Chapter 13

Accelerators

Particles physics, also known as 'high energy physics' is the study of the fundamental forces of nature and the particles that can be found at very high energies.

The most massive particles that has been discovered so far are W-boson with a mass of 80.4 GeV/c^2, Z-boson with a mass of 91.2 GeV/c^2, and *top*-quark with a mass of 172.0 GeV/c^2. All these particles are 100 times heavier than the proton. So we need a really high energy to produce these particles.

Another way of seeing that we need high energies is to note that we wish to probe very short distances. At the very least we want to probe distances which are small compared with a typical nuclear radius, i.e.

$$x \ll 1\,\mathrm{fm} = 10^{-15}\,\mathrm{m}$$

In order to do this the uncertainty in the position, Δx must be much smaller than 1 fm, and by Heisenberg's uncertainty principle

$$\Delta x \Delta p \geq \hbar/2$$

the uncertainty in momentum Δp must obey the inequality

$$\Delta p \gg \frac{\hbar}{1\,\mathrm{fm}} = 197\,\mathrm{MeV/c}.$$

This in turn means that the momenta of the particle used as a probe must have a momentum much larger than this, and hence an energy large compare with $\approx$ 200 MeV.

In fact, the weak interactions have a range which is more than two orders of magnitude shorter than this and so particles used to investigate the mechanism of weak interactions have to have energies of at least 100 GeV.

In order to achieve these very high energies particles are accelerated in "accelerators". Incident particles are accelerated to these high energies and scattered against another particle. There is enough energy to smash the initial particles up and produce many other particles in the final state, some of them with considerably higher masses than the incident particles. Such scattering is called "inelastic scattering" (conversely a scattering event in which the final state particles are the same as the initial particles is called "elastic scattering". Rutherford scattering or Mott scattering are examples of elastic scattering.) The word 'elastic' here means that none of the incoming energy is used up in the production of other particles.

In elastic scattering we talk about a differential cross-section (with respect to solid angle), which is the number of particles per incident flux in a given element of solid angle. For inelastic events we can talk about the total cross-section for a particular process. For example, at the LEP accelerator (electron-positron scattering) at CERN one possible process was

$$e^+ + e^- \rightarrow W^+ + W^-,$$

in which the electron and positron annihilate each other and produce two W-bosons instead. The W-boson has a mass of 80.4 GeV/c^2, so that total centre-of-mass energies of over 160 GeV are required for this process to take place. The cross-section $\sigma_{(e^+e^- \rightarrow W^+W^-)}$ is the total number of events in which two W-bosons are produced per unit incident flux (i.e. the number of W-boson pairs produced divided by the number of particle scatterings per unit area)

It is now believed that there exist particles with masses which are an order of magnitude larger than this and modern accelerators can achieve energies of up to 1 TeV (10^{12} eV). This new energy frontier and respectively new small distances can be probed by presently the most powerful accelerator in the world – the Large Hadron Collider (LHC) – at CERN which has resumed running in November 2009 with energy 7.5 TeV.

13.1 Fixed Target Experiments vs. Colliding Beams

The total energy of a projectile particle plus the target particle depends on the reference frame. The frame that is relevant for the production of high mass particles is the centre-of-mass frame for which the projectile and target have equal and opposite momentum p. For simplicity let us suppose that the projectile and target particle are the same, or possibly particle antiparticle (e.g. proton-proton, proton-antiproton, or electron-positron) so that their masses, m are the same. This means that in this frame both the particle have the same energy, E_{CM} (since we are usually dealing with relativistic particles, this means kinetic *plus* rest energy.)

Let us construct the quantity

$$s = \left(\sum_{i=1,2} E_i \right)^2 - \left(\sum_{i=1,2} \mathbf{p}_i \right)^2 c^2$$

In the centre-of-mass frame, where the momenta are equal and opposite the second term vanishes and we have

$$s = 4E_{CM}^2,$$

i.e. s is the square of the total incoming energy in the centre of mass frame - this is a quantity that is often used in particle physics and the notation s is always used. For one particle we know that $E^2 - p^2c^2$ is equal to m^2c^4 and is therefore the same in any frame of reference even though the quantities E and $\mathbf{p}$ will be different in the two frames. Likewise the above quantity, s, is the same in any frame of reference (we say that 'it invariant under Lorentz transformations.')

In the frame in which the target particle is at rest, its energy is mc^2 and its momentum is zero, whereas the projectile has energy E_{LAB} and momentum $\mathbf{p}_{LAB}$ so that we have

$$s \;=\; \left(E_{LAB} + mc^2\right)^2 - \mathbf{p}_{LAB}^2 c^2 \;=\; E_{LAB}^2 + m^2c^4 + 2mc^2 E_{LAB} - \mathbf{p}_{LAB}^2 c^2 \;=\; 2m^2c^4 + 2mc^2 E_{LAB},$$

where in the last step we have used the relativity relation

$$E_{LAB}^2 \;-\; \mathbf{p}_{LAB}^2 c^2 \;=\; m^2c^4.$$

Equating the two expressions for s (and taking a square root we obtain the relation

$$\sqrt{s} \;=\; 2E_{CM} \;=\; \sqrt{2m^2c^4 + 2mc^2 E_{LAB}}.$$

For non-relativistic incident particles with kinetic energy $T \ll mc^2$ for which $E_{LAB} = mc^2 + T$, this gives

$$\sqrt{s} \;=\; 2E_{CM} \;=\; 2mc^2 \;+\; T,$$

as expected, but for relativistic particles the centre-of-mass energy is considerably reduced. For example, taking the proton mass be be approximately 1 GeV/c^2, the if we have an accelerator that can accelerate protons up to an energy of 100 GeV, the total centre-of-mass energy achieved is only about 15 GeV - far less than the energy required to produce a particle of mass 100 GeV/c^2.

The solution to this problem is to use colliding beams of particles. In these experiments both the initial particles involved in the scattering emerge from the accelerator and are then stored in storage rings, in which the particles move in opposite directions around the ring, with their high energies maintained by means of a magnetic field. At various point around the rings the beams intersect and scattering takes place. In this way the laboratory frame *is* the centre-of-mass frame and the full energy delivered by the accelerator can be used to produce high mass particles.

13.2 Luminosity

The luminosity $\mathcal{L}$ is the number of particle collisions per unit area (usually quoted in cm^2) per second. The number of events of a particular type which occur per second is the cross-section multiplied by the luminosity. In the example of two W-boson production at LEP the cross-section, $\sigma_{(e^+e^- \to W^+W^-)}$ is 15 pb (p=pico means 10^{-12}) and the luminosity of LEP was 10^{32} per cm^2 per second. The number of these pairs of W-bosons produced per second is given by

$$\frac{dN_{W^+W^-}}{dt} \;=\; (15 \times 10^{-12} \times 10^{-28}) \times (10^{32} \times 10^4) \;=\; 1.5 \times 10^{-3},$$

where the first term in parenthesis is the cross-section converted to m^2 and the second is the luminosity converted to $\mathrm{m}^{-2}\,\mathrm{sec}^{-1}$. So, the general formula for reaction rate, $R = dN/dt$ is

$$R = \sigma \times \mathcal{L}$$

while for integrated luminosity over the time $L = \int \mathcal{L} dt$ the number of events, N, we will observe is given by

$$N = \sigma \times L$$

.

$\mathcal{L}$ is proportional to the number 'bunches' of particles in each beam, n (typically 5-100), the revolution frequency, f(kHz-MHz), N_1, N_2 – the number of particles in each bunch ($\simeq 10^{10} - 10^{11}$) and inversely proportional to the bean cross section, A ($\mu\mathrm{m}^2$):

$$\mathcal{L} = \frac{nfN_1N_2}{A}$$

As in the case of radioactivity the cross-section is a probability for a particular event and the actual number of events observed is a random distribution with that probability. If a cross-section predicts N events over a given time-period, the error on that number is $\sqrt{N}$ (this means that there is a 68% probability that the number of events observed will lie in the region $N - \sqrt{N}$ to $N = \sqrt{N}$). To be able to measure the above cross-section at LEP to an accuracy of 1% it was necessary to collect 10000 such W-pairs, which, at a rate of 1.5×10^{-3} per sec., took about three months.

We pay a price for colliding beam experiments in terms of luminosity. For a fixed target experiment we can make an estimate of the luminosity in the case of proton-proton scattering from the fact that the incident particles are travelling almost with the speed of light. The luminosity is given by the number of protons in a column of the target of unit area and length c. For a solid whose density is $10^4\,\mathrm{kg\,m}^{-3}$, and assuming that about one half of the target material consists of protons of mass 1.67×10^{-27} kg, this comes out to about 10^{35} per cm^2 per sec. In colliding beams it is necessary to focus the incident beams as tightly as possible using magnetic fields, in order to maximize the luminosity. So far, luminosities of 10^{32} per cm^2 per sec. have been achieved, which means the reaction rate is down by three orders of magnitude compared with a fixed target experiment. However, the LHC is designed to reach a luminosity of 10^{34} per cm^2 per sec. - i.e luminosities within an order of magnitude of that obtained in fixed target experiments.

13.3 Types of accelerators

As we have discussed, the general aim of accelerators is to collide two particles at high(est) energy and create new particles from combined energy and quantum numbers or to probe inside one of the particles to see what it is made of.

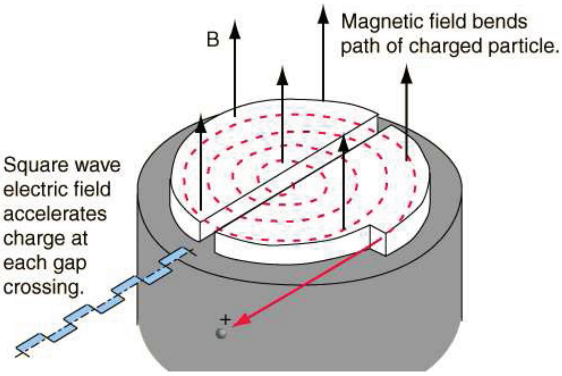
Only stable charged particles can be accelerated: such as electrons, positrons, protons, anti-proton and some ions. Potentially, the long-lived particles such as muon ($\tau \simeq 2 \times 10^{-6}$s) were discussed to be used in the future muon collider.

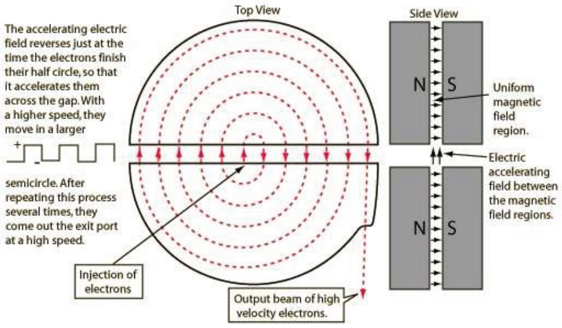
Single DC stage accelerators such as the Van de Graaff Generator can accelerate electrons and protons upto about 20 MeV.

There are two general types of modern accelerators – Circular (Cyclic) and Linear.

13.3.1 Cyclotrons

The prototype design for all circular accelerators is the cyclotron.





This is a device in which the (charged) particles to be accelerated move in two hollow metallic semi-disks (D's) with a large magnetic field B applied normal to the plane of the D's. The particles move in a spiral from the center and an alternating electric field is applied between the D's whose frequency is equal to the frequency of rotation of the charged particles, such that when the particles crosses from one of the D's to the other the electric field always acts in the direction which accelerates the particles.

A charged particle with charge e moving with velocity $\mathbf{v}$ in a magnetic field $\mathbf{B}$ experiences a force $\mathbf{F}$, where

$$\mathbf{F} \;=\; q\mathbf{v} \times \mathbf{B}.$$

When the magnetic field is perpendicular to the plane of motion of the charged particle, this force is always towards the centre and gives rise to centripetal acceleration, so that at the moment when the particles are moving in a circle of radius r

$$F \;=\; B\,e\,v \;=\; m\frac{v^2}{r}.$$

We see immediately that the angular velocity $\omega = v/r$ is constant, so that the frequency of the alternating electric field remains constant. The maximum energy that the particles can acquire depends on the radius, R, for which the velocity has its maximum value v_{max},

$$v_{max} \;=\; \frac{BeR}{m},$$

leading to a maximum kinetic energy

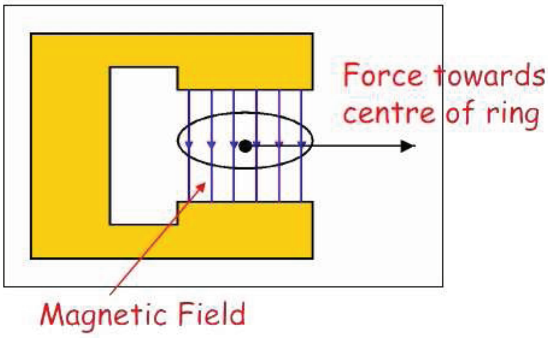
$$T_{max} \;=\; \frac{1}{2}mv_{max}^2 \;=\; \frac{B^2e^2R^2}{2m}.$$

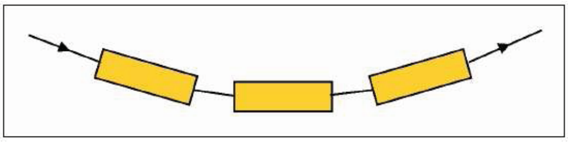
This works fine if the energy of the particle remains non-relativistic. However, in high energy accelerators the particles are accelerated to energies which are extremely relativistic - the particles are travelling very nearly with the velocity of light (at the LHC v/c will be $1 - 10^{-15}$!). Taking relativistic effects into account The angular velocity is now

$$\omega \;=\; \sqrt{1 - v^2/c^2}\frac{Be}{m}.$$

This means that as the particles accelerate, either the frequency of the applied electric field must vary - such machines are called "synchrocyclotrons" - or the applied magnetic field must be varied (or both) - such machines are called "synchrotrons".

At Synchrotron dipole magnets keep particles in circular orbit using $p = 0.3 \times B \times R$ (p in GeV/c, B in Tesla, R in meters), while quadrupole magnets used to focus the beam.





Since the bending field B is limited then the maximum energy is limited by the size of the ring. The CERN SPS (Super Proton Synchrotron) has a radius R = 1.1Km and a momentum of 450 GeV/c. Particles are accelerated by resonators (RF Cavities). The bending field B is increased with time as the energy (momentum p) increases so as to keep R constant [$p = 0.3BR$]. Electron synchrotrons are similar to proton synchrotrons except that the energy losses are greater.

One of the main limiting factors of synchrotron accelerators is the Synchrotron Radiation. A charged particle moving in a circular orbit is accelerating (even if the speed is constant) and therefore radiates. The energy radiated per turn per particle is:

$$\Delta E = \frac{4\pi e^2 \beta^2 \gamma^4}{3R}$$

where e is the charge, $\beta = v/c$ and $\gamma = 1/\sqrt{1-\beta^2} = E/m$, from which follows that

$$\Delta E \propto 1/m^4$$

For relativistic electrons and protons of the same momentum the ratio of energy losses are very large for electrons versus protons:

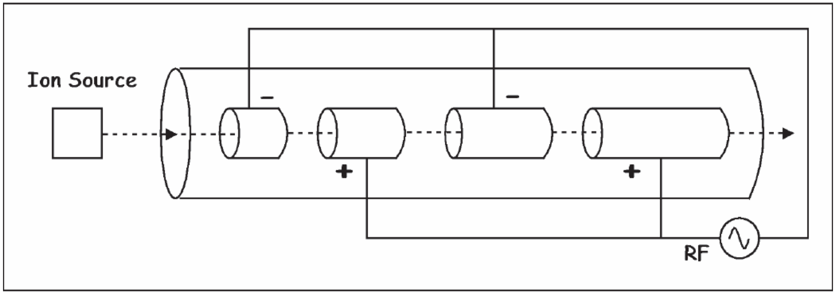
$$\frac{\Delta E_e}{\Delta E_p} = \left(\frac{m_p}{m_e}\right)^4 \simeq 10^{13}$$

.

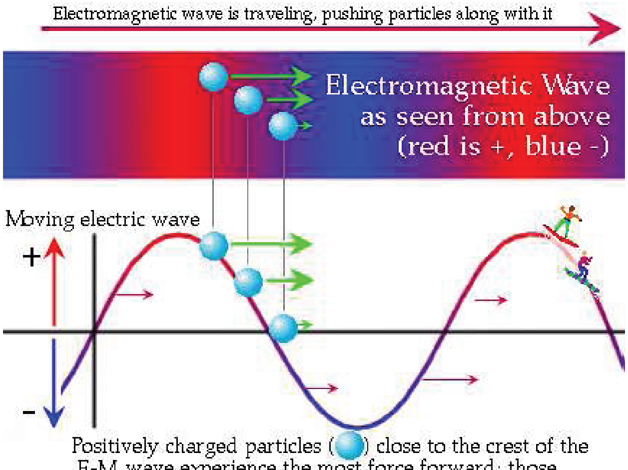
13.3.2 Linear Accelerators

The energy loss due to synchrotron radiation, can be avoided in a linear accelerator. In such a machine the particles are accelerated by means of an applied electric field along a long a tube.

Proton Linear Accelerators (Linacs) use a succession of drift tubes of increasing length (to compensate for increasing velocity).

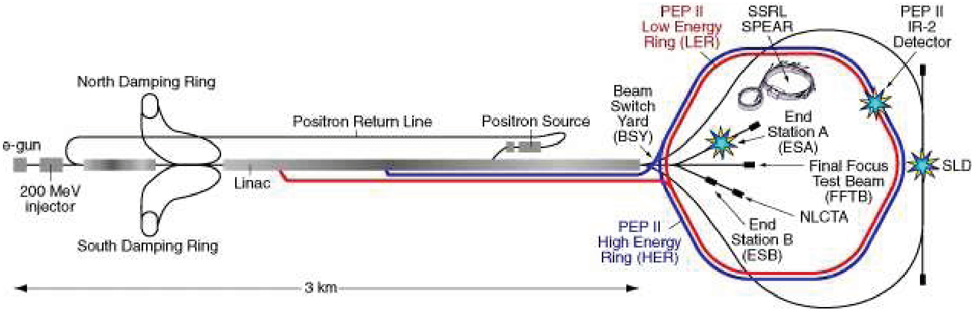


Particles always travel in vacuum. There is no field inside the drift tubes. External field between ends of tubes changes sign so proton always sees $-ve$ in front and $+ve$ behind. Proton linacs of 10-70m give energies of 30 to 200 MeV. Usually used as injectors for higher energy machines. Above a few MeV, electrons travel at speed of light. The 'tubes' become uniform in length and microwaves provide by Klystrons provide accelerating potential.



Positively charged particles (●) close to the crest of the E-M wave experience the most force forward; those closer to the center experience less of a force. The result is that the particles tend to move together with the wave.

The largest linear collider in existence is SLAC (Stanford Linear Collider Center) in California. This is 3 km. long and accelerates both electrons and positrons up to energies of 50 GeV. It is able to accelerate both electrons and positrons simultaneously by sending an electromagnetic wave in the microwave band along the beam pipe and injecting bunches of electrons and positrons which are precisely one half wavelength apart, so that the electric field acting on the positrons is in the forward direction and so accelerates the positrons in the forward direction, whereas the electric field acting on the electrons is in the backwards direction, but because the electrons have negative charge they are also accelerated in the forward direction.



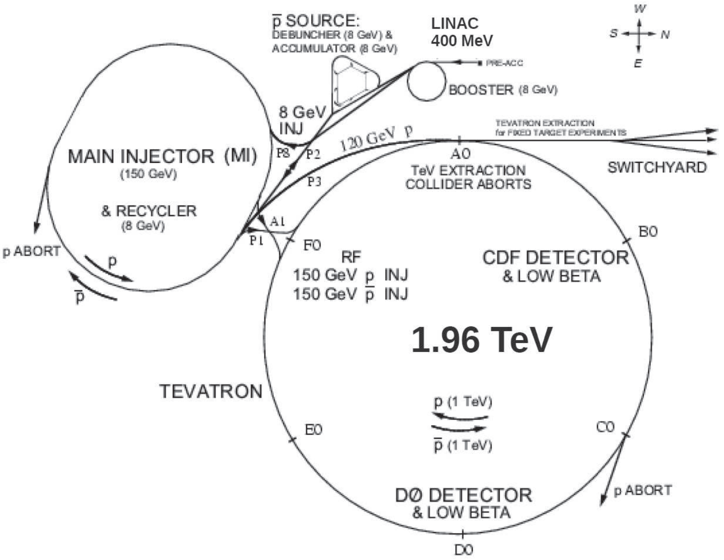
At the end of the tube the electrons and positrons are stored in a storage ring (they go around the storage ring in opposite directions under the influence of the same magnetic field) and there are intersection points where electron-positron scatterings occur.

There are plans (awaiting international approval) to build a much larger linear collider (known as ILC - the International Linear Collider) which will have a total centre-of-mass energy of 500 GeV (or perhaps even 1 TeV).

13.4 Main Recent and Present Particle Accelerators

Here are some of today's main accelerator laboratories.

- **FermiLab:**

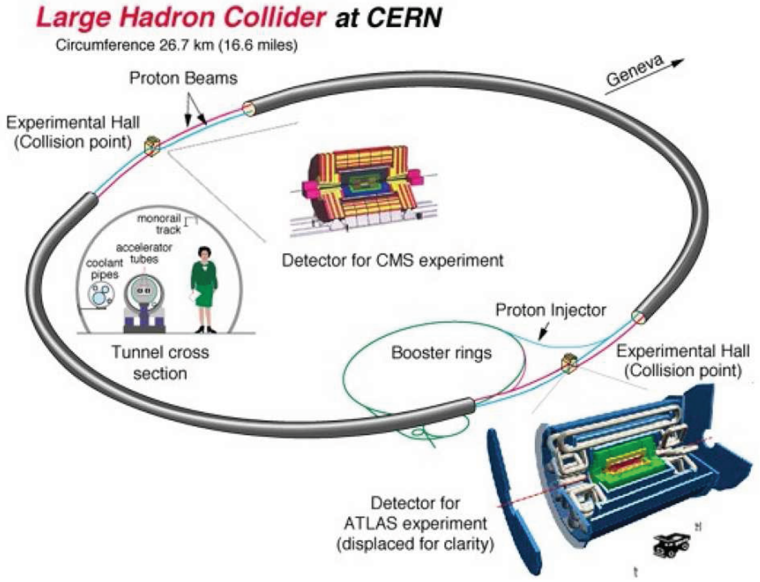


Situated just outside Chicago this is now running the Tevatron in which protons and antiprotons are each accelerated to an energy of 1.96 TeV and then move around a ring of circumference 6 km. This is a synchrotron in which very high magnetic fields are achieved using superconducting (electro-)magnets, which are capable of maintaining very large currents thereby producing large magnetic fields. The luminosity is

$10^{32}\,\mathrm{cm}^{-2}\,\mathrm{sec}^{-1}$

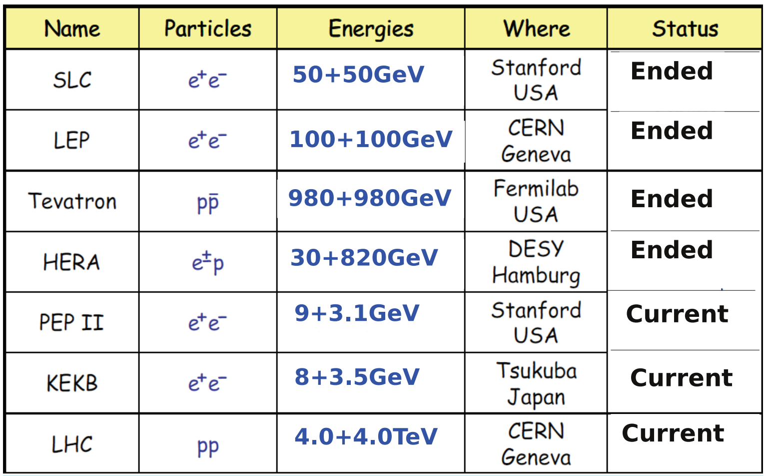
- **CERN:**
  Situated just outside Geneva, until 2001 the main experiment was LEP in which electrons and positrons were each accelerated to an energy of about 100 GeV, and had a luminosity of $10^{32}\,\mathrm{cm}^{-2}\,\mathrm{sec}^{-1}$. This was the largest electron synchrotron in the world with a circumference of 27 km.

  The next project at CERN is the LHC has started in September 2008. After an accident in October 2008, LHC has resumed its operation in November 2009 and now it is colliding protons against protons with energies $3.5\mathrm{TeV}\times 3.5\mathrm{TeV}$ resulting to a total cms energy of 7 TeV. Using a specially designed magnetic field configuration, two beams of protons moving in opposite direction around the same ring is possible. In the future, protons will each be accelerated to 7 TeV and the design luminosity is $10^{34}\,\mathrm{cm}^{-2}\,\mathrm{sec}^{-1}$.

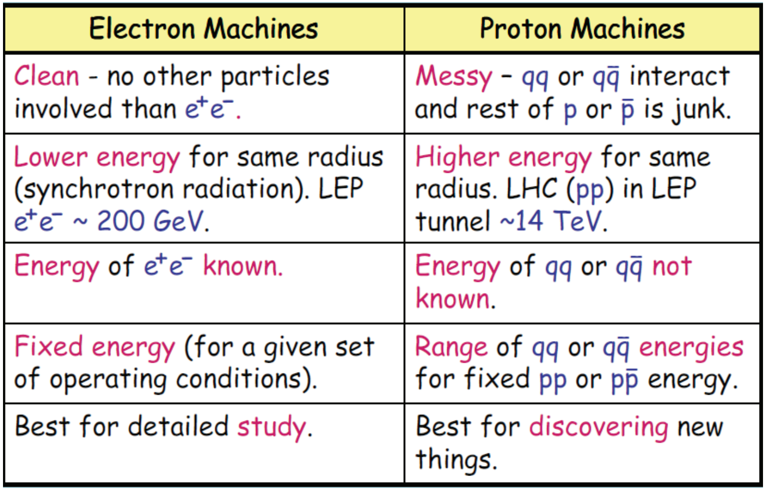


- **DESY:**
  Situated just outside Hamburg, this laboratory is running the HERA accelerator which accelerated protons to an energy of 820 GeV and electrons (or positrons) to 27 GeV. It is the only accelerator in which the initial particles are not the same - or particle-antiparticle pairs.

Table below presents summary on present and recent colliders as well as comparison of electron and proton accelerators.



Name	Particles	Energies	Where	Status
SLC	e^+e^-	**50+50GeV**	Stanford USA	**Ended**
LEP	e^+e^-	**100+100GeV**	CERN Geneva	**Ended**
Tevatron	$p\bar{p}$	**980+980GeV**	Fermilab USA	**Ended**
HERA	$e^{\pm}p$	**30+820GeV**	DESY Hamburg	**Ended**
PEP II	e^+e^-	**9+3.1GeV**	Stanford USA	**Current**
KEKB	e^+e^-	**8+3.5GeV**	Tsukuba Japan	**Current**
LHC	pp	**4.0+4.0TeV**	CERN Geneva	**Current**



Electron Machines	Proton Machines
Clean - no other particles involved than e^+e^-.	Messy – qq or $q\bar{q}$ interact and rest of p or $\bar{p}$ is junk.
Lower energy for same radius (synchrotron radiation). LEP e^+e^- ~ 200 GeV.	Higher energy for same radius. LHC (pp) in LEP tunnel ~14 TeV.
Energy of e^+e^- known.	Energy of qq or $q\bar{q}$ not known.
Fixed energy (for a given set of operating conditions).	Range of qq or $q\bar{q}$ energies for fixed pp or $p\bar{p}$ energy.
Best for detailed study.	Best for discovering new things.