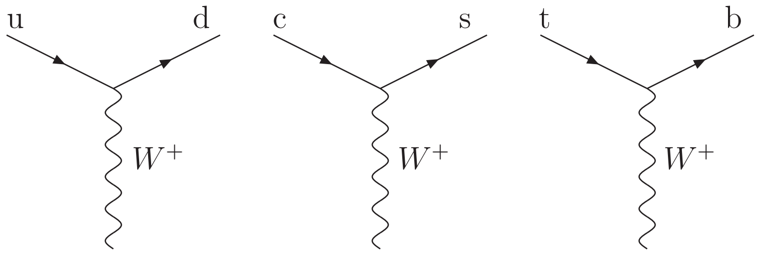
Chapter 17

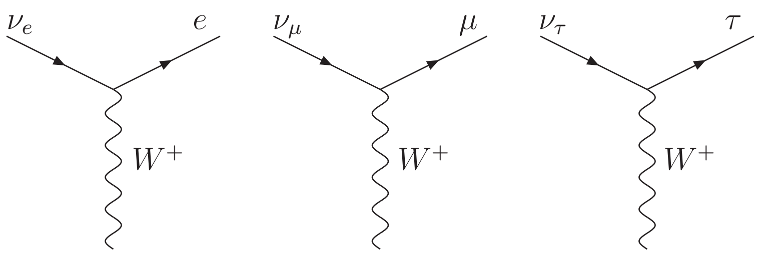
Weak Interactions

The weak interactions are mediated by $W^{\pm}$ or (neutral) Z exchange. In the case of $W^{\pm}$, this means that the flavours of the quarks interacting with the gauge boson can change.

$W^{\pm}$ couples to quark pairs (u, d). (c, s), (t, b) with vertices

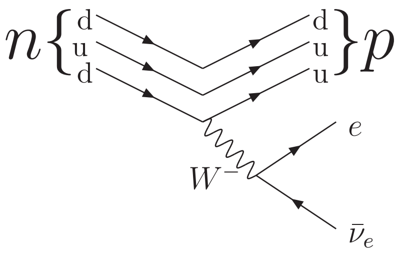


as well as to leptons (ν_e, e). (ν_μ, μ), (ν_τ, τ) with vertices



Note that in these interactions both quark number (baryon number) and lepton number are conserved.

It is this process that is responsible for β-decay. Neutron decays into a proton because a d-quark in the neutron converts into a u-quark emitting a W^- which then decays into an electron and anti-neutrino.



The amplitude for such a decay is proportional to

$$\frac{g_W^2}{(q^2 - M_W^2 c^2)},$$

where g_W is the strength of the coupling of the W^- to the quarks or leptons and $q^2 = E_q^2/c^2 - |\mathbf{q}|^2$, where $\mathbf{q}$ is the momentum transferred between the neutron and proton and E_q is the energy transferred. This momentum is of order 1 MeV/c and so we can neglect it in comparison with $M_W c$ which is 80.4 GeV/c. Thus the amplitude is proportional to

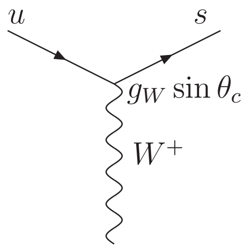
$$\frac{g_W^2}{M_W^2 c^2}.$$

The coupling g_W is not so small. In fact it is twice as large as the electron charge e. Weak interactions are weak because of the large mass term in the denominator.

At modern high energy accelerators, it is possible to produce weak interaction processes in which $|\mathbf{q}| \sim M_W c$ or even $|\mathbf{q}| \gg M_W c$. In such cases weak interactions are larger than electromagnetic interactions and almost comparable with strong interactions.

17.1 Cabibbo Theory

Particles containing strange quarks, e.g. $K^\pm$, K^0, Λ etc. cannot decay into non-strange hadrons via the strong interactions, which have to conserve flavour, but they can decay via the weak interactions. This is possible because $W^\pm$ not only couples a u-quark to a d-quark but can also (with a weaker coupling) couple a u-quark to an s-quark so we have a vertex



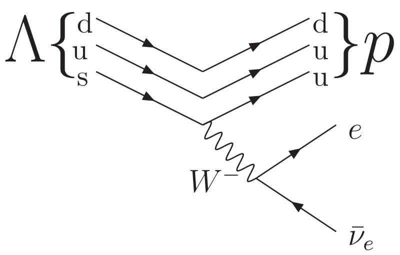
with coupling $g_W \sin\theta_C$, whereas the $u - d - W$ coupling is actually $g_w \cos\theta_C$. θ_C is called the "Cabibbo angle" and its numerical value is $\sin\theta_C \approx 0.22$.

This coupling allows a strange hadron to decay into non-strange hadrons and (sometimes) leptons.

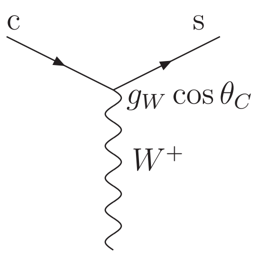
Thus, for example the decay

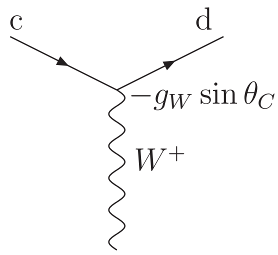
$$\Lambda \to p + e^- + \bar{\nu}_e$$

occurs when an s-quark converts into a u quark and emits a W^- which then decays into an electron and anti-neutrino. The Feynman graph is



Likewise, the c-quark has a coupling to the s-quark with coupling $g_W \cos\theta_C$ and a coupling to a d-quark with coupling $-g_W \sin\theta_C$.





This implies that charm hadrons are more likely to decay into hadrons with strangeness, because the coupling between a c-quark and a s-quark is larger than between a c-quark and a d-quark.

We can piece this together in a matrix form as follows

$$g_W \begin{pmatrix} d & s \end{pmatrix} \begin{pmatrix} \cos\theta_C & \sin\theta_C \\ -\sin\theta_C & \cos\theta_C \end{pmatrix} \begin{pmatrix} u \\ c \end{pmatrix}$$

This 2×2 matrix is called the “Cabbibo matrix”. It is described in terms of a single parameter, the Cabibbo angle.

Since we know that there are, in fact, three generations of quarks this matrix is extended to a general 3×3 matrix as follows

$$g_W \begin{pmatrix} d & s & b \end{pmatrix} \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} u \\ c \\ t \end{pmatrix}$$

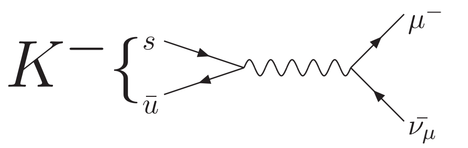
The 3×3 matrix is called the "CKM" (Cabibbo, Kobayashi, Maskawa) matrix. Quantum-mechanical constraints lead to the conclusion that of the nine elements there are only four independent parameters. Comparing the CKM matrix with the Cabibbo matrix we see that to a very good approximation, $V_{ud} \approx V_{cs} \approx \cos\theta_C$ and $V_{us} \approx -V_{cd} \approx \sin\theta_C$.

17.2 Leptonic, Semi-leptonic and Non-Leptonic Weak Decays

Because the $W^\pm$ couples either to quarks or to leptons, decays of strange mesons can either be leptonic, meaning that the final state consists only of leptons, semi-leptonic, meaning that the final state consists of both hadrons and leptons, or non-leptonic, meaning that the final state consists only of hadrons. For strange baryons only semi-leptonic and non-leptonic decays are possible because baryon number is strictly conserved - so there must be a baryon in the final state. Lepton number is also strictly conserved which means that a charged lepton is always accompanied by its anti-neutrino (or vice versa) in the final state.

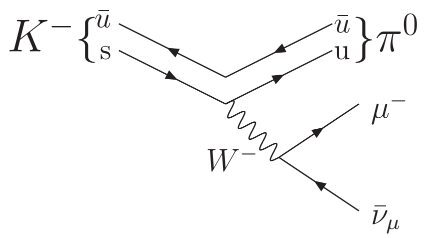
For mesons, examples are:

Leptonic decay $K^- \rightarrow \mu^- + \bar{\nu}_\mu$

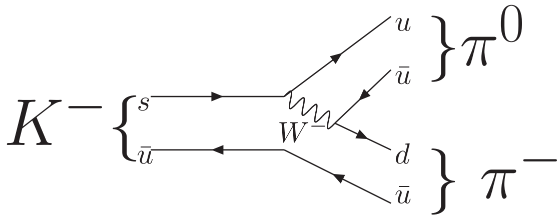


As well as converting an s-quark into a u-quark to emit a W^-, it is also possible to create a W^- from the annihilation of an s-quark with a $\bar{u}$ anti-quark.

Semi-leptonic decay $K^- \rightarrow \mu^- + \bar{\nu}_\mu + \pi^0$



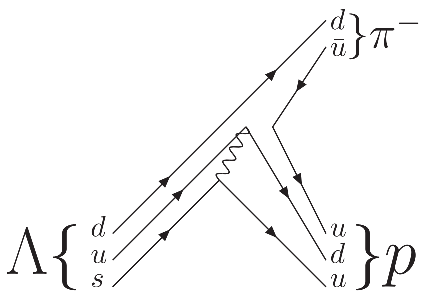
Non-leptonic decay $K^- \rightarrow \pi^0 + \pi^-$



Note that $m_K > 2m_\pi$ which is why this non-leptonic decay mode is energetically allowed.

In the case of baryons, we have already seen an example of a semi-leptonic decay, $\Lambda \to p\, e^- \; \bar{\nu}_e$. An example of a non-leptonic decay is

$$\Lambda \; \to \; p\, \pi^-$$



A W^- is exchanged between the s-quark and the u-quark in the Λ, converting them into a u-quark and a d-quark respectively. A $u - \bar{u}$ quark-antiquark pair is created in the process in order to make up the final state hadrons of a proton and a negative pion.

17.3 Flavour Selection Rules in Weak Interactions

Since in the exchange of a single $W^\pm$ an s-quark can be converted into a non-strange quark, it is highly unlikely that two strange quarks would be converted into non-strange quarks in the same decay process. We therefore have a selection rule for weak decay processes

$$\Delta S \; = \; \pm 1$$

Therefore, hadrons with strangeness -2 which decay weakly must first decay into a hadron with strangeness -1 (which in turn decays into non-strange hadrons). Thus, for example, we have

$$\Xi^0 \; \to \; \Lambda + \pi^0$$

The same selection rules apply for changes in other flavours (charm, bottom).

17.4 Parity Violation

The parity violation observed in β-decay arises because the $W^{\pm}$ tends to couple to quarks or leptons, which are left-handed (negative helicity), i.e. states in which the component of spin in their direction of motion is $-\frac{1}{2}\hbar$.

$W^{\pm}$ always couple to left-handed neutrinos. For quarks and massive leptons the $W^{\pm}$ *can* couple to positive helicity (right-handed) states, but the coupling is suppressed by a factor

$$\frac{mc^2}{E},$$

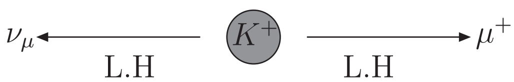
where m is the particle mass and E is its energy. The suppression is much larger for relativistically moving particles

In the case of nuclear β-decay, the nucleus is moving non-relativistically, but the electron typically has energy of a few MeV (and a mass of 0.511 MeV/c^2), so there is a significant suppression of the coupling to right-handed electrons. This is what was observed in the experiment by C.S. Wu on ^{60}Co.

For the coupling of $W^{\pm}$ to anti-quarks or anti-leptons, the helicity is reversed -i.e. the $W^{\pm}$ always couples to positive helicity anti-neutrinos and usually to positive helicity e^+, μ^+, τ^+ or to antiquarks, with a suppressed coupling to left-handed antileptons or anti-quarks.

A striking example of the consequence of this preferred helicity coupling can be seen in the leptonic decay of K^+.

$$K^+ \rightarrow \mu^+ + \nu_\mu$$



In the rest frame of the K^+ the momentum is zero, so the μ^+ and the ν_μ must move in opposite directions. The K^+ has zero spin, so by conservation of angular momentum, the two decay particles must have opposite spin component in any one chosen direction (e.g. the direction of the μ^+. This means that they have the *same* helicity. This means that the $W^{\pm}$ couples to the left-helicity anti-muon, μ^+ and such a coupling is suppressed by

$$\frac{m_\mu c^2}{E_\mu}$$

If we look at the decay mode

$$K^+ \rightarrow e^+ + \nu_e,$$

the same argument would lead to a suppression (of the decay amplitude) of

$$\frac{m_e c^2}{E_e}.$$

Since $m_e \ll m_\mu$ we expect the decay into a positron to be heavily suppresses. In fact we expect the ratio of the partial widths

$$\frac{\Gamma(K^+ \to \mu^+ \nu_\mu)}{\Gamma(K^+ \to e^+ \nu_e)} = \frac{m_\mu^2}{m_e^2} \approx 4 \times 10^4$$

This coincides very closely to the experimentally observed ratio.

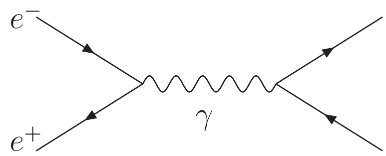
17.5 Z-boson interactions

As well as exchange of $W^\pm$ in which flavour is changed, the weak interactions are also mediated by a neutral gauge-boson, Z. This couples to both quarks and leptons but does not change flavour.

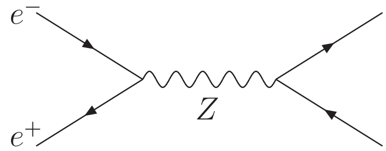
In that sense the interactions of the Z are similar to that of the photon, but there are some important differences.

- The Z couples to neutrinos whereas the photon does not (neutrinos have zero electric charge).
- The Z has a mass of 91.1 Gev/c^2, so the interactions are short range - like the interactions of the $W^\pm$.
- The Z also has a coupling of different strength to left-handed (negative helicity) and right-handed (positive helicity) quarks and leptons and so these interactions also violate parity.

Nevertheless, in any process where there can be photon exchange, there can also be Z exchange. In terms of Feynman diagrams for $e^+ e^-$ scattering into any pair of final state particles, we have



but also



The first diagram (photon exchange) has a propagator

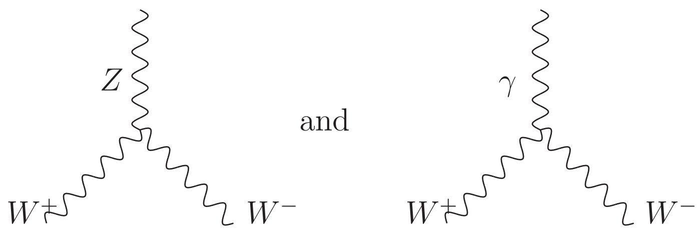
$$1/s,$$

where $\sqrt{s}$ is the centre-of-mass energy, whereas the second diagram (Z exchange) has a propagator

$$\frac{1}{s - M_Z^2 c^4}.$$

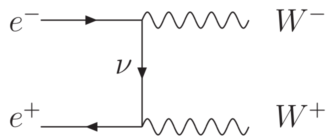
For relatively low centre-of-mass energies for which $\sqrt{s} \ll M_Z c^2$, the second diagram may be neglected and the second diagram gives a negligible contribution. But as $\sqrt{s}$ grows to become comparable (or greater than) $M_Z c^2$ both of these diagrams are equally important.

The Z and photon can both couple to $W^\pm$, so we get interaction vertices

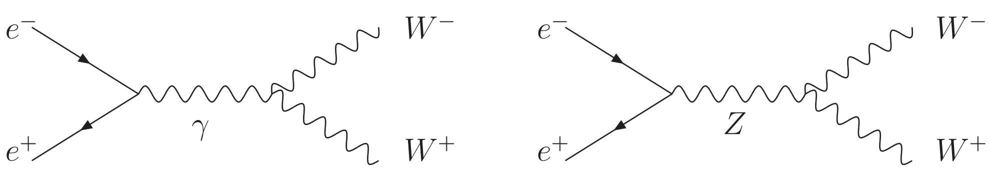


The interaction between the photon and $W^\pm$ is not surprising since the $W^\pm$ are charged and we would expect them to interact with photons, with coupling e. The interaction of $W^\pm$ with the Z is similar but has a different coupling.

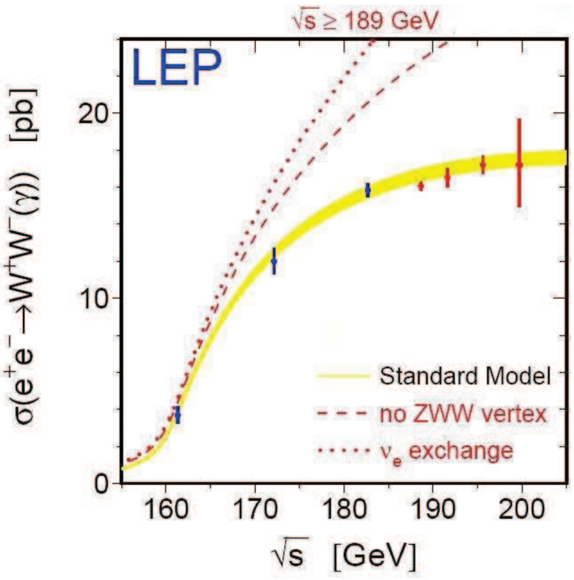
The coupling of the Z and photon to the $W^\pm$ was confirmed at the LEPII experiment at CERN where it was possible to accelerate electrons and positrons to sufficient energies to produce a W^+ and a W^- in the final state. From the coupling of the W to electron and neutrino the Feynman diagram for this process is



but because of the coupling of the Z and photon to $W^\pm$ we also have diagrams



The data from LEPII clearly show that these graphs have to be taken into account



It turns out that the Standard Model of weak and electromagnetic ("electroweak") interactions, developed in the 1960's by Glashow, Weinberg, and Salam, gives a relation between the weak coupling g_W, the (magnitude of the) electron charge, e and the masses of the Z and $W^{\pm}$

$$\frac{M_W}{M_Z} = \cos\theta_W$$

where θ_W is known as the weak mixing angle.

$$e = g_W \sin\theta_W = g_W \sqrt{1 - \frac{M_W^2}{M_Z^2}}$$

This enables us to make an order of magnitude estimate of the rates for weak processes at low energies.

At energies $\ll M_W c^2$, the amplitude for a $W^{\pm}$ exchange process is proportional to

$$\frac{g_W^2}{4\pi\epsilon_0 M_W^2 c^4},$$

so that the rate is proportional to

$$\left(\frac{g_W^2}{4\pi\epsilon_0 M_W^2 c^4}\right)^2.$$

Now for a weak decay rate we want dimensions of inverse time, so we need to multiply this by something with dimensions of the fourth power of energy divided by time. The only quantity proportional to the energy is the Q value of the decay, Q_β and to get inverse time we can divide by $\hbar$ so we get an estimate

$$\text{Rate} \sim \left(\frac{g_W^2}{4\pi\epsilon_0 \hbar c M_W^2 c^4}\right)^2 . \frac{Q_\beta^5}{\hbar}$$

The pre-factor is actually quite small. For example, for muon decay $Q_\beta \approx m_\mu c^2$, and the muon decay rate is actually

$$\frac{1}{\tau_\mu} = \frac{1}{768\pi^3}\left(\frac{g_W^2}{4\pi\epsilon_0\hbar c}\right)^2 \frac{m_\mu^4}{M_W^4}\frac{m_\mu c^2}{\hbar}.$$

We know

$$\frac{g_W^2}{4\pi\epsilon_0\hbar c} = \frac{e^2}{4\pi\epsilon_0\hbar c\sin^2\theta_W} = \frac{\alpha}{\sin^2\theta_W}$$

and

$$\sin^2\theta_W = 1 - \frac{M_W^2}{M_Z^2}.$$

Therefore from the measured masses of the W and Z we can determine the muon lifetime.

17.6 The Higgs mechanism

There is one further particle predicted by the Standard Model of electroweak interactions which has not yet been discovered.

This arises from the mechanism, discovered by P.Higgs, by which particles acquire their mass. The basic idea is that there exists a field, ϕ called the "Higgs field" which has a constant non-zero value everywhere in space. This constant value is called the "vacuum expectation value", $\langle\phi\rangle$.

In the absence of this field it is assumed that all particles would be massless and would travel with velocity c. But because of their interaction with the background Higgs field they are slowed down - thereby acquiring a mass, M

$$M = \frac{1}{2}\frac{g_H}{\sqrt{\epsilon_0\hbar c}}\langle\phi\rangle,$$

where g_H is the coupling of the particle to the Higgs field (the denominator factor $\sqrt{\epsilon_0\hbar c}$ gives it the correct dimensions.) This mechanism is part of the Standard Model.

The Higgs field couples to $W^\pm$ with coupling g_W so that

$$M_W = \frac{1}{2}\frac{g_W}{\sqrt{\epsilon_0\hbar c}}\langle\phi\rangle.$$

Inserting $g_W = e/\sin\theta_W$ with $\cos\theta_W = M_W/M_Z$ and $M_W = 80.4\,\mathrm{GeV/c^2}$, and $M_Z = 91.2\,\mathrm{GeV/c^2}$, we get the value of the vacuum expectation value

$$\langle\phi\rangle = 250\,\mathrm{GeV/c^2}$$

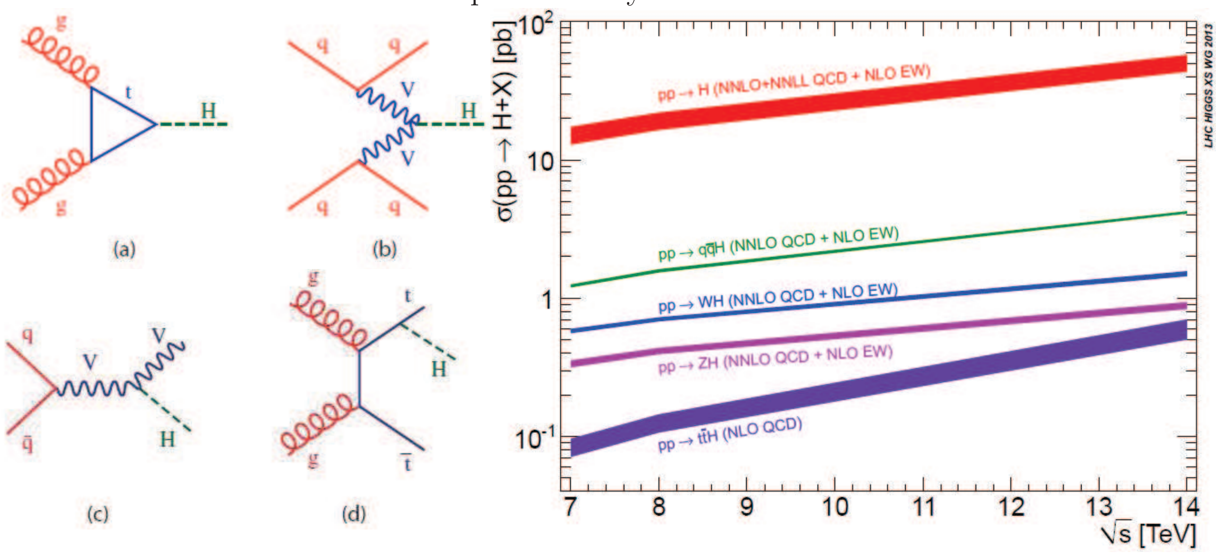
Other particles couple to the Higgs field with couplings that are proportional to their mass.

In the same way that there are quanta of the electromagnetic field which are particles (photons), so there must be quanta of the Higgs field. These are called “Higgs particles”. They must necessarily exist if the Higgs mechanism for generating masses for particles is to be consistent with quantum physics.

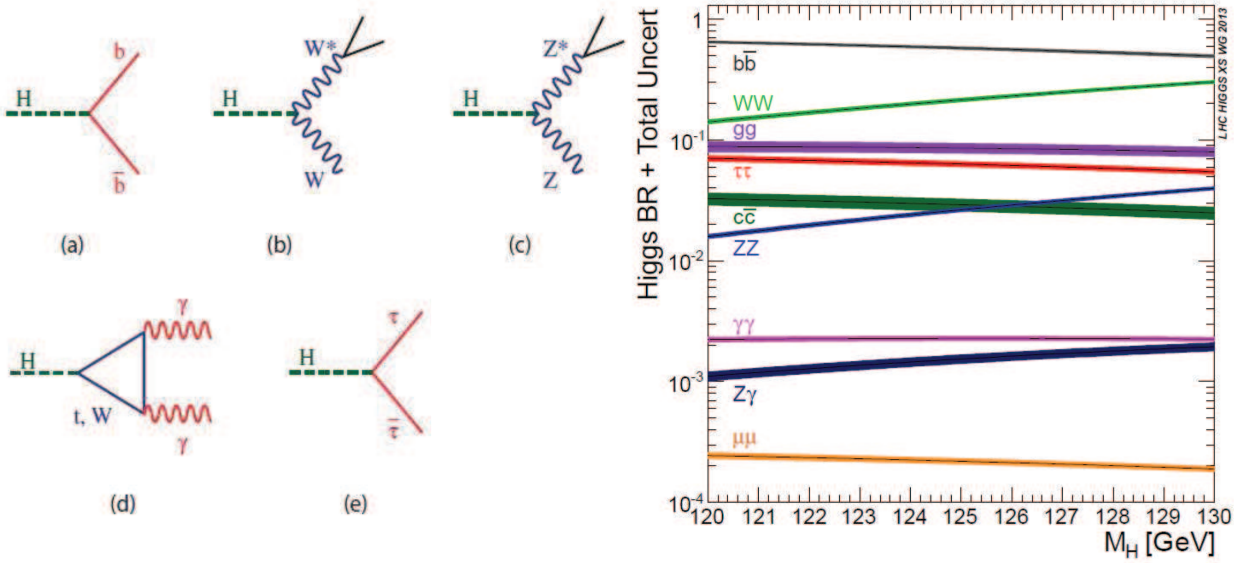
As it was mentioned in the introduction, the Higgs boson was discovered on the 4th of July 2012 by ATLAS and CMS collaborations at the LHC, completing the set of particles of the Standard Model. With a high confidence level this particle is confirmed to have the following properties:

1. It has a spin zero. This is consistent with the theoretical predictions since the vacuum expectation value has to be invariant under Lorentz transformations - so that it is the same in all frames of reference.
2. Higgs boson couples to $W^{\pm}$ and Z (which are consequently massive).
3. It does *not* directly couple to photons (which are massless) so it is uncharged.
4. it does not not couple directly to gluons (which are massless) and so it does not take part in the strong interactions.
5. Its coupling to massive particles is proportional to the particle mass.
6. Its mass is measured to be about 125 GeV/c^2.

Diagrams for production mechanisms of the Higgs boson at the LHC are shown below (left) together with the respective cross sections (right). They include: (a) gluon fusion, (b) weak-boson fusion, (c) Higgs-strahlung (or associated production with a gauge boson) and (d) associated production with top quarks processes. Note, that the first process is the loop-induce one: while Higgs boson does not interact directly with massless gluons, it actually can interact with gluons via virtual massive quarks (e.g. top-quarks which the strongest coupling to the Higgs boson) in the triangle loop diagram. Actually gluon fusion is the main production process of the Higgs boson, while the weak-boson fusion plays the next to leading role. Theoretical uncertainties are represented by the widths of the cross section bands.



Higgs boson decay is dominated by the most massive particles allowed by its mass because its coupling to particles is proportional to the mass. The t-quark mass is 175 GeV/c^2 so it cannot decay into a $t - \bar{t}$ pair. The next most massive quark is the b-quark so Higgs boson predominantly decays into a $b - \bar{b}$ pair, shown in diagram (a) below. Higgs boson is not sufficiently massive to decay into real W^+W^- or two real Z particles, however it can decay to one real and another virtual W or Z boson (W* and Z*) followed by their decay into fermion-antifermion pair as shown in diagrams (b) and (c). As in case of gluon fusion production process, Higgs boson can also decay into photon pair via its interactions with virtual top quark and W-boson as shown in diagram (d). Higgs boson also decays to $\tau^+\tau^-$ pair, the dominant leptonic decay channel since τ-lepton is the most massive amongst the leptons. The respective branching ratios for Higgs boson decay channels are shown in the right frame of the figure below as a function of the Higgs boson mass.



Higgs boson discovery was based not on the process with the highest production and decay rates, which would be naively the $gg \to H \to b\bar{b}$ process. Actually it was was based on the processes with optimal signal-to-background ratio and the highest signal significance. In particular, one of the most significant and cleanest signatures comes from $H \to \gamma\gamma$ decay for which Standard Model background is relatively low. Another very important and significant signature is based on $H \to ZZ^* \to 4$leptons decay which also provide clean 4-lepton signature.