Chapter 16

Constituent Quark Model

Quarks are fundamental spin-$\frac{1}{2}$ particles from which all hadrons are made up. Baryons consist of three quarks, whereas mesons consist of a quark and an anti-quark. There are six types of quarks called "flavours". The electric charges of the quarks take the value $+\frac{2}{3}$ or $-\frac{1}{3}$ (in units of the magnitude of the electron charge).

Symbol	**Flavour**	**Electric charge (e)**	**Isospin**	$\mathbf{I_3}$	**Mass Gev/c^2**
u	up	$+\frac{2}{3}$	$\frac{1}{2}$	$+\frac{1}{2}$	≈ 0.33
d	down	$-\frac{1}{3}$	$\frac{1}{2}$	$-\frac{1}{2}$	≈ 0.33
c	charm	$+\frac{2}{3}$	0	0	≈ 1.5
s	strange	$-\frac{1}{3}$	0	0	≈ 0.5
t	top	$+\frac{2}{3}$	0	0	≈ 172
b	bottom	$-\frac{1}{3}$	0	0	≈ 4.5

These quarks all have antiparticles which have the same mass but opposite I_3, electric charge and flavour (e.g. anti-strange, anti-charm, etc.)

16.1 Hadrons from u,d quarks and anti-quarks

Baryons:

Baryon	**Quark content**	**Spin**	**Isospin**	$\mathbf{I_3}$	**Mass Mev/c^2**
p	uud	$\frac{1}{2}$	$\frac{1}{2}$	$+\frac{1}{2}$	938
n	udd	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	940
Δ^{++}	uuu	$\frac{3}{2}$	$\frac{3}{2}$	$+\frac{3}{2}$	1230
Δ^{+}	uud	$\frac{3}{2}$	$\frac{3}{2}$	$+\frac{1}{2}$	1230
Δ^{0}	udd	$\frac{3}{2}$	$\frac{3}{2}$	$-\frac{1}{2}$	1230
Δ^{-}	ddd	$\frac{3}{2}$	$\frac{3}{2}$	$-\frac{3}{2}$	1230

- Three spin-$\frac{1}{2}$ quarks can give a total spin of either $\frac{1}{2}$ or $\frac{3}{2}$ and these are the spins of the baryons (for these 'low-mass' particles the orbital angular momentum of the quarks is zero - excited states of quarks with non-zero orbital angular momenta are also possible and in these cases the determination of the spins of the baryons is more complicated).
- The masses of particles with the same isospin (but different I_3) are almost the same - the differences being due to the electromagnetic interactions which distinguish members of the isospin multiplet with different electric charge. If it were possible to 'switch off' the electromagnetic interactions these masses would be exactly equal.
- The baryons which consist of three u-quarks or three d-quarks only occur for spin $\frac{3}{2}$ (we return to this later)

Mesons:

Meson	**Quark content**	**Spin**	**Isospin**	$\mathbf{I_3}$	**Mass Mev/c^2**
π^+	$u\bar{d}$	0	1	$+1$	140
π^0	$\frac{1}{\sqrt{2}}\left(u\bar{u} - d\bar{d}\right)$	0	1	0	135
π^-	$d\bar{u}$	0	1	-1	140
ρ^+	$u\bar{d}$	1	1	$+1$	770
ρ^0	$\frac{1}{\sqrt{2}}\left(u\bar{u} - d\bar{d}\right)$	1	1	0	770
ρ^-	$d\bar{u}$	1	1	-1	770
ω	$\frac{1}{\sqrt{2}}\left(u\bar{u} + d\bar{d}\right)$	1	0	0	782

- A spin-$\frac{1}{2}$ quark and an anti-quark with the same spin can combine (when the orbital angular momentum is zero) to give mesons of spin-0 or spin-1.

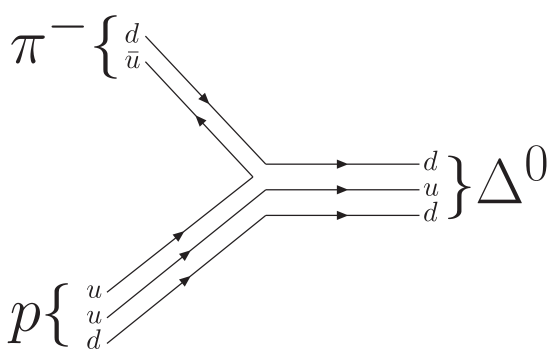
- The neutral mesons are not pure $u\bar{u}$, or $d\bar{d}$ states, but quantum superpositions of these.

- The neutral mesons have $I_3 = 0$. They could be in an isospin $I = 1$ state, (π^0, ρ^0), in which case their masses are similar to those of their charged counterparts, or $I = 0$ (ω) in which case their masses are somewhat different.

The strong interactions conserve flavour. There a d-quark cannot be converted into an s-quark (or vice versa), even though the electric charge is the same.

However, in a scattering process a quark *can* annihilate against an anti-quark of the same flavour, giving some energy which can be converted into mass and used to create a more massive particle. An example of this is

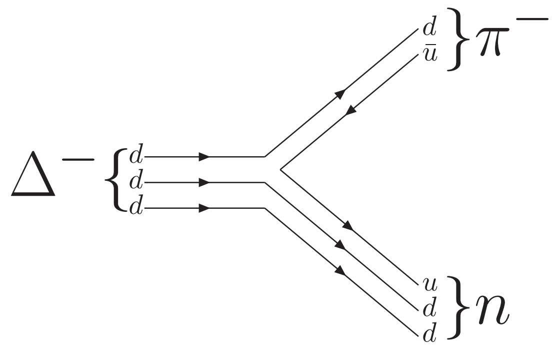
$$\begin{array}{ccccc} \pi^- & + & p & \rightarrow & \Delta^0 \\ (d\bar{u}) & & (uud) & & (udd) \end{array}$$



A u-quark from the proton and a $\bar{u}$ anti-quark from the pion have annihilated and the extra energy goes into the extra mass of the Δ^0, which is very short-lived and appears as a resonance in the $\pi^- p$ scattering cross-section.

Likewise in a decay process it is possible for some of the mass of the decaying particle to create a quark and anti-quark pair of the same flavour which go to forming the decay products, e.g.

$$\begin{array}{ccccc} \Delta^- & \rightarrow & \pi^- & + & n \\ (ddd) & & (ddu) & & (d\bar{u}) \end{array}$$



Here a u-quark and $\bar{u}$ anti-quark pair are created when the Δ^- decays and the $\bar{u}$ anti-quark binds with one of the d-quarks in the decaying Δ^- to make a π^-, whereas the u-quark binds with the other two d-quarks in the decaying Δ^- in order to make a neutron.

Quark and anti-quark pair creation is possible in any particle particle scattering process provided there is sufficient energy to create the final state particles. Thus for example it is possible to have the inelastic process

$$\begin{array}{ccccccccccc} p & + & p & \rightarrow & n & + & n & + & \pi^+ & + & \pi^+ \\ (uud) & & (uud) & & (udd) & & (udd) & & (u\bar{d}) & & (u\bar{d}) \end{array}$$

In this process two pairs of d-quarks and $\bar{d}$ anti-quarks are created. The d-quarks bind with the u and d quarks from the incoming protons to form neutrons, whereas the $\bar{d}$ anti-quarks bind with the remaining u-quarks in the incoming protons to form pions. In the centre-of-mass frame in which the total momentum is zero, so that the outgoing particles can be at rest - this is the lowest energy that they can have and is equal to sum of the masses of two neutrons and two pions (times by c^2), which is therefore the lowest total centre-of-mass energy of the incoming protons.

16.2 Hadrons with s-quarks (or $\bar{s}$ anti-quarks)

Baryons:

Baryon	**Quark content**	**Spin**	**Isospin**	$\mathbf{I_3}$	**Mass Mev/c^2**
Σ^+	uus	$\frac{1}{2}$	1	$+1$	1189
Σ^0	uds	$\frac{1}{2}$	1	0	1193
Σ^-	dds	$\frac{1}{2}$	1	-1	1189
Ξ^0	uss	$\frac{1}{2}$	$\frac{1}{2}$	$+\frac{1}{2}$	1314
Ξ^-	dss	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	1321
Λ	uds	$\frac{1}{2}$	0	0	1115
Σ^{*+}	uus	$\frac{3}{2}$	1	$+1$	1385
Σ^{*0}	uds	$\frac{3}{2}$	1	0	1385
Σ^{*-}	dds	$\frac{3}{2}$	1	-1	1385
Ξ^{*0}	uss	$\frac{3}{2}$	$\frac{1}{2}$	$+\frac{1}{2}$	1530
Ξ^{*-}	dss	$\frac{3}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	1530
Ω^-	sss	$\frac{3}{2}$	0	0	1672

- For historical reasons the s-quark was assigned strangeness equal to -1, so these baryons have strangeness -1, -2 or -3 for one, two, or three strange quarks respectively. (likewise the b-quark has bottom flavour -1, whereas the c-quark has flavour charm=+1, and the t-quark has flavour top=+1)
- As in the case of Δ^- and Δ^{++}, the Ω^- which has three s-quarks (strangeness=-3) has spin-$\frac{3}{2}$.

The Ω^- had not discovered when the Quark Model was invented - its existence was a prediction of the Model. Furthermore its mass was predicted from the observation

$$M_{\Sigma^*} - M_\Delta \approx M_{\Xi^*} - M_{\Sigma^*} \approx 150\,\mathrm{MeV/c^2}$$

giving a predicted value for M_Ω of

$$M_\Omega = M_{\Xi^*} + 150 = 1680\,\mathrm{MeV/c^2}.$$

Mesons:

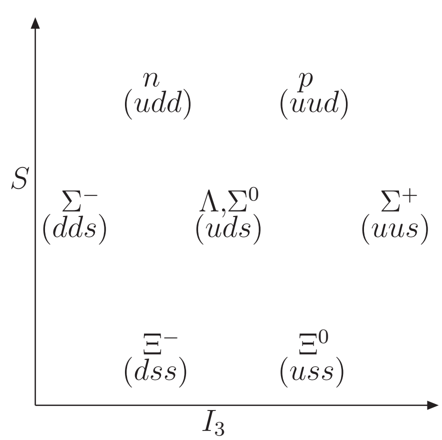
Meson	Quark content	Spin	Isospin	$\mathbf{I_3}$	Mass Mev/$\mathbf{c^2}$
K^+	$u\bar{s}$	0	$\frac{1}{2}$	$+\frac{1}{2}$	495
K^0	$d\bar{s}$	0	$\frac{1}{2}$	$-\frac{1}{2}$	495
$\overline{K^0}$	$s\bar{d}$	0	$\frac{1}{2}$	$+\frac{1}{2}$	495
K^-	$s\bar{u}$	0	$\frac{1}{2}$	$-\frac{1}{2}$	495
η	$(u\bar{u}$, $d\bar{d}$, $s\bar{s})$	0	0	0	547
K^{*+}	$u\bar{s}$	1	$\frac{1}{2}$	$+\frac{1}{2}$	892
K^{*0}	$d\bar{s}$	1	$\frac{1}{2}$	$-\frac{1}{2}$	896
$\overline{K^{*0}}$	$s\bar{d}$	1	$\frac{1}{2}$	$+\frac{1}{2}$	896
K^{*-}	$s\bar{u}$	1	$\frac{1}{2}$	$-\frac{1}{2}$	892
ϕ	$s\bar{s}$	1	0	0	1020

16.3 Eightfold Way:

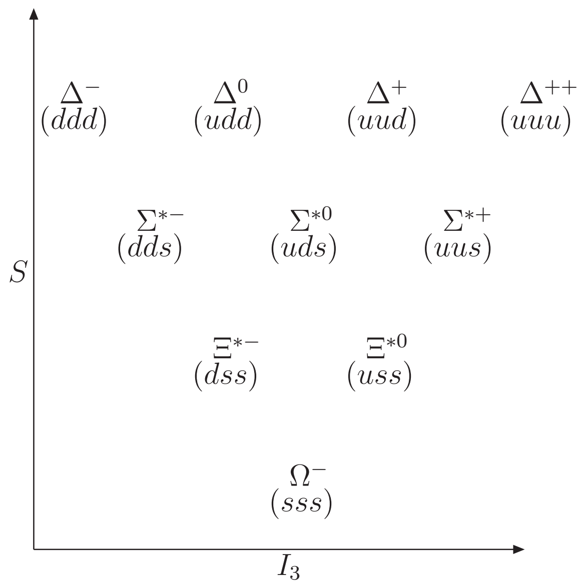
There is a method of classifying hadrons made up from u, d and s quarks and their anti-quarks by plotting particles with the same spin on a plot of strangeness against I_3.

For the lightest mesons and baryons there are eight particles on each plot. For this reason this classification method is known as the "Eightfold Way".

Spin-$\frac{1}{2}$ Baryons:

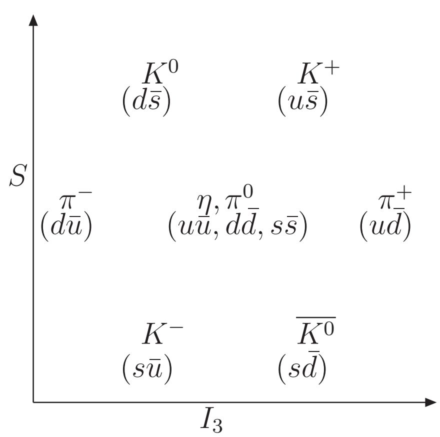


Spin-$\frac{3}{2}$ Baryons:

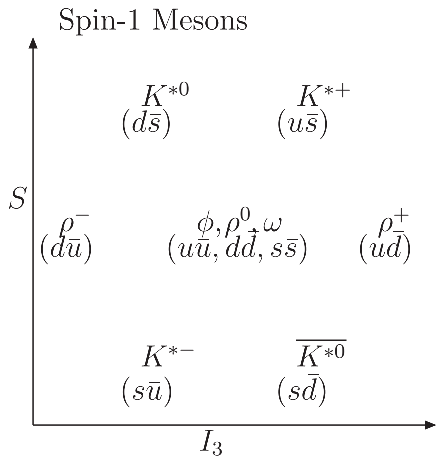


The rows contain the isospin multiplets. However, in the case of the row for $I = 1$, there can also be states with $I = 0$, $I_3 = 0$, so that the point in the middle can have two (or more) entries.

<u>Spin-0 Mesons:</u>



<u>Spin-1 Mesons:</u>



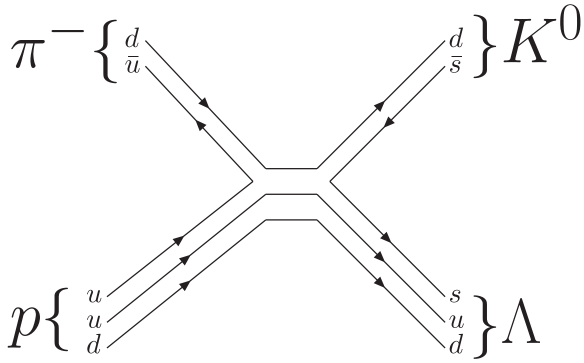
These meson multiplets contain the mesons and their antiparticles (obtained by replacing each quark by its anti-quark and vice versa), whereas the baryon multiplets have separate antiparticle multiplets which are bound states of three anti-quarks.

Some mesons, such as π^0, ρ^0, η are their own antiparticle, because they are bound states of a quark and an anti-quark of the same flavour so that replacing a quark by its anti-quark with the same flavour (and vice versa) produces the same particle. Other charged or neutral particles have separate antiparticles which have opposite electric charge and/or strangeness.

16.4 Associated Production and Decay

In strong interaction processes, quark flavour is conserved. s-quarks cannot be created or destroyed by the strong interactions (they can be created or destroyed by the weak interactions). This means that in a scattering experiment (e.g proton-proton or pion-proton scattering) one can only create a particle containing a strange quark if at the same time there is a particle containing an $\bar{s}$ anti-quark, so that the total strangeness is conserved. An example of such a process is

$$\begin{array}{ccccccc} \pi^- & + & p & \to & \Lambda & + & K^0 \\ (d\bar{u}) & & (duu) & & (dus) & & (\bar{s}d) \end{array}$$



What happens is that a u-quark annihilates against a $\bar{u}$ anti-quark and an s-quark $\bar{s}$ anti-quark pair has been created. This reaction is only possible above a threshold energy. In the centre-of-mass frame, the lowest total energy of the incoming particles is the sum of the masses of the Λ and the K^0, i.e.

$$\sqrt{s} \;=\; E_{CM}^{TOT} \;=\; (M_\Lambda \,+\, M_{K^0})\, c^2,$$

(here E_{CM}^{TOT} means to the *total* energy of the incoming (or outgoing) particles in the centre-of-mass frame - as the particles are not of the same mass, the individual energies of the particles will be different). In a (proton) fixed target experiment the pions must be accelerated to sufficient energy such that the centre-of-mass energy is greater than this value.

On the other hand the process

$$\begin{array}{ccccccccc} \pi^- & + & p & \rightarrow & \Lambda & + & \pi^0 \\ (d\bar{u}) & & (duu) & & (dus) & & (\bar{d}d, \bar{u}u) \end{array}$$

is forbidden because the number of strange quarks in the initial and final states is not the same.

It is possible to scatter charged kaons ($K^\pm$) against nucleons. The K^- contains an s-quark. it is therefore possible to produce strange baryons in this process, such as

$$\begin{array}{ccccccccc} K^- & + & n & \rightarrow & \Lambda & + & \pi^- \\ (s\bar{u}) & & (udd) & & (dus) & & (\bar{u}d) \end{array}$$

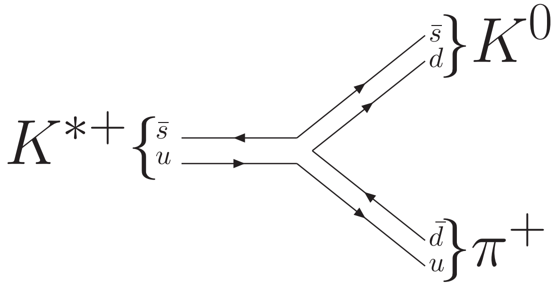
All flavours have been conserved in this reaction. However, $K^+ - n$ scattering will *not* produce a strange baryon because a strange baryon contains s-quarks but no $\bar{s}$ anti-quarks, whereas the K^+ contains a $\bar{s}$ anti-quark, but no s-quark.

Recent evidence has suggested that there is a resonance in $K^+ - n$ scattering at a centre-of-mass energy of 1.5 GeV. This suggests that there is an (unstable) particle with mass $1.5\,\mathrm{GeV/c^2}$. If confirmed this would be a new type of particle called a "pentaquark" since it must be a bound state of four quarks and an anti-quark ($\bar{s}uudd$). Such a particle does *not* fit in with the usual quark model picture of hadrons.

Similarly, in the decays of particles containing s-quarks (or $\bar{s}$ anti-quarks), the decay can proceed via the strong interactions and will be very rapid - leading to a large width - only if the decay products have a total strangeness which is equal to the strangeness of the decaying particle. For such a process to occur, the mass of the decaying particle must be larger than the combined mass of the decay products.

An example is

$$\begin{array}{ccccc} K^{*+} & \rightarrow & K^0 & + & \pi^+ \\ (u\bar{s}) & & (d\bar{s}) & & (u\bar{d}) \end{array}$$



A d-quark and $\bar{d}$ anti-quark pair have been created but the initial and final states both contain an $\bar{s}$ anti-quark - so flavour is conserved.

$$m_{K^*} = 842, \quad m_{K^*} = 495, \quad m_\pi = 135 \text{ (MeV/c}^2\text{)}$$

so there is enough energy in the decaying K^* to produce a kaon (K) and a pion, since the mass of the K^* exceeds the sum of the masses of the kaon and pion. This decay therefore can proceed via the strong interactions which means that the K^* has a very short lifetime. It is only seen as a resonance in the centre-of-mass frame of the $K - \pi$ system width a width of 50 MeV.

Likewise the Ξ^* has enough mass to decay into a Ξ plus a pion - the inital and final states both having strangeness -2, and similarly the Σ^* can decay into a Σ plus a pion, or into a Λ plus a pion - conserving strangeness. These decays are therefore very rapid as they proceed though strong interactions.

Most of the lighter strange particles do not have enough energy to decay into other strange particles. They therefore decay through the weak interactions - and therefore have a much longer lifetime. The usually can leave a track in a detector.

Combining associated production and decay one can have an event such as

$$\pi^+ + n \rightarrow K^{*+} + \Lambda \rightarrow K^0 + \pi^+ + \Lambda.$$

The observed particles are the K^0, π^+, Λ but if we look at the energies and momenta of the K^0 and π^+ and construct the quantity

$$P_{K\pi}^2 = (E_{K^0} + E_{\pi^+})^2 / c^2 - (\mathbf{p}_{K^0} + \mathbf{p}_{\pi^+})^2,$$

we would get a resonance peak at

$$P_{K\pi} = 842 \,\text{MeV/c},$$

indicating that at such momenta a K^* particle is produced for a very short time.

16.5 Heavy Flavours

When the quark model was invented only $u-$, $d-$ and s-quarks were postulated and all known hadrons could be built out of these three quarks and their anti-quarks. Since then three new quarks, c, b and t gave been discovered. They are much more massive than the $u-$, $d-$ and s-quarks, so they were not discovered until sufficiently large accelerators had been built and were in use. In the same way that there are hadrons containing one or more $s-$quarks (or $\bar{s}$ anti-quarks), there are hadrons which contain these heavy quarks. So far, only hadrons containing one c-quark or one b-quark (or their antiparticles) have been observed. It is believed that a hadron which contained a t-quark would be too unstable to form a bound state.

There are also bound states of $c\,\bar{c}$ and $b\bar{b}$. These are neutral - like the ϕ meson which is a bound state of $s\,\bar{s}$. These heavy quarks were first observed by observing these neutral bound states.

16.6 Quark Colour

There is a difficulty within the quark model when applied to baryons. This can be seen if we look at the Δ^{++} or Δ^{-} or Ω^{-}, which are bound states of three quarks of the same flavour. For these low-mass states the orbital angular momentum is zero and so the spatial parts of the wavefunctions for these baryons is symmetric under interchange of the position of two of these (identical flavour) quarks.

We know that the total wavefunction for a baryon must be antisymmetric as baryons have half-odd-integer spin, so the spin part of the wave function should be antisymmetric. On the other hand these baryons have spin-$\frac{3}{2}$ which means that the spin part of the wavefunction is symmetric (for example the $S_z = +\frac{3}{2}$ state is the state in which all three quarks have $s_z = +\frac{1}{2}$ and this is clearly symmetric under the interchange of two spins).

This is solved by assuming that quarks come in three possible "colour" states - R, G or B. The antisymmetry of the baryon wavefunction is restored by the assumption that the baryon wavefunction is antisymmetric under the interchange of two colours. If a baryon is composed of three quarks with flavours f_1, f_2 and f_3 the these should also have a colour index, e.g. f_1^R, f_1^G or f_1^B etc. The colour antisymmetric wavefunction is written

$$\frac{1}{\sqrt{6}}\left(|f_1^R f_2^G f_3^B\rangle + |f_1^B f_2^R f_3^G\rangle + |f_1^G f_2^B f_3^R\rangle - |f_1^B f_2^G f_3^R\rangle - |f_1^R f_2^B f_3^G\rangle - |f_1^G f_2^R f_3^B\rangle\right)$$

We can see that this changes sign if we interchange any two colours. This means that in order to have a totally antisymmetric wavefunction (including the colour part), the spin and spatial part must be *symmetric* so that a particle in which all three quarks have the same flavour (and zero orbital angular momentum) must be symmetric under the interchange of any two of the spins - and this is the spin-$\frac{3}{2}$ state.

A state of three different colours which is antisymmetric under the interchange of any two of the colours is called a "colour singlet" state - we can think of it as a colourless state.

The quarks themselves are a colour triplet - meaning that they can be in any one of three colour states.

It is assumed that all physically observable particles (i.e all hadrons) are colour singlets (colourless particles). This means that it is not possible to isolate individual quarks and observe them. indeed no individual quark has ever been observed. This is called "quark confinement" and it is the explanation of why the strong interactions are short-range, despite the fact that the gluons, which are the strong-interaction carriers, are massless - you can't pull a quark too far away from the other quarks or antiquarks in the hadron to which it is bound.

For mesons we also require that the quarks and anti-quarks bind in such a way that the meson is a colour singlet. in the case of a quark and ant-quark bound state this means that the wavefunction is a superposition of R with $\bar{R}$, G with $\bar{G}$, and B with $\bar{B}$. Thus, for example, the wavefunction for the π^+ is written

$$|\pi^+\rangle \quad = \quad \frac{1}{\sqrt{3}}\left(|u^R\overline{d^R}\rangle + |u^G\overline{d^G}\rangle + |u^B\overline{d^B}\rangle\right)$$

The colourless property is achieved by requiring that a quark of a given colour binds with an antiquark which is the antiparticle of a quark of the same colour - then we have to 'average' over all the colours by taking a superposition of all three possible colour pairs.