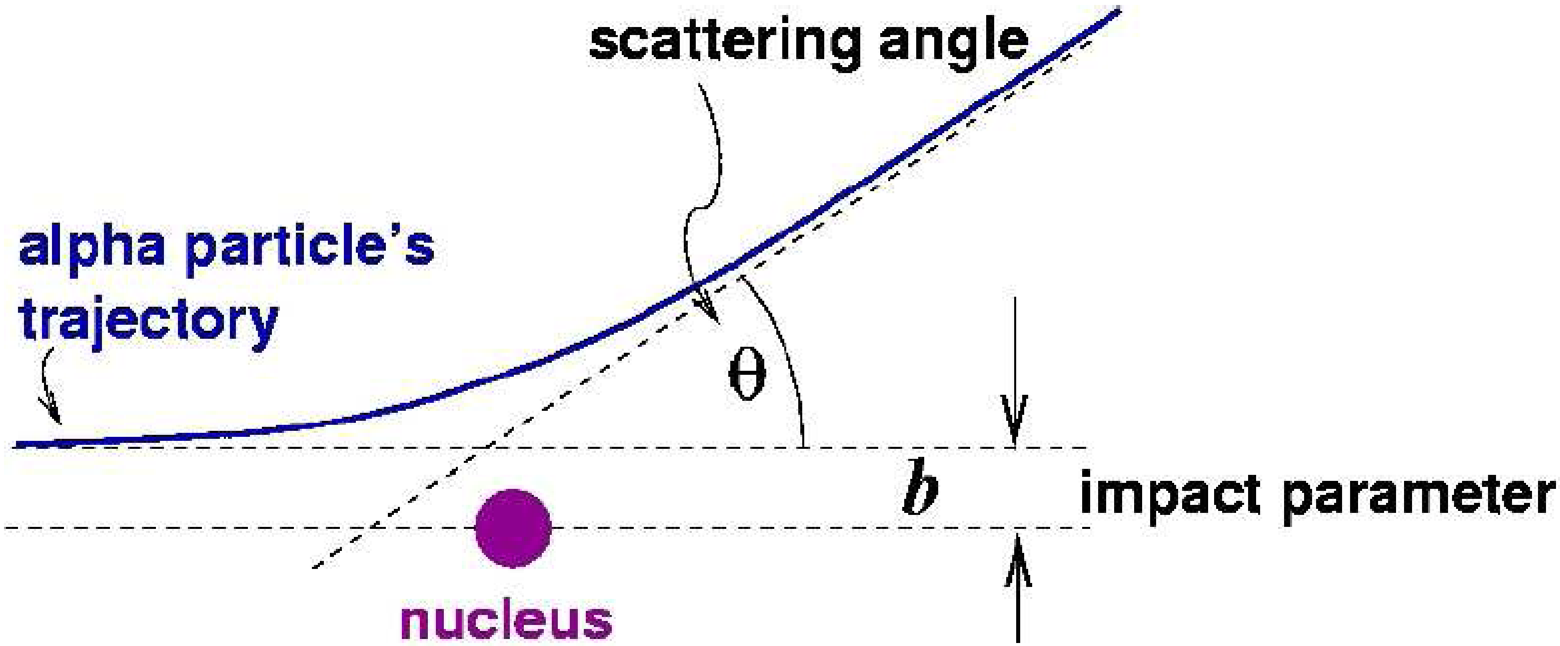
PHYS3002. Nuclei and Particles. Synoptic.

1 Models of Nuclei

2 Radioactivity and nuclear reactions

Rutherford Scattering and cross section

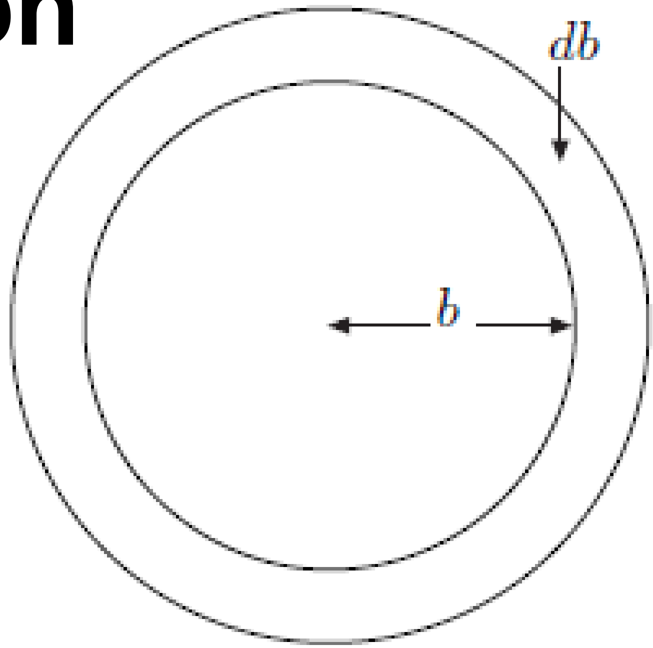


The relation between b (impact parameter) and θ (scattering angle) is given by

$$\tan\left(\frac{\theta}{2}\right) = \frac{D}{2b}$$

Cross section

The "flux", F of incident particles is defined as the number of incident particles arriving per unit area per second at the target.
The number of particles, $dN(b)$ per second, with impact parameter between b and $b + db$ is this flux multiplied by the area between two concentric circles of radius b and $b + db$.



$$dN(b) = F\, 2\pi b\, db$$

Differentiating equation above we derive

$$dN(\theta) = F\pi \frac{D^2}{4} \frac{\cos(\theta/2)}{\sin^3(\theta/2)} d\theta$$

We define the "differential cross-section", $d\sigma/d\theta$, with respect to the scattering angle is the number of scatterings between θ and $\theta + d\theta$ per unit flux, per unit range of angle, i.e.

$$\frac{d\sigma}{d\theta} = \frac{dN(\theta)}{F d\theta} = \pi \frac{D^2}{4} \frac{\cos(\theta/2)}{\sin^3(\theta/2)}.$$

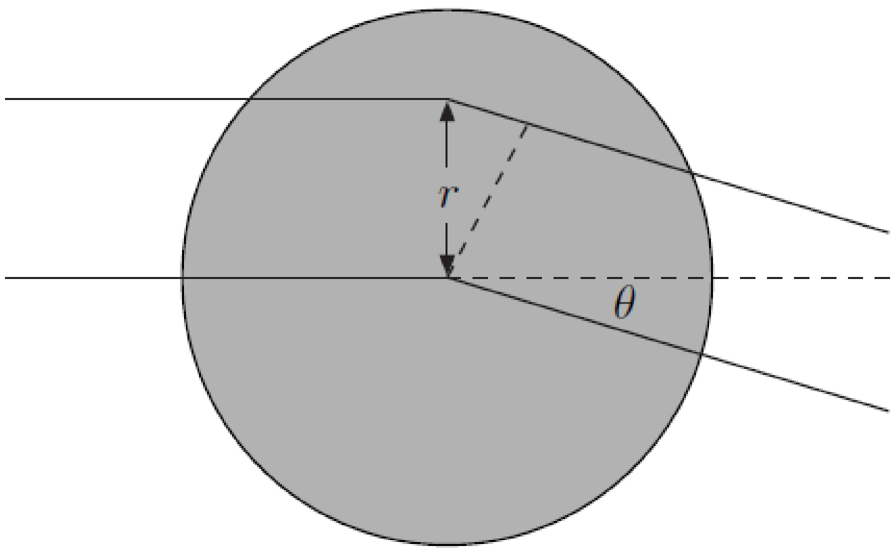
Using $d\Omega = \sin\theta d\theta d\phi$ we define differential cross-section with respect to a given solid angle Ω:

$$\frac{d\sigma}{d\Omega} = \frac{D^2}{8} \frac{\cos(\theta/2)}{\sin^3(\theta/2)} \frac{1}{2\sin(\theta/2)\cos(\theta/2)} = \frac{D^2}{16\sin^4(\theta/2)}.$$

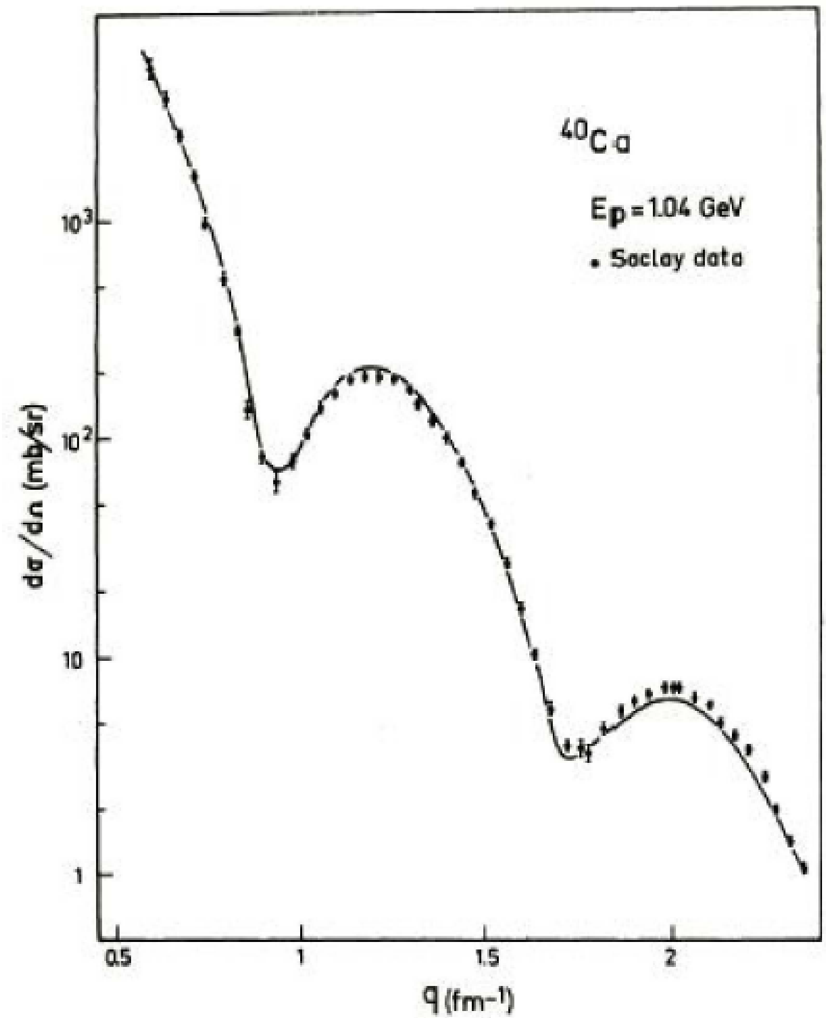
Diffraction

: $$\frac{d\sigma}{d\Omega}_{|\text{Mott}} = \frac{d\sigma}{d\Omega}_{|\text{Rutherford}} \left(1 - \frac{v^2}{c^2}\sin^2\left(\frac{\theta}{2}\right)\right)$$

The structure of "electric form-factor" has a diffractive nature



$$\frac{d\sigma}{d\Omega} = \frac{d\sigma}{d\Omega}_{|\text{Mott}} |F(q^2)|^2$$



and the first minimum of the diffractive pattern occurs when

$$\frac{qR}{\hbar} \approx \pi$$

Liquid drop model

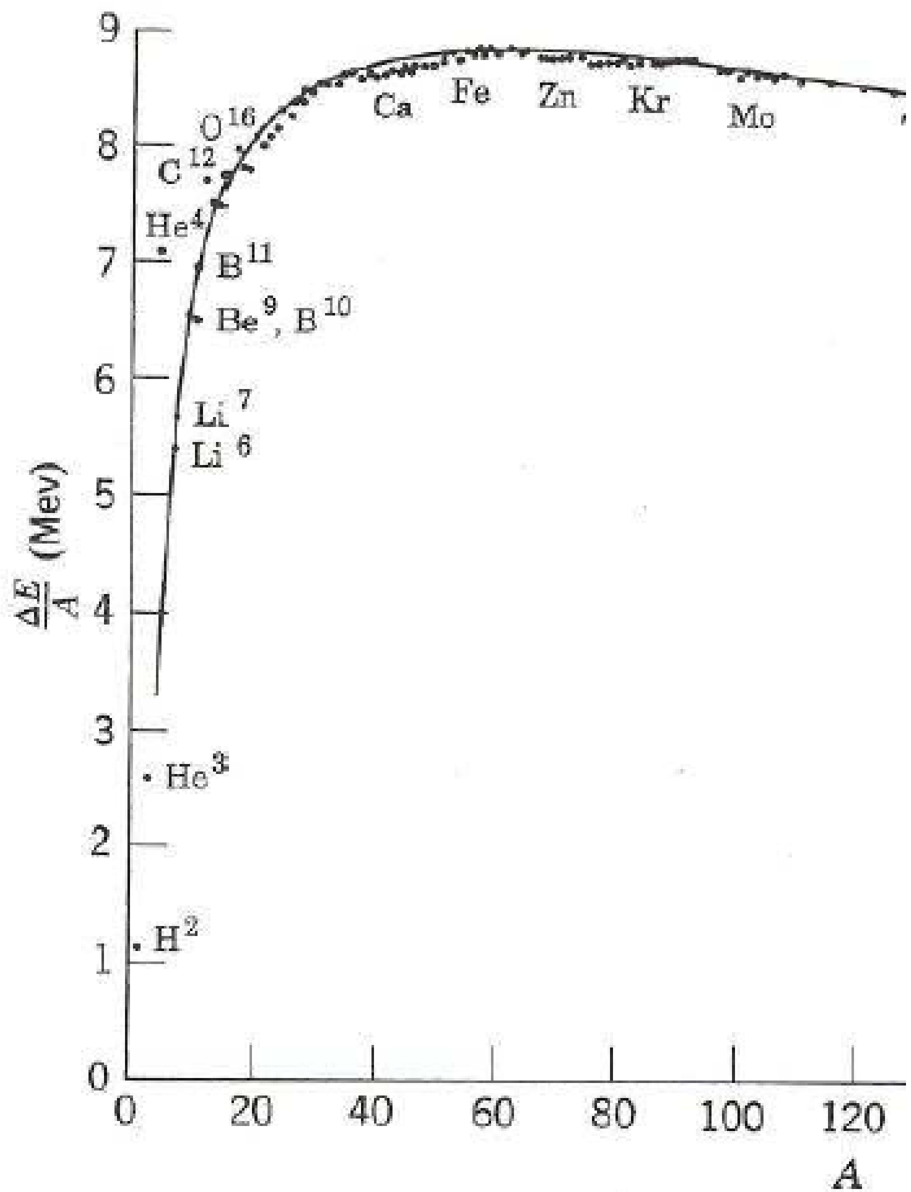
Binding Energy

The complete formula is

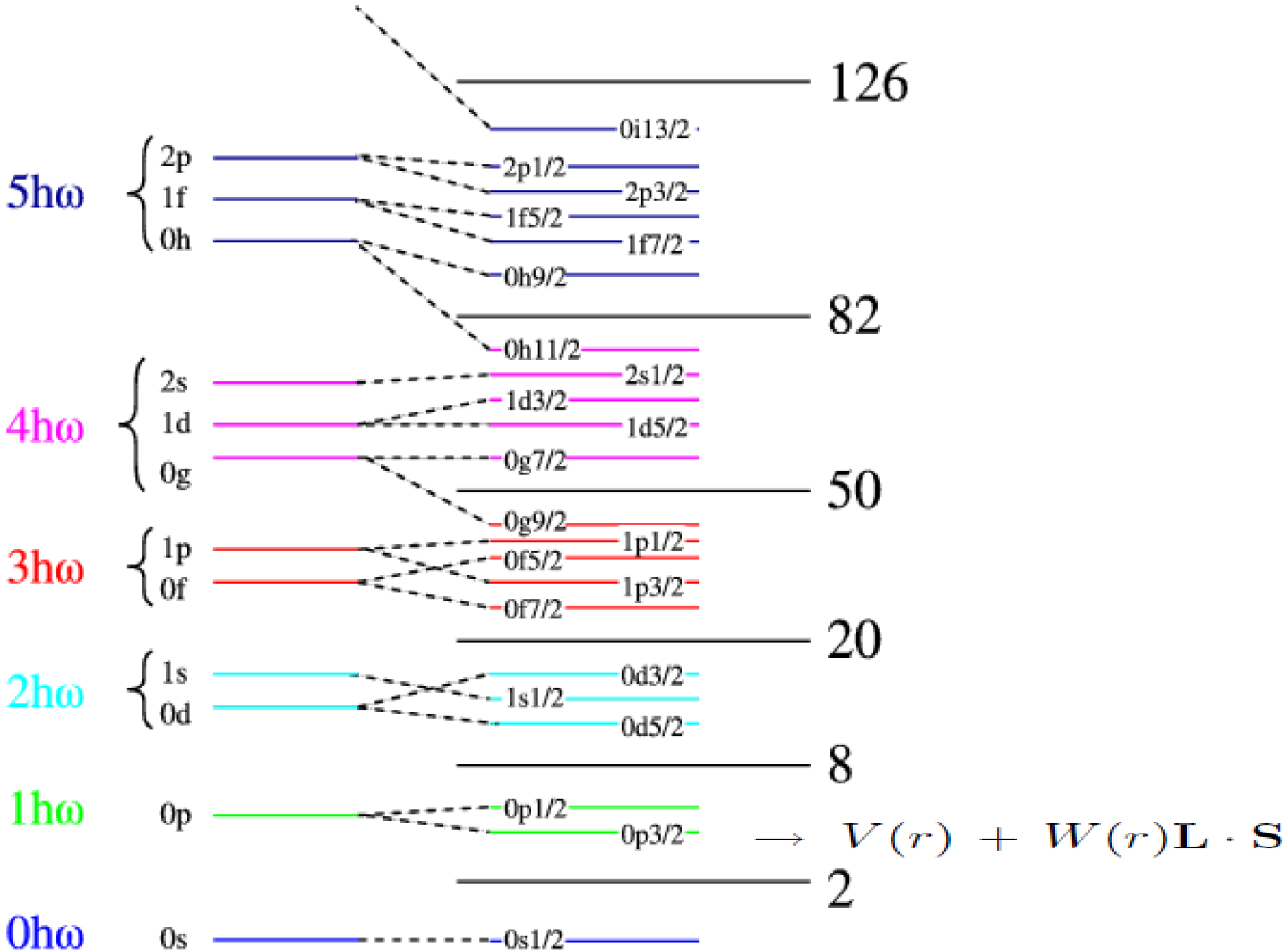
$$B(A,Z) = a_V\,A - a_S\,A^{2/3} - a_C\frac{Z^2}{A^{1/3}} - a_A\frac{(Z-N)^2}{A} + \frac{\left((-1)^Z + (-1)^N\right)}{2}\frac{a_P}{A^{1/2}}$$

where the values of fitted parameters a_V, a_S, a_C, a_A, a_P are

$$\begin{aligned} a_V &= 15.56\text{ MeV} \\ a_S &= 17.23\text{ MeV} \\ a_C &= 0.697\text{ MeV} \\ a_A &= 23.285\text{ MeV} \\ a_P &= 12.0\text{ MeV} \end{aligned}$$



Nuclear Shell Model



Decay Rates

$$\frac{dN(t)}{dt} = -\lambda N(t)$$

$$N(t) = N_0 e^{-\lambda t}$$

$$\tau = \frac{1}{\lambda} \qquad \tau_{\frac{1}{2}} = \frac{\ln 2}{\lambda} = \ln 2 \, \tau$$

Alpha Decay

$$^{(A+4)}_{(Z+2)}\{P\} \rightarrow {}^{A}_{Z}\{D\} + \alpha.$$

$$Q_\alpha = (m_P - m_D - m_\alpha)c^2.$$

Beta Decay

$$^{A}_{Z}\{P\} \rightarrow {}^{A}_{(Z+1)}\{D\} + e^- + \bar{\nu}, \quad \text{or} \quad {}^{A}_{Z}\{P\} \rightarrow {}^{A}_{(Z-1)}\{D\} + e^+ + \nu.$$

$$n \longrightarrow p + e^- + \bar{\nu}.$$

$$^{60}_{27}\text{Co} \rightarrow {}^{60}_{28}\text{Ni} + e^- + \bar{\nu}.$$

$$\langle \mathbf{s} \cdot \mathbf{p_e} \rangle \neq 0$$

Gamma Decay

The total angular momentum change, L in a nuclear transition can take the values

$|Li - Lf| \leq L \leq |Li + Lf|$, **L>0**

This is also subject to selection rules for the parity difference between initial and final states:

$P = (-1)^L$, for electric transitions

and

$P = (-1)^{L-1}$, for the (even further suppressed) magnetic transitions.

Here are some examples:

$2+ \longrightarrow 1-$(E1)

$2+ \longrightarrow 1+$(M1)

$3+ \longrightarrow 1-$(M2)

$3+ \longrightarrow 1+$(E2)

The Mőssbauer Effect

$$E_0 = E_\gamma + \frac{E_\gamma^2}{2M_N c^2}, \quad \text{and} \quad E_\gamma \simeq E_0 \left(1 - \frac{E_0}{2M_N c^2}\right)$$

$$\Gamma = \frac{\hbar}{\tau} \approx 10^{-3} \text{ eV}$$

$$\frac{\Delta\lambda}{\lambda} = \frac{\breve{v}}{c} = \frac{\Delta E}{E}$$

$$\Gamma = 2E\frac{v_{1/2}}{c}$$

Accelerators and particle kinematics

$$s = \left(\sum_{i=1,2} E_i\right)^2 - \left(\sum_{i=1,2} \mathbf{p}_i\right)^2 c^2$$

CM frame: $$s = 4E_{CM}^2$$

Fixed target: $$s = \left(E_{LAB} + mc^2\right)^2 - \mathbf{p}_{LAB}^2 c^2 =$$

$$E_{LAB}^2 + m^2c^4 + 2mc^2E_{LAB} - \mathbf{p}_{LAB}^2 c^2 = 2m^2c^4 + 2mc^2E_{LAB}$$

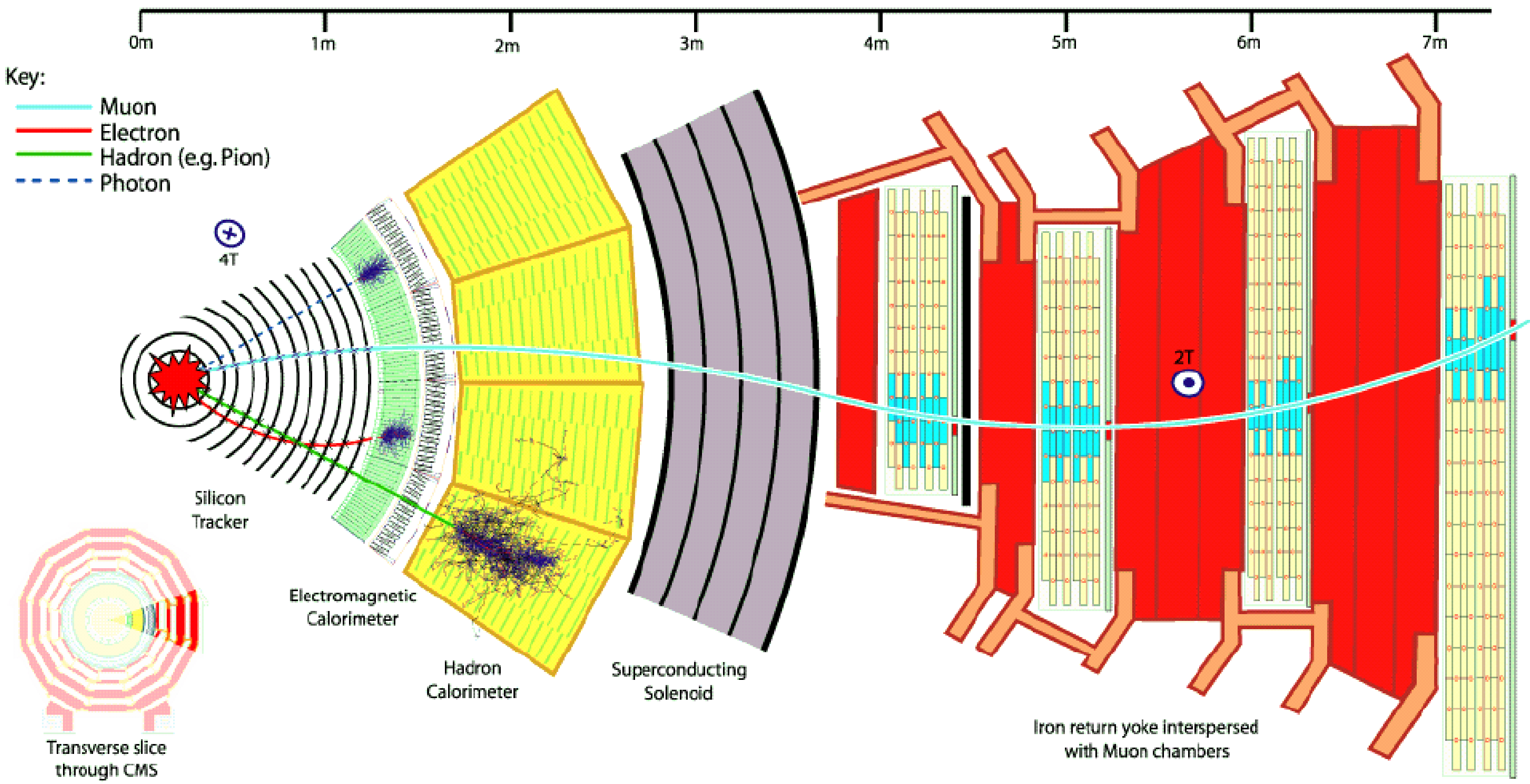
5 Fundamental Interactions (Forces) of Nature

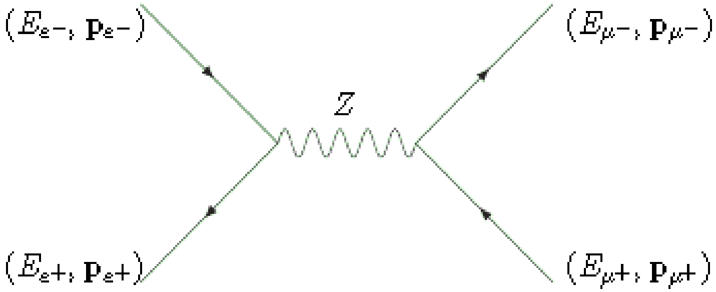
Interaction	**Gauge Boson** (Force carrier)	**Gauge Boson Mass**	**Interaction Range**
Strong	Gluon	0	short-range (a few fm)
Weak	$W^{\pm}$, Z	$M_W = 80.4$ GeV/c^2 $M_Z = 91.2$ GeV/c^2	short-range ($\sim 10^{-3}$ fm)
Electromagnetic	Photon	0	long-range
Gravity	Graviton	0	long-range

Leptons			Electric Charge
ν_e	ν_μ	ν_τ	0
e	μ	τ	-1

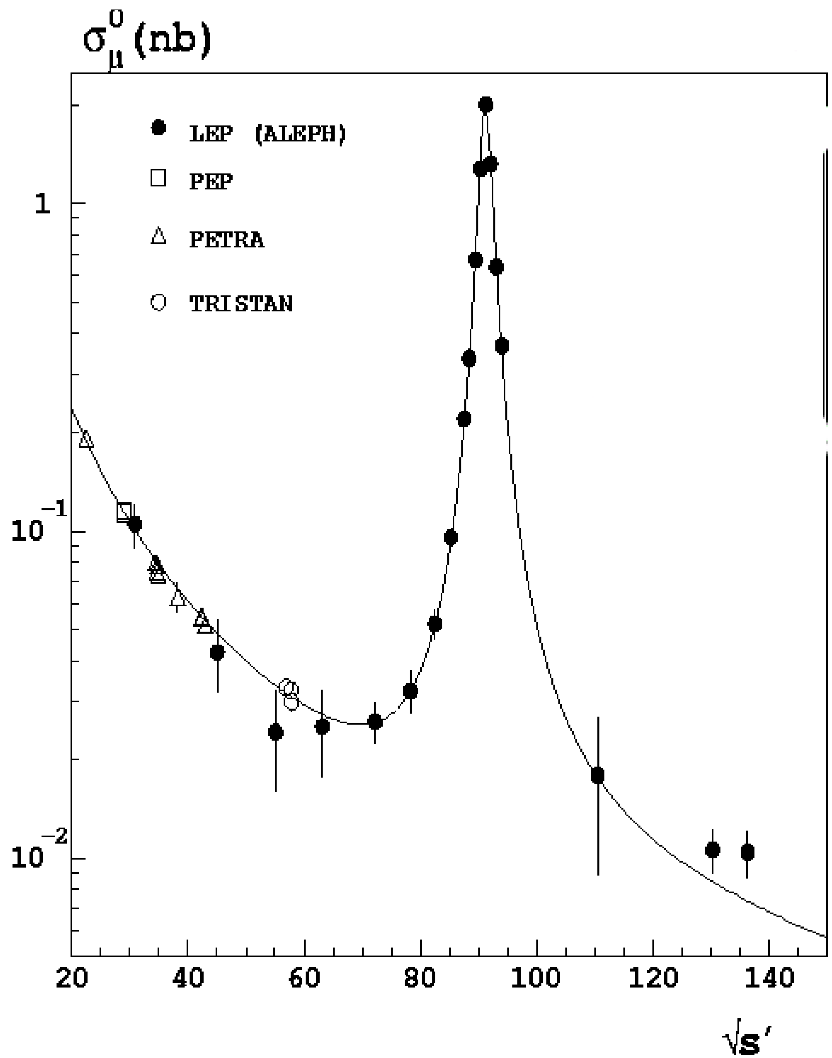
Symbol	**Flavour**	**Electric charge (e)**	**Isospin**	**I$_3$**	**Mass Gev/c^2**
u	up	$+\frac{2}{3}$	$\frac{1}{2}$	$+\frac{1}{2}$	≈ 0.33
d	down	$-\frac{1}{3}$	$\frac{1}{2}$	$-\frac{1}{2}$	≈ 0.33
c	charm	$+\frac{2}{3}$	0	0	≈ 1.5
s	strange	$-\frac{1}{3}$	0	0	≈ 0.5
t	top	$+\frac{2}{3}$	0	0	≈ 172
b	bottom	$-\frac{1}{3}$	0	0	≈ 4.5

CMS DETECTOR



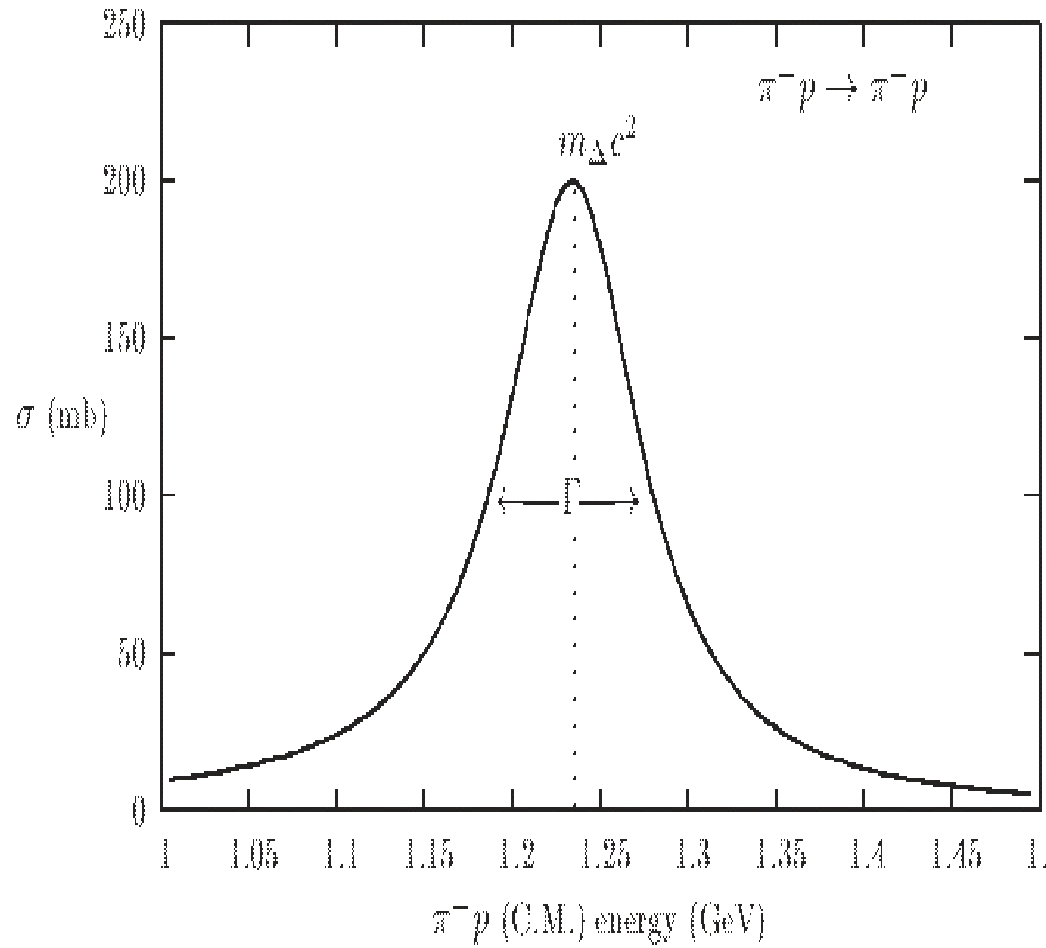


$$e^+ e^- \to Z \to \mu^+ \mu^-$$

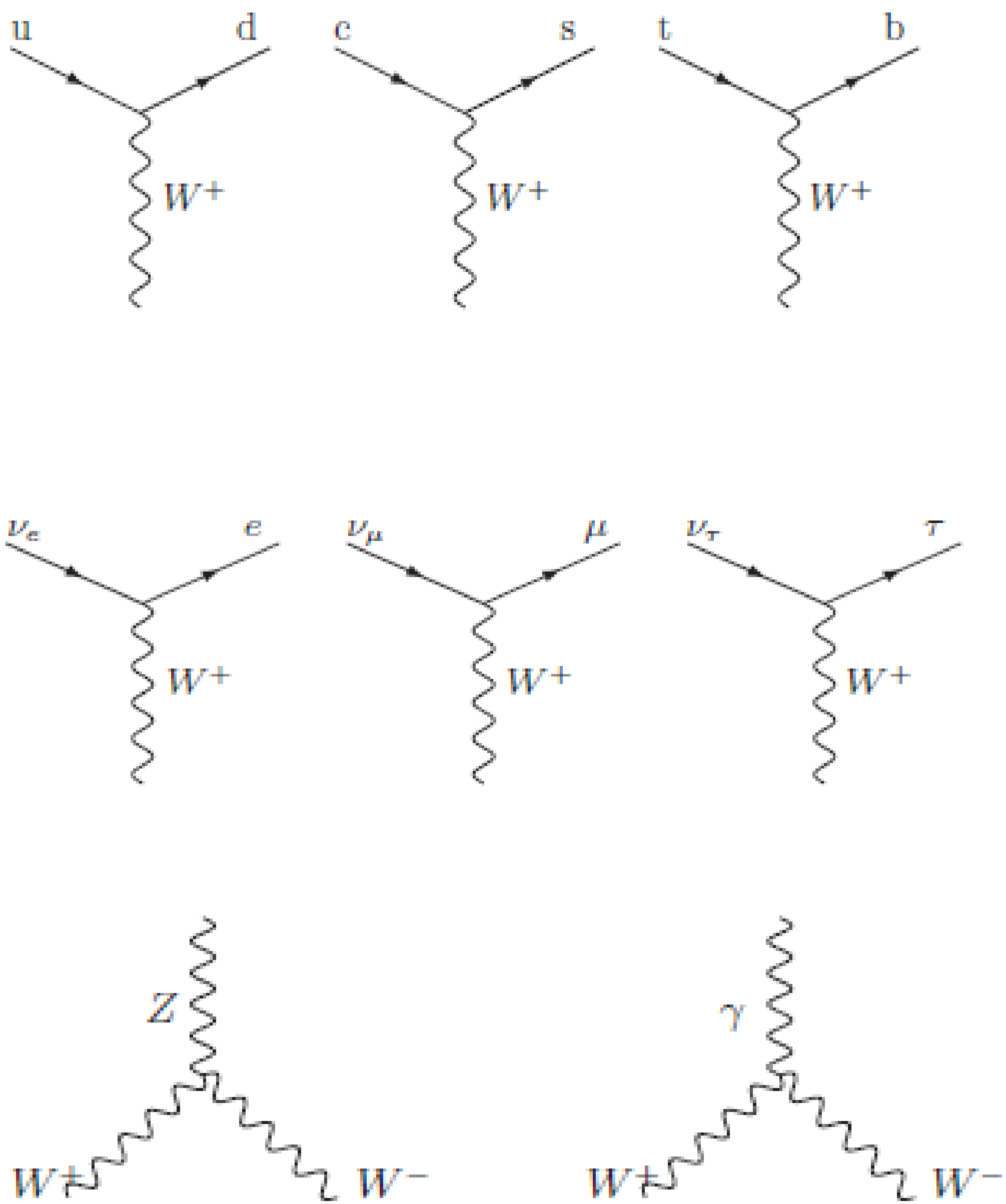


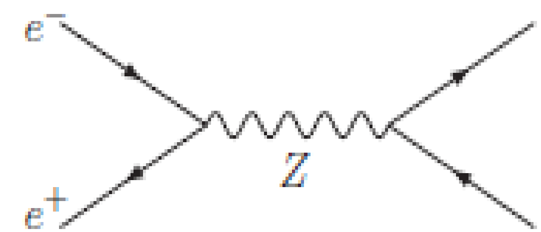
$$\frac{1}{(s - m_\rho^2 c^4 + i m_\rho \Gamma_\rho c^2)}$$

$$p + \pi^- \to \Delta^0 \to p + \pi^-$$



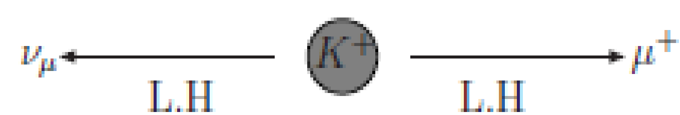
Weak interactions



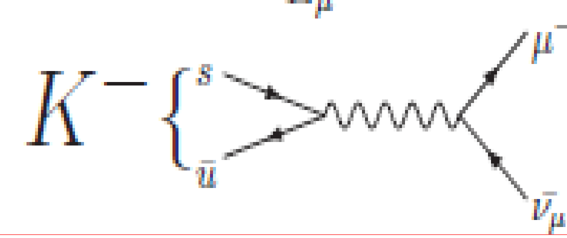


$$\frac{1}{s - M_Z^2 c^4}$$

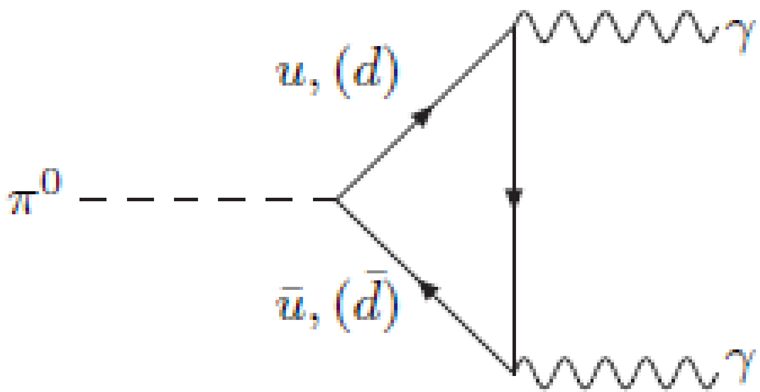
$$K^+ \rightarrow \mu^+ + \nu_\mu$$



$$\frac{m_\mu c^2}{E_\mu}$$



Electromagnetic Interactions



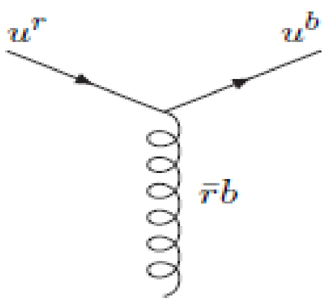
$$R = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)} = 3\left(Q_u^2 + Q_d^2 + Q_s^2\right) = 3\left(\left(\frac{2}{3}\right)^2 + \left(\frac{-1}{3}\right)^2 + \left(\frac{-1}{3}\right)^2\right) = 2$$

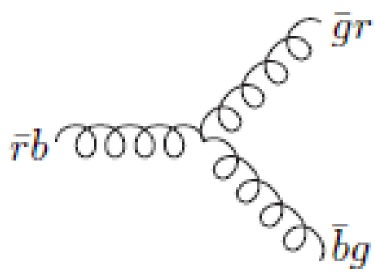
QCD

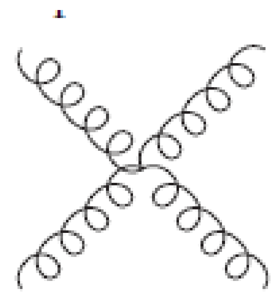
Δ^{++} (uuu)

$$\frac{1}{\sqrt{6}}\left(|f_1^R f_2^G f_3^B\rangle + |f_1^B f_2^R f_3^G\rangle + |f_1^G f_2^B f_3^R\rangle - |f_1^B f_2^G f_3^R\rangle - |f_1^R f_2^B f_3^G\rangle - |f_1^G f_2^R f_3^B\rangle\right)$$

$$|\pi^+\rangle = \frac{1}{\sqrt{3}}\left(|u^R\overline{d^R}\rangle + |u^G\overline{d^G}\rangle + |u^B\overline{d^B}\rangle\right)$$



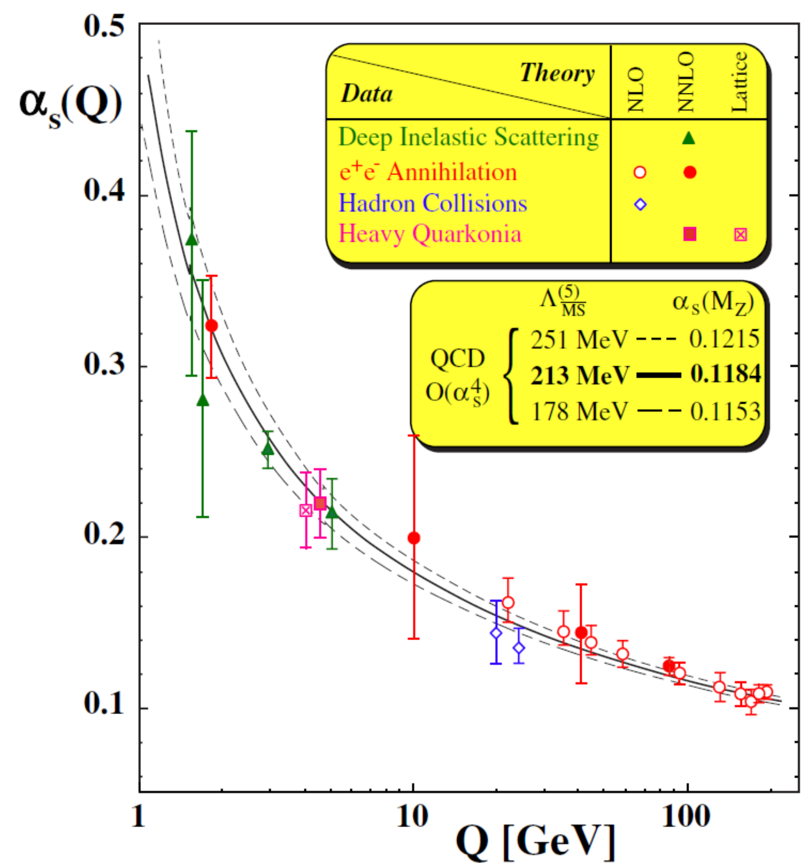




$$\alpha_s(Q) = \frac{\alpha_s(\mu)}{(1-\beta_0\alpha_s(\mu)\ln(Q^2/\mu^2))}$$

$$\beta_0 = -\frac{1}{4\pi}\left(11 - \frac{2}{3}n_f\right)$$

$$\alpha_s(M_Z c) = 0.12$$



Summary of Conservation laws

- **Baryon number:** baryons=+1, antibaryons=-1, mesons, leptons=0.
- **Lepton number:**
  - electron number: $e^-, \nu_e = 1,\ e^+, \overline{\nu_e} = -1$
  - muon number: $\mu^-, \nu_\mu = 1,\ \mu^+, \overline{\nu_\mu} = -1$
  - τ number: $\tau^-, \nu_\tau = 1,\ \tau^+, \overline{\nu_\tau} = -1$

	Strong Interactions	**Electromagnetic Interactions**	**Weak Interactions**
Baryon number	yes	yes	yes
Lepton number (all)	yes	yes	yes
Angular momentum	yes	yes	yes
Isospin	yes	no	no
Flavour	yes	yes	no
Parity	yes	yes	no
Charge conjugation	yes	yes	no
CP	yes	yes	almost