Chapter 14

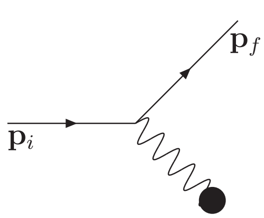
Fundamental Interactions (Forces) of Nature

Interaction	**Gauge Boson** (Force carrier)	**Gauge Boson Mass**	**Interaction Range**
Strong	Gluon	0	short-range (a few fm)
Weak	$W^{\pm}$, Z	$M_W = 80.4$ GeV/c^2 $M_Z = 91.2$ GeV/c^2	short-range ($\sim 10^{-3}$ fm)
Electromagnetic	Photon	0	long-range
Gravity	Graviton	0	long-range

Gravity is by far the weakest interaction. The gravitational force between two protons is about 10^{-9} of the electromagnetic force between them. We shall not discuss gravity further in these lectures. [Experiments designed to investigate the theory of gravity (General Relativity), are astronomical observations - the other end of the magnitude scale from particle physics.]

14.1 Relativistic Approach to Interactions

Electromagnetic Interaction:



Potential of a particle of charge e located at $\mathbf{r}$, due to another charge e' (fixed) at the origin is

$$V(r) \ = \ \frac{ee'}{4\pi\epsilon_0 r}$$

If this charge has initial momentum $\mathbf{p}_i$ and final momentum $\mathbf{p}_f$, its initial and final (time-independent) wavefunctions are given by

$$\Psi_i \ \propto \ e^{i\mathbf{p_i}\cdot\mathbf{r}/\hbar}$$

$$\Psi_f \ \propto \ e^{i\mathbf{p_f}\cdot\mathbf{r}/\hbar}$$

The amplitude for such a transition is

$$\mathcal{A} \ = \ \int \Psi_f^* V(r) \Psi_i d^3\mathbf{r} \ \propto \ ee' \int e^{i(\mathbf{p_i}-\mathbf{p_f})\cdot\mathbf{r}/\hbar} \frac{1}{r} d^3\mathbf{r}.$$

Performing the integral (this is a Fourier transform) we get

$$\mathcal{A} \ \propto \ \frac{ee'}{-|\mathbf{q}|^2},$$

where $\mathbf{q} = \mathbf{p}_f - \mathbf{p}_i$, is the momentum transferred from the scattered charged particle to the charge at the origin. (We have seen this in the Rutherford scattering formula where the cross-section is proportional to $1/|\mathbf{q}|^4$, so that the amplitude is proportional to $1/|\mathbf{q}|^2$).

For the scattering of a relativistic particle, this expression is modified to

$$\mathcal{A} \ \propto \ \frac{ee'}{(q_0^2 - |\mathbf{q}|^2)},$$

where $q_0 = (E_f - E_i)/c$, with E_i and E_f being the initial and final energy of the scattered particle. (In the non-relativistic limit $(E_f - E_i)/c \ll |\mathbf{p}_f - \mathbf{p}_i|$ so q_0^2 is negligible.)

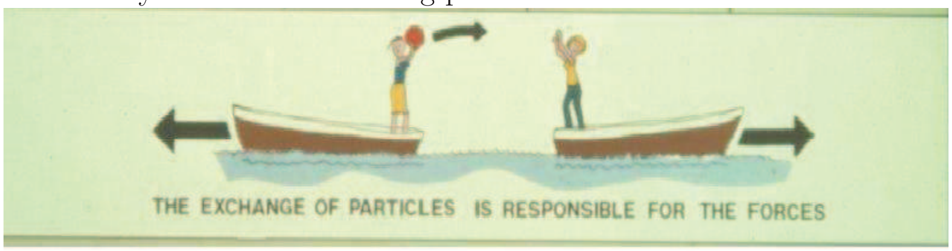
The interpretation of this process is that a photon, which is the 'carrier' of the electromagnetic interactions, with energy $c\,q_0$ and momentum $\mathbf{q}$, is exchanged between the two charged particles. The electric charges e and e' measure the strengths of the coupling of the charged particles to the photon, and the quantity

$$D(q_0, \mathbf{q}) \ = \ \frac{1}{q_0^2 - |\mathbf{q}|^2} \tag{14.1.1}$$

is the amplitude for the propagation of a photon whose energy is $c\,q_0$ and whose momentum is $\mathbf{q}$. This is known as a "propagator".

Recall that $(E_f - E_i)^2 - |\mathbf{p}_f - \mathbf{p}_i|^2 c^2 = c^2(q_0^2 - |\mathbf{q}|^2)$ is invariant under Lorentz transformations and so this propagator is the same in any reference frame.

In relativity, we cannot really talk about a 'potential' since this implies some sort of instantaneous action at a distance. The relativistic approach to all interactions is via the exchange of a "gauge boson" which carries the interaction between the particles that are interacting. The interaction between the interacting particles and the gauge bosons are always local in space and time. Particles can only influence each other at a distance because gauge bosons are emitted by one of the particles, they propagate through space-time, and are then absorbed by the other interacting particle.



There is a classical picture of this. Two people who throw a ball to each other back and forth will experience a repulsive force - there is no action at a distance but only an exchange of 'particles' that carry momentum.

14.2 Virtual particles

For a photon of energy $c\,q_0$ and momentum $\mathbf{q}$ we have the relation, $q_0 = |\mathbf{q}|$, so one would think that the propagator defined in eq.(14.1.1) would diverge.

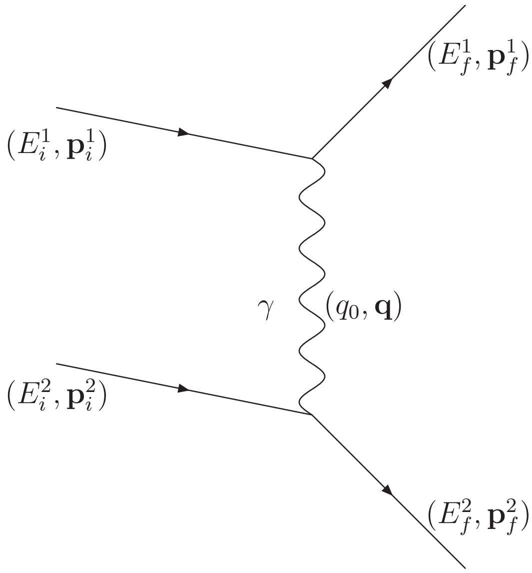
We are rescued by Heisenberg's uncertainty principle that tells us that over a sufficiently short period of time there is an uncertainty in energy. This means that if a particle only exists for a very short time we no longer have the usual relation between energy and momentum

$$E^2 = |\mathbf{p}|^2 c^2 + m^2 c^4$$

and in the case of the photon this means $q_0 \neq |\mathbf{q}|$. Such particles, which are exchanged rapidly between other particles, are called "virtual particles" and because their energy and momentum do not obey the relativistic energy-momentum relation they are said to be "off mass-shell".

14.3 Feynman Diagrams

We can 'unpin' the charge at the origin and use these techniques to calculate the scattering amplitude for the scattering of a particle of charge e_1 and incoming energy and momentum, $(E_i^1, \mathbf{p}_i^1)$ against a particle with charge e_2 and incoming energy and momentum $(E_i^2, \mathbf{p}_i^2)$ to form final state energies and momenta $(E_f^1, \mathbf{p}_f^1)$ and $(E_2^2, \mathbf{p}_f^2)$. We represent this process diagrammatically as



This is known as a "Feynman diagram" (or "Feynman graph"). The amplitude for the process is obtained by applying a set of "Feynman rules" for each vertex and internal line. The full set of Feynman rules takes into account the spins of the external and internal particles (gauge bosons, such as photons have spin one) - these are beyond the scope of these lectures.

Some of the Feynman rules in the case of electromagnetic interactions are:

- A factor of the charge at each vertex between a charged particle and a photon.
- Energy and momentum are conserved at each vertex.
- A factor of
$$D(q_0, \mathbf{q}) \;=\; \frac{1}{(q_0^2 - |\mathbf{q}|^2)}$$
for the propagation of an internal gauge boson with energy $c\, q_0$ and momentum $\mathbf{q}$.

In the above example we have (by conservation of energy and momentum at each vertex)

$$q_0 \;=\; (E_f^1 - E_i^1)/c \;= (E_f^2 - E_i^2)/c$$

$$\mathbf{q} \;=\; \mathbf{p}_f^1 - \mathbf{p}_i^1 \;= \mathbf{p}_f^2 - \mathbf{p}_i^2,$$

so that the amplitude is proportional to

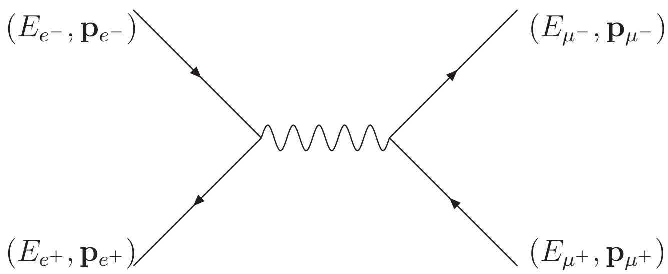
$$\frac{e_1 e_2}{(q_0^2 - |\mathbf{q}|^2)} \;=\; \frac{e_1 e_2}{(E_f^1 - E_i^1)^2/c^2 - |(\mathbf{p}_f^1 - \mathbf{p}_i^1)|^2}.$$

We can turn these diagrams on their side and consider the annihilation of a particle and its antiparticle. For example, the process

$$e^+ + e^- \rightarrow \mu^+ + \mu^-,$$

(the muons, $\mu^\pm$ are more massive copies of the electron or positron, with the same electric charge, e)

The Feynman diagram for this process is



Note the convention that the direction of the arrow on the antiparticles, e^+ and μ^+ are drawn against the direction of motion of the particles.

In this case the energy and momentum carried by the gauge boson are the sum of the initial (or final) energies and momenta

$$q_0 = (E_{e^-} + E_{e^+})/c = (E_{\mu^-} + E_{\mu^+})/c$$

$$\mathbf{q} = (\mathbf{p}_{e^-} + \mathbf{p}_{e^+}) = (\mathbf{p}_{\mu^-} + \mathbf{p}_{\mu^+})$$

so that the amplitude is proportional to

$$\frac{e^2}{(E_{e^-} + E_{e^+})^2/c^2 - |(\mathbf{p}_{e^-} + \mathbf{p}_{e^+})|^2}.$$

In the centre-of-mass frame of the incoming electron-positron pair (e.g. in the lab. frame of LEP) $(\mathbf{p}_{e^-} + \mathbf{p}_{e^+}) = 0$ and $(E_{e^-} + E_{e^+})$ is the total centre-of-mass energy, which we denote as $\sqrt{s}$ so that (absorbing a factor of c^2 into the constant of proportionality) we have

$$\mathcal{A} \propto \frac{e^2}{s}.$$

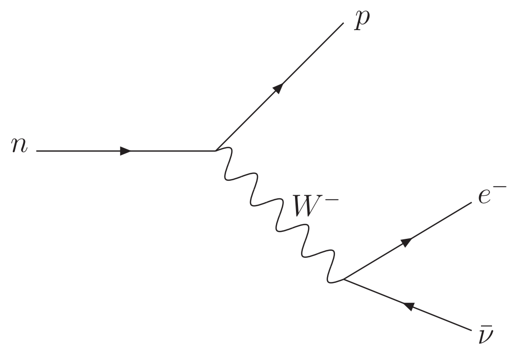
14.4 Weak Interactions

The gauge bosons (interaction carriers) of the weak interactions are $W^\pm$ and Z. the fact that the W-bosons carry electric charge tells us that electric charge can be exchanged in weak interaction processes, which is how we get β-decay.

The Feynman diagram for the process

$$n \rightarrow p + e^- + \bar{\nu}$$

is



(note that once again the direction of the arrow on the antineutrino is opposite to the direction of the antineutrino.)

What is happening is that the neutron emits a W^- and is converted into a proton (we will see later that the neutron and proton are not point particles but are made up of quarks, so that it is actually a d-quark, whose electric charge is $-\frac{1}{3}e$, that is converted into a u-quark, whose charge is $+\frac{2}{3}e$). At the other end (after the W^- has propagated) the W^- decays into an electron and an antineutrino.

The equivalent of the electric charge in weak interactions is a coupling, g_W, which indicates the strength of the coupling of the weakly interacting particles to the W-bosons and is approximately twice the electron charge (the coupling to the neutral Z-boson is almost equal to this value).

The W^- has a mass $M_W = 80.4\,\mathrm{GeV/c^2}$ and for the propagation of a massive particle, the propagator is

$$D^W(q_0, \mathbf{q}) \;=\; \frac{1}{q_0^2 - |\mathbf{q}|^2 - M_W^2 c^2},$$

where again $c\,q_0$ and $\mathbf{q}$ are the energy and momentum difference between the incoming neutron and outgoing proton and are transferred to the electron antineutrino pair. ($c\,q_0$ is the Q-value of the decay). Once again if the W-boson were a real particle we would have $q_0^2 - |\mathbf{q}|^2 - M_W^2 c^2 = 0$ and the propagator would diverge. However, in this process the W-boson propagates for a very short time before decaying into the electron and antineutrino- it is a virtual particle and therefore off mass-shell so that this energy momentum relation is violated by virtue of the uncertainty principle.

The amplitude for this decay is therefore proportional to

$$\frac{g_W^2}{q_0^2 - |\mathbf{q}|^2 - M_W^2 c^2}$$

If we take the non-relativistic limit we may neglect q_0^2 compared with $|\mathbf{q}|^2$ and this amplitude can be viewed as the matrix element of a weak potential, V^{wk}, between the initial (neutron) state with momentum $\mathbf{p}_n$ and final (proton) state momentum $\mathbf{p}_p$, with $\mathbf{q} = \mathbf{p}_p - \mathbf{p}_n$ i.e.

$$\frac{g_W^2}{-|\mathbf{q}|^2 - M_W^2 c^2} \;=\; \int e^{-i\mathbf{p}_p\cdot\mathbf{r}/\hbar} V^{wk}(r) e^{i\mathbf{p}_n\cdot\mathbf{r}/\hbar} d^3\mathbf{r}$$

The potential for which this relation is obeyed is

$$V^{wk}(r) \;=\; \frac{g_W^2}{r}\exp(-M_W cr/\hbar)$$

Such a potential is called a Yukawa potential (originally proposed by Yukawa as a description of the strong interactions mediated by pions - the mass of the W- in the above formula would then be replaced by the pion mass - this picture of the strong interactions is now obsolete).

As well as decreasing as $1/r$ (like the Coulomb potential) this has an exponentially suppressed term for large values of r. The effective force therefore has a range R, where

$$R \sim \frac{\hbar}{M_W c}.$$

At distances much larger than this the potential is rapidly suppressed. This is an extremely short range (about 10^{-3} fm.)

Returning to the amplitude in terms of the energy and momentum transferred, in the case of β-decay the momentum transferred (a few MeV/c) is very small compared with $M_W c$ which is 80.4 GeV/c so we can also neglect the momentum term and approximate the amplitude by

$$\frac{-g_W^2}{M_W^2 c^2}$$

Because of the very large mass of the W-boson this is extremely small and it is the reason that weak interactions are actually so weak.

We see that in general an interaction for which the gauge boson is massive has a range which is inversely proportional to the mass of the gauge boson, whereas interactions for which the gauge boson is massless are long range - meaning that the potential only falls as $1/r$.

14.5 Strong Interactions

The strong interaction is an exception to this rule. The gauge bosons (gluons) are massless and yet the strong interactions have a range of only a fm. The reason for this is due to a phenomenon known as "quark confinement", which will be discussed later. The essential idea is that the couplings of the strongly interacting particles to the gluons, which binds these strongly interacting particles together, grows as the distance between the particles increases - making it impossible to separate the particles to very large distances.