Chapter 12

Charge Independence and Isospin

If we look at mirror nuclei (two nuclides related by interchanging the number of protons and the number of neutrons) we find that their binding energies are almost the same.

In fact, the only term in the Semi-Empirical Mass formula that is *not* invariant under Z $\leftrightarrow$ (A-Z) is the Coulomb term (as expected).

$$B(A,Z) \;=\; a_V\, A \;-a_S\, A^{2/3} \;-a_C \frac{Z^2}{A^{1/3}} \;-a_A \frac{(Z-N)^2}{A} \;+\frac{\left((-1)^Z+(-1)^N\right)}{2}\frac{a_P}{A^{1/2}}$$

Inside a nucleus these electromagnetic forces are much smaller than the strong inter-nucleon forces (strong interactions) and so the masses are very nearly equal despite the extra Coulomb energy for nuclei with more protons.

Not only are the binding energies similar - and therefore the ground state energies are similar but the excited states are also similar.

As an example let us look at the mirror nuclei (Fig. 12.2) ${}^7_3$Li and ${}^7_4$Be, where we see that for all the states the energies are very close, with the ${}^7_4$Be states being slightly higher because it has one more proton than ${}^7_3$Li.

All this suggests that whereas the electromagnetic interactions clearly distinguish between protons and neutrons the strong interactions, responsible for nuclear binding, are 'charge independent'.

Let us now look at a pair of mirror nuclei whose proton number and neutron number differ by two, and also the nuclide between them. The example we take is ${}^6_2$He and ${}^6_4$Be, which are mirror nuclei. Each of these has a closed shell of two protons and a closed shell of two neutrons. The unclosed shell consists of two neutrons for ${}^6_2$He and two protons ${}^6_4$Be. the nuclide 'between' is ${}^6_3$Li which has one proton and one neutron in the outer shell.

From the principle of charge independence of the strong interactions we might have expected all three nuclides to display the same energy-level structure. We see that although there are states in ${}^6_3$Li which are close to the states of the mirror nuclei ${}^6_2$He and ${}^4_2$Be, there are also states in ${}^6_3$Li which have no equivalent in the two mirror nuclei.

We can understand this from the Pauli exclusion principle. In the case of ${}^6_2$He and ${}^4_2$Be

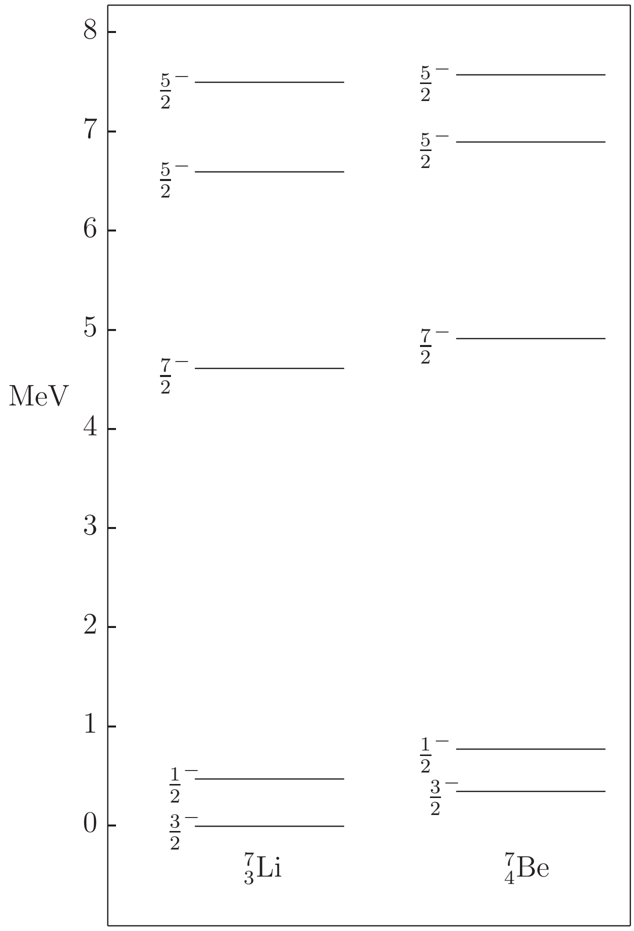

Figure 12.1: Energy states for the mirror nuclei (Fig. 12.2) $^7_3$Li and $^7_4$Be.

which have either two protons or two neutrons in the outer shell, these cannot be in the same state (with the same spin), whereas in the case of $^6_3$Li for which the nucleons in the outer shell are not identical, this principle does not apply and there are extra states, in which the neutron and proton are in the same state.

12.1 Isospin

We can express this is a more formal (mathematical), but useful way by introducing the concept of "Isospin".

If we have two electrons with z- component of their spin set to $s_z = +\frac{1}{2}$ and $s_z = -\frac{1}{2}$ (in units of $\hbar$) then we can distinguish them by applying a (non-uniform) magnetic field in the z-direction - the electrons will move in opposite directions. But in the absence of this

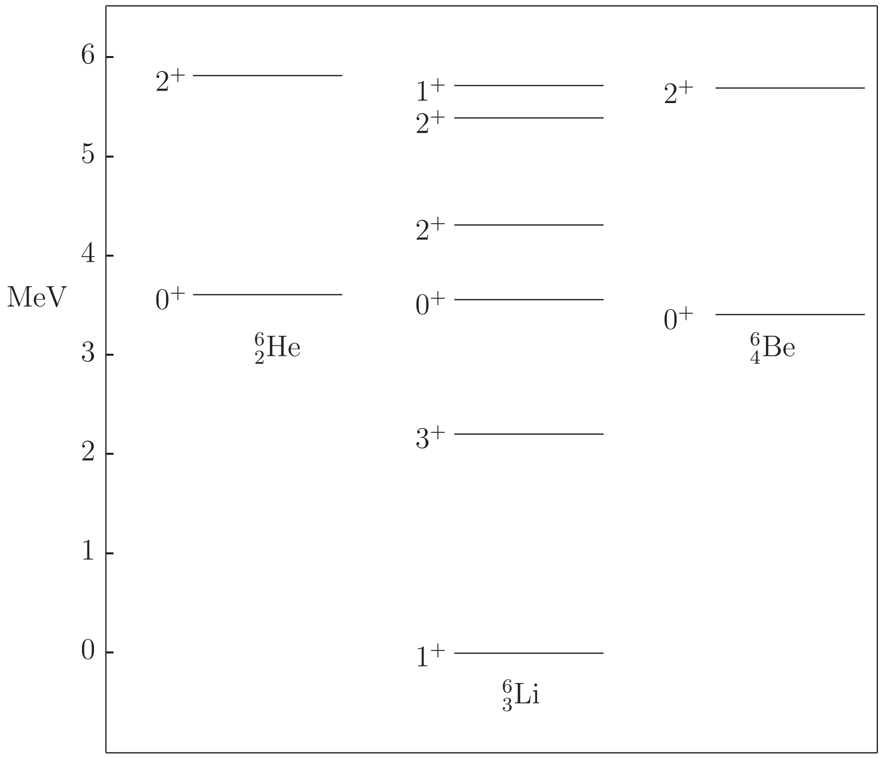

Figure 12.2: Energy states for $^6_2$He, $^7_3$Li and $^6_4$Be.

external field these two cannot be distinguished and we are used to thinking of these as two states of the same particle.

Similarly, if we could 'switch off' electromagnetic interactions we would not be able to distinguish between a proton and a neutron. As far as the strong interactions are concerned these are just two states of the same particle (a nucleon).

We therefore think of an imagined space (called an 'internal space') in which the nucleon has a property called "isospin", which is mathematically analogous to spin. The proton and neutron are now considered to be a nucleon with different values of the third component of this isospin.

Since this third component can take two possible values, we assign $I_3 = +\frac{1}{2}$ for the proton and $I_3 = -\frac{1}{2}$ for the neutron. The nucleon therefore has isospin $I = \frac{1}{2}$, in the same way that the electron has spin $s = \frac{1}{2}$, with two possible values of the third component.

As far as the strong interactions are concerned this just represents two possible quantum states of the same particle. If there were no electromagnetic interactions these particles would be totally indistinguishable in all their properties - mass, spin etc.

In the same way that angular momentum is conserved, isospin is conserved in any transition mediated by the strong interactions. This is an example of an approximate symmetry - inside the nucleus the strong forces between nuclei do not distinguish between particles with different third component of isospin and would lead to identical energy levels, but there are electromagnetic interactions which break this symmetry and lead to small differences in the energy levels of mirror nuclei.

The electromagnetic interactions couple to the electric charge, Q, of the particles and in the case of nucleons this electric charge is related to the third component of isospin by

$$Q = I_3 + \frac{1}{2}$$

Other particles can also be classified as isospin multiplets. For example there are three pions, $\pi^+_, \pi^0$, π^-, which have almost the same mass and zero spin etc. There are three of them with different charges but which behave in the same way under the influence of the strong interactions. Therefore they form an isospin multiplet with $I = 1$ and three possible third components, namely $+1$, $0\ -1$. In the case of pions the electric charges are equal to I_3.

Particles which are members of an isospin multiplet have the same properties, with the exception of their electric charge, i.e. they have the same spin and almost the same mass (the small mass differences being due to the electromagnetic interactions which are not isospin invariant. We will see later that particles can have other properties (call "strangeness", "charm" etc.) and members of an isospin multiplet will have the same values of these proerties as well.

In the same way that two electrons can have a total spin $S = 0$ or $S = 1$, two nucleons can have a total isospin $I = 0$ or $I = 1$, and (systems of n nucleons can have isospins up to $n/2$). For two electrons we may write the total wavefunction as

$$\Psi_{12} = \Psi(\mathbf{r}_1, \mathbf{r}_2)\chi(s_1, s_2),$$

where $\chi(s1, s2)$ is the spin part of the wavefunction. For $S = 1$ we have

$$\begin{aligned} \chi(s_1, s_2) &= (\uparrow\uparrow), \quad S_z = +1 \\ \chi(s_1, s_2) &= \frac{1}{\sqrt{2}}(\uparrow\downarrow + \downarrow\uparrow), \quad S_z = 0 \\ \chi(s_1, s_2) &= (\downarrow\downarrow), \quad S_z = -1 \end{aligned}$$

which is symmetric under interchange of the two spins, which means that by fermi statistics the spatial part of the wavefunction must be antisymmetric under the interchange of the positions of the electrons,

$$\Psi(\mathbf{r}_1, \mathbf{r}_2) = -\Psi(\mathbf{r}_2, \mathbf{r}_1),$$

or for the case of $S = 0$,

$$\chi(s_1, s_2) = \frac{1}{\sqrt{2}}(\uparrow\downarrow - \downarrow\uparrow),$$

which is antisymmetric under interchange of spins so it must be accompanied by a symmetric spatial part of the wavefunction

$$\Psi(\mathbf{r}_1, \mathbf{r}_2) \;=\; +\Psi(\mathbf{r}_2, \mathbf{r}_1).$$

In the case of two nucleons we also have a total isospin part of the wavefunction, so the complete wavefunction is

$$\Psi_{12} \;=\; \Psi(\mathbf{r}_1, \mathbf{r}_2)\chi_S(s_1, s_2)\chi_I(I_1, I_2),$$

where $\chi_I(I_1, I_2)$ is the isospin part of the wavefunction. For total isospin $I = 1$ we have

$$\begin{aligned}
\chi_I(I_1, I_2) &= (p\,p)\,, \quad I_3 = +1 \\
\chi_I(I_1, I_2) &= \frac{1}{\sqrt{2}}\,(p\,n\,+\,n\,p)\,, \quad I_3 = 0 \\
\chi_I(I_1, I_2) &= (n, n)\,, \quad I_3 = -1,
\end{aligned}$$

which is symmetric under the interchange of the isospins of the two nucleons, so that (as in the case of two electrons) it must be accompanied by a combined spatial and spin wavefunction that must be antisymmetric under simultaneous interchange of the two positions *and* the two spins. But we also have the $I = 0$ state

$$\chi_I(I_1, I_2) \;=\; \frac{1}{\sqrt{2}}\,(p\,n\,-\,n\,p)\,,$$

which is antisymmetric under the interchange of the two isospins and therefore when the nucleons are combined in this isospin state they must be accompanied by a combined spatial and spin wavefunction which is *symmetric* under simultaneous interchange of the two positions *and* the two spins.

Returning to the three nuclei ${}^6_2$He and ${}^6_4$Be and ${}^6_3$Li, the closed shells of neutrons and protons have a total isospin zero so we do not need to consider these in determining the isospin of the nuclei. We note that ${}^6_2$He has two neutrons in the outer shell so its isospin must be $I = 1$, with $I_3 = -1$ whereas the ${}^6_4$Be has two protons in the outer shell so its isospin must be $I = 1$, with $I_3 = +1$ implying that for these two nucleons the remaining part of the wavefunction (spatial and spin parts) must be antisymmetric under simultaneous interchange of the two positions and the two spins. On the other hand, the nucleus ${}^6_3$Li has one proton and one neutron in the outer shell and can therefore be either in an $I = 1$ state like the other two nuclei or in an $I = 0$ state which is not possible for the other two. The strong interactions will give rise to different energy levels depending on the total isospin of the nulceons in the outer shell (in the same way that atomic energy levels depend on the total angular momentum J). Thus we see that two of the states shown for ${}^6_3$Li can be identified as $I = 1$ states and they approximately match states for the other two nuclei, but the others are $I = 0$ states and have no counterpart in ${}^6_2$He or ${}^6_4$Be, and which have wavefunctions that are symmetric under the simultaneous interchange of the positions and the spins of the two nucleons in the outer shell.

The fact that the ground states of ${}^6_2$He and ${}^6_4$Be have spin zero and the ground state of ${}^6_3$Li has spin one, can be deduced from the isospin of these ground states. For ground state wavefunctions the orbital angular momentum, l, is zero and since the symmetry of the spatial part of the wavefunction is given by $(-1)^l$, this means that the spatial part of the wavefunction is symmetric under the interchange of the positions of the two nucleons in the outer shell. Since we know that the overall wavefunction for the two nucleons in the outer shell must be antisymmetric under interchange, because the nucleons are fermions, it follows that the isospin part and the spin part of the wavefunction must have opposite symmetry. Thus for the ground states of ${}^6_2$He and ${}^6_4$Be, which are in I=1 (symmetric) isospin states, the spin part of the wavefunction must be antisymmetric and therefore the spins of the two outer shell nucleons must combine to give spin $S = 0$, whereas for the ground state of ${}^6_3$Li which is **from the experiment known to be** in an I=0 (antisymmetric) isospin state, the spin part of the wavefunction must be symmetric and therefore the spins of the two outer shell nucleons must combine to give spin $S = 1$.