NON-RELATIVISTIC CLASSICAL MECHANICS REVISION NOTES:

1 Momentum and Kinetic Energy

A particle of mass m and velocity $\mathbf{v}$ (vector quantity) has momentum (vector quantity), $\mathbf{p}$

$$\mathbf{p} = m\mathbf{v},$$

and kinetic energy T,

$$T = \frac{1}{2}mv^2 = \frac{p^2}{2m}$$

2 Newton's Second law of Motion:

The force $\mathbf{F}$ (vector quantity) acting on a system of particles is equal to the rate of change of momentum

$$\mathbf{F} = \frac{d\mathbf{p}}{dt}$$

3 Angular Momentum

A particle with momentum $\mathbf{p}$ at a point $\mathbf{r}$ relative to some origin, has angular momentum $\mathbf{L}$ (vector quantity) given by

$$\mathbf{L} = \mathbf{r} \times \mathbf{p},$$

about that origin.

The moment of inertia of a system of particles with masses m_i and coordinates $\mathbf{r}_i$ about an axis in the direction $\hat{\mathbf{n}}$ is given by

$$\mathcal{I} = \sum_i m_i |\hat{\mathbf{n}} \times \mathbf{r}|^2$$

A particle with mass m moving in a plane at a point described by plane polar coordinates (r, θ) has angular momentum (normal to the plane) given by

$$L = mr\dot{\theta}^2.$$

Rotational Energy: The energy of rotation of a rigid body with moment of inertia $\mathcal{I}$ about an axis passing through the centre-of-mass of the body, and angular momentum (vector quantity) $\mathbf{L}$ about a point on that axis, is given by

$$T_{rot} = \frac{L^2}{2\mathcal{I}}$$

4 Laboratory and Centre-of-Mass Frames:

Laboratory Frame:
Particle of mass m_1 with velocity v_1 collides with a particle of mass m_2 at rest.

$$\text{Total magnitude of momentum, } p^{LAB} = m_1 v_1$$

$$\text{Total kinetic energy, } T^{LAB} = \frac{1}{2} m_1 v_1^2$$

Centre-of-Mass Frame:
Target is moving with velocity $-v_2$ such that the total incident momentum is zero. In this frame particle 1 has velocity $v_1 - v_2$, so that setting the total momentum to zero gives

$$m_1(v_1 - v_2) + m_2(-v_2) = 0,$$

or

$$v_2 = \frac{m_1}{m_1 + m_2} v_1$$

Total kinetic energy in this frame is

$$T^{C.M.} = \frac{1}{2} m_1 (v_1 - v_2)^2 + \frac{1}{2} m_2 v_2^2 = \frac{1}{2} v_1^2 \left(\frac{m_1 m_2^2}{(m_1 + m_2)^2} + \frac{m_2 m_1^2}{(m_1 + m_2)^2} \right) = \frac{1}{2} m_1 v_1^2 \frac{m_2}{(m_1 + m_2)}$$

In the limit $m_2 \gg m_1$ the kinetic energies become equal in both frames.