PHYS3002.
Nuclei and Particles.
Synoptic.

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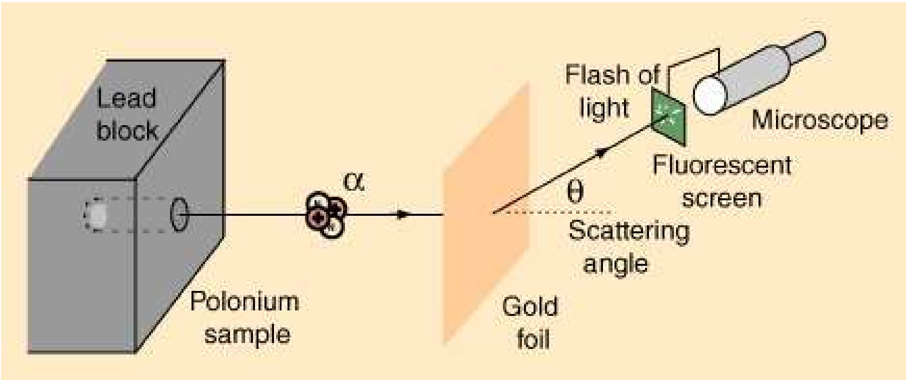
Contents

1 Models of Nuclei

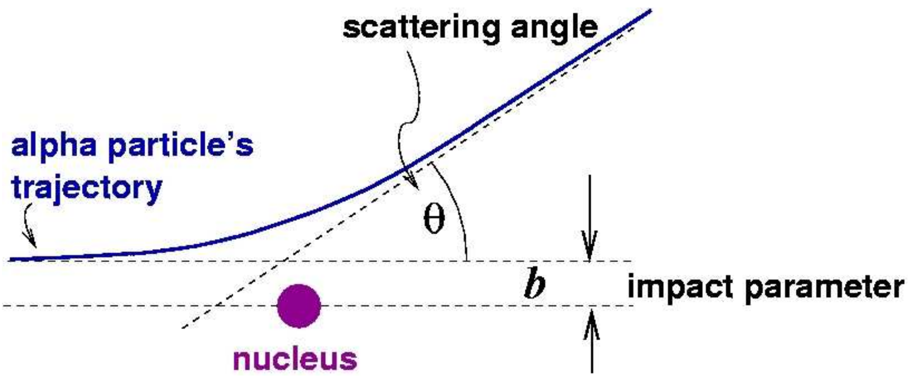
1.1 Rutherford Scattering and cross section



In 1911, Rutherford discovered the nucleus by analysing the data of Geiger and Marsden on the scattering of α-particles against a very thin foil of gold.

The data were explained by making the following assumptions:

- The atom contains a nucleus of charge Ze, where Z is the atomic number of the atom (i.e. the number of electrons in the neutral atom),
- The nucleus can be treated as a point particle,
- The nucleus is sufficiently massive compared with the mass of the incident α-particle that the nuclear recoil may be neglected,
- That the laws of classical mechanics and electromagnetism can be applied and that no other forces are present,
- That the collision is elastic.

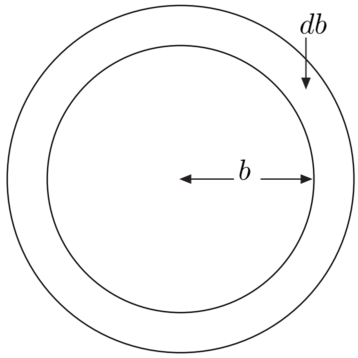


The relation between b (impact parameter) and θ (scattering angle) is given by

$$\tan\left(\frac{\theta}{2}\right) = \frac{D}{2b} \tag{1}$$

where D is the closest approach ($D = \frac{z\,Z\,e^2}{4\pi\epsilon_0 T}$).

1.2 Cross section



The "flux", F of incident particles is defined as the number of incident particles arriving per unit area per second at the target.
The number of particles, $dN(b)$ per second, with impact parameter between b and $b + db$ is this flux multiplied by the area between two concentric circles of radius b and $b + db$.

$$dN(b) = F\, 2\pi b\, db$$

Differentiating equation above we derive

$$dN(\theta) = F\pi \frac{D^2}{4} \frac{\cos(\theta/2)}{\sin^3(\theta/2)} d\theta$$

We define the "differential cross-section", $d\sigma/d\theta$, with respect to the scattering angle is the number of scatterings between θ and $\theta + d\theta$ per unit flux, per unit range of angle, i.e.

$$\frac{d\sigma}{d\theta} = \frac{dN(\theta)}{Fd\theta} = \pi \frac{D^2}{4} \frac{\cos(\theta/2)}{\sin^3(\theta/2)}.$$

Using $d\Omega = \sin\theta d\theta d\phi$ we define differential cross-section with respect to a given solid angle Ω:

$$\frac{d\sigma}{d\Omega} = \frac{D^2}{8} \frac{\cos(\theta/2)}{\sin^3(\theta/2)} \frac{1}{2\sin(\theta/2)\cos(\theta/2)} = \frac{D^2}{16\sin^4(\theta/2)}.$$

1.3 Units

The unit of length that is often used in nuclear physics is the "fermi" (fm) which is defined to be 10^{-15} m and energies are usually quoted in electron volts (Kev, MeV, or GeV). A cross-section of 1 fm^2 corresponds ot 10 mb.

For the purposes of numerical calculations, it is worth noting that

$$\hbar\, c = 197.3\,\mathrm{MeV\,fm},$$

so that

$$\frac{e^2}{4\pi\epsilon_0} = \alpha\,\hbar\, c = \frac{1}{137} \times 197.3\,\mathrm{MeV\,fm}$$

For example, the distance of closest approach is therefore given by

$$D = \frac{197.3}{137} \frac{zZ}{T}\,\mathrm{fm},$$

where the kinetic energy T is given in MeV.

1.4 Diffraction

There are deviations from the Rutherford scattering formula when the energy of the incident α-particle becomes too large, so that the distance of closest approach is of order a few fermi's. This happens because the size of the nucleus has a finite radius of the order of fm.

The Rutherford formula can generalised for the case of relativistic particles (Mott formula):

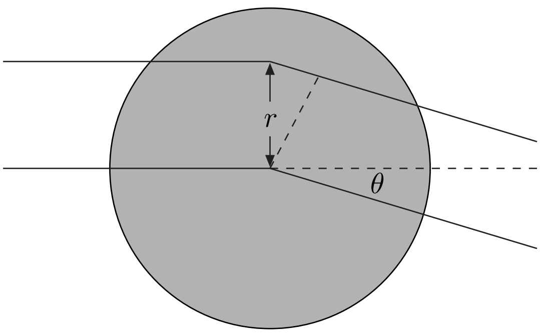
$$\frac{d\sigma}{d\Omega}_{|\mathrm{Mott}} = \frac{d\sigma}{d\Omega}_{|\mathrm{Rutherford}} \left(1 - \frac{v^2}{c^2}\sin^2\left(\frac{\theta}{2}\right)\right)$$

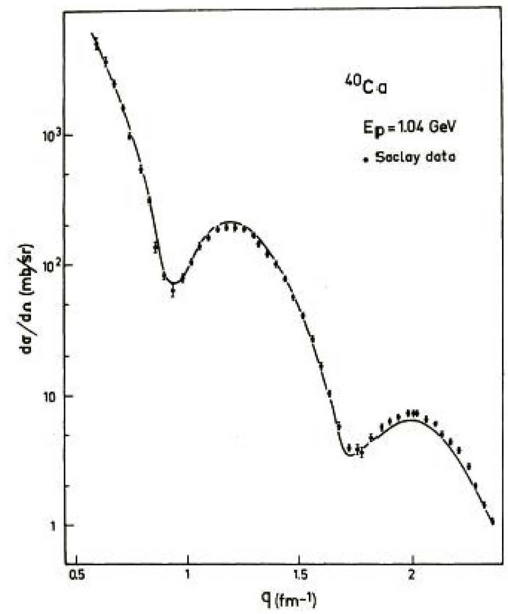
We account for the charge distribution of the nucleus by writing the differential cross-section as

$$\frac{d\sigma}{d\Omega} = \frac{d\sigma}{d\Omega}_{|\mathrm{Mott}} |F(q^2)|^2. \tag{2}$$

The correction factor $F(q^2)$ is called the "electric form-factor", where $\mathbf{q}$ is the momentum transferred by the electron in the scattering and its magnitude is related to the scattering angle by $q = 2p\sin\left(\frac{\theta}{2}\right)$, where p is the momentum of the incident electron.

The structure of "electric form-factor" has a diffractive nature





and the first minimum of the diffractive pattern occurs when

$$\frac{qR}{\hbar} \approx \pi$$

1.4.1 Electric Quadrupole Moments

So far, we have assumed that the charge distribution is spherically symmetric. If that were the case we would have

$$< x^2 > = < y^2 > = < z^2 > = \frac{1}{3} < r^2 >,$$

where

$$< x^2 > = \frac{1}{Ze}\int x^2 \rho(\mathbf{r}) d^3\mathbf{r}$$

etc.

However, for many nuclei this is not the case and they possess an "electric quadrupole moment" defined (with respect to an axis z) as

$$\mathcal{Q} = \int (3z^2 - r^2)\rho(\mathbf{r}) d^3\mathbf{r}$$

The $\mathcal{Q}/e$ has dimensions of area and is therefore usually quoted in barnes.

Nuclei that possess and electric quadrupole moment have a shape which is an oblate spheroid for $\mathcal{Q} < 0$ and a prolate spheroid for $\mathcal{Q} > 0$.

On the other hand, the electric dipole moment, which is a vector defined by

$$\mathbf{d} = \int \mathbf{r}\rho(\mathbf{r}) d^3\mathbf{r},$$

is almost zero. The reason for this that to a very good approximation, the wavefunction of a proton in a nucleus is a parity eigenstate, i.e. $\Psi(\mathbf{r}) = \pm\Psi(-\mathbf{r})$ which implies $\rho(\mathbf{r}) = \rho(-\mathbf{r})$, so that the electric dipole moment vanishes by symmetric integration.

1.5 Liquid drop model

1.5.1 Some Nuclear Nomenclature

- **Nucleon:** A proton or neutron.
- **Atomic Number, Z:** The number of protons in a nucleus.
- **Atomic Mass number, A:** The number of nucleons in a nucleus.
- **Nuclide:** A nucleus with a specified value of A and Z. This is usually written as $^A_Z\{Ch\}$ where Ch is the Chemical symbol. e.g. $^{56}_{28}$Ni means Nickel with 28 protons and a further 28 neutrons.
- **Isotope:** Nucleus with a given atomic number but different atomic mass number, i.e. different number of neutrons. Isotopes have very similar atomic and chemical behaviour but may have very different nuclear properties.
- **Isotone:** Nulceus with a given number of neutrons but a different number of protons (fixed (A-Z)).
- **Isobar:** Nucleus with a given A but a different Z.
- **Mirror Nuclei:** Two nuclei with odd A in which the number of protons in one nucleus is equal to the number of neutrons in the other and vice verse.

1.5.2 Binding Energy

The mass of a nuclide is given by

$$m_N = Z\, m_p + (A - Z)\, m_n - B(A, Z)/c^2,$$

where $B(A, Z)$ is the binding energy of the nucleons and depends on both Z and A. The binding energy is due to the strong short-range nuclear forces that bind the nucleons together. Unlike

Coulomb binding these cannot even in principle be calculated analytically as the strong forces are much less well understood than electromagnetism.

There are different terms contributing to the binding energy:

1. **Volume term:** $a_V\, A$ Each nucleon has a binding energy which binds it to the nucleus. The fact that it turns out to be proportional to A indicates that a nucleon only interact with its nearest neighbours.
2. **Surface term:**$-a_S\, A^{2/3}$. The nucleons at the surface of the 'liquid drop' only interact with other nucleons inside the nucleus, so that their binding energy is reduced.
3. **Coulomb term:**$-a_C\frac{Z^2}{A^{1/3}}$ Although the binding energy is mainly due to the strong nuclear force, the binding energy is reduced owing to the Coulomb repulsion between the protons. We expect this to be proportional to the square of the nuclear charge, Z, and by Coulomb's law it is expected to be inversely proportional to the nuclear radius.
4. **Asymmetry term:**$\frac{\left((-1)^Z+(-1)^N\right)}{2}\frac{a_P}{A^{1/2}}.-a_A\frac{(Z-N)^2}{A}$ This is a quantum effect arising from the Pauli exclusion principle which only allows two protons or two neutrons (with opposite spin direction) in each energy state. The upshot of this is that nuclides with $Z = N = (A - Z)$ have a higher binding energy, whereas for nuclei with different numbers of protons and neutrons (for fixed A) the binding energy decreases as the square of the number difference.
5. **Pairing term:**$\frac{\left((-1)^Z+(-1)^N\right)}{2}\frac{a_P}{A^{1/2}}$. It is found experimentally that two protons or two neutrons bind more strongly than one proton and one neutron.

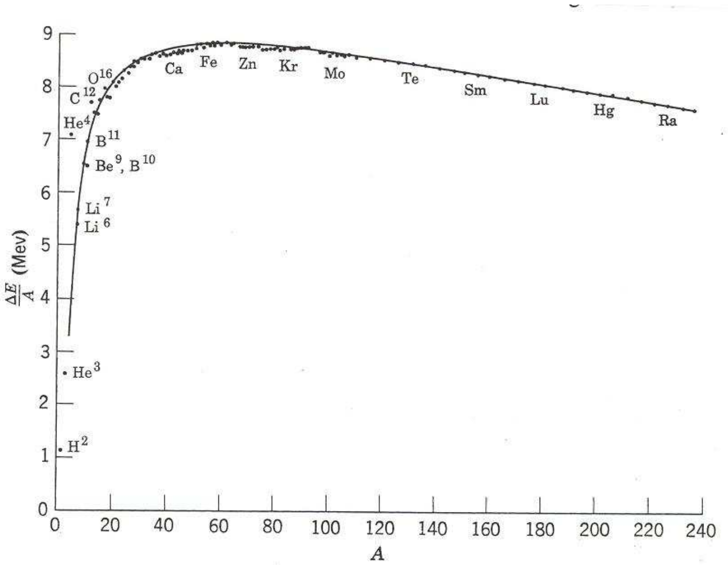
The complete formula is

$$B(A,Z) \;=\; a_V\, A \,-a_S\, A^{2/3} \,-a_C\frac{Z^2}{A^{1/3}}$$

$$-a_A\frac{(Z-N)^2}{A} + \frac{\left((-1)^Z+(-1)^N\right)}{2}\frac{a_P}{A^{1/2}}$$

where the values of fitted parameters a_V, a_S, a_C, a_A, a_P are

$$\begin{aligned} a_V &= 15.56 \text{ MeV} \\ a_S &= 17.23 \text{ MeV} \\ a_C &= 0.697 \text{ MeV} \\ a_A &= 23.285 \text{ MeV} \\ a_P &= 12.0 \text{ MeV} \end{aligned}$$



1.6 Nuclear Shell Model

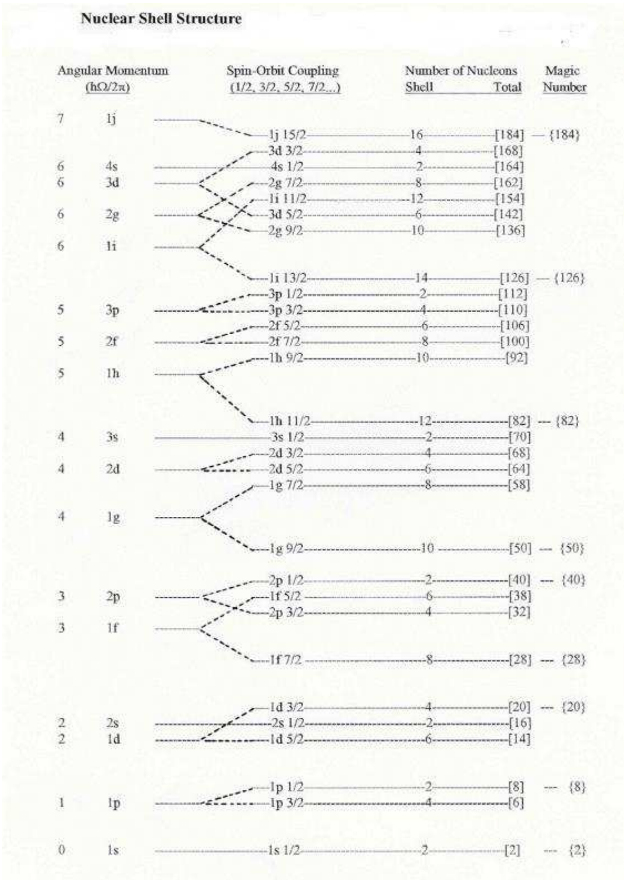
The binding energies predicted by the Liquid Drop Model underestimate the actual binding energies of "magic nuclei" for which either the number of neutrons $N = (A - Z)$ or the number of protons, Z is equal to one of the following "magic numbers"

$$2,\; 8,\; 20,\; 28,\; 50,\; 82,\; 126.$$

This is particularly the case for "doubly magic" nuclei in which *both* the number of neutrons and the number of protons are equal to magic numbers.

Several more features of the magic nuclei: there are more stable isotopes if Z is a magic number, and more stable isotones if N is a magic number and elements with Z equal to a magic number have a larger natural abundance than those of nearby elements; if N is magic number then the cross-section for neutron absorption is much lower than for other nuclides; the energies of the excited states are much higher than the ground state if either N or Z or both are magic numbers. These magic numbers can be explained in terms of the Shell Model of the nucleus, which considers each nucleon to be moving in some potential and classifies the energy levels in terms of quantum numbers $n\,l\,j$, in the same way as the wavefunctions of individual electrons are classified in Atomic Physics.
Spin-orbit coupling plays an important role for the shells distribution. In the case of nuclear binding the effect is about 20 times larger then in the case of electromagnetic spin-orbit coupling of the atomic physics. It comes from a term in the nuclear potential itself which is proportional to $\mathbf{L} \cdot \mathbf{S}$, i.e.

$$V(r) \ \rightarrow \ V(r) \,+\, W(r)\mathbf{L} \cdot \mathbf{S}.$$



As in the case of Atomic Physics (j-j coupling scheme) the orbital and spin angular momenta of the nucleons combine to give a total angular momentum j which can take the values $j = l + \frac{1}{2}$ or $j = l - \frac{1}{2}$. The spin-orbit coupling term leads to an energy shift proportional to

$$j(j+1) - l(l+1) - s(s+1), \quad (s = 1/2).$$

A further feature of this spin-orbit coupling in nuclei is that the energy split is in the opposite sense from its effect in Atomic Physics, namely that states with higher j have *lower energy.*

The large spin-orbit effect leads to crossing over of energy levels into different shells. For example, the state above the $2p$ state is $1g$ (l=4), which splits into $1g_{\frac{9}{2}}$, $(j = \frac{9}{2})$ and $1g_{\frac{7}{2}}$, $(j = \frac{7}{2})$. The energy of the $1g_{\frac{9}{2}}$ state is sufficiently low that it joins the shell below, so that this fourth shell now consists of $1f_{\frac{7}{2}}, 2p_{\frac{3}{2}}$, $1f_{\frac{5}{2}}$, $2p_{\frac{1}{2}}$ and $1g_{\frac{9}{2}}$. The maximum occupancy of this state ($(2j{+}1)$ protons (neutrons) for each j) is now 8+4+6+2+10=30, which added to the previous magic number, 20, gives the next observed magic number of 50.

1.6.1 Spin and Parity of Nuclear Ground States.

Nuclear states have an intrinsic spin and a well defined parity, $\eta = \pm 1$, defined by the behaviour of the wavefunction for all the nucleons under reversal of their coordinates with the centre of the nucleus at the origin.

$$\Psi(-\mathbf{r_1}, -\mathbf{r_2} \cdots - \mathbf{r_A}) \ = \ \eta \Psi(\mathbf{r_1}, \mathbf{r_2} \cdots \mathbf{r_A})$$

.

The spin and parity of nuclear ground states can usually be determined from the shell model. Protons and neutrons tend to pair up so that the spin of each pair is zero and each pair has even parity ($\eta = 1$). Thus we have: **Even-even** nuclides (both Z and A even) have zero intrinsic spin and even parity; **Odd A** nuclei have one unpaired nucleon, and the he spin of the nucleus is equal to the j-value of that unpaired nucleon and the parity is $(-1)^l$ (where l is the orbital angular momentum of the unpaired nucleon); **Odd-odd** nuclei, where is an unpaired proton whose total angular momentum is j_1 and an unpaired neutron whose total angular momentum is j_2 and the total spin of the nucleus is the (vector) sum of these angular momenta and can take values between $|j_1 - j_2|$ and $|j_1 + j_2|$ (in unit steps).

Example: ${}^{47}_{22}\mathrm{Ti}$ (titanium) has an even number of protons and 25 neutrons. 20 of the neutrons fill the shells up to magic number 20 and there are 5 in the $1f_{\frac{7}{2}}$ state ($l = 3,\ j = \frac{7}{2}$) Four of these form pairs and the remaining one leads to a nuclear spin of $\frac{7}{2}$ and parity $(-1)^3 = -1$.

1.7 The Collective Model

The Shell Model has its shortcomings. This is particularly true for heavier nuclei. The Shell Model does not predict magnetic dipole moments or the spectra of excited states very well as well as it does not predict well electric quadrupole moments.

The failure of the Shell Model to correctly predict electric quadrupole moments arises from the assumption that the nucleons move in a spherically symmetric potential. The Collective Model generalises the result of the Shell Model by considering the effect of a non-spherically symmetric potential, which leads to substantial deformations for large nuclei and consequently large electric quadrupole moments. The Collective Model explains the low-lying excited states of heavy nuclei with rotational and vibrational excitations. For example, an even-even nucleus whose ground state has zero spin, rotationally excited states have energies

$$E_{\mathrm{rot}} = \frac{I(I+1)\,\hbar^2}{2\mathcal{I}}$$

where $\mathcal{I}$ is the moment of inertia of the nucleus about an axis through the centre perpendicular to the axis of symmetry. It turns out that the rotational energy levels of an even-even nucleus can only take even values of I.

For odd-A nuclides for which the spin of the ground state I_0 is non-zero, the rotational levels have excitation levels of

$$E_{\mathrm{rot}} = \frac{1}{2\mathcal{I}}\left(I(I+1) - I_0(I_0+1)\right)\hbar^2,$$

where I can take the values $I_0 + 1,\ I_0 + 2$ etc.

2 Radioactivity and nuclear reactions

2.1 Radioactivity

Some nuclides have a far higher binding energy than some of its neighbours. When this is the case it is often energetically favourable for a nuclide with a low binding energy ("parent nucleus") to decay into one with a higher binding energy ("daughter nucleus"), giving off either an α-particle, which is the a ${}^4_2$He (helium) nucleus (α-decay) or an electron (positron) and another very low mass particle called a "antineutrino" ("neutrino"). This is called "β-decay". One more source of radioactivity arises when a nucleus in a metastable excited state ("isomer") decays directly or indirectly to its ground state emitting one or more high energy photons (γ-rays).

The difference in the binding energies is equal to the kinetic energy of the decay products.

2.2 Decay Rates

The probability of a parent nucleus decaying in one second is called the "decay constant", (or "decay rate") λ. If we have $N(t)$ nuclei then the number of 'expected' decays per second is $\lambda\, N(t)$. The number of parent nuclei decreases by this amount and so we have

$$\frac{dN(t)}{dt} = -\lambda N(t). \tag{3}$$

This differential equation has a simple solution - the number of parent nuclei decays exponentially -

$$N(t) = N_0 e^{-\lambda t},$$

where N_0 is the initial number of parent nuclei at time $t = 0$.

The time taken for the number of parent nuclei to fall to $1/e$ of its initial value is called the "mean lifetime", τ of the radioactive nucleus, and we can see from eq.(3) that

$$\tau = \frac{1}{\lambda}.$$

Quite often one talks about the "half-life", $\tau_{\frac{1}{2}}$ of a radioactive nucleus, which is the time taken for the number of parent nuclei to fall to one-half of its initial value. From eq.(3) we can also see that

$$\tau_{\frac{1}{2}} = \frac{\ln 2}{\lambda} = \ln 2\, \tau.$$

It was stated above that the "expected" number of decays per second would be $\lambda N(t)$. This does not mean that there will always be precisely this number of decays per second.

Radioactive decay is a random process with a *probability* λ that any one nucleus will decay in one second.

The laws of random distributions tell us that if the expected number of events in a given period of time is ΔN, then the 'error' on this number is $\sqrt{\Delta N}$. More precisely there is a 68% probability that the number of events will be in the range

$$\Delta N - \sqrt{\Delta N} \to \Delta N + \sqrt{\Delta N}.$$

This means that if we want to measure the decay constant (lifetime, half-life) to within an accuracy of ϵ, we need to collect at least $1/\epsilon^2$ decays.

2.3 Alpha Decay

α- decay is the radioactive emission of an α-particle which is the nucleus of ${}^4_2$He, consisting of two protons and two neutrons. This is a very stable nucleus as it is doubly magic. The daughter nucleus has two protons and four nucleons fewer than the parent nucleus.

$$ {}^{(A+4)}_{(Z+2)}\{P\} \rightarrow {}^{A}_{Z}\{D\} + \alpha. $$

The "Q-value" of the decay, Q_α is the difference of the mass of the parent and the combined mass of the daughter and the α-particle, multiplied by c^2.

$$ Q_\alpha = (m_P - m_D - m_\alpha)\, c^2. $$

It is equal to the sum of the kinetic energies of the α particle and the daughter: $Q_\alpha = T_\alpha + T_D$. From the momentum conservation $p_\alpha + p_D = 0$ we derive $T_D = \frac{m_\alpha}{m_D} T_\alpha$ and (neglecting the binding energies), $T_\alpha = \frac{A}{(A+4)} Q_\alpha$.

The α-decay mechanism is the quantum tunnelling through the potential which is the superposition of the strong well potential and the Coulomb potential. A range of lifetimes is from e^{-15} to e^{45} secs.

2.4 Beta Decay

β-decay is the radioactive decay of a nuclide in which an electron or a positron is emitted.

$$ {}^{A}_{Z}\{P\} \rightarrow {}^{A}_{(Z+1)}\{D\} + e^- + \bar{\nu}, \quad \text{or} \quad {}^{A}_{Z}\{P\} \rightarrow {}^{A}_{(Z-1)}\{D\} + e^+ + \nu. $$

The atomic mass number is unchanged so that these reactions occur between "isobars".

The electron (or positron) does not exist inside the nucleus but is created in the reaction

$$ n \rightarrow p + e^- + \bar{\nu}. $$

As in the case of α-decay the difference between the mass of the parent nucleus, m_P and the mass of the daughter, m_D plus the electron is the Q-value for the decay, Q_β,

$$ Q_\beta = (m_P - m_D - m_e) c^2, $$

and in this case the recoil of the daughter can be neglected because the electron is so much lighter than the nuclei.

We would expect this Q-value to be equal to the kinetic energy of the emitted electron, but what is observed is a spectrum of electron energies up to a maximum value which is equal to this Q-value. There is a further puzzle. Since the number of spin-$\frac{1}{2}$ nucleons is the same in the parent and daughter nuclei, the difference in the spins of the parent and daughter nuclei must be an integer. The solution to both of these puzzles was provided in 1930 by Pauli who postulated the existence of a massless neutral particle with spin-$\frac{1}{2}$ which always accompanies the electron in β-decay. This was called a neutrino.

β-decay exhibits a further peculiarity. It violates parity. his was discovered in 1957 by C.S. Wu who observed the decay of radioactive cobalt

$$ {}^{60}_{27}\mathrm{Co} \;\rightarrow\; {}^{60}_{28}\mathrm{Ni} + e^- + \bar{\nu}. $$

Under parity conservation one would expect $\langle \mathbf{s}\cdot\mathbf{p_e}\rangle = 0$ since $\mathbf{s}\cdot\mathbf{p_e} \;\rightarrow\; -\mathbf{s}\cdot\mathbf{p_e}$ under the space reflection, but experiment has clearly demonstrated that $\langle \mathbf{s}\cdot\mathbf{p_e}\rangle \neq 0$

2.5 Gamma Decay

As we have seen γ-decay is often observed in conjunction with α- or β-decay when the daughter nucleus is formed in an excited state and then makes one or more transitions to its ground state, emitting a photon whose energy is equal to the energy difference between the initial and final nuclear state. These energy differences are usually of order 100 KeV so the photon is well in the γ-ray region of the electromagnetic spectrum.

The lifetime of excited nuclear states is usually of the order of $10^{-13} - 10^{-12}$ s., so the lifetime is far too short to be measured.

As in the case of Atomic Physics the transition amplitude is proportional to the matrix element of the electric field between the initial and final wavefunctions of the nucleon that makes the transition. This electric field has a space dependence that may be written $E = E_0 e^{i\mathbf{p}\cdot\mathbf{r}/\hbar}$, where $\mathbf{p}$ is the momentum of the emitted photon. The transition amplitude is therefore proportional to

$$ A \;\propto\; \int \Psi_f^*(\mathbf{r})\mathbf{k}\cdot\mathbf{r}\Psi_i(\mathbf{r})d^3\mathbf{r}, $$

where Ψ_i and Ψ_f are the initial and final wavefunctions of the proton that makes the transition. This is called "electric dipole" transition. We can estimate

$$ \mathbf{p}\cdot\mathbf{r}/\hbar = \frac{10^5 eV\cdot 1fm}{1.973\cdot 10^8 eV\cdot 1fm} \ll 1 $$

so the each power of $\mathbf{p}\cdot\mathbf{r}/\hbar$ in the amplitude is greatly suppressed. Furthermore since $\mathbf{r}$ is odd under parity reversal, we require the initial and final sates to be of opposite parity, which means that the orbital angular momentum changes by one unit. If the parity of the initial and final states are the same then the transition is still allowed, then the photon carries away the angular momentum by flipping the spin of the nucleon that makes the transition. This is called a "magnetic dipole transition".

The total angular momentum change, L in a nuclear transition can take the values $|I_i - I_f| \leq L \leq |I_i + I_f|$,. This is also subject to selection rules for the parity difference between initial and final states, namely $\Delta P = (-1)^L$, for electric transitions (written E{L}) with angular momentum L, and and $\Delta P = (-1)^{L-1}$, for the (even further suppressed) magnetic transitions. Here are some examples: $2^+ \rightarrow 1^-$(E1), $2^+ \rightarrow 1^+$(M1), $3^+ \rightarrow 1^-$(M2), $3^+ \rightarrow 1^+$(E2).

The Mössbauer Effect

In case of gamma decay in nuclear physics the nuclear recoil takes place. The recoil energy is $T = \frac{E_\gamma^2}{2M_N c^2}$, from momentum conservation. The de-excitation energy E_0 is the sum of the photon energy plus this kinetic energy

$$E_0 \;=\; E_\gamma + \frac{E_\gamma^2}{2M_N c^2}, \quad \text{and} \quad E_\gamma \;\simeq\; E_0 \left(1 - \frac{E_0}{2M_N c^2}\right)$$

For a photon of energy 100 KeV and a nucleus with A=100, this recoil energy is about 0.05 eV. Furthermore if we now use the emitted photon to bombard a similar nuclide with the hope of exciting it, we find that the target nucleus also recoils so that the energy that it can absorb in its own rest frame, E_0' is given by

$$E_0' \;=\; E_\gamma \left(1 - \frac{E_0}{2M_N c^2}\right) \approx E_0 \left(1 - \frac{E_0}{M_N c^2}\right),$$

so that E_0' falls short of E_0 by about 0.1 eV for $A = 100$ and $E_0 = 100KeV$. On the other hand, even for fast decaying excited states with lifetimes, τ, of about 10^{-12} s., the line-width is given by

$$\Gamma \;=\; \frac{\hbar}{\tau} \approx 10^{-3} \text{ eV},$$

The way out of this was discovered by Mössbauer. If the source and target nuclei are both fixed in a crystal lattice then the recoil momentum can be taken up by the entire crystal (who mass is many orders of magnitude larger than that of the nucleus) and the recoil energy is negligible.

This is called the Mössbauer effect and it provides an extremely accurate method for measuring the widths of nuclear transitions using Doppler effect: $\frac{\Delta\lambda}{\lambda} \;=\; \frac{v}{c} = \frac{\Delta E}{E}$ one finds $\Gamma \;=\; 2E\frac{v_{1/2}}{c}$, where $v_{1/2}$ is the velocity for which the absorption falls to one-half of its peak value.

2.6 Nuclear Fission

If we look again at the binding energies (per nucleon) for different nuclei, we note that the heavier nuclei have a smaller binding energy than those in the middle of the Periodic Table.

This means that it is energetically favourable for a heavy nucleus (with A greater than about 100) to split into two fragments of smaller nuclei, thereby releasing energy which goes into the kinetic energy of the fragments. This process is called "nuclear fission". An example of nuclear fission is

$${}^{238}_{92}\mathrm{U} \;\rightarrow\; {}^{145}_{57}\mathrm{La} + {}^{90}_{35}\mathrm{Br} + 3n$$

The neutrons which are emitted in a fission reaction can be absorbed by another parent nucleus which then itself undergoes induced fission. This is the principle of the "chain reaction".

Let k be the number of neutrons produced in a sample of fissile material at stage n of this chain divided by the number of neutrons produced at stage $n-1$. This number will depend on how many of the neutrons produced at stage $n-1$ are absorbed by a nucleus that can undergo induced fission.

- If $k < 1$ the the chain reaction will simply fizzle out and the process will halt very quickly.
- If $k > 1$, then the chain reaction will grow until all the fissile material is used up (atomic bomb).
- If $k = 1$ then we have a controlled reaction. This is needed in a nuclear reactor. The absorption is controlled by interspersing the uranium with cadmium or boron rods that absorb most of the neutrons .

2.7 Nuclear Fusion

If we look again at the binding energies (per nucleon) for different nuclei, we note also that the lightest nuclei have a much smaller binding energy per nucleon than those in the middle of the Periodic Table.

Much more energy per nucleon can be released by fusion of two of these light nuclei to form a heavier nucleus, than in the case of fission.

The example is the fusion of a deuteron and a hydrogen nucleus into helium

$$ {}^2_1\mathrm{H} + {}^1_1\mathrm{H} \rightarrow {}^3_2\mathrm{He} + \gamma $$

which is the part of the fusion cycle at the Sun.

$$
\begin{aligned}
{}^1_1\mathrm{H} + {}^1_1\mathrm{H} &\rightarrow {}^2_1\mathrm{H} + e^+ + \nu \quad (\times 2) \\
{}^1_1\mathrm{H} + {}^2_1\mathrm{H} &\rightarrow {}^3_2\mathrm{He} + \gamma \quad (\times 2) \\
{}^3_2\mathrm{He} + {}^3_2\mathrm{He} &\rightarrow {}^4_2\mathrm{He} + {}^1_1\mathrm{H} + {}^1_1\mathrm{H}
\end{aligned}
$$

The total energy released by this process is 25.3 MeV.

3 Charge Independence and Isospin

If we look at mirror nuclei (two nuclides related by interchanging the number of protons and the number of neutrons) we find that their binding energies are almost the same.

We can express this is a more formal (mathematical), but useful way by introducing the concept of "Isospin".

If we have two electrons with z- component of their spin set to $s_z = +\frac{1}{2}$ and $s_z = -\frac{1}{2}$ (in units of $\hbar$) then we can distinguish them by applying a (non-uniform) magnetic field in the z-direction - the electrons will move in opposite directions. But in the absence of this external field these two cannot be distinguished and we are used to thinking of these as two states of the same particle.

Similarly, if we could 'switch off' electromagnetic interactions we would not be able to distinguish between a proton and a neutron. As far as the strong interactions are concerned these are just two states of the same particle (a nucleon). We therefore think of an imagined space (called an 'internal space') in which the nucleon has a property called "isospin", which is mathematically analogous to spin. The proton and neutron are now considered to be a nucleon with different values of the third component of this Since this third component can take two possible values, we assign $I_3 = +\frac{1}{2}$ for the proton and $I_3 = -\frac{1}{2}$ for the neutron. The nucleon therefore has isospin $I = \frac{1}{2}$, in the same way that the electron has spin $s = \frac{1}{2}$, with two possible values of the third component.

In the case of two nucleons we also have a total isospin part of the wavefunction, so the complete wavefunction is

$$ \Psi_{12} = \Psi(\mathbf{r}_1, \mathbf{r}_2)\chi_S(s_1, s_2)\chi_I(I_1, I_2), $$

where $\chi_I(I_1, I_2)$ is the isospin part of the wavefunction. For total isospin $I = 1$ we have

$$ \chi_I(I_1, I_2) = (p\,p)\,, \quad I_3 = +1 $$

$$\chi_I(I_1,I_2) \;=\; \frac{1}{\sqrt{2}}\,(p\,n\,+\,n\,p)\,,\quad I_3=0$$
$$\chi_I(I_1,I_2) \;=\; (n,n)\,,\quad I_3=-1,$$

which is symmetric under the interchange of the isospins of the two nucleons, so that (as in the case of two electrons) it must be accompanied by a combined spatial and spin wavefunction that must be antisymmetric under simultaneous interchange of the two positions *and* the two spins. But we also have the $I=0$ state

$$\chi_I(I_1,I_2) \;=\; \frac{1}{\sqrt{2}}\,(p\,n\,-\,n\,p)\,,$$

which is antisymmetric under the interchange of the two isospins and therefore when the nucleons are combined in this isospin state they must be accompanied by a combined spatial and spin wavefunction which is *symmetric* under simultaneous interchange of the two positions *and* the two spins.

4 Accelerators and particle kinematics

Particles physics, also known as ‘high energy physics’ is the study of the fundamental forces of nature and the particles that can be found at very high energies.

In order to resolve small distances we need a high energy (momentum) according to Heisenberg’s uncertainty principle

$$\Delta p \;\gg \frac{\hbar}{1\,\mathrm{fm}} \;=\; 197\,\mathrm{MeV/c}.$$

We also need accelerators to produce new massive particles predicted by theories beyond the Standard Model. The Large Hadron Collider (LHC) at CERN will aim for this resuming its run in September 2009 with energy 10 TeV for the first year and then up to 14 TeV later on.

4.1 Fixed Target Experiments vs. Colliding Beams

Let us construct the quantity

$$s \;=\; \left(\sum_{i=1,2} E_i\right)^2 \;-\; \left(\sum_{i=1,2} \mathbf{p}_i\right)^2 c^2$$

In the centre-of-mass frame, where the momenta are equal and opposite the second term vanishes and we have

$$s \;=\; 4E_{CM}^2,$$

i.e. s is the square of the total incoming energy in the centre of mass frame - this is a quantity that is often used in particle physics and the notation s is always used. For one particle we know that $E^2-p^2c^2$ is equal to m^2c^4 and is therefore the same in any frame of reference even though the quantities E and $\mathbf{p}$ will be different in the two frames. Likewise the above quantity, s, is the same in any frame of reference (we say that ‘it invariant under Lorentz transformations.’)

In the frame in which the target particle is at rest, its energy is mc^2 and its momentum is zero, whereas the projectile has energy E_{LAB} and momentum $\mathbf{p}_{LAB}$ so that we have

$$s \;=\; \left(E_{LAB}+mc^2\right)^2 - \mathbf{p}_{LAB}^2c^2 \;=\; E_{LAB}^2+m^2c^4+2mc^2E_{LAB}-\mathbf{p}_{LAB}^2c^2 \;=\; 2m^2c^4+2mc^2E_{LAB},$$

where in the last step we have used the relativity relation

$$E_{LAB}^2 - \mathbf{p}_{LAB}^2 c^2 = m^2 c^4.$$

Equating the two expressions for s (and taking a square root we obtain the relation

$$\sqrt{s} = 2E_{CM} = \sqrt{2m^2c^4 + 2mc^2 E_{LAB}}.$$

For collider experiment we have collider energy $\sqrt{s} = 2E_{CM}$ while for the fixed target experiment $\sqrt{s} \simeq \sqrt{2mc^2 E_{LAB}}$. For example, taking the proton mass be be approximately 1 GeV/c^2, the if we have an accelerator that can accelerate protons up to an energy of 100 GeV, the total centre-of-mass energy achieved is only about 15 GeV - far less than the energy required to produce a particle of mass 100 GeV/c^2.

4.2 Luminosity

The luminosity $\mathcal{L}$ is the number of particle collisions per unit area (usually quoted in cm^2) per second. The number of events of a particular type which occur per second is the cross-section multiplied by the luminosity. In the example of two W-boson production at LEP the cross-section, $\sigma_{(e^+e^- \to W^+W^-)}$ is 15 pb (p=pico means 10^{-12}) and the luminosity of LEP was 10^{32} per cm^2 per second. The number of these pairs of W-bosons produced per second is given by

$$\frac{dN_{W^+W^-}}{dt} = \sigma_{(e^+e^- \to W^+W^-)} \times \mathcal{L} = (15 \times 10^{-12} \times 10^{-28}) \times (10^{32} \times 10^4) = 1.5 \times 10^{-3},$$

where the first term in parenthesis is the cross-section converted to m^2 and the second is the luminosity converted to m^{-2} sec^{-1}.

One can also define integrated luminosity L as the number of particle collisions per unit area which is $\mathcal{L}$ integrated over the time. The the total number of expected events, N, is

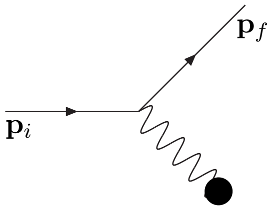
$$N = L \times \sigma.$$

5 Fundamental Interactions (Forces) of Nature

Interaction	**Gauge Boson** (Force carrier)	**Gauge Boson Mass**	**Interaction Range**
Strong	Gluon	0	short-range (a few fm)
Weak	$W^\pm$, Z	$M_W = 80.4$ GeV/c^2 $M_Z = 91.2$ GeV/c^2	short-range ($\sim 10^{-3}$ fm)
Electromagnetic	Photon	0	long-range
Gravity	Graviton	0	long-range

5.1 Photon Propagator

Potential of a particle of charge e located at $\mathbf{r}$, due to another charge e' (fixed) at the origin is



$$V(r) = \frac{ee'}{4\pi\epsilon_0 r}$$

If this charge has initial momentum $\mathbf{p}_i$ and final momentum $\mathbf{p}_f$, its initial and final (time-independent) wavefunctions are given by

$$\Psi_i \propto e^{i\mathbf{p_i}\cdot\mathbf{r}/\hbar}, \quad \Psi_f \propto e^{i\mathbf{p_f}\cdot\mathbf{r}/\hbar}$$

The amplitude for such a transition is

$$\mathcal{A} = \int \Psi_f^* V(r)\Psi_i d^3\mathbf{r} \propto ee' \int e^{i(\mathbf{p_i}-\mathbf{p_f})\cdot\mathbf{r}/\hbar}\frac{1}{r}d^3\mathbf{r}.$$

Performing the integral (this is a Fourier transform) we get

$$\mathcal{A} \propto \frac{ee'}{-|\mathbf{q}|^2},$$

where $\mathbf{q} = \mathbf{p}_f - \mathbf{p}_i$, is the momentum transferred from the scattered charged particle to the charge at the origin.

For the scattering of a relativistic particle, this expression is modified to

$$\mathcal{A} \propto \frac{ee'}{(q_0^2 - |\mathbf{q}|^2)},$$

where $q_0 = (E_f - E_i)/c$, with E_i and E_f being the initial and final energy of the scattered particle.

$$D(q_0, \mathbf{q}) = \frac{1}{q_0^2 - |\mathbf{q}|^2} \tag{4}$$

is the amplitude for the propagation of a photon whose energy is $c\, q_0$ and whose momentum is $\mathbf{q}$. This is known as a "propagator".

5.2 Virtual particles

$$E^2 \neq |\mathbf{p}|^2c^2 + m^2c^4$$

for the short-lived particles. These particles called "virtual particles" and because their energy and momentum do not obey the relativistic energy-momentum relation they are said to be "off mass-shell".

5.3 Feynman Diagrams

Feynman rules we study in this course in the case of electromagnetic interactions are:

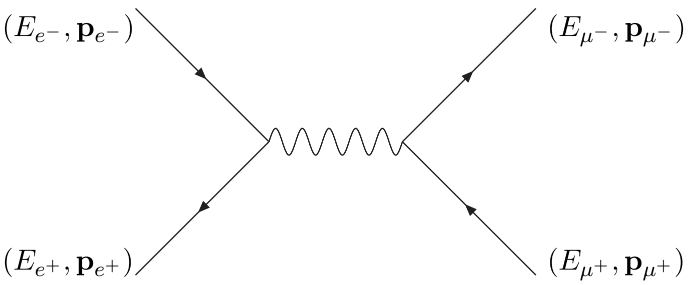
- A factor of the charge at each vertex between a charged particle and a photon.
- Energy and momentum are conserved at each vertex.
- A factor of

$$D(q_0, \mathbf{q}) \;=\; \frac{1}{(q_0^2 - |\mathbf{q}|^2)}$$

for the propagation of an internal gauge boson with energy $c\,q_0$ and momentum $\mathbf{q}$.

For example, the process

$$e^+ + e^- \;\to \mu^+ + \mu^-,$$



Note the convention that the direction of the arrow on the antiparticles, e^+ and μ^+ are drawn against the direction of motion of the particles.

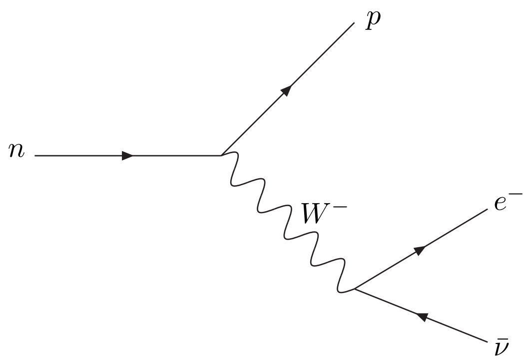
5.4 Weak Interactions

The gauge bosons (interaction carriers) of the weak interactions are $W^\pm$ and Z. the fact that the W-bosons carry electric charge tells us that electric charge can be exchanged in weak interaction processes, which is how we get β-decay.

The Feynman diagram for the process

$$n \;\to\; p + e^- + \bar{\nu}$$

is



The W^- has a mass $M_W = 80.4\,\mathrm{GeV/c^2}$ and for the propagation of a massive particle, the propagator is

$$D^W(q_0, \mathbf{q}) \;=\; \frac{1}{q_0^2 - |\mathbf{q}|^2 - M_W^2 c^2},$$

where again $c\,q_0$ and $\mathbf{q}$ are the energy and momentum difference between the incoming neutron and outgoing proton and are transferred to the electron antineutrino pair. The amplitude for this decay is therefore proportional to

$$\frac{g_W^2}{q_0^2 - |\mathbf{q}|^2 - M_W^2 c^2}$$

.

The potential for which the relation

$$\frac{g_W^2}{-|\mathbf{q}|^2 - M_W^2 c^2} = \int e^{-i\mathbf{p}_p\cdot\mathbf{r}/\hbar} V^{wk}(r) e^{i\mathbf{p}_n\cdot\mathbf{r}/\hbar} d^3\mathbf{r}$$

is obeyed is

$$V^{wk}(r) = \frac{g_W^2}{r}\exp(-M_W c r/\hbar)$$

. The effective force therefore has a range R, where

$$R \sim \frac{\hbar}{M_W c}.$$

At distances much larger than this the potential is rapidly suppressed. This is an extremely short range (about 10^{-3} fm.)

In the case of β-decay the momentum transferred (a few MeV/c) is very small compared with $M_W c$ which is 80.4 GeV/c so we can also neglect the momentum term and approximate the amplitude by

$$\frac{-g_W^2}{M_W^2 c^2}$$

Because of the very large mass of the W-boson this is extremely small and it is the reason that weak interactions are actually so weak.

6 Classification of matter Particles

Particle type		**Strong interaction**	**Weak interaction**	**Electromagnetic interaction**	**Spin**
Leptons		No	Yes	Some	$\frac{1}{2}$
	Mesons	Yes	Yes	Yes	integer
Hadrons	Baryons	Yes	Yes	Yes	half-integer

6.1 Leptons

Nature gives us three copies of each "family" or "generation" of particles. There are, therefore, two particles with similar properties to the electron (electric charge $-e$, spin-$\frac{1}{2}$, weakly interacting but not strongly interacting). These are called the muon (μ) and the tau (τ). Each of these has its own neutrino, ν_μ and ν_τ respectively.

Thus the six leptons are

Leptons			Electric Charge
ν_e	ν_μ	ν_τ	0
e	μ	τ	-1

6.2 Hadrons

These are particles that partake in the strong interactions.

Hadrons with integer spins (bosons) are called "mesons", whereas hadrons with half (odd-)integer spins (fermions) are called "baryons".

Mesons are bound states of a quark and an anti-quark and can therefore have integer spin. Baryons are bound states of three quarks and can have spin-$\frac{1}{2}$ or spin-$\frac{3}{2}$.

The proton is the only hadron which is absolutely stable (the lifetime is known to be greater than 10^{32} years!). All other hadrons decay eventually into protons, leptons and photons.

Hadrons participate in all interactions since quarks from which they consist participate in all interactions. As for leptons, there are also 3 generations for quarks

Quarks			Electric Charge
u	c	t	+2/3
d	s	b	-1/2

6.3 Short-lived particles - resonances

Particles with a lifetime of less than about 10^{-11} s. do not live long enough to leave a track in a detector. They are observed as "resonances" - peaks in production cross-sections or in decay channels when the centre-of-mass energy of the incident particles in a scattering experiment is equal to the mass of the resonance particle (times c^2), or if the centre-of-mass energy of some subset of the final state particles is to the mass of the resonance particle (times c^2).

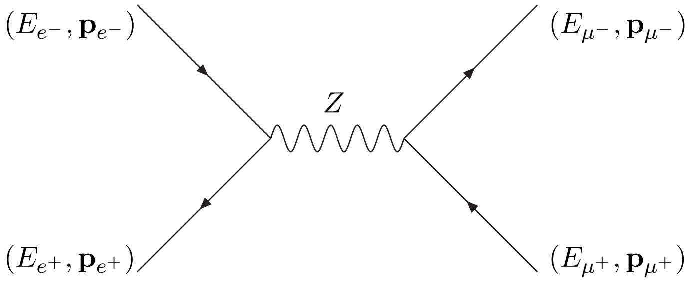
These peaks have a width Γ, corresponding to the uncertainty in their energy due to the fact that they have a short lifetime, τ. According to Heisenberg's uncertainty relation

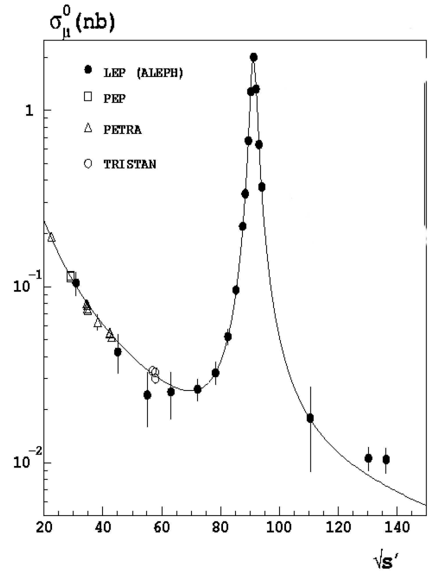
$$\Gamma = \frac{\hbar}{\tau}$$

For example, consider the Z-boson. The Feynman graph for a typical production and decay process

$$e^+ e^- \to Z \to \mu^+ \mu^-$$

is





The amplitude for the exchange of the Z is (up to an overall constant)

$$\frac{1}{(s - M_z^2 c^4)}$$

where $s = (E_{e^-} + E_{e^+})^2 - (\mathbf{p}_{e^-} + \mathbf{p}_{e^+})^2 c^2$, is the square of the centre-of-mass energy of the incident electron-positron pair.

We see that if we tune $\sqrt{s}$ to be exactly equal to $M_Z c^2$ this diverges. The above formula neglects the fact that the particle is unstable and has a width, Γ and the modified amplitude is

$$A \propto \frac{1}{(s - M_z^2 c^4 + i\Gamma M_z c^2)}$$

so that the scattering cross-section is proportional to

$$\sigma \propto \frac{1}{(s - M_z^2 c^4)^2 + \Gamma^2 M_Z^2 c^4}.$$

6.4 Partial Widths and branching ratios

An unstable particle can usually decay into several different possible "channels". The fraction of the decays into a particular channel is called the "branching ratio"

For example the branching ratio for a Z to decay into a $\mu^+ \mu^-$ pair, $B_{Z \to \mu\mu}$ is 3.4 %. The width of the resonance in the process

$$e^+ e^- \to Z \to \mu^+ \mu^-$$

is the "partial width", $\Gamma_{Z \to \mu\mu}$ and in this case it is 0.084 GeV. The total width Γ_{tot} is the sum of all the partial widths. The branching ratio is the ratio of the partial width for a particular channel and the total width

$$B_X = \frac{\Gamma_X}{\Gamma_{tot}}.$$

7 Constituent Quark Model

Quarks are fundamental spin-$\frac{1}{2}$ particles from which all hadrons are made up. Baryons consist of three quarks, whereas mesons consist of a quark and an anti-quark. There are six types of quarks called “flavours”. The electric charges of the quarks take the value $+\frac{2}{3}$ or $-\frac{1}{3}$ (in units of the magnitude of the electron charge).

Symbol	**Flavour**	**Electric charge (e)**	**Isospin**	**I_3**	**Mass Gev/c^2**
u	up	$+\frac{2}{3}$	$\frac{1}{2}$	$+\frac{1}{2}$	≈ 0.33
d	down	$-\frac{1}{3}$	$\frac{1}{2}$	$-\frac{1}{2}$	≈ 0.33
c	charm	$+\frac{2}{3}$	0	0	≈ 1.5
s	strange	$-\frac{1}{3}$	0	0	≈ 0.5
t	top	$+\frac{2}{3}$	0	0	≈ 172
b	bottom	$-\frac{1}{3}$	0	0	≈ 4.5

7.1 Hadrons from u,d quarks and anti-quarks

Baryon	**Quark content**	**Spin**	**Isospin**	**I_3**	**Mass Mev/c^2**
p	uud	$\frac{1}{2}$	$\frac{1}{2}$	$+\frac{1}{2}$	938
n	udd	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	940
Δ^{++}	uuu	$\frac{3}{2}$	$\frac{3}{2}$	$+\frac{3}{2}$	1230
Δ^{+}	uud	$\frac{3}{2}$	$\frac{3}{2}$	$+\frac{1}{2}$	1230
Δ^{0}	udd	$\frac{3}{2}$	$\frac{3}{2}$	$-\frac{1}{2}$	1230
Δ^{-}	ddd	$\frac{3}{2}$	$\frac{3}{2}$	$-\frac{3}{2}$	1230

Meson	**Quark content**	**Spin**	**Isospin**	**I_3**	**Mass Mev/c^2**
π^+	$u\bar{d}$	0	1	+1	140
π^0	$\frac{1}{\sqrt{2}}(u\bar{u}-d\bar{d})$	0	1	0	135
π^-	$d\bar{u}$	0	1	-1	140
ρ^+	$u\bar{d}$	1	1	+1	770
ρ^0	$\frac{1}{\sqrt{2}}(u\bar{u}-d\bar{d})$	1	1	0	770
ρ^-	$d\bar{u}$	1	1	-1	770
ω	$\frac{1}{\sqrt{2}}(u\bar{u}+d\bar{d})$	1	0	0	782

The strong interactions conserve flavour. There a d-quark cannot be converted into an s-quark (or vice verse), even though the electric charge is the same. For example,

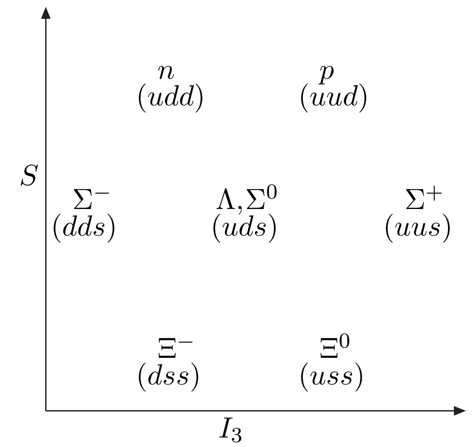
$$\begin{array}{ccccc} \Delta^- & \rightarrow & \pi^- & + & n \\ (ddd) & & (ddu) & & (d\bar{u}) \end{array}$$

7.2 Hadrons with s-quarks (or $\bar{s}$ anti-quarks)

Baryon	Quark content	Spin	Isospin	I$_3$	Mass Mev/c^2
Σ^+	uus	$\frac{1}{2}$	1	$+1$	1189
Σ^0	uds	$\frac{1}{2}$	1	0	1193
Σ^-	dds	$\frac{1}{2}$	1	-1	1189
Ξ^0	uss	$\frac{1}{2}$	$\frac{1}{2}$	$+\frac{1}{2}$	1314
Ξ^-	dss	$\frac{1}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	1321
Λ	uds	$\frac{1}{2}$	0	0	1115
Σ^{*+}	uus	$\frac{3}{2}$	1	$+1$	1385
Σ^{*0}	uds	$\frac{3}{2}$	1	0	1385
Σ^{*-}	dds	$\frac{3}{2}$	1	-1	1385
Ξ^{*0}	uss	$\frac{3}{2}$	$\frac{1}{2}$	$+\frac{1}{2}$	1530
Ξ^{*-}	dss	$\frac{3}{2}$	$\frac{1}{2}$	$-\frac{1}{2}$	1530
Ω^-	sss	$\frac{3}{2}$	0	0	1672

Meson	Quark content	Spin	Isospin	I$_3$	Mass Mev/c^2
K^+	$u\bar{s}$	0	$\frac{1}{2}$	$+\frac{1}{2}$	495
K^0	$d\bar{s}$	0	$\frac{1}{2}$	$-\frac{1}{2}$	495
$\overline{K^0}$	$s\bar{d}$	0	$\frac{1}{2}$	$+\frac{1}{2}$	495
K^-	$s\bar{u}$	0	$\frac{1}{2}$	$-\frac{1}{2}$	495
η	$(u\bar{u},\ d\bar{d},\ s\bar{s})$	0	0	0	547
K^{*+}	$u\bar{s}$	1	$\frac{1}{2}$	$+\frac{1}{2}$	892
K^{*0}	$d\bar{s}$	1	$\frac{1}{2}$	$-\frac{1}{2}$	896
$\overline{K^{*0}}$	$s\bar{d}$	1	$\frac{1}{2}$	$+\frac{1}{2}$	896
K^{*-}	$s\bar{u}$	1	$\frac{1}{2}$	$-\frac{1}{2}$	892
ϕ	$s\bar{s}$	1	0	0	1020

Eightfold Way



Example of the reaction involving s-quarks:

$$\begin{array}{ccccccccc} \pi^- & + & p & \to & \Lambda & + & K^0 \\ (d\bar{u}) & & (duu) & & (dus) & & (\bar{s}d) \end{array}$$

7.3 Quark Colour

There is a difficulty within the quark model when applied to baryons. This can be seen if we look at the Δ^{++} or Δ^- or Ω^-, which are bound states of three quarks of the same flavour. For these low-mass states the orbital angular momentum is zero and so the spatial parts of the wavefunctions for these baryons is symmetric under interchange of the position of two of these (identical flavour) quarks.

On the other hand these baryons have spin-$\frac{3}{2}$ which means that the spin part of the wavefunction is symmetric (for example the $S_z = +\frac{3}{2}$ state is the state in which all three quarks have $s_z = +\frac{1}{2}$ and this is clearly symmetric under the interchange of two spins). But the total wavefunction for a baryon must be antisymmetric since it is a fermion!

This is solved by assuming that quarks come in three possible "colour" states - R, G or B. The antisymmetry of the baryon wavefunction is restored by the assumption that the baryon wavefunction is antisymmetric under the interchange of two colours. If a baryon is composed of three quarks with flavours f_1, f_2 and f_3 the these should also have a colour index, e.g. f_1^R, f_1^G or f_1^B etc. The colour antisymmetric wavefunction is written

$$\frac{1}{\sqrt{6}}\left(|f_1^R f_2^G f_3^B\rangle + |f_1^B f_2^R f_3^G\rangle + |f_1^G f_2^B f_3^R\rangle - |f_1^B f_2^G f_3^R\rangle - |f_1^R f_2^B f_3^G\rangle - |f_1^G f_2^R f_3^B\rangle\right)$$

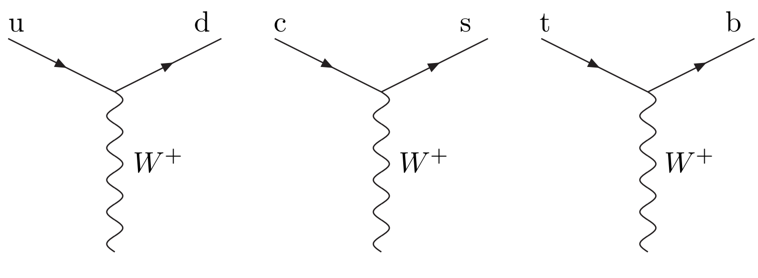
A state of three different colours which is antisymmetric under the interchange of any two of the colours is called a "colour singlet" state - we can think of it as a colourless state. The quarks themselves are a colour triplet - meaning that they can be in any one of three colour states.

For mesons we also require that the quarks and anti-quarks bind in such a way that the meson is a colour singlet. in the case of a quark and ant-quark bound state this means that the wavefunction is a superposition of R with $\bar{R}$, G with $\bar{G}$, and B with $\bar{B}$. Thus, for example, the wavefunction for the π^+ is written

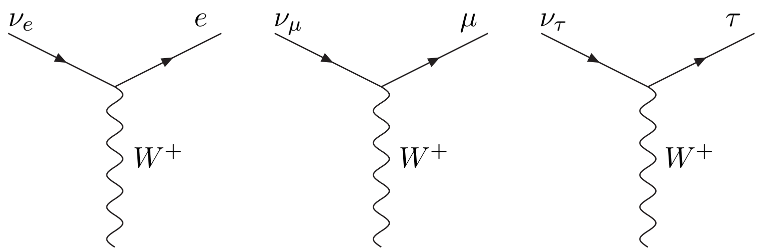
$$|\pi^+\rangle = \frac{1}{\sqrt{3}}\left(|u^R\overline{d^R}\rangle + |u^G\overline{d^G}\rangle + |u^B\overline{d^B}\rangle\right)$$

8 Weak Interactions

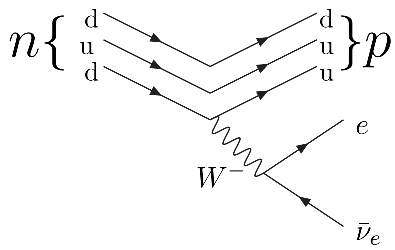
$W^{\pm}$ couples to quark pairs (u,d). (c,s), (t,b) with vertices



as well as to leptons (ν_e, e). (ν_μ, μ), (ν_τ, τ) with vertices

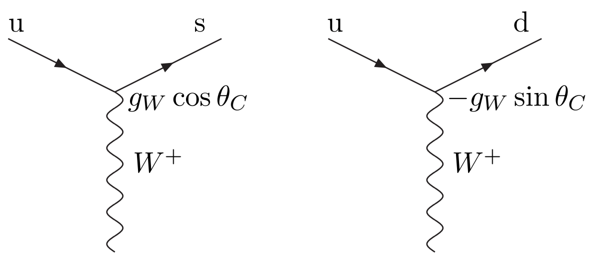


It is this process that is responsible for β-decay. Neutron decays into a proton because a d-quark in the neutron converts into a u-quark emitting a W^- which then decays into an electron and anti-neutrino.



8.1 Cabibbo Theory

Particles containing strange quarks, e.g. $K^{\pm}$, K^0, Λ etc. cannot decay into non-strange hadrons via the strong interactions, which have to conserve flavour, but they can decay via the weak interactions. This is possible because $W^{\pm}$ not only couples a u-quark to a d-quark but can also (with a weaker coupling) couple a u-quark to an s-quark so we have a vertex



We can piece this together in a matrix form as follows ($\sin\theta_c \simeq 0.22$))

$$g_W \begin{pmatrix} d & s \end{pmatrix} \begin{pmatrix} \cos\theta_C & \sin\theta_C \\ -\sin\theta_C & \cos\theta_C \end{pmatrix} \begin{pmatrix} u \\ c \end{pmatrix}$$

This 2×2 matrix is called the "Cabbibo matrix". It is described in terms of a single parameter, the Cabibbo angle. Since we know that there are, in fact, three generations of quarks this matrix is extended to a general 3×3 "CKM" (Cabibbo, Kobayashi, Maskawa) matrix.

8.2 Parity Violation

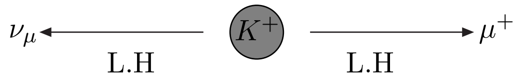
$W^{\pm}$ always couple to left-handed neutrinos. For quarks and massive leptons the $W^{\pm}$ *can* couple to positive helicity (right-handed) states, but the coupling is suppressed by a factor

$$\frac{mc^2}{E},$$

where m is the particle mass and E is its energy. The suppression is much larger for relativistically moving particles.

A striking example of the consequence of this preferred helicity coupling can be seen in the leptonic decay of K^+.

$$K^+ \rightarrow \mu^+ + \nu_\mu$$



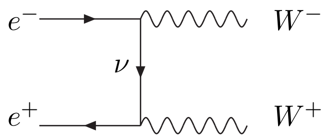
We expect the ratio of the partial widths

$$\frac{\Gamma(K^+ \rightarrow \mu^+ \nu_\mu)}{\Gamma(K^+ \rightarrow e^+ \nu_e)} = \frac{m_\mu^2}{m_e^2} \approx 4 \times 10^4$$

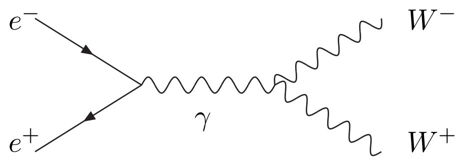
This coincides very closely to the experimentally observed ratio.

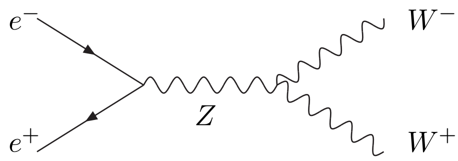
8.3 Z-boson interactions

The coupling of the Z and photon to the $W^{\pm}$ was confirmed at the LEPII experiment at CERN where it was possible to accelerate electrons and positrons to sufficient energies to produce a W^+ and a W^- in the final state. From the coupling of the W to electron and neutrino the Feynman diagram for this process is



but because of the coupling of the Z and photon to $W^{\pm}$ we also have diagrams





8.4 The Higgs mechanism

Higgs boson It arises from the mechanism, discovered by P.Higgs, by which particles acquire their mass. The basic idea is that there exists a field, ϕ called the "Higgs field" which has a constant non-zero value everywhere in space. This constant value is called the "vacuum expectation value", $\langle\phi\rangle$.

In the absence of this field it is assumed that all particles would be massless and would travel with velocity c. But because of their interaction with the background Higgs field they are slowed down - thereby acquiring a mass, M

$$M = \frac{1}{2}\frac{g_H}{\sqrt{\epsilon_0\hbar c}}\langle\phi\rangle,$$

where g_H is the coupling of the particle to the Higgs field (the denominator factor $\sqrt{\epsilon_0\hbar c}$ gives it the correct dimensions.) This mechanism is part of the Standard Model.

The Higgs field couples to $W^{\pm}$ with coupling g_W so that

$$M_W = \frac{1}{2}\frac{g_W}{\sqrt{\epsilon_0\hbar c}}\langle\phi\rangle.$$

Inserting $g_W = e/\sin\theta_W$ with $\cos\theta_W = M_W/M_Z$ and $M_W = 80.4\,\mathrm{GeV/c^2}$, and $M_Z = 91.2\,\mathrm{GeV/c^2}$, we get the value of the vacuum expectation value

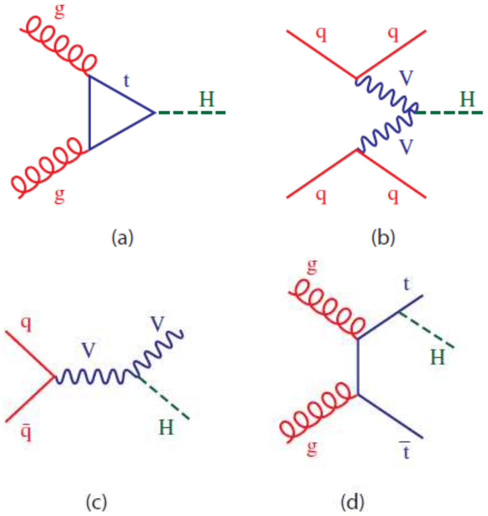
$$\langle\phi\rangle = 250\,\mathrm{GeV/c^2}$$

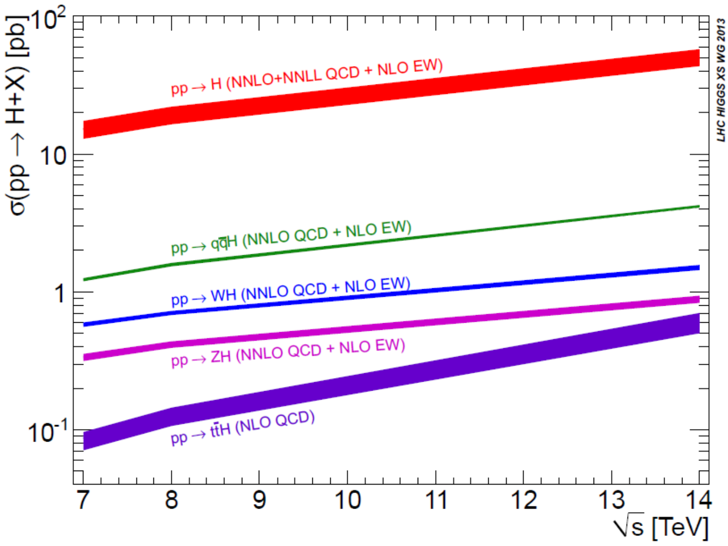
Other particles couple to the Higgs field with couplings that are proportional to their mass.

This was quite a historical event on the 4th of July 2012, when the Higgs boson was discovered by ATLAS and CMS collaborations at the LHC, completing the set of particles of the Standard Model. With a high confidence level this particle is confirmed to have the following properties:

1. It has a spin zero. This is consistent with the theoretical predictions since the vacuum expectation value has to be invariant under Lorentz transformations - so that it is the same in all frames of reference.
2. Higgs boson couples to $W^{\pm}$ and Z (which are consequently massive).
3. It does *not* directly couple to photons (which are massless) so it is uncharged.
4. it does not not couple directly to gluons (which are massless) and so it does not take part in the strong interactions.
5. Its coupling to massive particles is proportional to the particle mass.
6. Its mass is measured to be about 125 GeV/c^2.

Diagrams for production mechanisms of the Higgs boson at the LHC are shown below (left) together with the respective cross sections (right). They include: (a) gluon fusion, (b) weak-boson fusion, (c) Higgs-strahlung (or associated production with a gauge boson) and (d) associated production with top quarks processes. Note, that the first process is the loop-induce one: while Higgs boson does not interact directly with massless gluons, it actually can interact with gluons via virtual massive quarks (e.g. top-quarks which the strongest coupling to the Higgs boson) in the triangle loop diagram. Actually gluon fusion is the main production process of the Higgs boson, while the weak-boson fusion plays the next to leading role. Theoretical uncertainties are represented by the widths of the cross section bands.





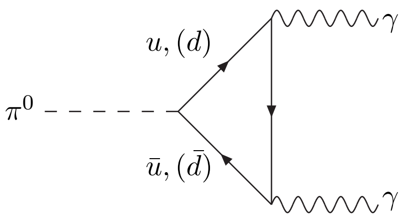
9 Electromagnetic Interactions

9.1 Electromagnetic Decays

There are a few cases of particles which could decay via the strong interactions without violating flavour conservation, but where the masses of the initial and final state particles are such that this decay is not energetically allowed.

$$|\pi^0\rangle \;=\; \frac{1}{\sqrt{2}}\left(|u\bar{u}\rangle \;-\; |d\bar{d}\rangle\right)$$

In either of these states, the quark can annihilate against the antiquark of identical flavour to produce two photons. In terms of Feynman diagrams we have



9.2 Electron-positron Annihilation

Another striking piece of evidence that quarks come in three colours comes from the study of the process

$$e^+ + e^- \;\to\; \text{hadrons}$$

(summed over all possible hadrons in the final state)

The only difference between this process and $e^+ + e^- \rightarrow \mu^+ + \mu^-$ one is the coupling of the final state quarks or final state muons to the photon, i.e. the electric charges of the quarks and the muons.

This means that for a quark of flavour i with electric charge Q_i (in units of e) the ratio of the amplitudes is

$$\frac{\mathcal{A}(e^+e^- \rightarrow q_i\bar{q}_i)}{\mathcal{A}(e^+e^- \rightarrow \mu^+\mu^-)} = Q_i.$$

In order to calculate the ratio of total cross-sections we square the amplitude and sum over all possible final state quarks that can be produced, so that

$$R = \frac{\sigma(e^+e^- \rightarrow \text{hadrons})}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)} = \sum_i Q_i^2.$$

How many quarks we sum over depends on the centre-of-mass energy $\sqrt{s}$. If $\sqrt{s} < 2m_c c^2$, then only u, d and s quarks can be produced in the final state and we have

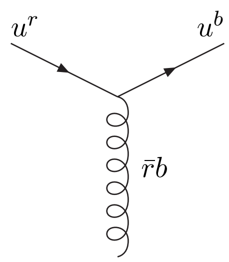
$$R = \frac{\sigma(e^+e^- \rightarrow \text{hadrons})}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)} = 3\left(Q_u^2 + Q_d^2 + Q_s^2\right) = 3\left(\left(\frac{2}{3}\right)^2 + \left(\frac{-1}{3}\right)^2 + \left(\frac{-1}{3}\right)^2\right) = 2.$$

The factor of 3 is needed because we can produce final state quarks in any of the three colours states (in principle these would be distinguishable at the quark level - so we multiply the cross-section by 3 and not the amplitude.)

10 Quantum Chromodynamics (QCD)

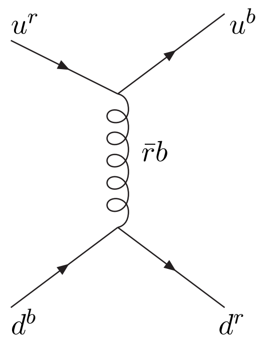
10.1 Gluons and Colour

In the same way that in weak interactions, the weak gauge bosons $W^\pm$ can effect changes of flavour when they interact with quarks, in the case of strong interactions the strong gauge bosons (gluons) can effect changes of colour of the quarks (but conserve flavour). Thus we get interaction vertices of the form



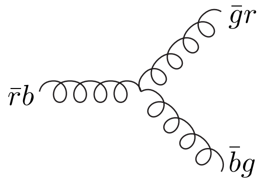
There are 6 such colour changing gluons and in addition two colour neutral gluons (like the Z in weak interactions) which do not change colour, making a total of 8 gluons

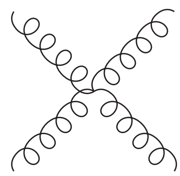
Strong interaction processes consist of quarks exchanging gluons and (usually) changing colour, e.g



This theory of strong interactions, developed in the 1970's is called Quantum Chromodynamics (QCD).

In the same way that the Z can couple to the $W^{\pm}$, so gluons can couple to each other





10.2 Running Coupling

In spite of the strong coupling QCD becomes calculable at sufficiently high energy/momentum scales, Q, the effective strong coupling becomes small.

The reason that this becomes small is 'negative screening'. When an electric charge is probed by another charge, the virtual photon exchanged between them can sometimes create a pair of charged particles (a particle and its antiparticle), which exist for a short while before annihilating each other again.

$$\alpha_s(Q) = \frac{\alpha_s(\mu)}{(1 - \beta_0 \alpha_s(\mu) \ln(Q^2/\mu^2))}$$

where $\alpha_s(\mu)$ is the value of α_s at some reference momentum scale (it serves as the integration constant for the differential equation) and

$$\beta_0 = -\frac{1}{4\pi}\left(11 - \frac{2}{3}n_f\right)$$

Usually this is taken to be $\mu = M_Z c$, since the value of α_s was measured very accurately at LEPI at this scale and its value was found to be

$$\alpha_s(M_Z c) = 0.12,$$

which is not too large.

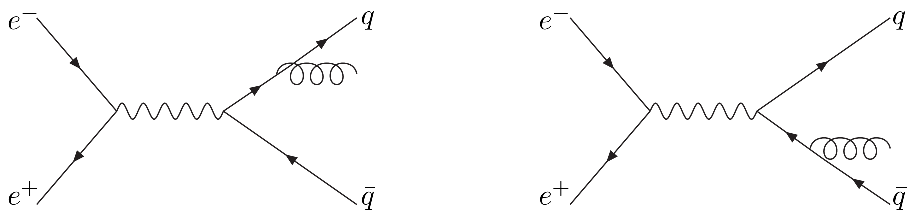
10.3 Quark Confinement

It is not possible to isolate a single quark or gluon. Consider a meson, which is a quark-antiquark state of the opposite colour (e.g. red and anti-red) bound together by a 'string' of gluons. As we try to pull the quark and antiquark apart, the tension in the string increases and eventually the string will 'snap' producing a quark a the end of the part of the string containing the antiquark (of opposite colour) and likewise and antiquark of opposite colour at the end of the part of the string containing the quark. So we end up with two mesons, both of which are colour singlets (colourless), but we do not succeed in isolating a single quark or antiquark.

The only hadron states that we can observe are colourless (colour singlet) states - either mesons which are superpositions of quark-antiquark pairs of opposite colours, or baryons which consist of three quarks but which are antisymmetric under the interchange of any two quark colours. This is known as "quark confinement". Its exact mechanism is not understood, but numerical studies in QCD confirm that this confinement does indeed take place.

10.4 Three Jets in Electron-positron Annihilation

Because quarks interact with gluons one can also have Feynman diagrams



which have a quark, an antiquark, and a gluon in the final state. The gluon also fragments into a hadron jet and so we get three jets of particles.

The first such events we observed at DESY in 1979. Since gluons cannot be isolated and observed directly, this was taken as the first piece of evidence that gluons existed and coupled to quarks.

11 Summary of Conservation laws

- **Baryon number:** baryons=+1, antibaryons=-1, mesons, leptons=0.

- **Lepton number:**

  - electron number: e^-, $\nu_e = 1$, e^+, $\overline{\nu_e} = -1$
  - muon number: μ^-, $\nu_\mu = 1$, μ^+, $\overline{\nu_\mu} = -1$
  - τ number: τ^-, $\nu_\tau = 1$, τ^+, $\overline{\nu_\tau} = -1$

	Strong Interactions	**Electromagnetic Interactions**	**Weak Interactions**
Baryon number	yes	yes	yes
Lepton number (all)	yes	yes	yes
Angular momentum	yes	yes	yes
Isospin	yes	no	no
Flavour	yes	yes	no
Parity	yes	yes	no
Charge conjugation	yes	yes	no
CP	yes	yes	almost