An introduction to noncooperative game theory

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Abstract

Game Theory has attracted a steadily growing interest since the '40s and has experienced an extraordinary development in various directions. Many disciplines like economics, social sciences, biology and engineering use Game Theory in modeling real-world situations. Its importance is witnessed by many Nobel laureates who received that prize for their contributions to Game Theory. Such a theory analyzes situations of strategic interaction between individuals (also called players) that generates an outcome affected by their collective choice. Traditionally, a game can be viewed as cooperative or noncooperative depending on whether the players can sign or not a binding agreement that forces them to follow a common strategy plan. In this course we focus only on the second type, which well describes many practical situations in the field of Operations Research. More specifically, the formal description of a game will be introduced distinguishing between games where the players move simultaneously and those where players move sequentially. Then, the basic solution concepts will be presented and sufficient conditions for the existence of those solutions will be discussed. Finally, some important classes of games will be considered and illustrated by examples.

Plan of the lectures

Lecture 1 (2 hours): first definitions and examples. Normal form representation of a game: definition and examples. Extensive form representation of a game: definition of a tree, perfect and imperfect information, perfect and imperfect recall. Relationship between extensive and normal form representation. Examples and applications.

Lecture 2 (2 hours): normal-form games, solution concepts. Dominant and dominated strategies and the process of iterated elimination of dominated strategies. Best responses. Mixed extension of a normal-form game. Nash equilibrium: definition and properties; existence theorem.

Lecture 3 (2 hours): normal-form games, further issues and examples. The notion of security: maxmin strategies and maxmin values. Some special classes of normal-form games widely used in literature like zero-sum games and potential games.

Lecture 4 (2 hours): extensive-form games, strategies and solution concepts. Pure strategies, mixed strategies and behaviour strategies. Subgames. Sequential rationality and backward induction, subgame perfect Nash equilibrium.

Lecture 5 (2 hours): extensive-form games, further issues and examples. Examples and applications; Stackelberg games. Extensive-form games with imperfect information: an introduction to the theory of refinements.

Some reference books

- T. Başar, G. J. Olsder, *Dynamic Noncooperative Game Theory*, SIAM, second edition (1998)
- P. Dutta, *Strategies and Games*, The MIT Press (1999)
- M. J. Osbourne, *An Introduction to Game Theory* Oxford University Press (2003)
- M. Maschler, E. Solan, S. Zamir *Game Theory.* Cambridge University Press, second edition (2020)