Dynamics of Disordered Periodic Systems

Vibration Analysis of Mistuned Blade Assemblies

When investigating the dynamics of engineering structures such as turbomachinery blade assemblies, the convenient assumption of perfect periodicity enables analysts to reduce the computational effort. The corresponding eigenvalue problem defined by block-circulant matrices can be block-diagonalized. Modelling and analyzing only a single sector, i.e., one blade and the corresponding portion of the disc allows the dynamics of the entire structure to be deduced. Unfortunately, the structural disorder or mistuning that always occurs in practice destroys this periodicity and provides some uncertainty in the blades' dynamics.

Mode Localization

The full dynamic problem of the entire structure must then be solved. Such parameter variations result in the splitting of multiple natural frequencies in each passband and in irregular mode shapes. In particular vibration may be localized around one or a few blades instead of being uniformly distributed among the blades.

The system studied is a finite element beam model of a continuously shrouded blade assembly. It is weakly or strongly coupled. Each blade accounts for many bending modes. The effects of 0.05% mistuning originating from changes in densities, Young’s Modulus, interblade coupling and blade root stiffnesses are separately analyzed and compared.

Reduced-Order Model Comparison of exact mode shapes with ROM mode shapes

When Finite Element Models are used to examine the responses of blade assemblies and mistuning is included in the dynamic analysis, a computational cost problem arises; each blade accounts for a considerable number of degrees-of-freedom (dof). Statistical methods involving numerical techniques, such as Monte Carlo Simulations are even more expensive especially when accurate statistics are to be obtained.

Randomly mistuned systems can require thousands of simulations. This problem can be tackled by building a reduced-order model (ROM) with a much lower size. Such models try to efficiently and cheaply capture the dynamics of the original Finite Element-based system, the natural frequencies and the mode shapes in particular. Then a full Monte Carlo Simulation can be performed at much reduced cost. The system considered is a finite element beam model of a coupled 20-blade system with uncertain root stiffnesses. The mistuning strength is 5%. For the system considered Monte Carlo Simulations of 100 randomly mistuned systems, each of 420 dof, take more than 3 hours, the Reduced-Order Model will take about 4 min. This considerable saving allows an accurate study of sensitivity of the coupled blades to mistuning and the production of accurate statistics. 10000 systems of 420 dof and 20 random, normally distributed and uncorrelated variables have been simulated.

Standard deviations of natural frequencies in the 2 first passbands

Comparison of the effects of various parameters on mode localization for a weakly coupled 7-blade system

The standard deviations obtained for natural frequencies in each passband form a U-shaped curve.