UNIVERSITY OF Southampton

School of Engineering Sciences

Using Generative Topographic Mapping for dimension reduction in design optimization

UTC for Computational Engineering

Asha Viswanath, Alexander Forrester and Andy Keane, CEDG, School of Engineering Sciences

Introduction

The need for dimension reduction in design optimization arises in design problems dealing with very high dimensions, which increase the computational burden of the design process because the sample space required for the design search varies exponentially with the dimensions. This work describes the application of a latent variable method called Generative Topographic Mapping (GTM) in dimension reduction of a data set by transformation into a low-dimensional latent space. The attraction it presents is that design variables are not removed, but only transformed and hence there is no risk of missing out on information relating to all the variables. The method is demonstrated on a 2D Branin function and applied to a problem in wing design. Apart from dimension reduction in optimization, we also used the tool to provide an efficient parametrization scheme for airfoils, the results of which are also discussed.

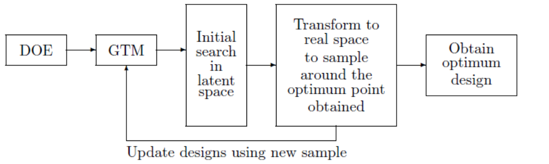
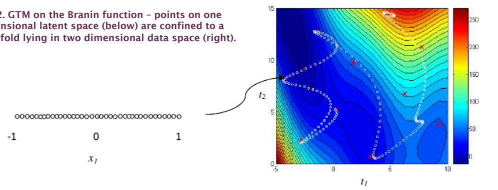


Fig. 1. GTM based design search workflow

Methodology

The method of GTM tries to model the probability density of the data in high dimensional *D*-space, $\mathbf{T} = \{t_1,\ldots, t_N\}$ in terms of a grid of latent variables $\mathbf{x} = \{x_1,\ldots,x_K\}$ with lower dimension *L* by training a *L* dimensional manifold which is embedded in the *D*-space. The parameters of the GTM are determined during training through the maximization of the log-likelihood of the model usually using a E-M algorithm, see figure 3. A detailed derivation and explanation of the method is available in Bishop *et.al* [1]. Our GTM-based optimization algorithm is shown above, see figure 1. This method follows a response-surface based approach, starting with an experimental design (DOE) and the trained GTM being used as the surrogate. GTM uses Bayesian inference for the training and hence we combine statistical analysis of data along with optimization methods. The low dimensional latent space is searched for the best point which is updated to the training data set before retraining the GTM. The iterations are continued till there is no further improvement in design. This follows the two-stage response surface based optimization approach. Here the DOE was also conducted for different samples and the optimum obtained averaged.

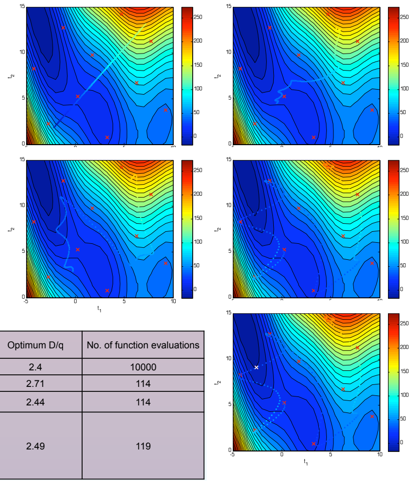
Fig. 2. GTM on the Branin function – points on one dimensional latent space (below) are confined to a manifold lying in two dimensional data space (right).



Methodology

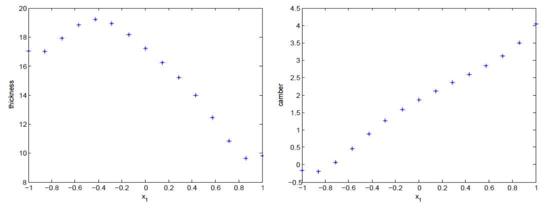
- Lastly, we applied GTM to study the characteristics of parameters in airfoil design. The NACA 4-digit representation of airfoils is well-established and depends on two main parameters - thickness to chord ratio and camber. By considering a different, and intentionally `clumsy', representation of an airfoil using co-ordinate points at various cross-sections of NACA airfoils, we can generate numerous variables governing the shape design. We then train a 2D GTM to reduce this high dimensional data to just two latent variables and hope that these latent space variables resemble camber and thickness to chord ratio. In that case, we can then apply GTM to more complex designs with high dimensions, for which we do not have prior information regarding the variables, to obtain meaningful representation of its objective function in terms of fewer variables and then conduct an optimization in the reduced dimension. For the experiment, NACA airfoil designations NACA 0006 to NACA 4430 are considered and having generated the co-ordinates from these airfoils, the lift drag (*Cl/Cd*) ratio is calculated using a full-potential VGK solver[2]. The parameters Reynolds number *Re*=1e7, Mach number *M*=0.12, initial angle of attack *a0*=0 and a target *Cl*=0.8 are considered. A data set of 105 airfoils is used to train a 2D GTM with 4 E-M cycles and *K*=15*15 latent grid points. The assumption that the latent space would represent the two most important variables in an airfoil, namely thickness to chord ratio and camber, did not perform as well as hoped. However the airfoils generated from 1D GTM show a variety of different thickness and camber and hence a 1D GTM could span the design space of airfoil shapes reasonably effectively, see figure 4. Though we could not attribute the two variables, camber and thickness-to-chord ratio to any one latent variable, we may have derived a geometric paramaterization scheme for airfoil shapes since a 1-dimensional GTM could represent a wide range of airfoils with different cambers and thicknesses.
- The method is applied to an aircraft wing design problem having 11 independent variables with the objective function of minimizing the wing drag *D/q*. The drag estimation tool is TADPOLE [4]. A 3D latent space could effectively model the 11D design space. The total computation time is 2 minutes as against a GA run of 20 population size for 500 generations which took 2 minutes and gave a optimum of 2.4! See table 1.
- Application to 100 dimension.
- Robust update methods.
- Application to constrained optimization problems.

Fig. 3. GTM training cycles of 2D Branin function



Method	Optimum D/q	No. of function evaluations
Genetic Algorithm	2.4	10000
3D GTM	2.71	114
Kriging	2.44	114
3D GTM with exploration update	2.49	119

Table 1.Comparison of different methods for TADPOLE wing design [4]



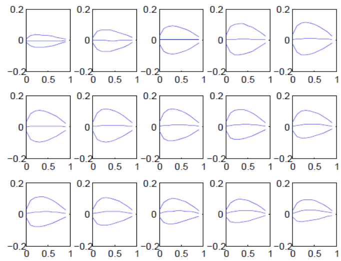


Fig. 4. Thickness and camber plots for different latent points for 1D GTM. A wide range of airfoils are regenerated by 1D GTM as shown

References

1.Bishop, C., M., Svensen, M., Williams, C., K., I.: GTM: The Generative Topographic Mapping. Neural Computation. 10, 215-234 (1998)
2.ESDU: VGK method for two-dimensional aerofoil sections, Technical Report 96028.(1996)
3.Forrester, A., I., J., Sobester, A. and Keane, A., J.: Surrogate Models in Engineering Design: A practical guide, Wiley and Sons. (2008)
4.Cousin, J. and Metcalfe, M.: The BAE Ltd transport aircraft synthesis and optimization program, AIAA paper 90-3295. (1990)
5.Keane, A., J. and Nair,P., B.: Computational approaches for aerospace design, Wiley and Sons. (2000)

www.soton.ac.uk/ses/research/ced/posters.html | email: ajk@soton.ac.uk

Computational Engineering and Design Group, University of Southampton, SO17 1BJ, UK