Stochastic Reduced Basis Methods for Uncertainty Quantification

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UTP for design

C E D G

Introduction

Computational modelling of engineering systems and natural phenomena often involve uncertainties arising out of lack of information (e.g. operating conditions, boundary conditions, etc.) and intrinsic variability (e.g. boundary definition, material properties, etc.). The idealized (nominal) values of uncertain parameters are often selected for numerical simulations as a result of which the correlation between the numerical results and reality remains unclear. With tremendous growth in computing power and availability of sophisticated numerical techniques in recent times, the prognoses of the numerical simulations are no longer limited by discretization errors involved in solving the governing equations but by input uncertainties. Hence reliable prognoses of the numerical simulations can be obtained by quantifying input uncertainties and propagating it to the response. If a probabilistic modelling framework is adopted for uncertainty quantification, then the system can be modelled by a set of Stochastic PDEs (SPDEs) and uncertainty propagation can be carried out by numerically solving the SPDEs.

Existing Methods

A brief overview of commonly used approaches for solving SPDEs is shown in Figure 1. In theory, simulation techniques such as the Monte Carlo method can be applied to approximate the output statistics to an arbitrary degree of accuracy, provided sufficient number of samples are used. Unfortunately, the computational cost incurred by direct simulation techniques can be prohibitive for high-fidelity models. Over the last decade, the Spectral Stochastic Finite Element Method (SSFEM) which employs Polynomial Chaos (PC) expansions has emerged as the method of choice for solving a wide class of SPDEs. However, the computational cost incurred by the SSFEM becomes prohibitive for large-scale problems with many random variables. This has motivated the development of advanced stochastic solvers that can cope with large-scale systems such as those encountered in engineering practice.

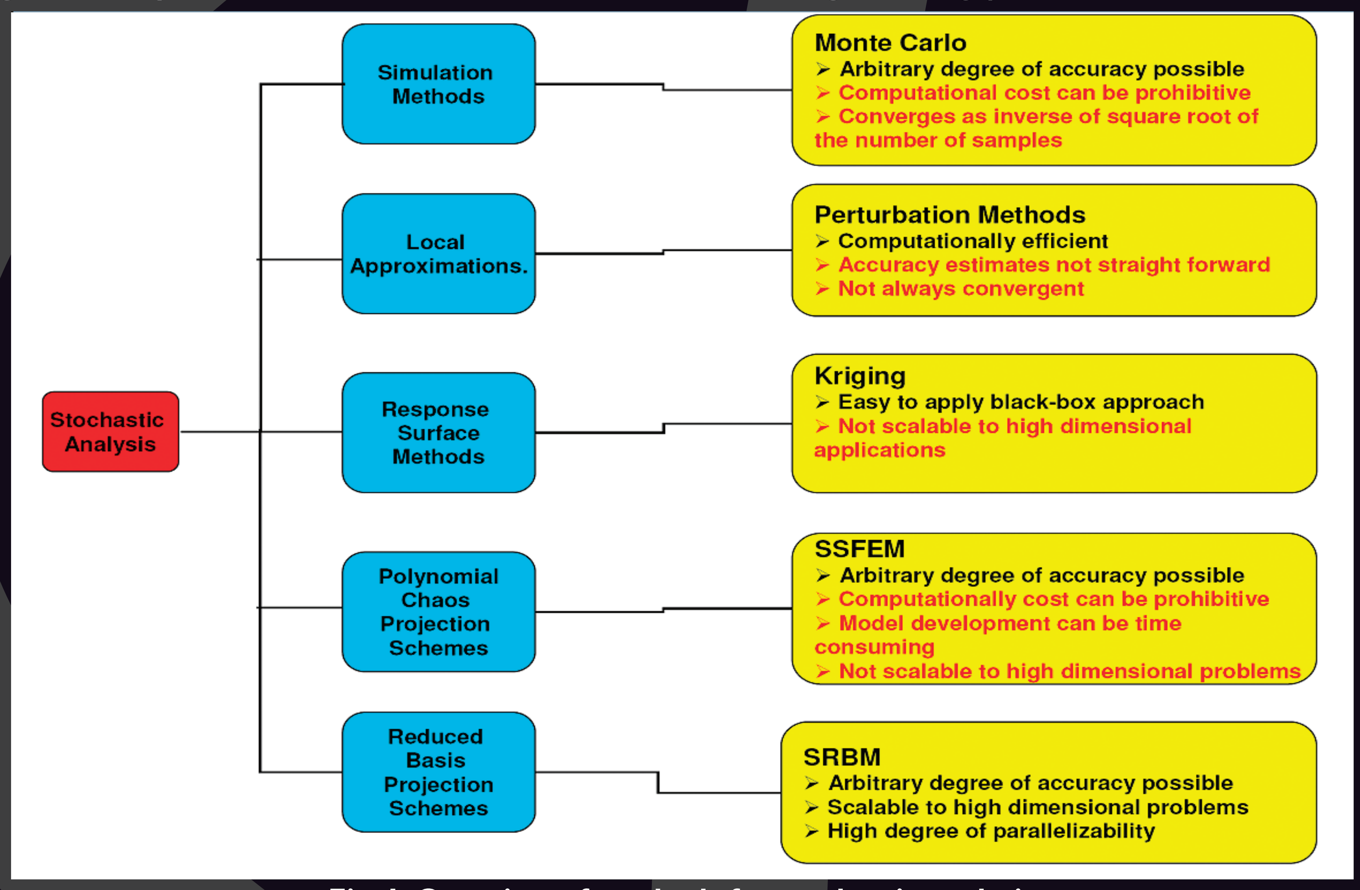

Fig. 1. Overview of methods for stochastic analysis.

Stochastic Reduced Basis Methods

Recently, Stochastic Reduced Basis Methods or SRBMs were developed (at the University of Southampton) for solving linear stochastic system(s) of equations obtained by fully discretizing SPDEs. The central idea underpinning this approach is to employ a set of basis vectors spanning a preconditioned stochastic Krylov subspace to approximate the response. Numerical studies on a range of problems have demonstrated that SRBMs perform better than existing methods both in terms of execution times and accuracy. In particular, the computational cost incurred by SRBMs can be orders of magnitude lower than direct Monte Carlo simulation and the SSFEM.

Figure 2 shows the convergence trends of percentage error in standard deviation for a model elasticity problem (shown in the inset) as the number of basis vectors is increased using SRBMs and SSFEM. The computational cost incurred by SRBMs and the SSFEM for this problem is shown in Figure 3. It can be noted from these trends that SRBMs can be more accurate than the SSFEM, while incurring significantly lower computational cost.

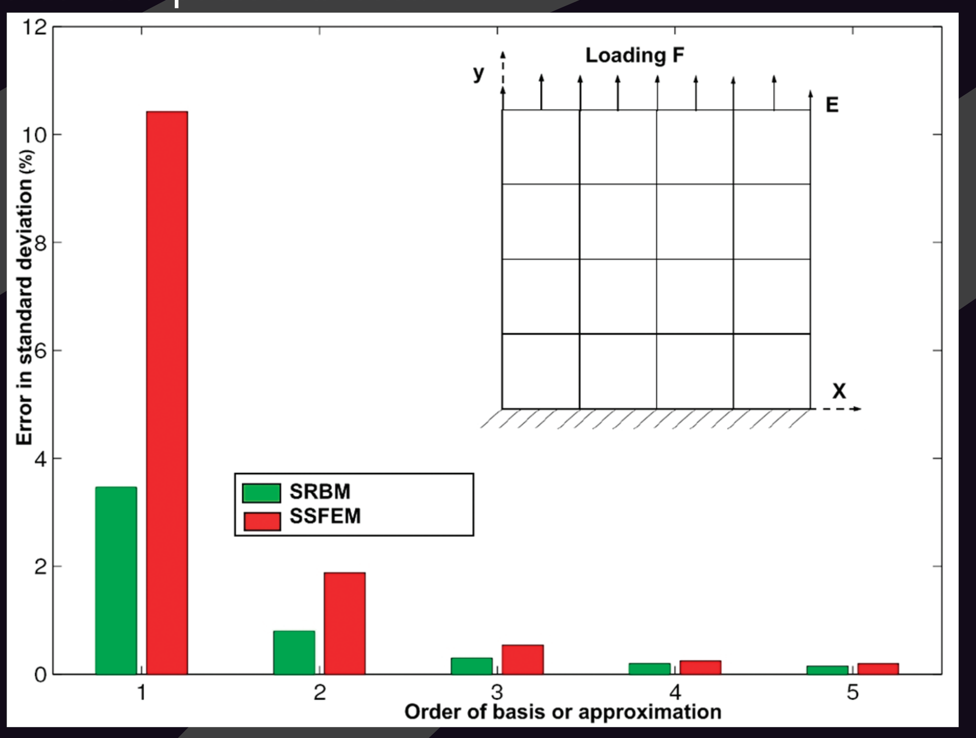

Fig. 2 Percentage error in standard deviation of displacement as a function of approximation order.

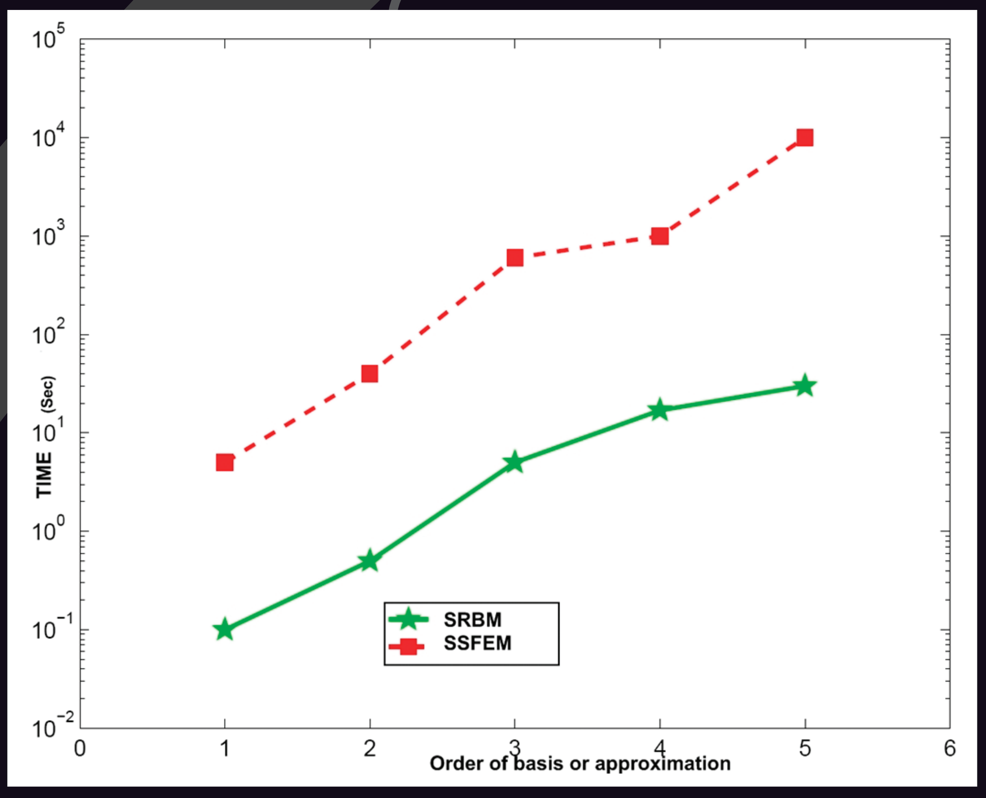

Fig 3. Comparison of computational cost as a function of approximation order.

Application to Gas Turbine Blades

In general, stochastic analysis (using SRBM) of any physical system involves two main steps: (1) identifying the random variables governing the phenomenon (uncertainty identification and modelling) and (2) propagating these uncertainties through the governing equations to obtain the statistics of the outputs of interest (uncertainty propagation). Figure 4 outlines the steps involved in quantifying the variability in the performance of a turbine blade in the presence of uncertainty. These blades operate at temperatures which are well above their melting points. Hence the blades are designed with a hollow core through which relatively cool air is channelled through to the surface via the cooling holes. Due to manufacturing uncertainties, the core positions change from blade-to-blade and from batch-to-batch resulting in uncertain geometry of the blades. This may result in uncertain cooling across the blades which in turn affects the life and maintenance costs of these components. Other sources of uncertainties include spatial variability in material properties and boundary conditions. Given a numerical solution of the set of SPDEs governing the performance of this system, it becomes possible to compute various measures of response variability in the post-processing phase.

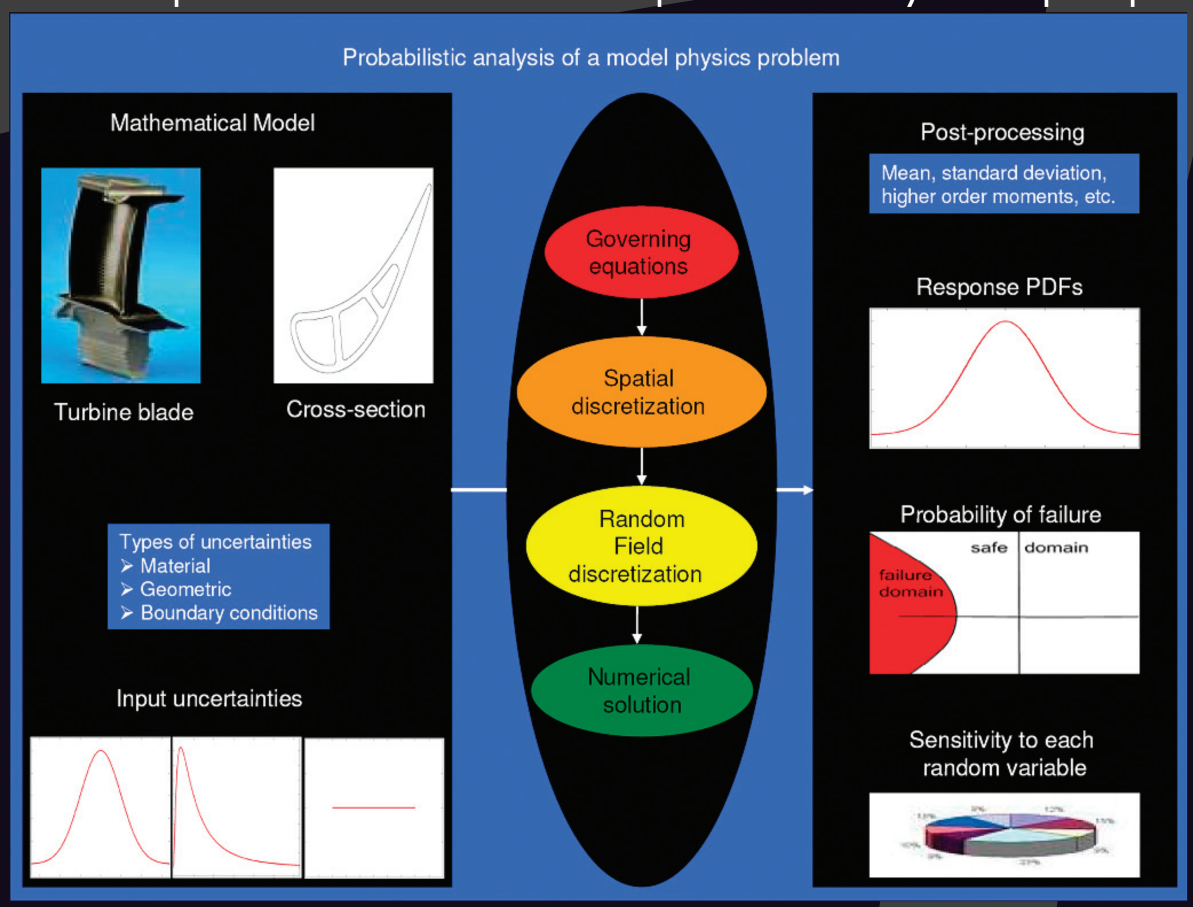

Fig 4. Steps involved in stochastic analysis of the performance of a gas turbine blade.

SRBMs have been successfully applied to study heat transfer in a two-dimensional profile of the turbine blade shown in Figure 4. The material property (thermal conductivity) is considered uncertain in this case and modelled by a Gaussian random field. The spatial distributions of the first two statistical moments of the temperature are shown in Figure 5.

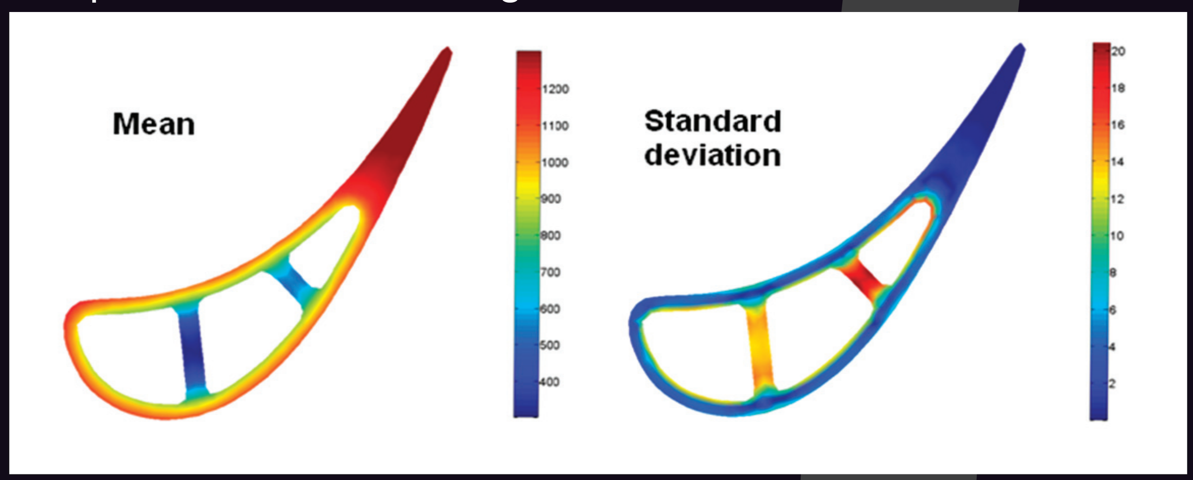

Fig 5. Spatial distribution of mean and standard deviation of temperature.

Ongoing Work

A novel and efficient method to quantify the impact of geometrical uncertainties arising from deviations in the position of the cores (uncertainty in boundary definition) is currently under development. Efforts are also currently underway to tackle the theoretical and computational challenges that arise when dealing with large-scale nonlinear systems.

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