2007 Section A of examination problems on Nuclei and Particles

Section A

A1. A fossil containing 1 gramme of carbon has a radioactivity of 0.03 disintegrations per second. A living organism has a concentration of $^{14}\mathrm{C}$ of one part in 10^{12} (the rest being almost all $^{12}\mathrm{C}$) and the half-life of $^{14}\mathrm{C}$ is 5370 years (=1.81×10^{11} seconds.)

Estimate the age of the fossil. [4]

A2. The nuclide $^{109}_{52}\mathrm{Te}$ decays to $^{105}_{50}\mathrm{Sn}$, emitting an α-particle with kinetic energy 3.11 MeV. The binding energy per nucleon of $^{109}_{52}\mathrm{Te}$ is 8.318 MeV, whereas that of $^{105}_{50}\mathrm{Sn}$ is 8.397 MeV.

Calculate the binding energy per nucleon of an α-particle. [4]

A3. The deuteron is a bound state of a proton and a neutron. There are *no* bound states of two proton or two neutrons.

Use this information and consideration of the symmetry properties of the deuteron wavefunction to deduce the spin of the deuteron in its ground-state. [4]

A4. What is meant by luminosity?

The LHC will have a luminosity of $10^{34}\ /\mathrm{cm}^2/\mathrm{sec}$. The cross-section for Higgs production is estimated to be 10 pb. After what period of running will this cross-section be known to an accuracy of 1% (assume that all Higgs particles produced are detected)? [4]

A5. The branching ratio for the decay of the η-meson into three pions is 32.5%. The partial width for this decay is 0.42 KeV. What is the lifetime of the η-meson? [4]

A1.

A fossil containing 1 gramme of carbon has a radioactivity of 0.03 disintegrations per second. A living organism has a concentration of ^{14}C *of one part in* 10^{12} *(the rest being almost all* ^{12}C*) and the half-life of* ^{14}C *is 5370 years (=*1.81×10^{11} *seconds.)*
Estimate the age of the fossil.

Decay constant of ^{14}C

$$\lambda = \frac{\ln(2)}{t_{1/2}} = 3.83 \times 10^{-12} \; sec^{-1}$$

1 gramme of carbon from living organism contains

$$\frac{10^{-3}}{12 \, m_p} \times 10^{-12} = 4.99 \times 10^{10} \;\; \text{nuclei of } {}^{14}\text{C}$$

and therefore has an activity of

$$A_0 = (4.99 \times 10^{10}) \times (3.83 \times 10^{-12}) = 0.191 \; Bq.$$

Age of fossil is

$$T = t_{1/2} \times \frac{\ln\left(\frac{0.191}{0.03}\right)}{\ln(2)} = 14000 \text{ yrs}$$

A2.

The nuclide ${}^{109}_{52}$Te *decays to* ${}^{105}_{50}$Sn*, emitting an* α*-particle with kinetic energy 3.11 MeV. The binding energy per nucleon of* ${}^{109}_{52}$Te *is 8.318 MeV, whereas that of* ${}^{105}_{50}$Sn *is 8.397 MeV. Calculate the binding energy per nucleon of an* α*-particle.*

Taking into account the recoil of the daughter nucleus, the Q-value of the α-decay is

$$Q = \frac{109}{105} \times 3.11 = 3.23 \; MeV$$

Binding energy of ${}^{109}_{52}$Te is 8.320 is

$$109 \times 8.318 = 906.66 \; MeV$$

Binding energy of ${}^{105}_{50}$Sn is 8.320 is

$$105 \times 8.397 = 881.69 \; MeV$$

Therefore the binding energy of the α-particle must be

$$906.66 - 881.69 + 3.23 = 28.2 \; MeV$$

Therefore the binding energy per nucleon is

$$\frac{28.2}{4} = 7.05 \ MeV$$

A3.

The deuteron is a bound state of a proton and a neutron. There are no bound states of two proton or two neutrons.
Use this information and consideration of the symmetry properties of the deuteron wavefunction to deduce the spin of the deuteron in its ground-state.

The wavefunction for a deuteron may be written

$$\Psi_d = \Psi(\mathbf{r})\chi_S\chi_I$$

This must be overall antisymmetric under interchange of the two nucleons.

The absence of a two-proton or two-neutron bound state implies that the deuteron is an isosinglet, $I = 0$. The symmetry of the isospin part of the wavefunction, χ_I, given by $(-1)^I$ is symmetric.

For a two-body system in its ground state, the orbital angular momentum, l, is zero, so the symmetry of the spatial part of the wavefunction $\Psi(\mathbf{r})$ given by $(-1)^l$ is symmetric.

This means that the spin part of the wavefunction, χ_S, given by $(-1)^S$, must be antisymmetric,

The ground state must therefore have spin $S = 1$

A4.

What is meant by luminosity?
The LHC will have a luminosity of 10^{34} /cm^2/sec. *The cross-section for Higgs production is estimated to be 10 pb. After what period of running will this cross-section be known to an accuracy of 1% (assume that all Higgs particles produced are detected)?*

Luminosity is the number of incident particles per unit area (usually measured in cm^2) per second.
No. of Higgs, N_H produced per second

$$N_H = \mathcal{L}\sigma = (10^{34} \times 10^4) \times (10 \times 10^{-12} \times 10^{-28}) = 0.1$$

For an accuracy of 1 part in 100 we need to collect 100^2 events. This will take a time t,

$$t = \frac{100^2}{0.1} = 10^5 \ secs$$

A5.

The branching ratio for the decay of the η-meson into three pions is 32.5%. The partial width for this decay is 0.42 KeV. What is the lifetime of the η-meson?

Total width of η is

$$\Gamma = \frac{0.42}{0.325} = 1.29\ KeV$$

Lifetime is given by

$$\tau = \frac{\hbar}{\Gamma} = \frac{1.05 \times 10^{-34}}{(1.29 \times 10^{3}) \times (1.6 \times 10^{-19})} = 5.1 \times 10^{-19}\ sec.$$

2008 Section A of examination problems on Nuclei and Particles

Section A

A1. Determine the age of ancient wooden items if it is known that the specific activity of C^{14} nuclide in them amounts to 3/5 of that in recently felled trees. The half-life of C^{14} nuclei is 5570 years. [4]

A2. Calculate the shift of the de-excitation energy line E_0 of the photon emitted by nuclei with atomic mass M. Why this photon can not be absorbed by this nuclei? Approximate your result with $E_0/(Mc^2) \ll 1$ [4]

A3. Briefly describe one piece of experimental evidence for each of

(a) the existence of quarks

(b) the existence of colour charge

(c) the existence of the gluon. [6]

A4. Which of the following processes are forbidden by the law of conservation of lepton charge:

(1) $n \to p + e^- + \nu_e$ (2) $\pi^+ \to \mu^+ + e^- + e^+$,

(3) $\pi^- \to \mu^- + \nu_\mu$, (4) $p + e^- \to n + \nu_e$ [2]

A5. The partial width for Z-boson decay to any charged lepton channel is 84 MeV, the Z-boson decay fraction to hadrons is 69.9%, while Z-boson decay fraction to neutrinos is 20%. What is the Z-boson width and lifetime? [4]

A1 Solution (seen on the problem sheet)

In old wooden atoms the number of C^{14} nuclei steadily decreases because of radioactive decay. (In live trees biological processes keep replenishing C^{14} nuclei maintaining a balance. This balance starts getting disrupted as soon as the tree is felled.) [1]

If $T_{1/2}$ is the half life of C^{14} then

$$e^{-t\frac{ln2}{T_{1/2}}} = \frac{3}{5} \quad [1]$$

Hence

$$t = T_{1/2}\frac{ln(5/3)}{ln2} = 4105 \text{ years} \simeq 4.1\times 10^3 \text{ years} \quad [2]$$

A2 Solution (seen at seminar)

The momenta conservation implies equal absolute momentum of the photon and recoiled nuclei:

$$\sqrt{2MT} = E_\gamma/c \quad [1]$$

where T s the kinetic energy of the recoiled nuclei. From energy conservation it follows that deexitation energy E_0 is equal to the summ of the photon energy and recoiled nuclei:

$$E_0 = E_\gamma + T \quad [1]$$

Solving quadratic equation from above two equations

$$E_\gamma^2/(2Mc^2) + E_\gamma - E_0 = 0$$

we find

$$E_\gamma \simeq E_0(1 - \frac{E_0}{2Mc^2})$$

so, the shift of the photon line is

$$\delta_\gamma = \frac{E_0^2}{2Mc^2}$$

. [1]

Due to this shift, which is typically bigger than the width of the line this photon can not be absorbed by the same nuclei at rest. [1]

A3 Solution (in course notes)

For each of these mark will be given for each pertinent point from amongst:

a) The main evidence supporting the quark theory is the non-uniform distribution of charge in proton and neutron as seen in the scattering of high energy electrons on nucleons, which shows that a nucleon has internal structure. The proposed quark theory assumed the existence of three types of quark, u, d, s and their antiparticles, which have fractional charges and certain quantum numbers, as constituents of hadrons: a baryon consists of three quarks; a meson, a quark and an antiquark. The quark theory was able to explain the structure, spin and parity of hadrons. [2]

(b) The main purpose of postulating the color quantum number was to overcome the statistical difficulty that according to the quark theory Δ^{++}, a particle of spin 3/2, should consist of three u quarks with parallel spins, while the Pauli exclusion principle forbids three ferminions of parallel spins in the same ground state. To get over the color dimension for quarks was proposed: each quark could have one of three colors. Although the three quarks of Δ^{++} have parallel spins, they have different colors, thus avoiding violation of the Pauli exclusion principle. The proposal of the color also explained the relative cross section R for producing hadrons in e^+e^- collisions. [2]

(c) Quantum chromodynamics predicts gluons as mediators of the strong force. Gluons can be emitted by quarks or gluons. In the electron-positron collider machine PETRA in DESY the three-jet phenomenon found in the hadronic final state provides strong evidence for the existence of gluons. The phenomenon is interpreted as an electron and a position colliding to produce a quark-antiquark pair, one of which then emits a gluon. The gluon and the two original quarks separately fragment into hadron jets, producing three jets in the final state. From the observed rate of three-jets events the coupling constant for strong interaction can be deduced. [2]

A4 Solution (problem sheet)

(1) The process $n \to p + e^- + \nu_e$ cannot occur as there are 2 more leptons on the right compared to zero on the left

(2) The process $\pi^+ \to \mu^+ + e^- + e^+$ is forbidden because this corresponds to a change of lepton number (0 on the left and -1 on the right)

(3) The process $\pi^- \to \mu^- + \nu_\mu$ is forbidden because μ^-, ν_μ being both leptons and $\Delta L = 2$ therefore

(4) is allowed. [2]

A5 Solution (problem sheet)

$\Gamma(Z \to all) = 3\Gamma(Z \to e^+e^-)/(1 - 0.699 - 0.2) = 3 \times 84 \text{ MeV}/0.101 = 2495$ MeV i.e. $\simeq 2.5$ GeV [2]

$\tau = \hbar/\Gamma = \frac{1.05\times10^{-34}}{(2.5\times10^9)\times(1.6\times10^{-19})} = 1.64 \times 10^{-24}$ sec [2]

2009 Section A of examination problems on Nuclei and Particles

Section A

A1. Given that first two shells in the Shell Model are 1s and 1p shells, determine spin and parities of the following nuclei in their ground state:
$^{10}_{5}$B, $^{11}_{5}$B, $^{12}_{6}$C, $^{13}_{6}$C. In case of several possible states, list all possibilities. [4]

A2. Find the third components of isospin of the nuclides $^{6}_{2}\mathrm{He}$, $^{6}_{3}\mathrm{Li}$, $^{6}_{4}\mathrm{Be}$. [3]

A3. Determine the age of ancient wooden items if it is known that the specific activity of C^{14} nuclide in them amounts to 1/5 of that in recently felled trees. The half-life of C^{14} nuclei is 5570 years. [4]

A4. A de-excitation transition in a nucleus with mass M has energy E_0. Compute the shift from E_0 in the energy of the emitted photon (you may assume that $E_0 \ll Mc^2$). [3]

The nuclei can not re-absorb a photon of this energy - why? [1]

A5. In proton-proton scattering, it is possible to produce an Ω^- (strangeness -3) along with a neutron and three other identical particles. What are these three other particles? [2]

A6. The B^+ meson (mass $5.3\,\mathrm{GeV/c^2}$) is a bound state of a u-quarks and a $\bar{b}$ anti-quark, with zero spin.

Calculate the ratio of the partial widths ($m_\tau = 1.8\ GeV/c^2$, $m_\mu = 0.106\ GeV/c^2$)

$$\frac{\Gamma(B^+ \to \tau^+ \nu_\tau)}{\Gamma(B^+ \to \mu^+ \nu_\mu)},$$

explaining your result. [3]

Section A

A1 Solution (Seen on the problem sheet)

${}^{10}_{5}$B: Z=5, N=5. There are five neutrons and protons filling shells as follows: $(1s_{1/2})^2(1p_{3/2})^3$. Therefore there is one unpaired proton in $1p_{3/2}$ state and one unpaired neutron in the same state which can form $(0)^+$, $(1)^+$, $(2)^+$ and $(3)^+$ states. [1]

${}^{11}_{5}$B: Z=5, N=6. All neutrons are paired, and there are five protons among which one is unpaired in $1p_{3/2}$ state with $l = 1$ and $j = 3/2$. So, the nuclei is in the $(3/2)^-$ state. [1]

${}^{12}_{6}$C: Z=6, N=6. All neutrons and protons are paired, so nuclei is in the $(0)^+$ state [1]

${}^{13}_{6}$C: Z=6, N=7. There is one unpaired neutron in $1p_{1/2}$ state according to $(1s_{1/2})^2(1p_{3/2})^4(1p_{1/2})^1$ shell structure. So $l = 1, j = 1/2$ and the nuclei is in the $(1/2)^-$ state. [1]

A2 Solution (Seen on the problem sheet)

${}^{6}_{2}$He has two neutrons in the outer shell, therefore $I_3 = -\frac{1}{2} - \frac{1}{2} = -1$ [1]

${}^{6}_{3}$Li has one proton and one neutron in the outer shell, so $I_3 = +\frac{1}{2} - \frac{1}{2} = 0$ [1]

${}^{6}_{4}$Be has two protons in the outer shell, therefore $I_3 = +\frac{1}{2} + \frac{1}{2} = +1$ [1]

A2 Solution (Seen on the problem sheet)

In old wooden atoms the number of C^{14} nuclei steadily decreases because of radioactive decay. [1]

In live trees biological processes keep replenishing C^{14} nuclei maintaining a balance. This balance starts getting disrupted as soon as the tree is felled. [1]

If $T_{1/2}$ is the half life of C^{14} then

$$e^{-t\frac{ln2}{T_{1/2}}} = \frac{1}{5} \qquad [1]$$

Hence

$$t = T_{1/2}\frac{ln(5)}{ln(2)} = 5570 \text{ years} \times \frac{ln(5)}{(ln2)} \simeq 13 \times 10^3 \text{ years} \qquad [1]$$

A3 Solution (Seen at seminar) The momenta conservation implies equal absolute momentum of the photon and recoiled nuclei:

$$\sqrt{2MT} = E_\gamma/c \qquad [1]$$

where T s the kinetic energy of the recoiled nuclei. From energy conservation it follows that deexitation energy E_0 is equal to the sum of the photon energy and recoiled nuclei:

$$E_0 = E_\gamma + T \qquad [1]$$

Solving quadratic equation from above two equations

$$E_\gamma^2/(2Mc^2) + E_\gamma - E_0 = 0 \quad \text{we find} \quad E_\gamma \simeq E_0(1 - \frac{E_0}{2Mc^2})$$

so, the shift of the photon line is

$$\delta_\gamma = \frac{E_0^2}{2Mc^2} \qquad [1]$$

Due to this shift, which is typically bigger than the width of the line this photon can not be absorbed by the same nuclei at rest. [1]

A5 Solution (Seen on the problem sheet)

We have the following quark content for given initial and final states:

$uud + uud \rightarrow sss + udd + 3x$ [1]

To conserve the strangeness we can write reaction as

$uud + uud \rightarrow sss + udd + 3(u\bar{s})$ where $u\bar{s}$ form K^+ meson. [1]

A6 Solution (Bookwork)

Spin components of B^+ decay products along any given direction are equal and opposite (B^+ has zero spin). Recalling that decay products are particle and antiparticle moving in the opposite directions one concludes that B^+ decay products have *the same helicities*. The coupling of W (decay takes place via the virtual W-boson) to right-handed charged fermions is suppressed as m_τ/E_τ or m_μ/E_μ. [1]

So the ratio of amplitudes equal to $\frac{A(B^+ \rightarrow \tau^+ \nu_\tau)}{A(B^+ \rightarrow \mu^+ \nu_\mu)} = \frac{m_\tau}{m_\mu}$, [1]

while

$$\frac{\Gamma(B^+ \rightarrow \tau^+ \nu_\tau)}{\Gamma(B^+ \rightarrow \mu^+ \nu_\mu)} = \left(\frac{A(B^+ \rightarrow \tau^+ \nu_\tau)}{A(B^+ \rightarrow \mu^+ \nu_\mu)}\right)^2 = \left(\frac{m_\tau}{m_\mu}\right)^2 = 288$$

[1]