Chapter 18

Electromagnetic Interactions

18.1 Electromagnetic Decays

There are a few cases of particles which could decay via the strong interactions without violating flavour conservation, but where the masses of the initial and final state particles are such that this decay is not energetically allowed.

For example the Σ^{*0} (mass 1385 MeV/c^2) can decay into a Λ (mass 1115 MeV/c^2) and a π^0 (mass 135 MeV/c^2). The quark content of the Σ^{*0} and Λ are the same and the π^0 consists of a superposition of quark-antiquark pairs of the same flavour. As required in strong interaction processes the isospin is conserved - the Σ^{*0} has isospin I=1, the Λ has isospin $I = 0$ and the pion has isospin $I = 1$.

On the other hand, the Σ^0 whose mass is 1189 MeV/c^2 does not have enough energy to decay into a Λ and a pion.

In such cases the decay can proceed via the electromagnetic interactions producing one or more photons in the final state. The dominant decay mode of the Σ^0 is

$$\Sigma^0 \rightarrow \Lambda + \gamma$$

The quark content of the Σ^0 and the Λ are the same, but one of the charged quarks emits a photon in the process. Note that in this decay the isospin is *not* conserved - the initial state has isospin I=1, whereas the final state has isospin $I = 0$. Electromagnetic interactions do not conserve isospin.

Because the electromagnetic coupling constant, e, is much smaller than the strong coupling constant the rates for such decays are usually much smaller than the rates for decays which can proceed via the strong interactions.The lifetime of the Σ^0 is 10^{-10} seconds, whereas the Σ^{*0} has a width of 36 MeV, corresponding to a lifetime of about 10^{-23} seconds.

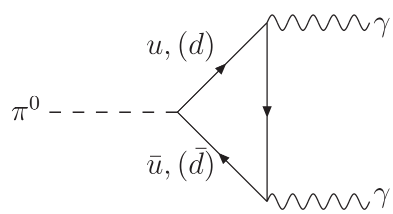
Another important example of electromagnetic decay is the decay of the π^0 into two photons.

$$\pi^0 \rightarrow \gamma + \gamma.$$

Note that to produce only one photon would not be possible by conservation of energy and momentum. For a π^0 decaying from rest momentum is conserved because the two photons have identical frequency and move in opposite directions. The π^0 is actually a superposition of a $u - \bar{u}$ quark-antiquark pair and a $d - \bar{d}$ quark-antiquark pair

$$|\pi^0\rangle \;=\; \frac{1}{\sqrt{2}}\left(|u\bar{u}\rangle \;-\; |d\bar{d}\rangle\right)$$

In either of these states, the quark can annihilate against the antiquark of identical flavour to produce two photons. In terms of Feynman diagrams we have



Using Quantum Field Theory, we can work out the decay rate for this process and summing over the u and d contributions we get an estimate for the π^0 lifetime of

$$\tau \;=\; 7.6 \times 10^{-16}\, s,$$

whereas the measured value if 8.4×10^{-17} seconds.

What has gone wrong is that we have forgotten about colour. In the calculation of the decay *amplitude* we must not only sum the contribution from the above Feynman diagram over u and d quarks but also over the three possible colours that these quarks can have. This gives us a further factor of 3 in the decay amplitude and so a factor of 9 in the decay rate.

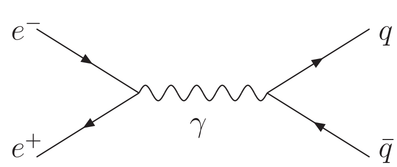
18.2 Electron-positron Annihilation

Another striking piece of evidence that quarks come in three colours comes from the study of the process

$$e^+ + e^- \;\rightarrow\; \text{hadrons}$$

(summed over all possible hadrons in the final state)

At the level of quarks, the Feynman diagram for this process is

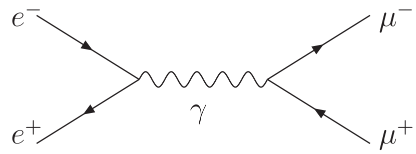


(provided $\sqrt{s} \ll M_Z c^2$ so that the Z-exchange diagram can be neglected.)

For the process

$$e^+ + e^- \rightarrow \mu^+ + \mu^-,$$

the Feynman diagram is



The only difference between these two graphs is the coupling of the final state quarks or final state muons to the photon, i.e. the electric charges of the quarks and the muons.

This means that for a quark of flavour i with electric charge Q_i (in units of e) the ratio of the amplitudes is

$$\frac{\mathcal{A}(e^+e^- \rightarrow q_i\bar{q}_i)}{\mathcal{A}(e^+e^- \rightarrow \mu^+\mu^-)} = Q_i.$$

In order to calculate the ratio of total cross-sections we square the amplitude and sum over all possible final state quarks that can be produced, so that

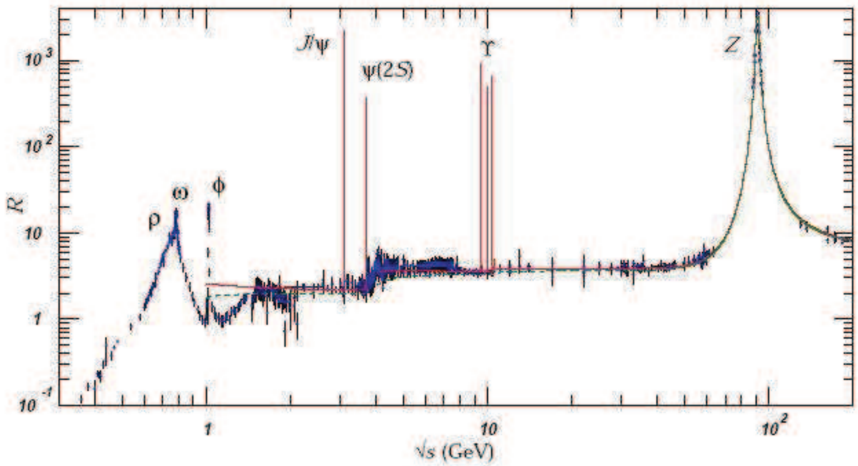
$$R = \frac{\sigma(e^+e^- \rightarrow \text{hadrons})}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)} = \sum_i Q_i^2.$$

How many quarks we sum over depends on the centre-of-mass energy $\sqrt{s}$. If $\sqrt{s} < 2m_c c^2$, then only u, d and s quarks can be produced in the final state and we have

$$R = \frac{\sigma(e^+e^- \rightarrow \text{hadrons})}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)} = 3\left(Q_u^2 + Q_d^2 + Q_s^2\right) = 3\left(\left(\frac{2}{3}\right)^2 + \left(\frac{-1}{3}\right)^2 + \left(\frac{-1}{3}\right)^2\right) = 2.$$

The factor of 3 is needed because we can produce final state quarks in any of the three colours states (in principle these would be distinguishable at the quark level - so we multiply the cross-section by 3 and not the amplitude.)

In the region $2m_c c^2 < \sqrt{s} < 2m_b c^2$ we can also produce a c- (charm) quark and this has to be added to the cross-section. Likewise, for $\sqrt{s} > 2m_b c^2$ we also have to include the production of b-quarks. Thus we expect this ratio, R, to have jumps as we cross thresholds in incoming energy which allow the production of more massive quarks.



Fragmentation

What we are interested in is the total cross-section for $e^+ \, e^-$ to annihilate to produce hadrons, whereas what we have calculated is the total cross-section to all quarks.

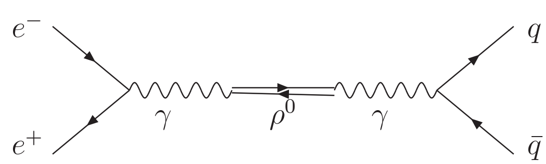
What happens is that the quarks, which cannot be observed directly, interact with gluons in a complicated way and are converted into sets of ordinary hadrons. This process is called "fragmentation". Its mechanism is not understood but several computer simulations have been developed which mimic this process fairly well.

In the centre-of-mass frame, the final state quark and antiquark are moving in opposite directions. What usually happens is that the process of fragmentation acting on the quark and antiquark separately leads to two narrow jets of particles moving in opposite directions.

Resonances

As well as the almost constant value for the ratio, R, between energy thresholds (with jumps near each threshold), the quantity, R, is populated with resonances wherever $\sqrt{s}/c^2$ is equal to the mass of a neutral, spin one, particle that can couple directly to a photon.

The spin has to be the same as the spin of the photon (spin 1), as there is a direct coupling between the photon and the resonant particle which must conserve angular momentum.



At low energies these are mesons such as ρ^0 which consist of a quark and antiquark of the same flavour (or superpositions like in the case of π^0). When $\sqrt{s}/c^2 = m_\rho$ the ρ propagator

$$\frac{1}{(s - m_\rho^2 c^4 + i m_\rho \Gamma_\rho c^2)}$$

gives rise to a resonance.

The thresholds for the production of more massive quarks are also indicated by thresholds. When $\sqrt{s} \approx 2m_c c^2$ it is possible to create a resonance of a particle called J/Ψ which is a bound state of a c-quarks and a $\bar{c}$ antiquark, with mass 3.1 GeV/c^2 - there are further resonances corresponding to excited states with the same quark content.

Likewise at the threshold $\sqrt{s} = 2m_b c^2$, there is a resonance called the Υ, which is a bound state of a b-quark and a $\bar{b}$ antiquark, with mass 9.5 GeV/c^2 - and also some further resonances corresponding to excited states.