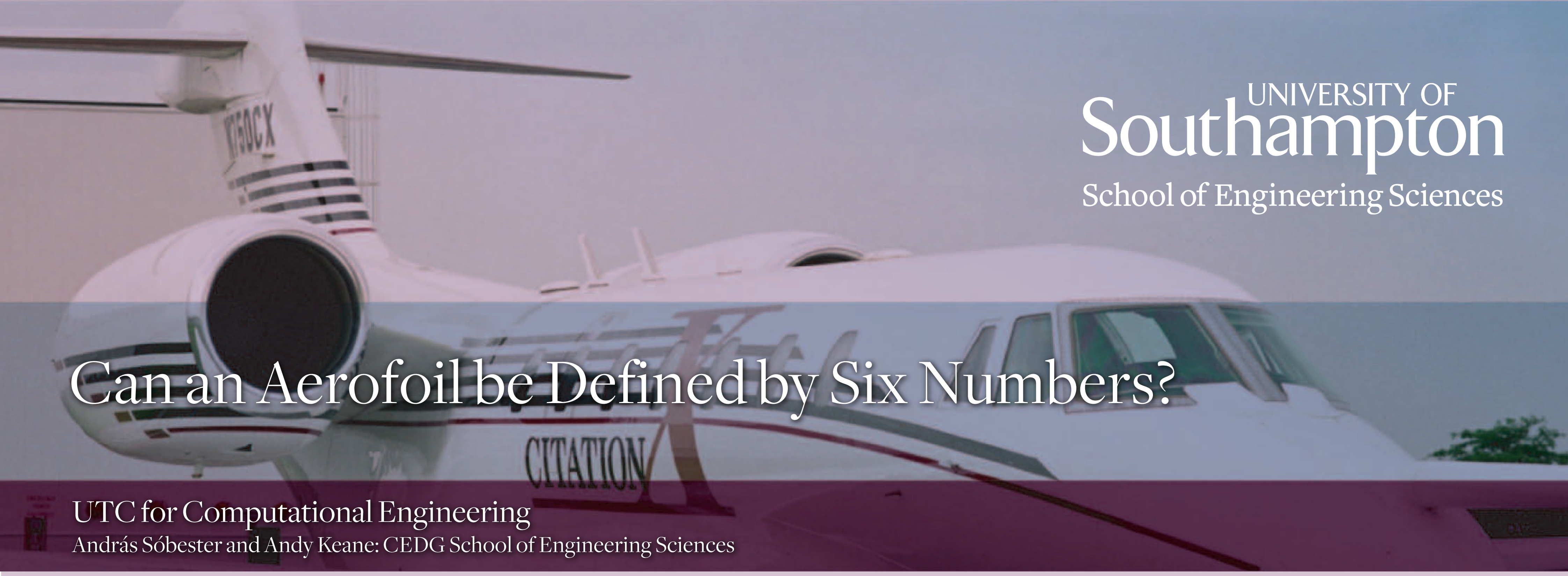

UNIVERSITY OF Southampton
School of Engineering Sciences

Can an Aerofoil be Defined by Six Numbers?

UTC for Computational Engineering
András Sóbester and Andy Keane: CEDG School of Engineering Sciences

Abstract

Here we revisit the Ferguson spline formulation, known since the 1960s, and we propose it as a means of airfoil parameterization, it being ideally suited to implementation in commercial Computer Aided Design (CAD) engines. The development providing the impetus: off-the-shelf CAD tools are taking a widening role in the design process even at its lowest, conceptual levels. We argue that, since similarly constructed splines lie at the heart of modern CAD modeling, the most natural way to describe, say, a wing geometry is via Ferguson-style cubic splines. Further, we show that in the interest of parameterization parsimony, adequate airfoil shape control can be achieved without knots (other than those on the leading and the trailing edge), at least at the conceptual level of any design process.

1. Ferguson's Parametric Splines

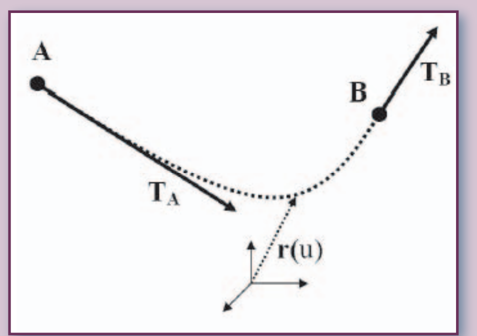

Fig.1 Ferguson spline and its boundary conditions.

We seek a parametric curve $\mathbf{r}(u)$ with $u \in [0;1]$, connecting two points $\mathbf{r}(0) = \mathbf{A}$ and $\mathbf{r}(0) = \mathbf{B}$. We impose two tangents on the curve: $d\mathbf{r}=du|_{u=0} = \mathbf{T}_A$ and $d\mathbf{r}=du|_{u=1} = \mathbf{T}_B$, as shown in **Fig. 1**. We define the curve as the polynomial

$$\mathbf{r}(u) = \sum_{i=0}^{3} \mathbf{a}_i u^i, u \in [0,1]. \qquad (1)$$

We find the four vectors required to define the curve by setting the endpoint conditions:

$$\begin{aligned} \mathbf{A} &= \mathbf{a}_0 \\ \mathbf{B} &= \mathbf{a}_0 + \mathbf{a}_1 + \mathbf{a}_2 + \mathbf{a}_3 \\ \mathbf{T}_A &= \mathbf{a}_1 \\ \mathbf{T}_B &= \mathbf{a}_1 + 2\mathbf{a}_2 + 3\mathbf{a}_3 \end{aligned} \qquad (2)$$

Re-arranging in terms of the vectors:

$$\begin{aligned} \mathbf{a}_0 &= \mathbf{A} \\ \mathbf{a}_1 &= \mathbf{T}_A \\ \mathbf{T}_A &= \mathbf{a}_1 \\ \mathbf{T}_B &= \mathbf{a}_1 + 2\mathbf{a}_2 + 3\mathbf{a}_3 \end{aligned} \qquad (3)$$

Substituting back into Equation (1) we obtain:

$$\mathbf{r}(u) = \mathbf{A}(1 - 3u^2 + 2u^3) + \mathbf{B}(3u^2 - 2u^3) + \mathbf{T}_A(u - 2u^2 + u^3) + \mathbf{T}_B(-u^2 + u^3) \qquad (4)$$

or in matrix form:

$$\mathbf{r}(u) = [1\ u\ u^2\ u^3] \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -3 & 3 & -2 & -1 \\ 2 & -2 & 1 & 1 \end{bmatrix} \begin{bmatrix} \mathbf{A} \\ \mathbf{B} \\ \mathbf{T}_A \\ \mathbf{T}_B \end{bmatrix} \qquad (5)$$

$\mathbf{r}(u)$ is, essentially, a Hermitian interpolant and the bracketed factors in equation (4) can be viewed as its basis functions. **Fig.2** conveys an intuitive understanding of their effect on the shape of the interpolant.

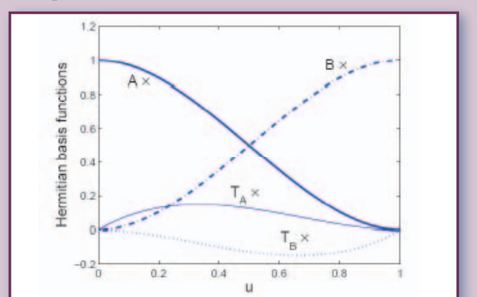

Fig.2 The four basis functions of equation (4) shown alongside their respective multipliers.

2. Aerofoil description

Using Ferguson's splines, the number of aerofoil design variables can be reduced to six (assuming a sharp trailing edge) while still maintaining a sufficiently broad coverage of the design space. We achieve this by not using any knots, other than the endpoints. The geometry is illustrated in **Fig.3**. The tangent of the upper surface $r^u(u)$ in **A** (at the leading edge) is denoted by $\mathbf{T}^A_{upper}$ and its tangent in **B** by $\mathbf{T}^B_{upper}$. The same logic is used for the nomenclature of the lower surface.

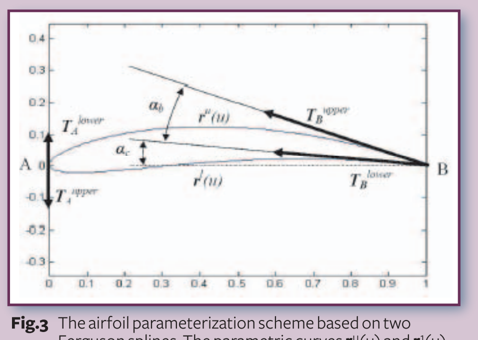

Fig.3 The airfoil parameterization scheme based on two Ferguson splines. The parametric curves $\mathbf{r}^u(u)$ and $\mathbf{r}^l(u)$ describe the upper and lower surfaces respectively.

The shape of the airfoil is thus defined by the orientation and the magnitude of the tangent vectors. T^A_{upper} and T^B_{lower} will always be pointing vertically downwards and upwards respectively, with their magnitude defining the tension in the spline, thus controlling the `bluntness' of the leading edge. α_c, which might be called the camber angle, defines the orientation of T^B_{lower}, while the boattail angle α_b determines the orientation of T^B_{upper}. The magnitudes (tangent tensions) of these vectors determine the shape of the middle section of the airfoil.

3. Drag Versus Structural Design Drivers – a Design Study

In **Fig.4** we demonstrate the flexibility of the airfoil parameterization scheme proposed here, as well as the coverage of the design space, by conducting such a trade study. We construct a Pareto front involving two objectives. For a range of airfoils we calculate their maximum depth, that is, the maximum distance between their upper and lower surface. This is an important consideration in single main spar designs. We also compute the drag (cd) for each airfoil, using VGK, a two-dimensional ow solver, which couples finite difference solutions of the inviscid ow about the airfoil with solutions for the displacement effects of the boundary layer and wake. The inviscid component is the solution of the full potential equations for steady, compressible flow. The viscous drag is estimated from the momentum thickness of the wake far downstream of the aerofoil.

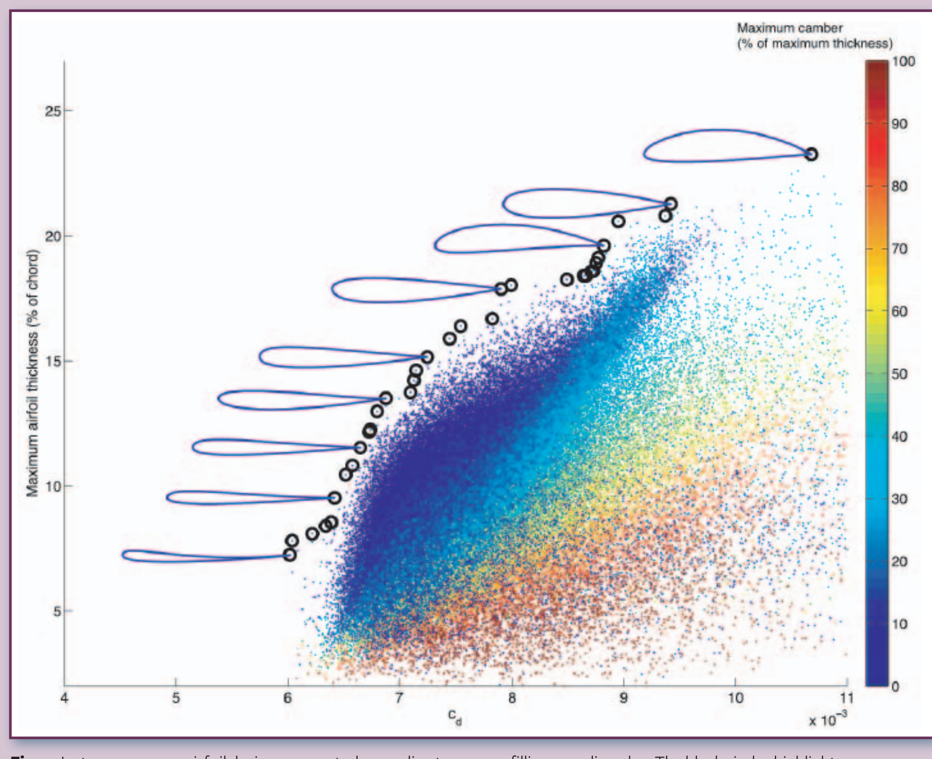

Fig.4 Just over 130,000 airfoil designs generated according to a space-filling sampling plan. The black circles highlight non-dominated (Pareto-optimal) solutions of the airfoil thickness versus drag at $c_l = 0.35$ trade-offs; some of these airfoils are also depicted, positioned with their trailing edges on the corresponding point. The colours represent maximum camber.

Conclusions

Using a pair of simple Ferguson splines to represent airfoils, while not a universally applicable scheme (less well suited to detail design, particularly of transonic applications), is a parsimonious way of describing airfoils, which, as shown in **Fig.5**, comes with the bonus of being natural and easy to implement in a CAD-based design system.

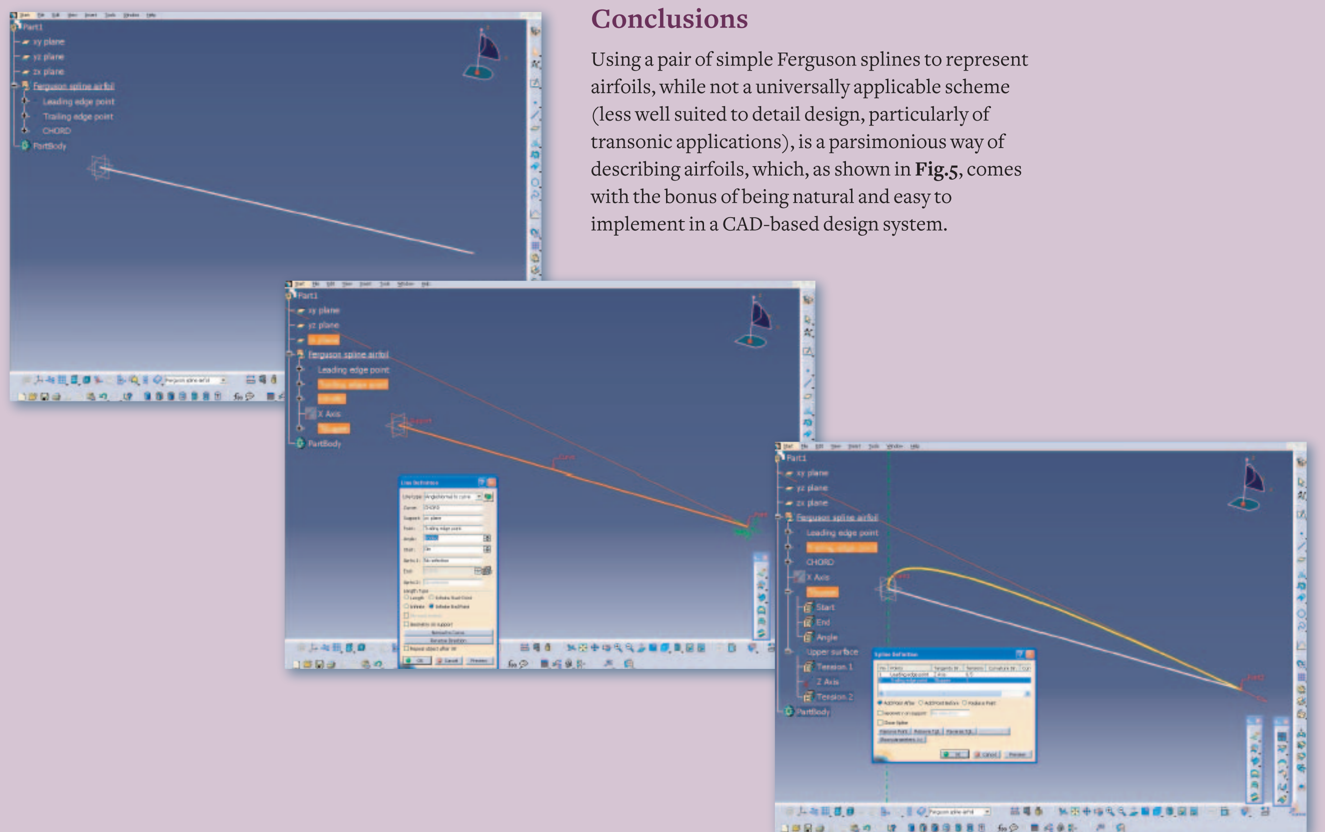

Fig.5 The Catia construction of the upper surface of an aerofoil based on Ferguson splines is a three-click process.

www.soton.ac.uk/ses/research/ced/posters.html | email: A.SOBESTER@soton.ac.uk
Computational Engineering & Design Group, University of Southampton, SO17 1BJ, U.K.