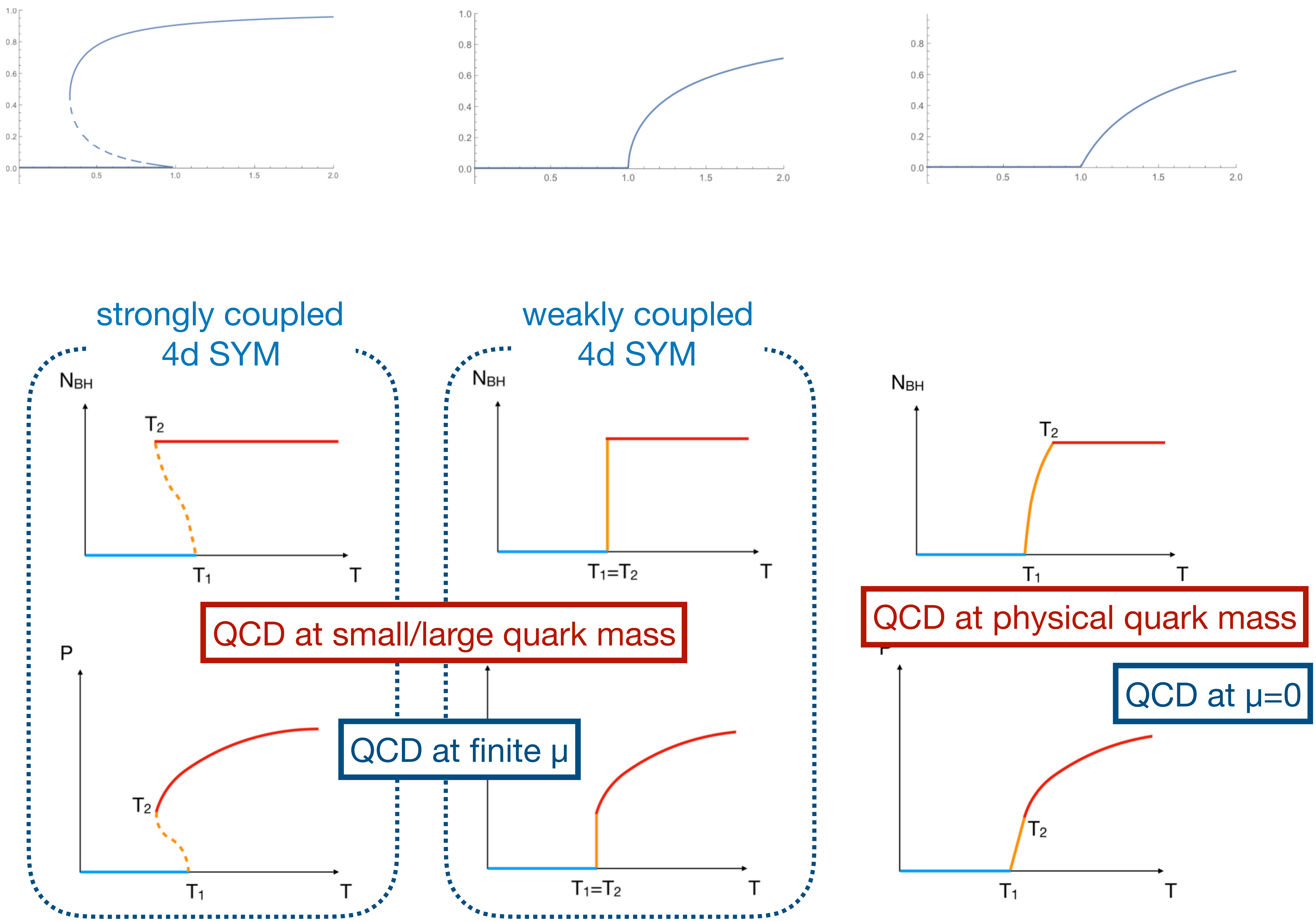

weakly coupled
4d SYM
N_BH
T1=T2
T
P
T1=T2
T



1.0
0.8
0.6
0.4
0.2
0.0
0.5
1.0
1.5
2.0
strongly coupled
4d SYM
weakly coupled
4d SYM
N_BH
T_2
T_1
T
T_1=T_2
P

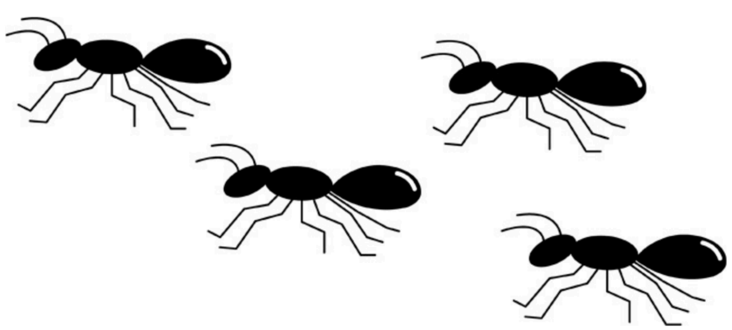


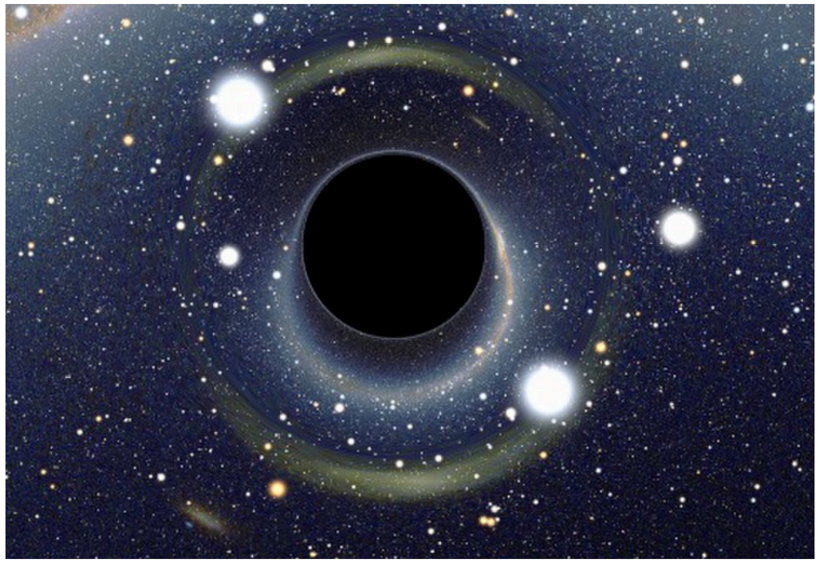
strongly coupled
4d SYM
weakly coupled
4d SYM
QCD at small/large quark mass
QCD at physical quark mass
N_{BH}
T_2
T_1
T
T_1=T_2
P



1.0
0.8
0.6
0.4
0.2
0.0
0.5
1.0
1.5
2.0
strongly coupled
4d SYM
weakly coupled
4d SYM
N_BH
T_2
T_1
T
T_1=T_2
QCD at physical quark mass
QCD at small/large quark mass
P
QCD at μ=0
QCD at finite μ

Testing the partial deconfinement





Cotler-MH-Ishiki-Watanabe, in preparation

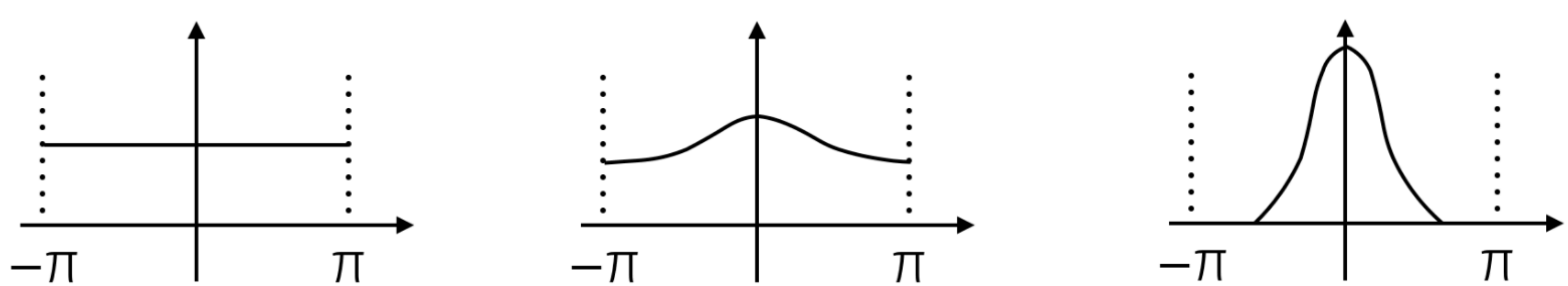
- ‘Polyakov loop’ is a useful order parameter.

$$P = \frac{1}{N}\sum_{j=1}^{N} e^{i\theta_j}$$

- Phase distribution:

confined phase
P=0

deconfined phase
P ≠ 0



‘partially’ deconfined **‘completely’ deconfined**

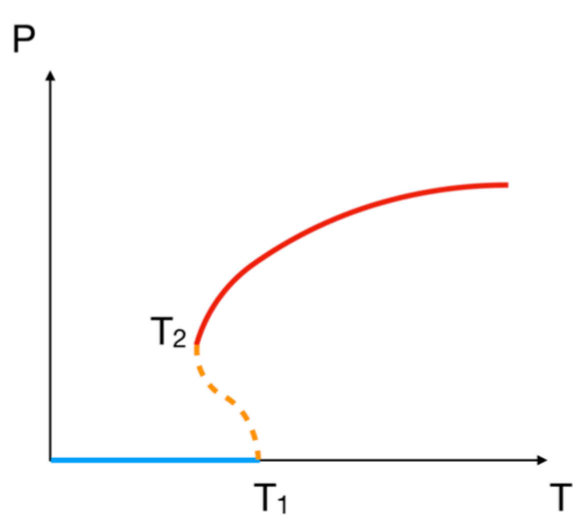
- Matrix model
- 4d YM
- 2d SYM

dimensional reduction

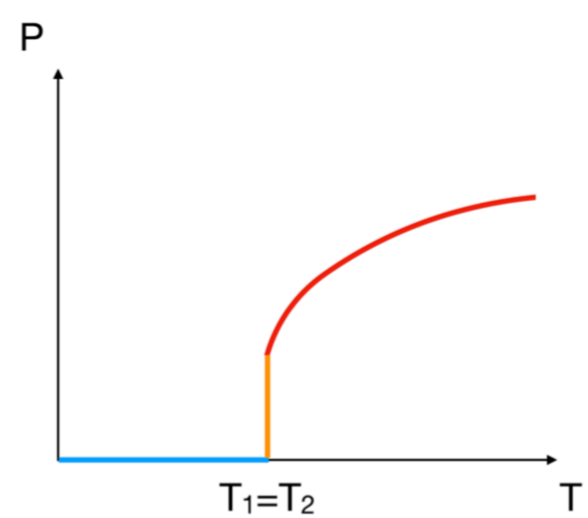
(1+3)-d N=4 SYM on S^3 → (1+0)-d matrix model

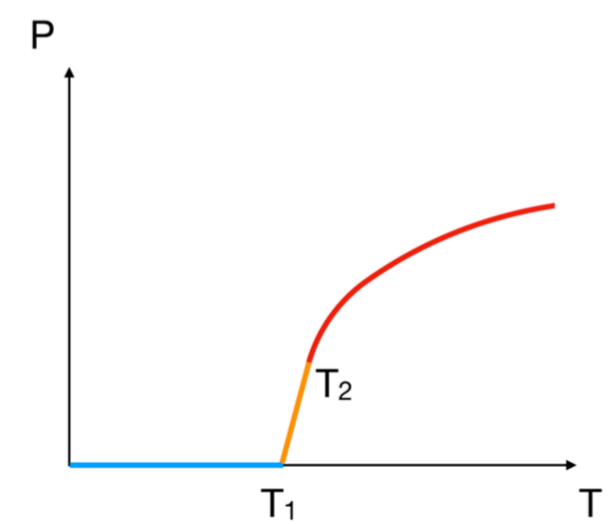
& keep only bosonic part:

$$L = N\mathrm{Tr}\left(\frac{1}{2}D_tX_I^2 + \frac{1}{4}[X_I, X_J]^2 - \frac{\mu^2}{2}\sum_{i=1}^{3}X_i^2 - \frac{\mu^2}{8}\sum_{a=4}^{9}X_a^2 - i\sum_{i,j,k=1}^{3}\mu\epsilon^{ijk}X_iX_jX_k\right)$$



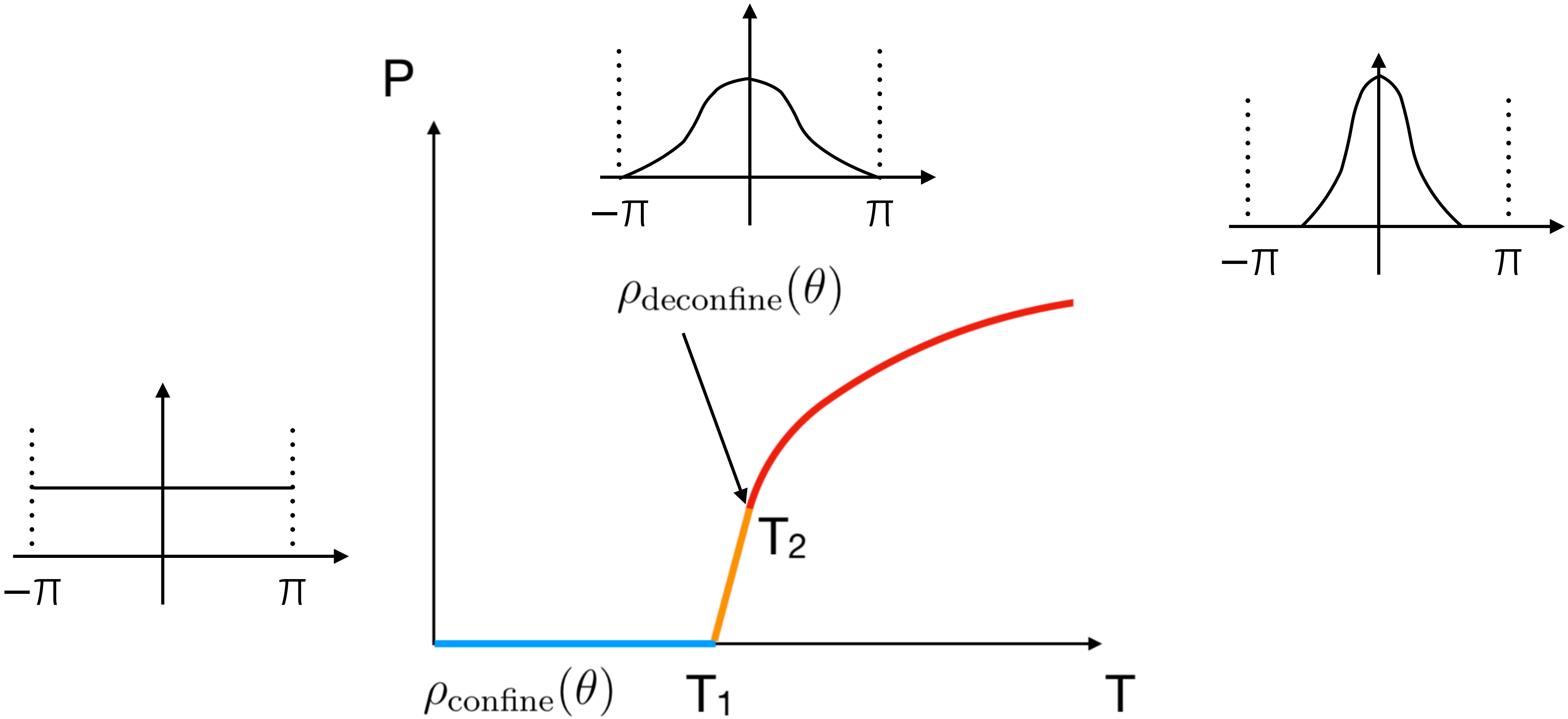
large μ



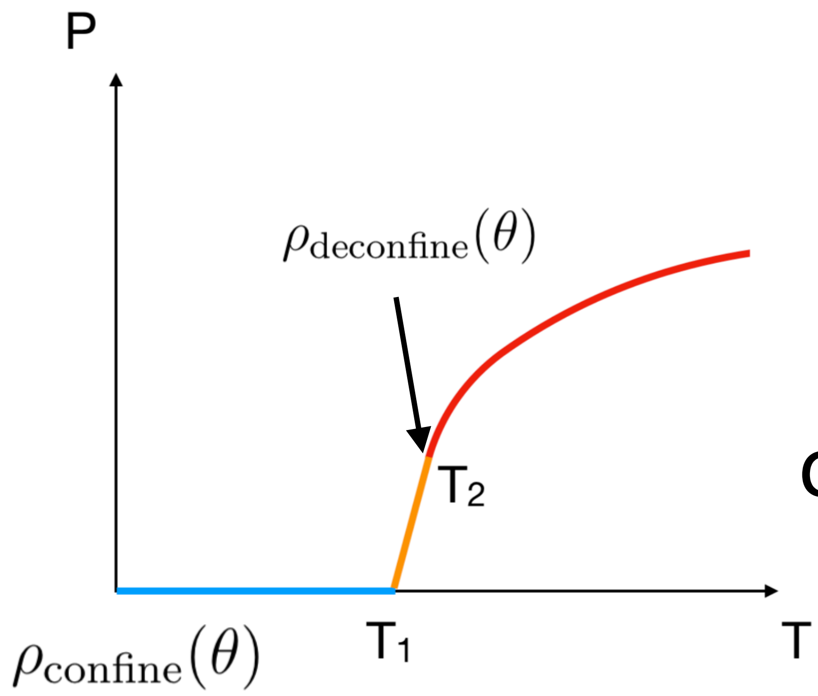


small μ

Let's start with this.



$$\rho(\theta) = \frac{N-M}{N}\rho_{\rm confine}(\theta) + \frac{M}{N}\rho_{\rm deconfine}(\theta) = \frac{N-M}{N}\cdot\frac{1}{2\pi} + \frac{M}{N}\rho_{\rm deconfine}(\theta)$$

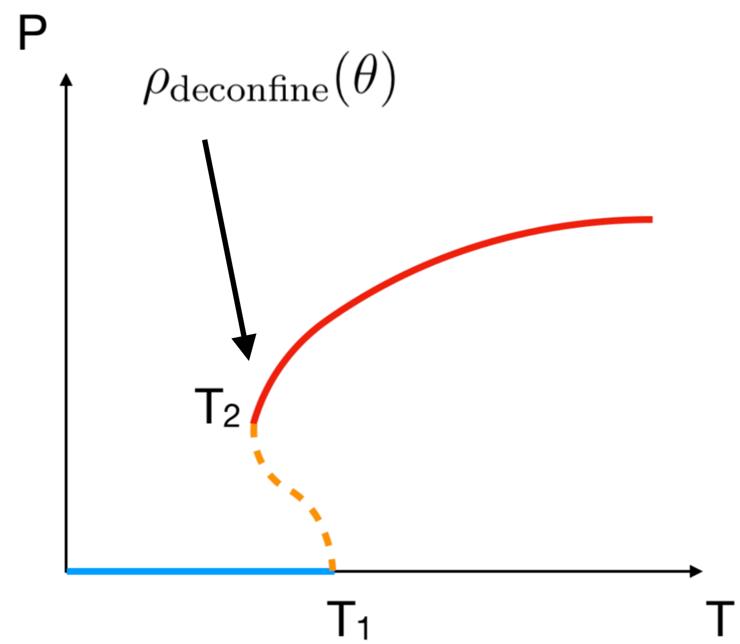


Gross-Witten-Wadia transition separates completely and partially deconfined phases.

$$\rho(\theta) = \begin{cases} \frac{1}{2\pi} & (T \le T_1) \\ \frac{1}{2\pi}\left(1 + \frac{2}{\kappa}\cos\theta\right) & (T_1 < T < T_2) \\ \frac{2}{\pi\kappa}\cos\frac{\theta}{2}\sqrt{\frac{\kappa}{2} - \sin^2\frac{\theta}{2}} & (T \ge T_2, |\theta| < 2\arcsin\sqrt{\kappa/2}) \end{cases}$$

$$\rho(\theta) = \frac{N-M}{N}\rho_{\rm confine}(\theta) + \frac{M}{N}\rho_{\rm deconfine}(\theta) = \frac{N-M}{N}\cdot\frac{1}{2\pi} + \frac{M}{N}\rho_{\rm deconfine}(\theta)$$

$$\frac{M}{N} = \frac{2}{\kappa}$$



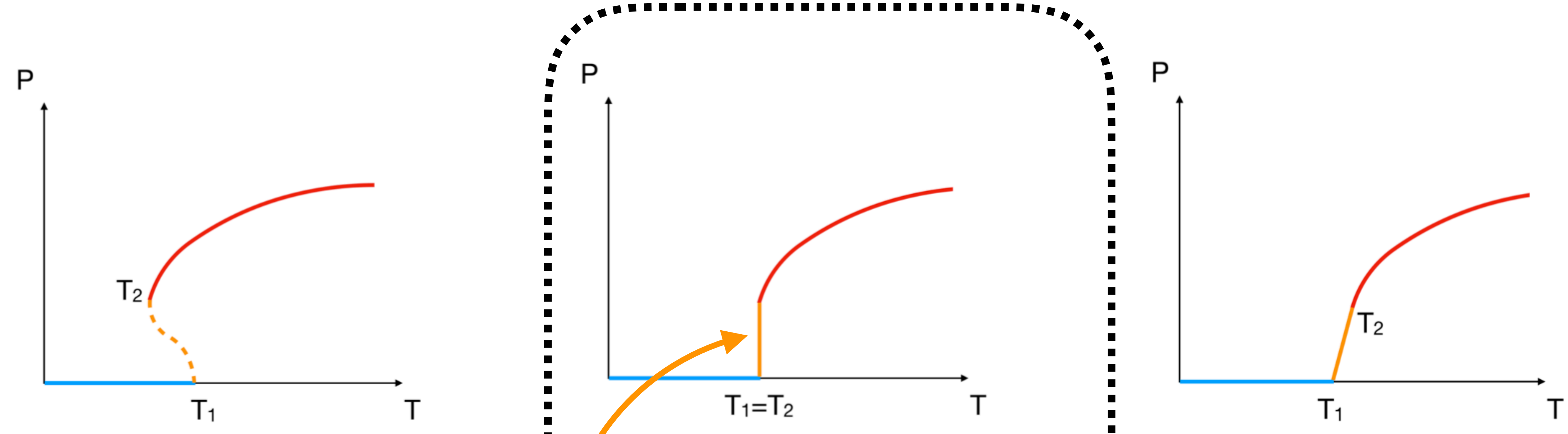
$$\rho(\theta) = \begin{cases} \frac{1}{2\pi} & (T \le T_1) \\ \frac{1}{2\pi}\left(1 + \frac{2}{\kappa}\cos\theta\right) & (T_1 < T < T_2) \\ \frac{2}{\pi\kappa}\cos\frac{\theta}{2}\sqrt{\frac{\kappa}{2} - \sin^2\frac{\theta}{2}} & (T \ge T_2,\ |\theta| < 2\arcsin\sqrt{\kappa/2}) \end{cases}$$

not tested yet

$T_2 < T_1$

It does hold.

- Matrix model
- 4d YM
- 2d SYM



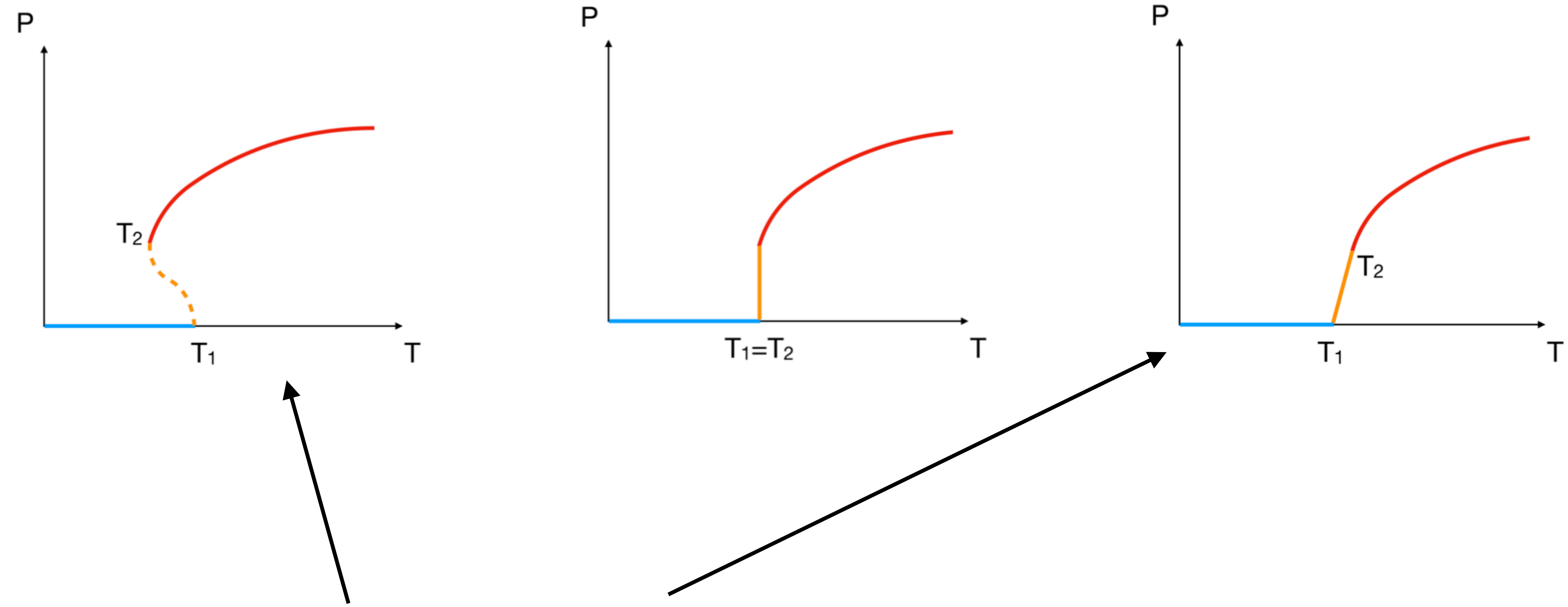
Strongly coupled 4d SYM on S^3 **Weakly coupled 4d SYM on S^3**

$$\rho(\theta) = \frac{1}{2\pi}\left(1 + \frac{2}{\kappa}\cos\theta\right)$$

(Aharony et al 2003)

$$\rho(\theta) = \frac{N-M}{N}\rho_{\text{confine}}(\theta) + \frac{M}{N}\rho_{\text{deconfine}}(\theta) = \frac{N-M}{N}\cdot\frac{1}{2\pi} + \frac{M}{N}\rho_{\text{deconfine}}(\theta)$$

$$\frac{M}{N} = \frac{2}{\kappa}$$



These two can also appear depending on the detail of the theory

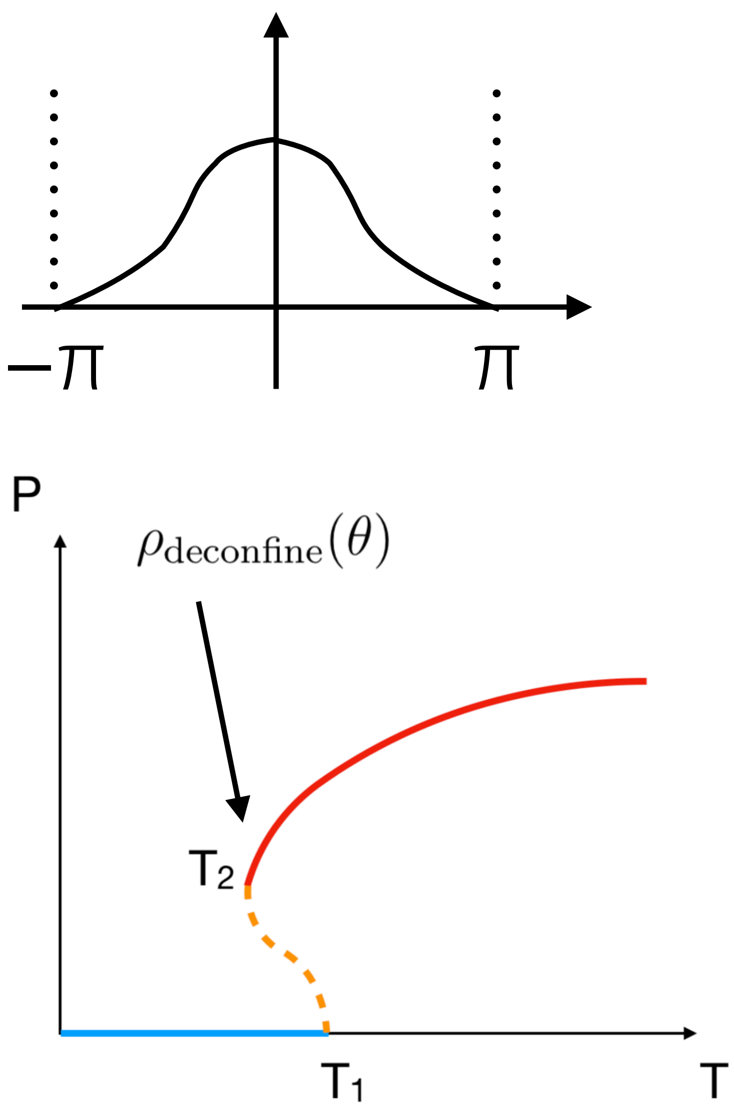
In all known cases, the same holds

$$\rho(\theta) = \frac{1}{2\pi}\left(1 + \frac{2}{\kappa}\cos\theta\right)$$

$$\rho(\theta) = \frac{N-M}{N}\rho_{\text{confine}}(\theta) + \frac{M}{N}\rho_{\text{deconfine}}(\theta) = \frac{N-M}{N}\cdot\frac{1}{2\pi} + \frac{M}{N}\rho_{\text{deconfine}}(\theta)$$

$$\frac{M}{N} = \frac{2}{\kappa}$$

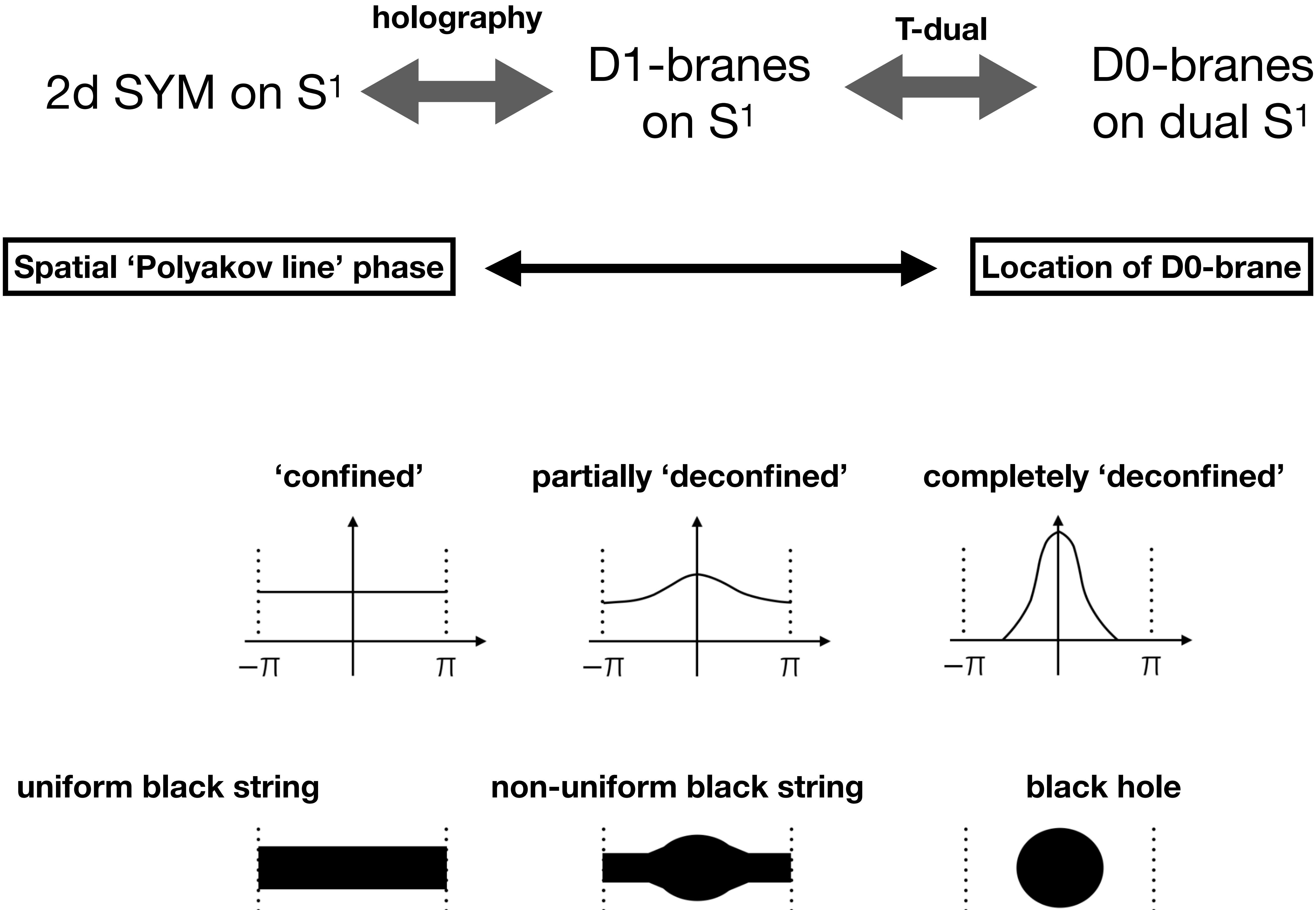
Pure YM on R³

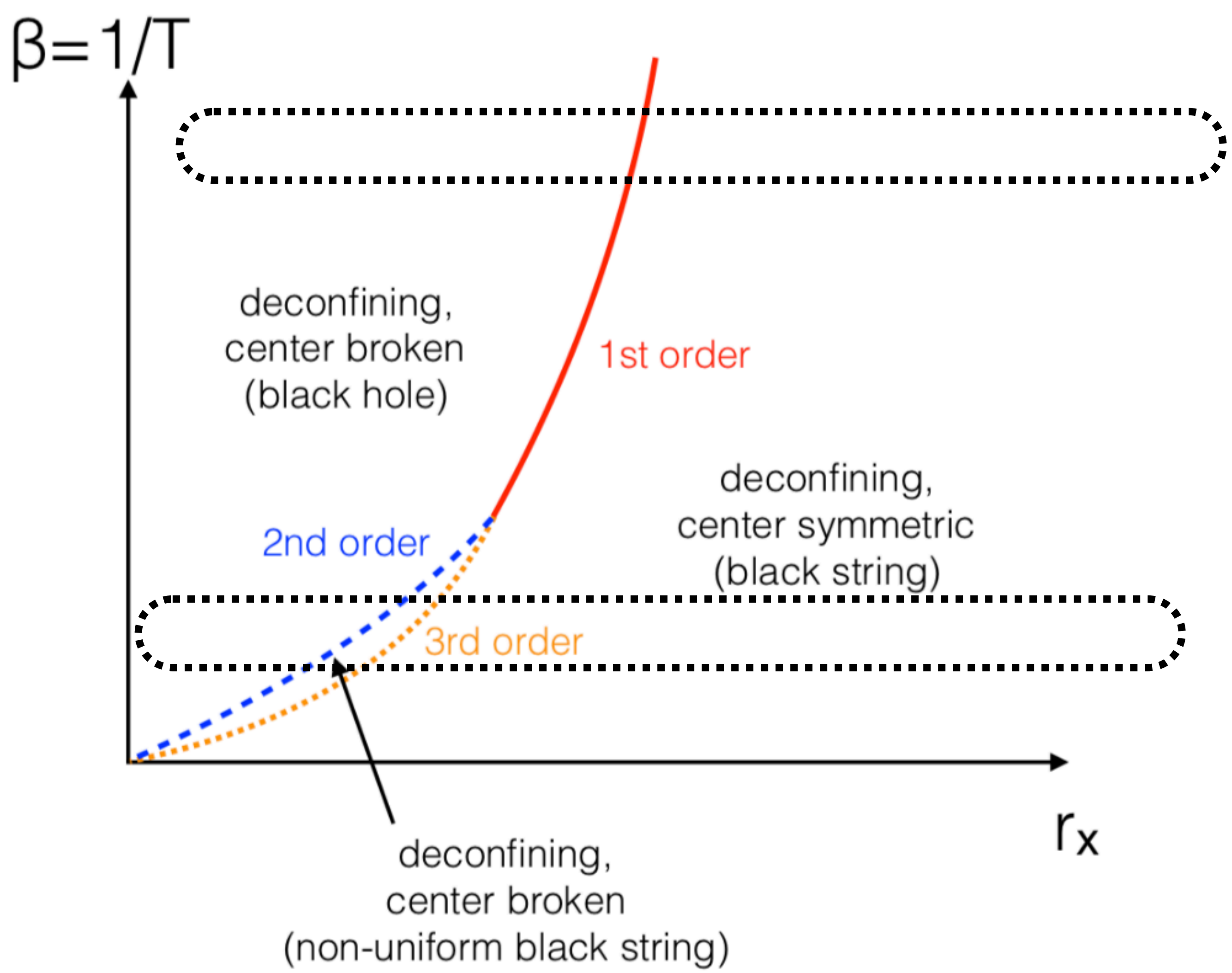


P=1/2

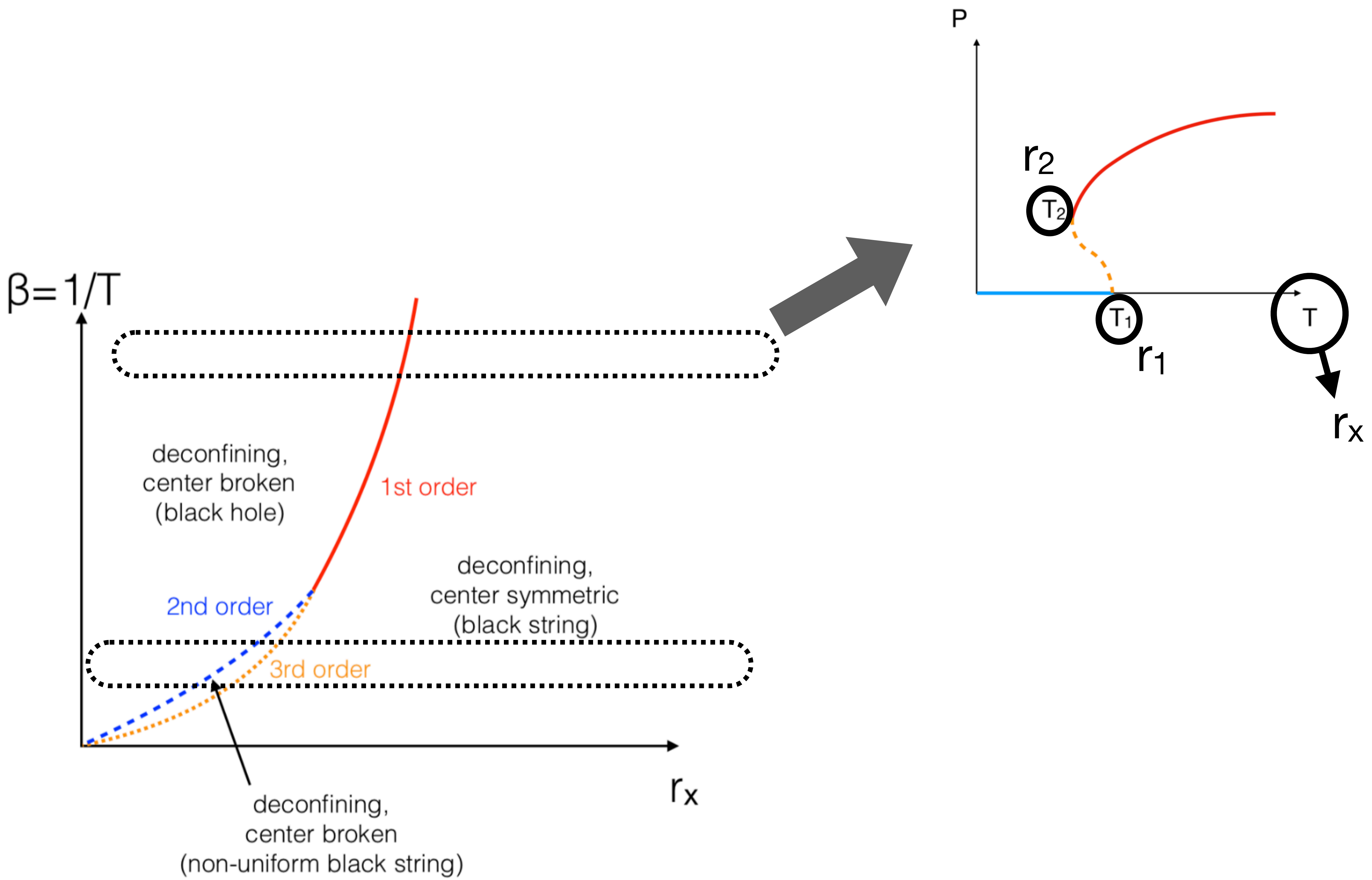
This value has been obtained for SU(3), SU(4), …, SU(6) by lattice Monte Carlo simulation

- Matrix model
- 4d YM
- 2d SYM

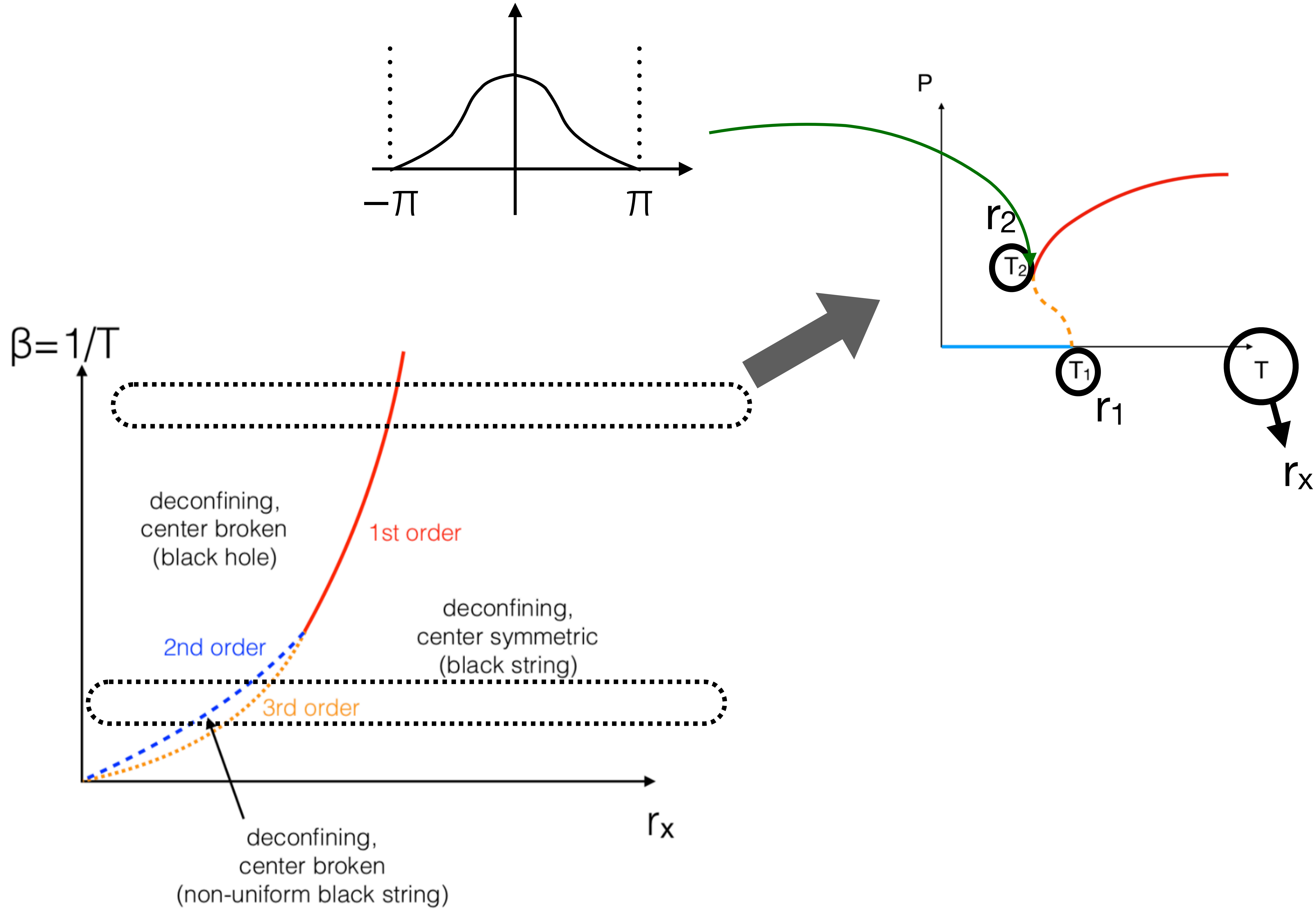

holography
T-dual
2d SYM on S1
D1-branes
on S1
D0-branes
on dual S1
Spatial ‘Polyakov line’ phase
Location of D0-brane
‘confined’
partially ‘deconfined’
completely ‘deconfined’
−π
π
−π
π
−π
π
uniform black string
non-uniform black string
black hole



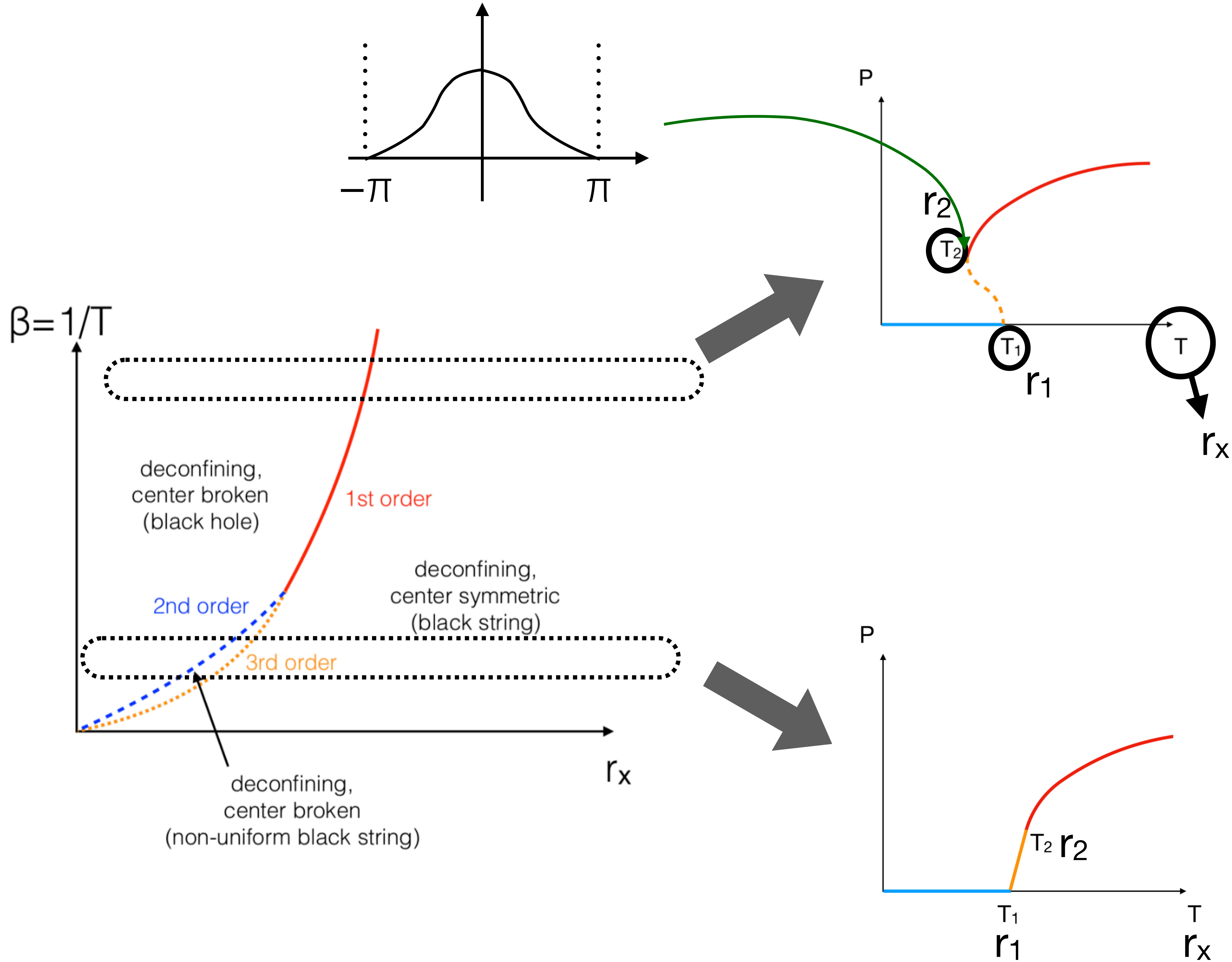
β=1/T
deconfining,
center broken
(black hole)
1st order
2nd order
deconfining,
center symmetric
(black string)
3rd order
r_x
deconfining,
center broken
(non-uniform black string)



β=1/T
deconfining,
center broken
(black hole)
1st order
deconfining,
center symmetric
(black string)
2nd order
3rd order
deconfining,
center broken
(non-uniform black string)
r_x
P
r_2
T_2
T_1
r_1
T
r_x



−π
π
P
r2
T2
T1
r1
T
rx
β=1/T
deconfining,
center broken
(black hole)
1st order
2nd order
deconfining,
center symmetric
(black string)
3rd order
rx
deconfining,
center broken
(non-uniform black string)



−π
π
P
r2
T2
T1
r1
T
rx
β=1/T
deconfining,
center broken
(black hole)
1st order
2nd order
deconfining,
center symmetric
(black string)
3rd order
rx
deconfining,
center broken
(non-uniform black string)
P
T2 r2
T1
r1
T
rx

10d Schwarzschild from 4d SYM

via

Partial Deconfinement

M.H., Maltz, 2016

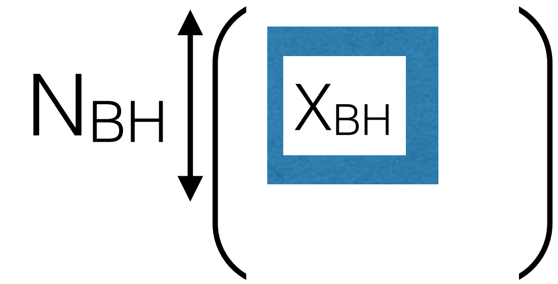
Heuristic Gauge Theory Derivation (1)

- Take radius of S^3 to be 1.
- At strong coupling, the interaction term $(N/\lambda)*\mathrm{Tr}[X_I,X_J]^2$ is dominant.

$\lambda = g_{YM}^2 N$

- Eigenvalues of $Y = \lambda^{-1/4}X$ are O(1) because the interaction is simply $N*\mathrm{Tr}[Y_I,Y_J]^2$.
- Hence eigenvalues of X are $O(\lambda^{1/4})$.

Heuristic Gauge Theory Derivation (2)



- When bunch size shrinks to $N_{BH}<N$, 't Hooft coupling effectively becomes $\lambda_{BH}=g_{YM}^2N_{BH}$

$\lambda=g_{YM}^2N$

- Hence eigenvalues of X_{BH} are $O(\lambda_{BH}^{1/4}) = O(g_{YM}^{1/2}N_{BH}^{1/4})$.

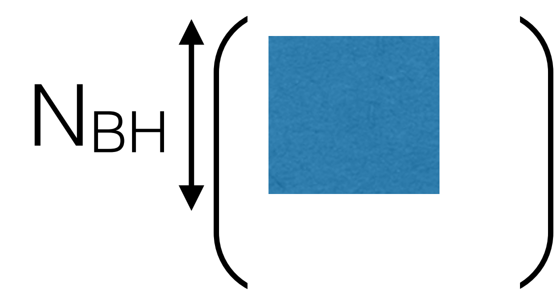
  - $E_{BH}\sim N_{BH}^2(N_{BH}/N)^{-1/4}$, $S_{BH}\sim N_{BH}^2$

  - $T_{BH}\sim(N_{BH}/N)^{-1/4}$

Heuristic Gauge Theory Derivation (3)

- $E_{BH} \sim N_{BH}^2 (N_{BH}/N)^{-1/4}$, $S_{BH} \sim N_{BH}^2$

- $T_{BH} \sim (N_{BH}/N)^{-1/4}$

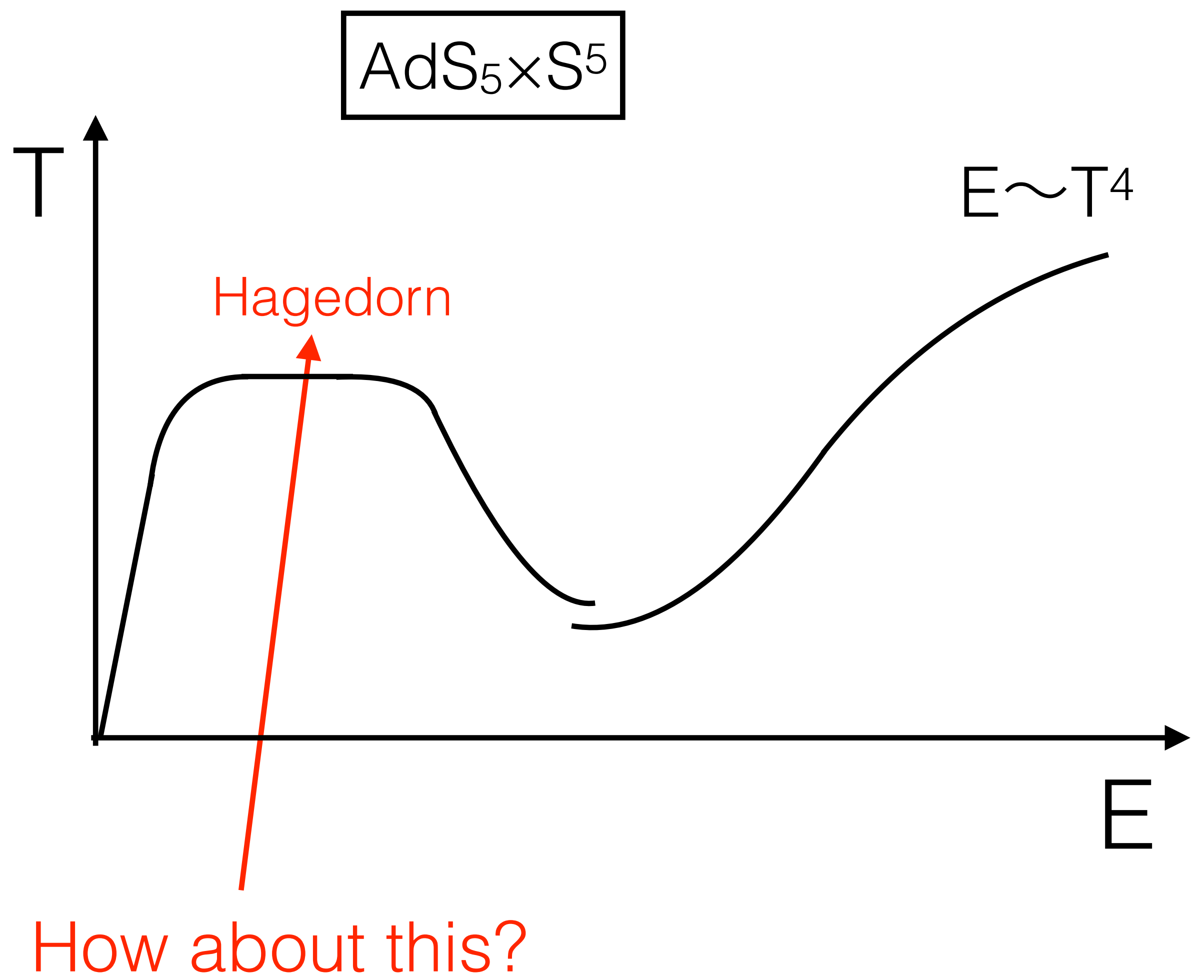


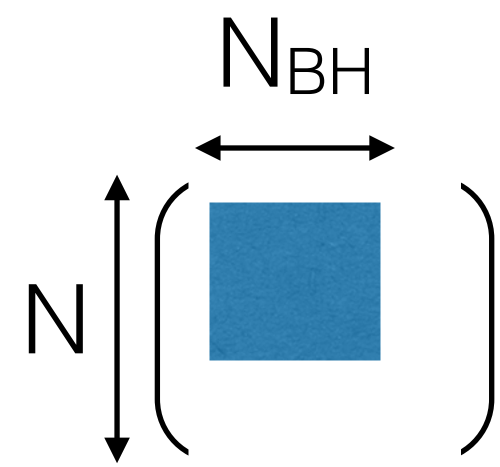
- $E_{BH} \sim N^2 (N_{BH}/N)^{7/4} \sim 1/(G_{N,10} T_{BH}^7)$

- $S_{BH} \sim N^2 (N_{BH}/N)^2 \sim 1/(G_{N,10} T_{BH}^8)$

10d Schwarzschild!

- The same logic applied to M-theory region of ABJM gives 11d Schwarzschild, $E \sim 1/G_{N,11} T^8$.





$E < E_{min}$

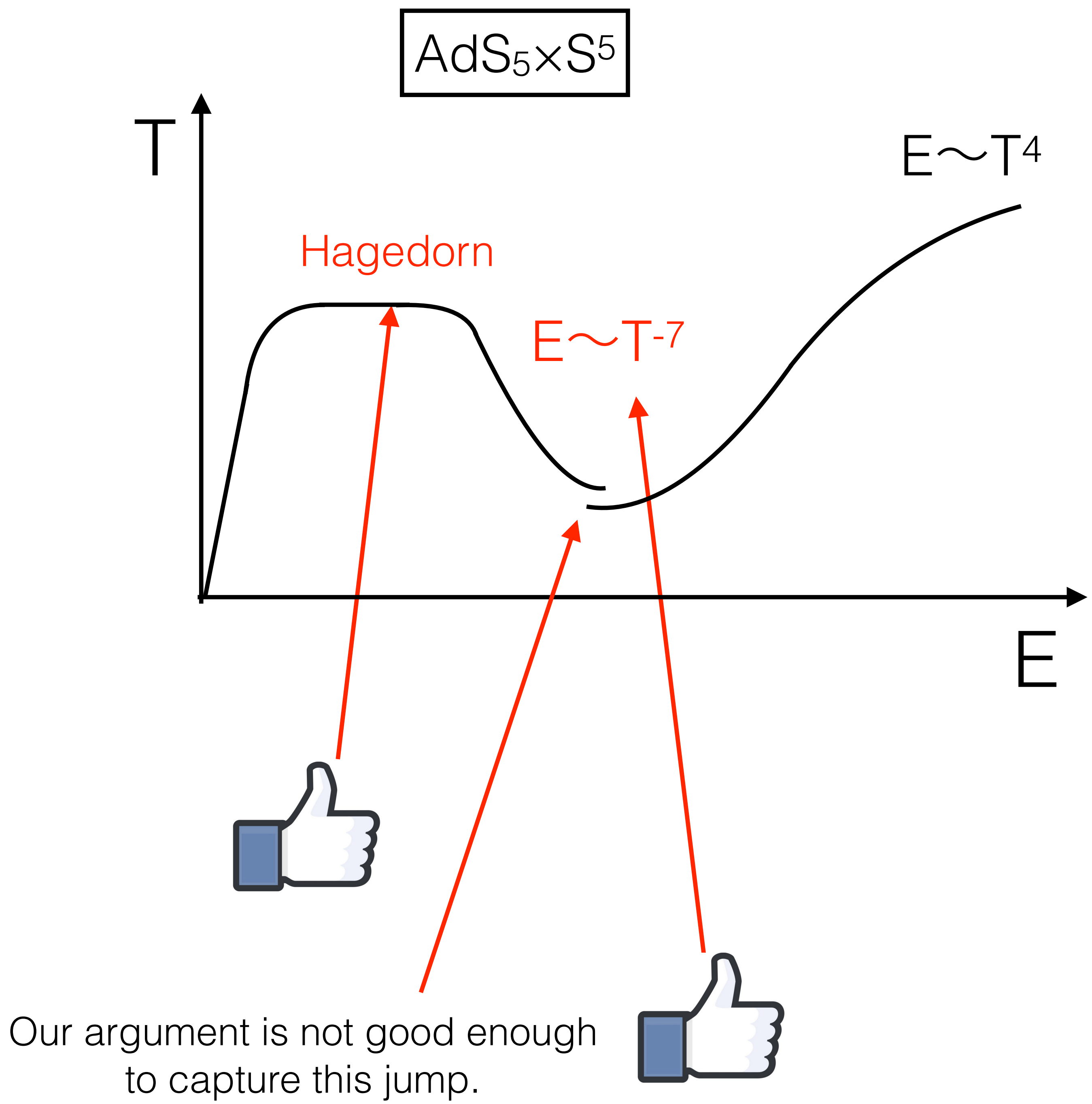
$T_{BH} = T_{Hagedorn} \sim 1$

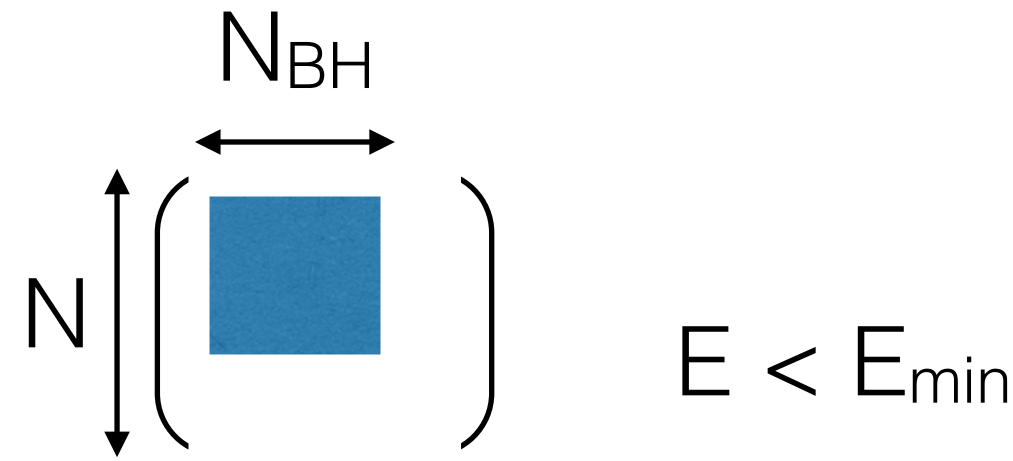
$E_{BH} \sim S_{min} \sim N_{BH}^2$

when $g_{YM}^2 N_{BH} << 1$

Just perturbative SYM.

$g_{YM}^2 N_{BH} << 1$



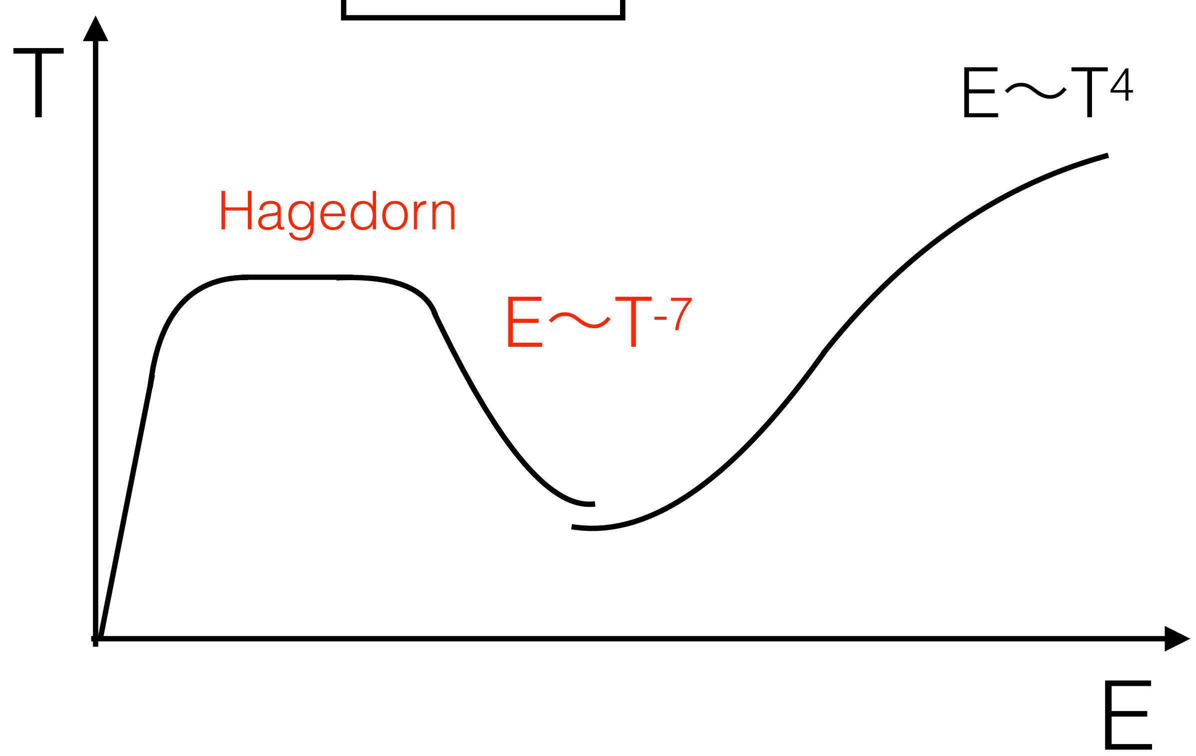


Large BH = 'Large' Matrices

Small BH = 'Small' Matrices

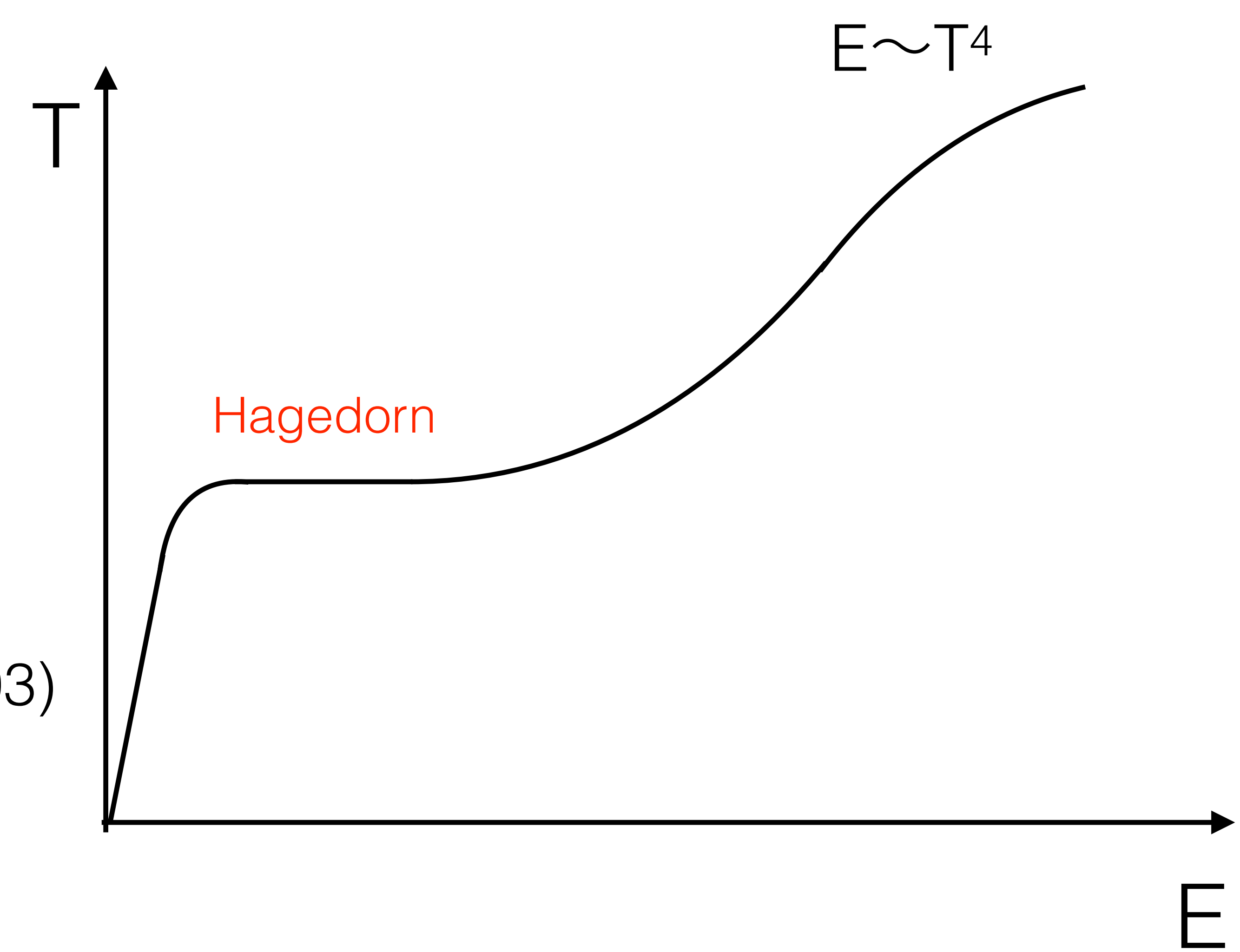
$\lambda >> 1$

$AdS_5 \times S^5$



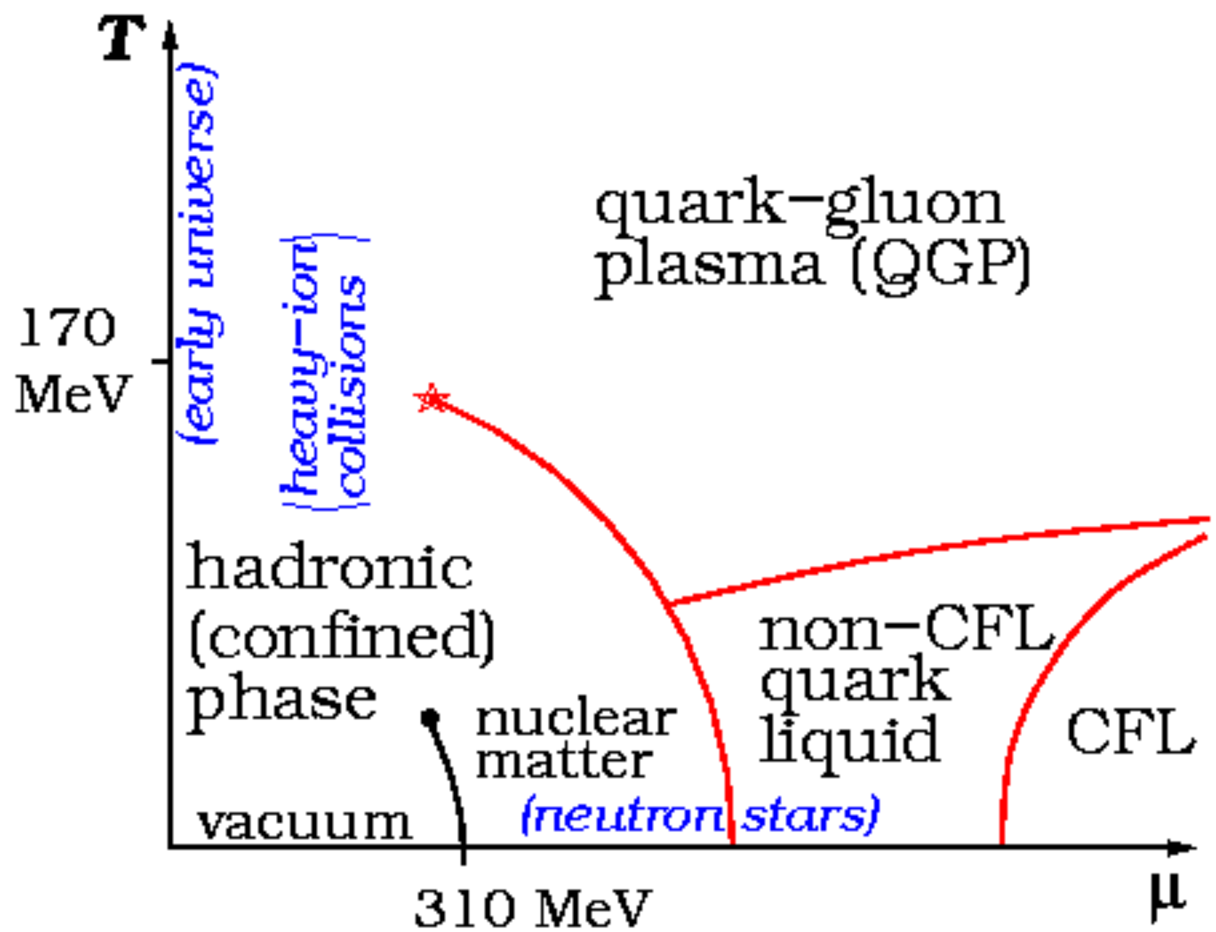
$\lambda << 1$

(see also Aharony et al 2003)



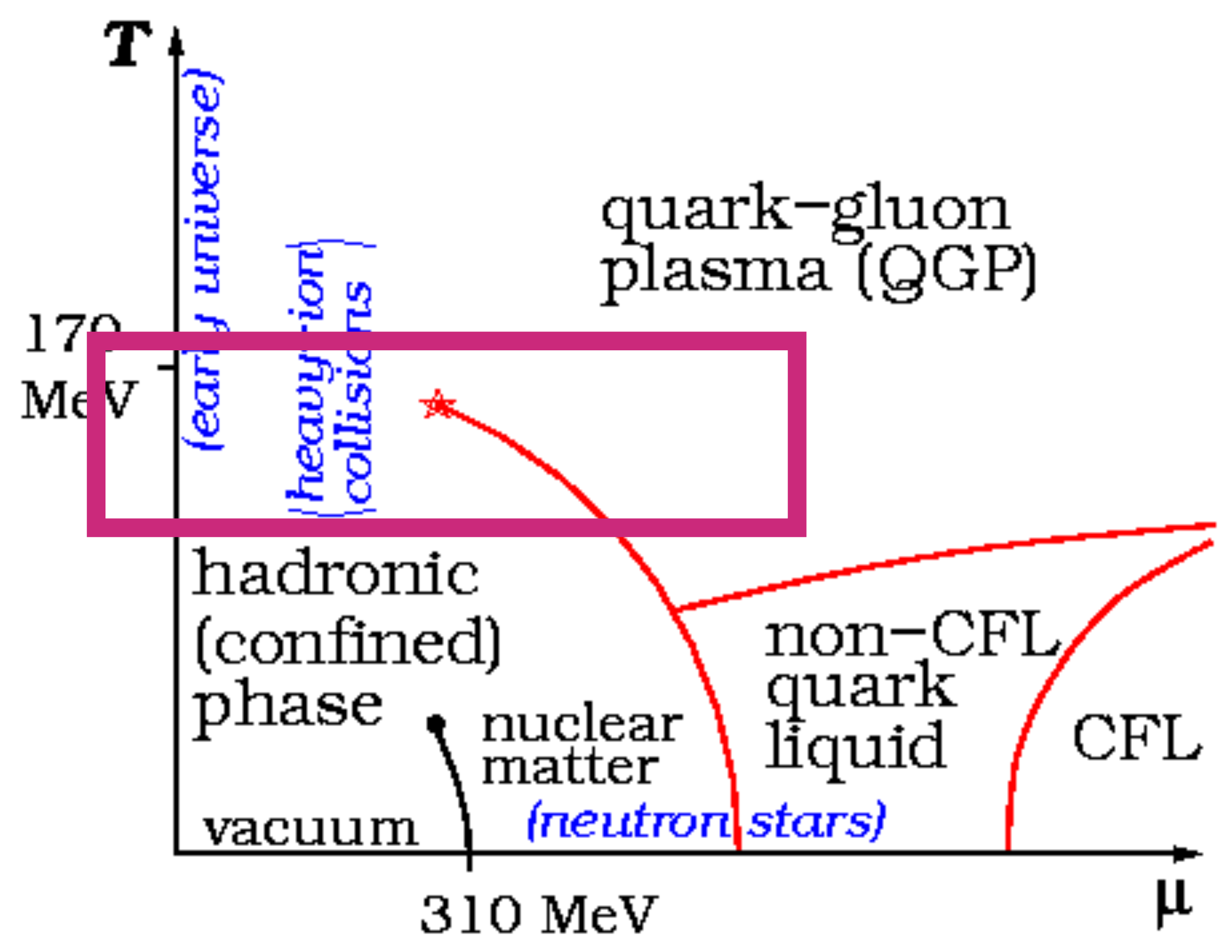
Finite density QCD
for
Hawking Evaporation?

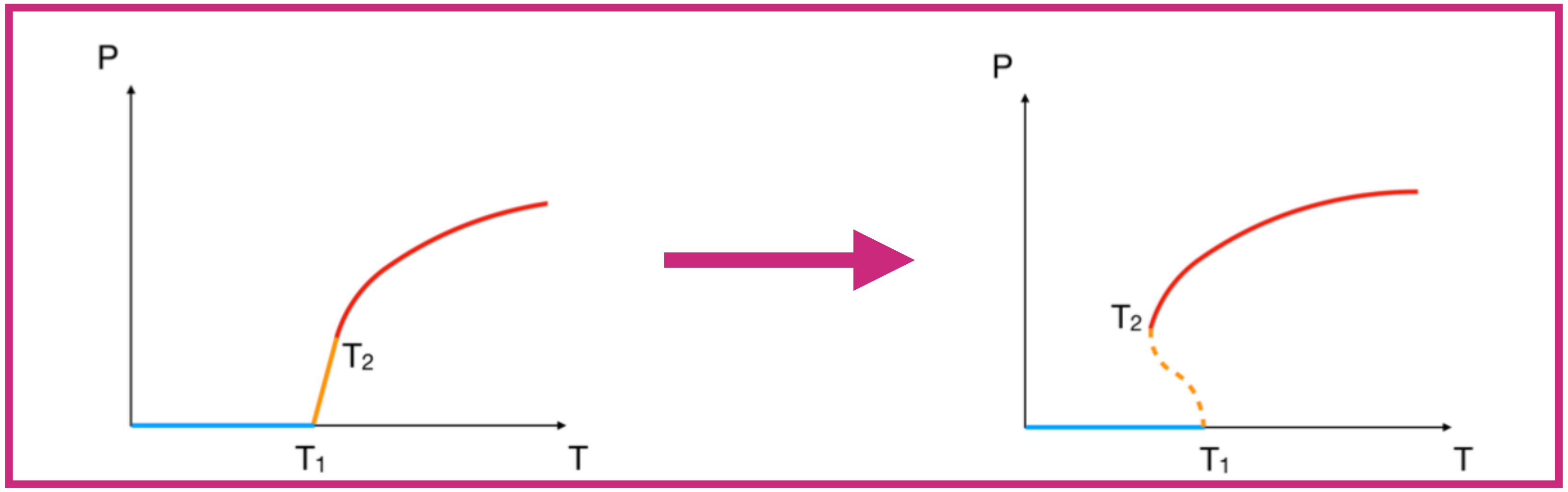
Conjectured QCD phase diagram



(from Wikipedia)

Conjectured QCD phase diagram



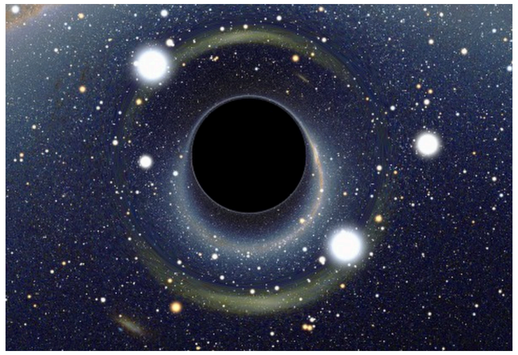


(from Wikipedia)

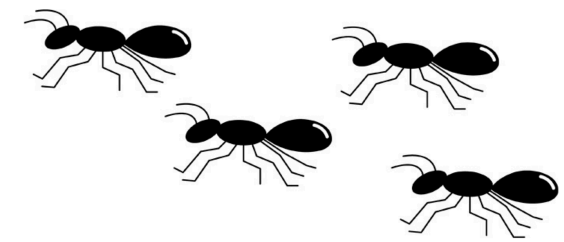
- ‘Evaporating black hole’ should be there.

disclaimer: ‘Gravity dual’ can be very stringy.

- What would be the experimental signal?
- ‘Applied holography’ should be a good tool.



Conclusion

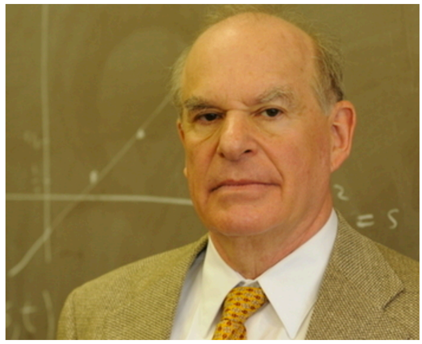


- Ants are smart. They know many things about black hole.

Lesson #2: Take "coincidences" seriously.

- 'Partial deconfinement' and 'Schwarzschild Black Hole' are rather generic in gauge theories.
- 'Hawking evaporation' in the heavy ion collision?
- It is important to study gauge theory, in order to understand quantum gravity. More should come, stay tuned.

backups

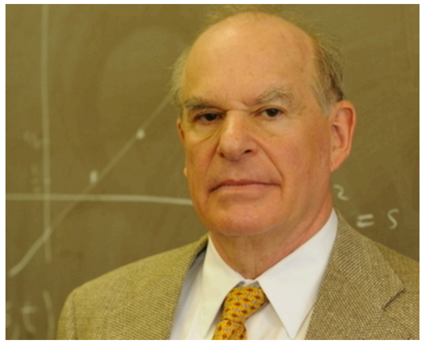


Lesson #1: If a theory developed for purpose A turns out to be better suited for purpose B, modify your goal accordingly.

The original goal of string theory was a theory of hadrons, but it turned out to work better as a theory of quantum gravity and unification. The massless particles should be identified as gauge particles and a graviton rather than vector mesons and a Pomeron.

When Yang and Mills formulated gauge theory in 1954, they identified SU(2) gauge fields with ρ mesons. 15 years later theorists developing dual models (the original name of string theory) made the same "mistake".

In 1974 we proposed to change the goal of string theory. It took another decade for the advantages of this interpretation to be widely appreciated. Perhaps there is a lesson in that, as well.



Lesson #3: **When working on hard problems explore generalizations with additional parameters.**

This lesson seems to be widely appreciated. There are many examples in the literature.

A couple of well-known examples are the Ω background for $\mathcal{N} = 2$ gauge theories and the $\mathbb{Z}_k$ orbifold generalization of $AdS_4 \times S^7$, which plays an important role in ABJM theory.