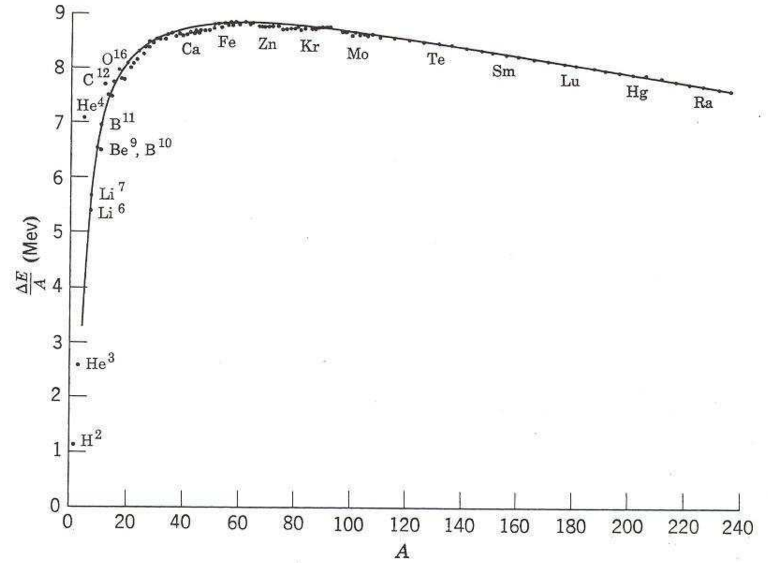
Chapter 11

Nuclear Fusion

If we look again at the binding energies (per nucleon) for different nuclei, we note also that the lightest nuclei have a much smaller binding energy per nucleon than those in the middle of the Periodic Table.



Much more energy per nucleon can be released by fusion of two of these light nuclei to form a heavier nucleus, than in the case of fission.

For example, if we consider the fusion of a deuteron and a hydrogen nucleus into helium

$$^2_1\mathrm{H} + {}^1_1\mathrm{H} \rightarrow {}^3_2\mathrm{He} + \gamma$$

The γ is emitted because the helium is formed in an excited state. The mass of a deuteron is 3.34358×10^{-27} kg, that of a proton is 1.67262×10^{-27} kg, and for $^3_4$He the mass is 5.00832×10^{-27} kg. Using $E = \Delta mc^2$ where Δm is the mass difference between the initial and final states, and converting into MeV, we find that this reaction releases 4.4 MeV

$$E = (3.34358 + 1.67262 - 5.00832) \times 10^{-27}\mathrm{kg} \times (3 \cdot 10^8 \mathrm{m/s})^2 / 1.6021^{-19}\mathrm{J/eV} \simeq 4.4\mathrm{MeV}$$

which is carried off in the energy of the γ and the kinetic energy of the helium nucleus. The energy released per fusion reaction is usually much less than that released in a typical fission reaction. However, the energy released per nucleon and therefore the energy released per unit mass is very much greater.

Many fusion reactions are a little more complicated than this, for the opposite reason that fission products are unstable against β-decay. The fusion products usually have *too few* neutrons to form a stable nucleus and one of the protons converts into a neutron, emitting a positron and a neutrino. For example, there is no bound state of two protons, i.e. ${}^2_2\mathrm{He}$ does not exist. Therefore for proton-proton fusion

$${}^1_1\mathrm{H} + {}^1_1\mathrm{H} \rightarrow {}^2_1\mathrm{H} + e^+ + \nu$$

Such fusion reactions do not occur spontaneously. The reason for this is that the nuclei are positively charged so that they repel each other and they have to overcome the Coulomb barrier in order to be able to get close enough to be able to fuse. An approximate estimate of the energy required for two protons to fuse is

$$E = \frac{e^2}{4\pi\epsilon_0 R}$$

where R is a typical nuclear radius. Inserting 4 fm for this radius we get an energy of around 350 KeV.

In order for protons to have an average energy associated with the degree of freedom corresponding to motion in the direction of the target proton, we would need a temperature T such that

$$E = \frac{1}{2}kT$$

Putting in numbers we get a required temperature of 4×10^9 K.

In practice, fusion can take place at temperatures which are considerably lower than this. Fusion takes place in the core of the sun whose temperature is a mere 1.3×10^7 K. The reason for this is twofold

1. The above calculation of temperature determines the *average* energy per proton. But we know that these energies are distributed according to the Maxwell-Boltzmann distribution, which has a tail (albeit exponentially suppressed) and this tail means that there are some particles whose energy is much larger than the average.
2. It is not necessary for the incident protons to have sufficient energy to overcome the Coulomb barrier entirely. The protons can also get through the barrier by quantum tunnelling, provided the barrier height is not too high above the kinetic energy of the incoming particle.

The fusion in the sun and other stars, which is their source of energy, works in cycles. A cycle is a series of stages of fusion in which the initial particles are protons, but in subsequent stages the product of a previous stage fusion can fuse with another proton to form yet another fusion product. At the end of the cycle all the intermediate fusion products have disappeared leaving a stable fusion product, usually ${}^4_2\mathrm{He}$.

The most common such cycle is the so-called proton cycle:

$$\begin{aligned} {}^1_1\mathrm{H} + {}^1_1\mathrm{H} &\rightarrow {}^2_1\mathrm{H} + e^+ + \nu \quad (\times 2) \\ {}^1_1\mathrm{H} + {}^2_1\mathrm{H} &\rightarrow {}^3_2\mathrm{He} + \gamma \quad (\times 2) \\ {}^3_2\mathrm{He} + {}^3_2\mathrm{He} &\rightarrow {}^4_2\mathrm{He} + {}^1_1\mathrm{H} + {}^1_1\mathrm{H} \end{aligned}$$

The third step in this cycle has as its initial state the result of two instances of the first two steps. At the end of the cycle there is no ${}^3_2\mathrm{He}$ and if we balance the number of initial and final protons we see that a net four protons have been turned into one ${}^4_2\mathrm{He}$ nucleus and two positrons, two neutrinos and two photons:

$$4{}^1_1\mathrm{H} \rightarrow^4_2 \mathrm{H} + 2e^+ + 2\nu + 2\gamma$$

The total energy released by this process is 24.7 MeV (one can calculate it using $m_{{}^4_2\mathrm{H}} = 6.64466 \times 10^{-27}\mathrm{kg}, \mathrm{m_e} = 9.109 \times 10^{-31}\mathrm{kg}$).

Other cycles also occur within the sun. An example is the carbon cycle

$$\begin{aligned} {}^{12}_6\mathrm{C} + {}^1_1\mathrm{H} &\rightarrow {}^{13}_7\mathrm{N} + \gamma \\ {}^{13}_7\mathrm{N} &\rightarrow {}^{13}_6\mathrm{C} + e^+ + \nu \\ {}^{13}_6\mathrm{C} + {}^1_1\mathrm{H} &\rightarrow {}^{14}_7\mathrm{N} + \gamma \\ {}^{14}_7\mathrm{N} + {}^1_1\mathrm{H} &\rightarrow {}^{15}_8\mathrm{O} + \gamma \\ {}^{15}_8\mathrm{O} &\rightarrow {}^{15}_7\mathrm{N} + e^+ + \nu \\ {}^{15}_7\mathrm{N} + {}^1_1\mathrm{H} &\rightarrow {}^{12}_6\mathrm{C} + {}^4_2\mathrm{He} \end{aligned}$$

Two steps are β-decays of fusion products which have too few neutrons for stability and therefore one of the protons is converted into a neutron in the β-decay process. The ${}^{12}_6\mathrm{C}$ is regenerated and the net effect is again four protons have been turned into one ${}^4_2\mathrm{He}$ nucleus and two positrons, two neutrinos and three photons, with total energy released by this process is 24.7 MeV. The carbon is initially produced by the fusion of three ${}^4_2\mathrm{He}$ nuclei. The fusion processes in the carbon cycle require more energy in order to overcome the Coulomb barrier, and is therefore more likely at higher temperatures. On the other hand, once these temperatures have been reached, this cycle is more likely than the proton cycle, because in the proton cycle it is necessary for two ${}^3\mathrm{He}$ nuclei to fuse together - which is unlikely because the ${}^3\mathrm{He}$ nuclei are produces from previous fusion processes and their density is low. The sun is a relatively cool star for which the proton cycle dominates.

The γ-rays are initially produced at energies of around 1 MeV. They scatter against other charged particles in the sun, losing energy at each scattering (Compton effect) and eventually "thermalize", i.e. they settle at an energy (wavelength) distribution which is the black-body distribution at the temperature of the surface of the sun - a distribution with a peak in the visible light range.

For over fifty years a great deal of effort has been put into trying to produce the equivalent environment of the core temperature of the sun, in order to be able to use fusion as an energy source. The necessary temperatures have been achieved but usually for too short a time for the fusion to take place.

In 1989 Pons and Fleischman (Southampton) announced that they had observed cold fusion in a chemistry laboratory. It is now widely accepted that their interpretation of their data was erroneous.

More recently it has been suggested that when small bubbles are adiabatically suppressed they can reach very high temperatures forming a plasma. Certainly a light flash can be observed from this plasma. In 2002 it was suggested by Taleyarkhan and collaborators that the temperature could reach that required for fusion and that a γ-ray emission was observed. The experiment has recently been repeated by Suslick et. al. who have cast doubt as to whether this γ-ray is indeed associated with fusion. More information about this can be found at the Website
http://www.nature.com/news/2005/050228/full/050228-7.html