Multi-fidelity global optimization using co-kriging

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Abstract
Optimization problems, where the set of conditions are sought which relate to an optimum scenario, are encountered in most branches of science. In this work we consider the class of optimization problems where there are a number of ways of obtaining values of the quantity to be optimized (the objective function). Data from fast but less accurate models and slower but more accurate calculations or measurements can be correlated to learn more about the objective function. We present an optimization strategy which uses correlated data to build a statistical model of the objective function to predict the location of the global optimum.

A one variable example
Imagine that our experience to compute data are calculated by the function $f(x) = x^2 + e(x)$ where $e(x)$ is a normal distribution with mean 0 and standard deviation 1. If we sample the design space extensively using the cheap function at $X_1 = 0$, $X_2 = 1$, $X_3 = 2$, and $X_4 = 3$, and a cheaper estimate of this data is given by $y(x) = x^2 + e(x)$, we can sample the design space extensively using both functions at $X_1 = 0$, $X_2 = 1$, $X_3 = 2$, and $X_4 = 3$. The kriging model or monte kriging model is then fitted to the data, and the estimated function is used to predict the unknown values at the design points.

An aerospace example
The benefits of co-kriging, and in particular representing co-kriging, are demonstrated through the optimization of a generic transonic aircraft wing. Our cheap analysis is in the form of an empirical design methodology (TPPP). The design is optimized using a series of pre-designed wings in two steps. First, our cheap analysis is used to create an initial design which is then evaluated using a more expensive analysis. The design is then modified to improve the performance of the wing.

Conclusions
Our results demonstrate that a co-kriging strategy can be used to optimize complex systems using a variety of different models, including complex simulations and expensive models. The co-kriging strategy was able to find optimal solutions more quickly than using traditional optimization methods. The co-kriging strategy was also able to find solutions that are more robust to changes in the model parameters.

References

Table 1: Performance comparison for the transonic aircraft wing problem (average over 100 trials).

<table>
<thead>
<tr>
<th>Method</th>
<th>CPU time</th>
<th>RMSE</th>
<th>FOM</th>
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<td>0.108</td>
<td>2.586</td>
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</table>

Fig. 1. A sample co-kriging example. The model prediction at point 0 has been significantly improved using extensive sampling from the cheap function.

Fig. 2. Estimated error in the co-kriging prediction of the function $f(x) = x^2 + e(x)$.

Fig. 3. Performance comparison for the transonic aircraft wing problem (average over 100 trials).

Fig. 4. Performance comparison for the transonic aircraft wing problem (average over 100 trials).