Money and Credit Coexistence, Excess Capacity, and the Size of Monetary Aggregates *

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Abstract

This paper develops a model where money is demanded in excess of spending needs. As a result, money coexists with large availabilities of credit and the model explains the levels of monetary aggregates held in modern economies via the endogenous creation of inside money. At the heart of the model there is a search friction in the goods market which generates spare production and spending capacity. As a consequence there is an endogenous productivity wedge, due to spare production capacity, and an endogenous money velocity, due to spare spending capacity.

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The use of esteemed articles as a store or medium for conveying value may in some cases precede their employment as currency [...] Such a generally esteemed substance as gold seems to have served, firstly, for ornamental purposes; secondly, as stored wealth; thirdly, as a medium of exchange.” Jevons (1875)

1 Introduction

An unresolved question in monetary theory is why money coexists with the amount of credit and financial sophistication observed in modern societies. This is a problem because, in monetary theories where a role of money is not assumed ad hoc, credit crowds money out rather quickly. However, advancements in information technology are such that a range of financial arrangements that require information and commitment, such as credit and insurance products, are now increasingly available. While notable contributions demonstrated that it is possible to explain some coexistence, monetary equilibrium remains fragile to credit: Gu et al. (2016) (GMW) find that the value of money and credit taken together cannot exceed that of transactions in a monetary equilibrium, so when there is enough credit to finance all equilibrium transactions, money has no value. The present paper develops a model where credit can be larger than transactions and yet money has value. A second contribution is to show that the information and commitment frictions necessary to give money a role can be weak enough to allow for futures contracts through which it is possible to have equilibrium with full insurance.

As a consequence of money coexisting with large availabilities of credit, the paper explains the apparently puzzling evidence that, as modern economies go increasingly cashless, more money is held by the private sector. To get a sense of the magnitudes, an aggregate measures of monetary holdings, such as M1, was 2.63 times larger than quarterly GDP during the COVID-19 pandemic; the number becomes 3.62 using either M2 or M3, and 4.19 using MZM. This is not just a recent phenomenon. For instance, M2 and M3 were larger than 2 times quarterly GDP between 1960 and 1990. It seems that accounting for these quantities, which appear to be in large excess of transaction needs, should be a primary goal of monetary theory. But how to explain these figures? Credit is important here because it expands the monetary base into broader money supply. Therefore, broad money expands as credit becomes more available. However, the credit needed to match the aforementioned monetary aggregates is too large for monetary equilibrium in the existing microfounded models where money demand is not assumed ad hoc, but emerges endogenously as a consequence of deep frictions. This paper develops a model where agents demand liquidity in excess of their spending per capita. As a result, money is valued even when the money stock, or credit
availabilities, are in excess of transactions per capita and it becomes possible to match the above mentioned monetary aggregates.

The framework is a standard neoclassical model augmented with a credit limit and with a search friction in the goods market. As a result, spending opportunities are hard to find and it is optimal not to spend all available funds. Money demand emerges as a need to store these unspent funds: the difficulty to spend generates “left over money”, or money as a residual store of value to carry unspent wealth into the future. The paper shows that with this motive, money is demanded even when it is dominated as a transaction means.

The model follows the new monetarist approach where, like here, there are information frictions that limit credit, and there is a search friction in the goods market. However, to accommodate the key mechanism that generates liquidity as a store of value, it proved convenient to develop a framework that departs from the standard new monetarist framework —Lagos and Wright (2005) (LW)— and in a sense goes back to the seminal paper of Kiyotaki and Wright (1989), where there is no centralized market. To understand it is useful to start with LW. In that set-up, the search problem that spending opportunities are hard to find is resolved through an ex-post walrasian market where agents are free to adjust their composition of money and goods. As a result, money is demanded in the walrasian market for its transaction value in the next period. The present paper does not include a frictionless goods market. This absence generates an additional motive to hold money thanks to which money is demanded even when it is dominated as a transaction means. To demonstrate this, I add a capital good that gives higher return than money and which can be turned into consumption at any time. With such a capital good, money would not be demanded in LW: an agent would leave the walrasian market with capital and with no money because capital guarantees more consumption in the next period. However, the paper shows that without the walrasian market, money is held not just for transaction purposes, but also as a consequence of the difficulty to turn liquid assets into goods: the lack of a centralized market makes capital hard to find. As a result, agents demand money to store value even if they prefer capital. So money is held not because easier to spend than capital (i.e. for transaction purposes), but because easier to acquire. Notice that this does not mean that money cannot then be used for transactions: if this was so then money would not have value. But the transaction motive alone would not be sufficient to demand money because, as mentioned, agents prefer capital. To paraphrase, money is not demanded in order to make transactions easily, but rather because one does not make transactions easily.\(^1\)

\(^1\)Generating money holdings through the difficulty to find better assets seems consistent with how Warren Buffett recently justified Berkshire’s large cash holdings: in his 2022 annual letter to shareholders he explained their $144 billion holdings of cash and cash equivalents as “a consequence of my failure to find entire companies or small portions thereof (that is, marketable stocks) which
Relating money and assets, the mechanism offers a resolution to the esteemed rate of return dominance puzzle voiced by Hicks: why money and more remunerative assets coexist? Because money is easier to find (and not because it is easier to spend). As a result, the transaction motive can be minimal, and yet money demand is large and very robust to credit. In particular, the value of money and credit can be larger than income. This way it is possible to match the magnitudes of aggregate monetary holdings. A general equilibrium perspective helps understand why money demand is larger than income: aggregate income equals aggregate spending, but since unmatched agents are not spending their income, matched agents must spend more than their income. And because agents do not know whether they are going to be in a match, everyone wants access to spending power (money + credit) in excess of their income. As a result, even if individual credit is equal to income, money demand is still positive.2

This mechanism is also akin to the Baumol-Tobin (BT) model where an ad hoc cost generates a portfolio friction in reallocating wealth between a liquid asset and goods: recent examples include Alvarez and Lippi (2009), Kaplan and Violante (2014), and Ragot (2014) who emphasises the role of money as a store asset in BT models. One can think of the search friction in this paper as a microfoundation of a resembling portfolio friction. From this perspective, a motivation for the microfoundation may be in order: while the BT framework proved useful for many applications, there the extent to which liquid assets play a role is a matter of degree arbitrarily set through a fixed cost. Instead here the portfolio problem is endogenous and related to the presence of information frictions and to the ability of the private sector to issue liabilities that play a role as a medium of exchange. As a result, the model speaks to the issue of coexistence of money and financial sophistication.3

To have credit it is necessary to have some commitment. Like in Gu et al. (2013) and meet our criteria for long-term holding.”

2That money is demanded in excess of average spending is akin the precautionary need for money in Bewley models. However, the mechanism and the implications differ. In precautionary models there is no search friction in buying goods but agents defer their spending for rainy days. Unlike the precautionary literature, here money is robust to insurance: even redistributing goods and money to insure against the matching risk, still there is money in excess of spending because, owing to the search friction and bilateral trading, each individual must carry the liquidity to make the initial transaction before redistribution can take place. Furthermore, money is robust to the presence of assets that pay higher dividends and that can be instantly turned into consumption. Instead, in Bewley models it is necessary to give to money the advantage of being more easily turned into goods to buffer idiosyncratic shocks: see Wen (2015) (alternatively, it is possible to have capital subject to uninsurable idiosyncratic shocks directly as in Brunnermeier and Sannikov 2016).

3Wallace (1998) points out that in The Wealth of Nations (1776, Book 2, Chap. 2) Smith discussed whether the ability of the private sector to issue liabilities that play a role as a medium of exchange should be regulated. This issue is up to date given the technological advances that favoured the introduction of Bitcoin. Wallace argues that such question cannot be convincingly addressed in a model where liquid assets are not microfounded.
GMW, it is shown how the level of commitment required can arise with participation constraints as in Kehoe and Levine (1993) and Alvarez and Jermann (2000), and imperfect monitoring. The information assumptions generate the incentives required to sustain some debt and preserve a role for money, see Kocherlakota (1998) and Wallace (2010). Furthermore, the paper shows that through the commitment generated, it is possible to have full insurance against the matching shocks and to sustain predetermined terms of trade. Being able to set the terms of trade in advance, it is possible to consider competitive search which internalizes the search externality. This way, the model isolates the key spending problem of turning liquid into illiquid assets due to the search friction, but it is otherwise very close to a neoclassical setup. Insurance is introduced in this model via futures contracts. These contracts are welfare improving as they remove idiosyncratic risk without violating the borrowing limit.\(^4\) That full insurance is implementable may seem surprising given that in Alvarez and Jermann (2000) among others, not enough endogenous commitment is generated despite perfect monitoring. Indeed the derivation of the result is involved but the intuition is simple and boils down to a key assumption: agents caught reneging on their contracts pay their period obligations. In Alvarez and Jermann (2000), agents can be punished by seizing all the assets that they may own, but the actual misappropriation cannot be garnished. As a result of the different assumption, in the present paper monitored default is always strictly worse than no default. Therefore, there is always a level of monitoring, which although imperfect, is high enough to ensure that to honour all obligations is incentive compatible.\(^5\) While insurance markets and predetermined terms of trade are not key for the main results, they make the model analytically tractable while allowing for wealth effects and embedding this framework in an otherwise standard representative agent neoclassical model with capital, amenable to quantitative work. Insurance markets also highlight how monetary equilibrium does not hinge on their absence.

Quantitatively, the extent to which the model generates large money demand and coexistence with credit depends on the severity of the search friction and in particular, on the probability of finding goods. E.g. with a probability of 0.9, the monetary aggregate is 1.1 times GDP; that number lies between M1 and M2 in pre-pandemic years.\(^6\)

\(^4\) The credit limit can be large but not too large for monetary equilibrium: if the credit limit does not bind, it is possible to pool all liquid assets and those in a match can spend it all, leaving no residual wealth to be stored in money. Under a binding credit limit, those in a match cannot borrow enough to spend all existing liquid assets.

\(^5\) Another difference is that attention is not restricted to the “not too tight” borrowing limits as in Alvarez and Jermann (2000). Like in Bethune et al. (2018), the incentive constraints can also be slack. This way it is possible to move the borrowing limit more freely and test the extent to which money is robust to credit.

\(^6\) It would be also possible to add goods with very little or no search friction at all that are demanded in bounded amounts, such as food and other daily needs. To the extent that agents do
The paper also shows that when credit is not perfect, a monetary equilibrium is welfare improving relative to a nonmonetary equilibrium. This result is not uncommon in monetary models but the mechanism here is different: the possibility to store value in the liquid asset makes agents less preoccupied about not finding goods, thereby looking for better trading opportunities relative to the nonmonetary equilibrium where the liquid asset has no value. Technically, money leads to a lower market tightness (firms over buyers) where the probability of selling goods is higher for firms relative to the nonmonetary equilibrium. So money is desirable because it increases firms’ ability to sell i.e. their measured productivity and overall transactions. Similarly, inflation erodes the willingness of buyers to hold money; this increases the velocity of money and induces the buyers to choose a market tightness where it is easier for them to find goods, but harder for firms to sell. As a result, inflation leads to a decline in measured aggregate productivity (TFP).

The link between endogenous TFP and velocity also has implications that relate to recent debates about a saving glut and secular stagnation. In particular, a quantitative paper that builds on this theory finds the suggested search mechanism to be a powerful source of recessions: as matching conditions deteriorate, buyers do not spend their money and firms do not sell their goods (surge in liquid savings, spare production capacity, and decline in productivity). This way the model not only generates a business cycle with an endogenous TFP wedge and the usual comovements in real variables, but it also accounts for the comovements of velocity and of capacity utilization with output, see Mennuni (2021). The paper proceeds as follows. Section 2 sets up the model, Section 3 shows the results, Section 4 includes a simple application to match empirical monetary aggregates, and Section 5 concludes. The appendices contain first order conditions, proofs, and the decentralization of the representative agent environment through futures contracts.

## 2 The Model

Time is discrete. The economy is populated by a continuum of measure one of households that live forever. In each period static firms produce goods for consumption and investment not want to spend all their wealth on these goods, the results would still go through.

The ratio of firms over buyers can also be rearranged as the value of supply over demand. So the presence of money increases the demand relative to a nonmonetary equilibrium.

There is a growing literature with search frictions in the goods market and endogenous TFP: examples include Bai et al. (2019), Huo and Ríos-Rull (2013), Petrosky-Nadeau and Wasmer (2011), Den Haan (2014), and Ghassibe and Zanetti (2019). The main contribution to this literature is to use that mechanism to construct a theory of money. This way, it is possible to relate TFP to money velocity. Furthermore, Mennuni (2021) shows that disciplining the model through monetary quantities has implications for the identification of the shocks that drive the business cycle.
purposes with a neoclassical production function of labour and capital. Similarly to the
standard neoclassical model, in each period firms sell goods to households while labour and
capital inputs are supplied by households and demanded by firms. These two latter input
markets are competitive. Instead, the market for goods is subject to a search friction.\footnote{It would be possible to consider search frictions for the inputs markets too. But to isolate the
key novelties, the model is kept as close as possible to the neoclassical one. However, since capital
goods are accumulated through final goods, it should be explained why capital goods are subject
to a search friction in the final goods market, but are rented without frictions in the capital input
market. A possible story is that households have heterogeneous storage technologies. But firms,
which operate within the period, do not have that problem and thus rent any variety of capital.}

Besides consumption and capital, there is money: a costlessly storable, divisible and
intrinsically useless object. Since this asset is homogeneous, it is not subject to the search
friction. The money supply, denoted $M$, grows through lump-sum injections $\Delta m$ made by
a government to each household at the beginning of each period.

Search is competitive as in Moen (1997). In particular, the way households and firms
meet and trade follows Menzio et al. (2013) and is described next. There is a continuum
of submarkets indexed by the terms of trade $(p, q) \in \mathbb{R}_+ \times \mathbb{R}_+$ where $p$ is the price per
unit of good paid by the household (the buyer) and $q$ the quantity sold by the firm (the
seller). So $pq$ is the payment of the buyer.\footnote{Terms of trade are usually expressed as the balance $pq$ and the quantity $q$ exchanged. Indexing
submarkets by $(p, q)$ is equivalent and will prove convenient later.} If a unit of production of size $q$ is not matched
with a household, it is destroyed. A firm chooses how many trading posts to create in each
submarket (i.e. how many units of size $q$ to put for sale in each submarket) and a household
chooses which submarket to visit. It is convenient to use one of these submarkets as the
numeraire. So $p$ is the price of a good in a submarket relative to the price of the same good
in the numeraire submarket.

The buyer cannot visit multiple submarkets in the same period and can at most find one
trading post. A household and a trading post meet in pairs; let the matching function $\mu$ be
concave and homogeneous of degree one in the number of trading posts $f$ and households $h$,
with continuous derivatives. In a sub-market with tightness $\theta = \frac{f}{h}$, let $\psi(\theta) = \mu(f, h)/h = \mu(\theta, 1)$ denote the probability with which a household or buyer finds a trading post, and
$\phi(\theta) = \mu(f, h)/f = \mu(1, 1/\theta)$ the probability with which a trading post is matched with a
buyer. The function $\psi$ is strictly increasing with $\psi(0) = 0$ and $\psi(\infty) = 1$. $\phi$ is strictly
decreasing with $\phi(0) = 1$ and $\phi(\infty) = 0$. $\psi$ and $\phi$ have continuous derivatives and such
that $1/(\phi\psi)$ is convex in $\theta$. As shown in the proof of Proposition 2, convexity of $1/(\phi\psi)$
ensures that consumers face a convex constraint set. Since $\psi$ and $\phi \in [0, 1]$, are monotone,
one with positive and one with negative slope, this property is easily met: as $\theta$ approaches
either zero or infinity $\phi\psi$ tends to zero, so its inverse goes to infinity. In between $\phi\psi$ is
larger, so its inverse is smaller. As a result $1/(\phi\psi)$ tends to be u-shaped.\footnote{11}

In order to direct search, the terms of trade are determined before a match is found and market tightness varies with the terms of trade across the sub-markets according to the equilibrium function $\theta(p, q)$, which is taken as given by firms and households. As a result, the probabilities $\phi$ and $\psi$ are endogenous functions of $(p, q)$. Because of firms free entry, the menu of submarkets identified by $\theta(p, q)$ makes firms indifferent. Then, households choices determine which submarket is going to be active over that menu.\footnote{12}

Similarly to the neoclassical model, the payment $pq$ need not take the form of money: firms also accept to deliver the good in exchange for private bonds. To clarify, it is useful to specify the following timing within the period.

- The input markets clear at the beginning but payment from firms to households is deferred to the end, after firms revenues are realized.
- After the input markets clear and before inputs are paid, households and firms meet in the frictional market. To pay, households can use money, bond holdings, or issue an intratemporal bond, a promise for later payment at the end of the period.
- Firms use the proceedings (money or bonds) to remunerate the inputs.

At maturity, bonds clear in money, or by clearance of net bond positions.\footnote{13} It is shown that, absent a credit limit on the ability to issue such bonds, money has no value. However, a key point of the paper is that the credit necessary for that case is very large. To clarify that the resilience of money to credit does not come from any other missing markets, I also consider an intertemporal bond to roll over any existing debt. This bond is perfectly liquid in the sense that firms are indifferent between it and money. However, it is shown that this bond is not traded in equilibrium.

\footnote{11}Notice that some of the matching functions used in practice, such as the Cobb-Douglass, do not have the probabilities bounded between 0 and 1. That notwithstanding, a Cobb-Douglas matching function $m = f^\gamma h^{1-\gamma}$ has $1/(\phi\psi)$ convex in $\theta$ as long as $\gamma > 0.5$. The intuition for $\gamma > 0.5$ goes as follows. In choosing the submarket, buyers face the following trade-off: a low $p$ implies a low $\psi$ and thus a low $\psi$. If the price is too elastic to $\theta$, a reduction in $\theta$ reduces the price so much that buyers are likely to choose the submarket with minimum price i.e. a corner solution. As shown later, $1 - \gamma$ is the elasticity of $p$ to $\theta$. So a high $\gamma$ ensures a sufficiently low elasticity, which in turn ensures convexity of the budget constraint and thus an interior solution.

\footnote{12}A motivation for adopting competitive search is that it does not add a bargaining inefficiency, thereby not introducing a further element of departure from the neoclassical framework. Furthermore, similarly to Menzio et al. (2013), directed search brings some tractability because the menu of all possible combinations $\theta, p, q$ is independent of the wealth distribution. However, because this model has capital, it is not fully block recursive, which motivates the introduction of insurance later.

\footnote{13}Bonds do not clear in goods because goods are subject to the search friction, so that would make bonds illiquid.
A further innovation is to introduce insurance against matching risk via futures contracts. This arrangement makes the model tractable and shows that the large demand for liquidity is not due to precautionary motives. Similarly to the big family assumption of Shi (1997), the purpose is to redistribute goods between people that like the same variety. However, the arrangement does not assume a big family but is decentralized through the futures. This way, all agents are individually rational.

The household has to honour the debt, the terms of trade agreed ex-ante in the competitive search protocol, and the futures. In Section 2.4.1 I show that all these contracts are honoured under the limited commitment that is enforceable with imperfect monitoring, which is required to make money essential as pointed out by Wallace (2010). These frictions call for a limit on credit and an upper-bound on the quantity $q$ sold in a submarket. These 2 limits enable equilibrium with no endogenous default under the following assumption: to enter a submarket one has to show proof of means, i.e. one has to show enough money holdings and credit availability to afford the purchase in the submarket, should a match occur.\textsuperscript{14} As in Gu et al. (2013), I model imperfect monitoring via a probability of being monitored.\textsuperscript{15} Let $p_{ToT}$, $p_b$, and $p_f$ be the probabilities of being caught defaulting on the terms of trade (ToT), debt, and futures respectively. Furthermore, I assume that $p_{ToT} = p_b$ and the probability of being caught does not increase if the agent defaults on the ToT and credit jointly. A justification for this restriction is that since households borrow from firms, default on the ToT and on the debt is a default on a firm. This restriction simplifies the exposition because, as a result, the firm never offers credit to someone that defaults on the ToT. This reduces the number of cases to consider but the results would hold more generally. Agents caught defaulting pay their period obligations and move to an unmonitored regime in which there are no consequences from defaulting. As a result they loose access to credit as they would ex-post default on it. However, it is shown that submarkets with low prices and a limited amount of insurance are incentive compatible even when there is no endogenous threat so they retain limited access to the futures market and to some submarkets.

Finally, firms have incentives to default on the ToT and on the payment to the inputs. To avoid this, firms are monitored with some probability and in case of monitored default all revenues are confiscated. The monitoring of the firm does not disclose the identity of the buyer. As I argue later, with enough firm monitoring, firms constraints are slack as they

\textsuperscript{14}Otherwise one could enter a submarket with $pq$ very large, and then default at the moment of paying. Proof of means are commonly required, for example, when booking hotel rooms and for travelling visas. Evidence of monetary holdings can be provided while remaining anonymous.

\textsuperscript{15}There are alternative ways to model imperfect record-keeping such as to have a fixed cost to access a public record, or to make the public record accessible only to a group of the population. I followed the approach of Gu et al. (2013) because this way credit is a perfect substitute to money up to the borrowing limit.
are looser than the ones of the household, and thus can be ignored. Since the emphasis in this paper is on the household, I assume that there is such a level of firms monitoring.

2.1 Planning problem under perfect monitoring

To understand how this model works, it is instructive to start with a planning problem with perfect monitoring. The main takeaway is that the efficient allocation is the one of the neoclassical model with no search frictions.

**Definition 1** Under perfect monitoring an allocation is efficient if it solves the following:

\[ \bar{V}(k) = \max_{\{q,c,k',\theta\}} u(c) + \beta \bar{V}(k') \]  

s.t. \[ \theta q \leq Ak^\alpha, \]  

\[ c + k' - k(1 - \delta) \leq \phi(\theta)\theta q, \]  

\[ q, c, k' \geq 0, \quad \theta \geq 0. \]

The planner chooses market tightness \( \theta \) (or equivalently the number of trading posts \( f \) as households have measure 1 so \( f = \theta \)).

Constraint (2) ensures that total production is not smaller than the quantity offered by each trading post \( q \) times the number of trading posts \( \theta \).\(^{16}\) Resource constraint (3) states that consumption and investment are smaller or equal to the amount of goods exchanged, which in turn is equal to the supply of the trading posts \( f q = \theta q \), times the probability that a trading post finds a match \( \phi(\theta) \).

Notice that there are no incentive constraints because of perfect monitoring. Also, there is no household’s heterogeneity resulting from the fact that some agents are matched and some are not because the preferences are such that it is efficient to redistribute all goods found equally across households. As a result, this planning problem assumes degenerate consumption and capital distributions without loss of generality.

It should be fairly straightforward to see that it is optimal to have \( \theta = 1 \). This is because Constraints (2) and (3) imply

\[ c + k' - k(1 - \delta) \leq \phi(\theta)Ak^\alpha. \]  

\(^{16}\)Strictly speaking this constraint holds at each trading post \( i \), not just summing up all trading posts: \( \theta q \leq Ak^\alpha_n^{1-\alpha} \) for all \( i \). However, because of the constant returns to scale production function, summing all trading posts up is without loss of generality.

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From this last equation it is evident that the lowest $\theta$ the better as this way $\phi$ is as large as possible and most of the production is either consumed or invested. As $\theta \to 0$, there is no “waste” because $\phi = 1$ and the allocation coincides with the one of the neoclassical model.\textsuperscript{17} To better understand why it is optimal to have $\theta$ as small as possible it is useful to draw a comparison with labour search models such as Mortensen and Pissarides (1994); there market tightness is given by the ratio between vacancies and unemployment. If vacancies were free to post, free entry would imply infinite vacancies. A previous version of the paper included effort costs associated to searching for $q$ which had the role played by vacancy posting costs in Mortensen and Pissarides (1994), leading to an efficient market tightness bounded away from zero.

The fact that this model, absent vacancy costs and information frictions, has the same allocation as the neoclassical model, will help isolate the effects of the information frictions in the decentralized economy, and the role of money.

From (4) it is also possible to appreciate how the matching probability $\phi$ is related to TFP: through the lens of a neoclassical model, a high $\phi(\theta)$ would be captured by a higher TFP parameter. So in this model the Solow residual is endogenous. Intuitively, the higher $\phi$ the less waste. Later it is shown that also money holdings and velocity are related to the matching probabilities.

\subsection{2.2 Households in the decentralized economy}

Households liquid funds at the beginning of a period are $p_m(m + \Delta m) + a$, where $m$ is money held from the previous period, $p_m$ is its price in terms of the numeraire sub market, and $a$ is the value in terms of the numeraire of the intertemporal bond holdings.\textsuperscript{18}

A household enters submarket $(p, q)$ such that

$$q \leq \bar{q}$$

and

$$pq \leq p_m(m + \Delta m) + a + B.$$\textsuperscript{17}

\textsuperscript{17}It should now be clear why one needs to restrict $\theta$ to a compact set: if the constraint simply was $\theta > 0$, then the problem would have a Sup for $\theta = 0$. Some of the propositions below work better if the problem has a Max.

\textsuperscript{18}Households also own the capital stock, which –like firms production– could in principle be put on sale in the frictional goods market. However, this market does not operate once insurance markets are introduced later. Clearly a household would never put on sale capital at the same price as the one in which she buys: this is because with the proceedings she may not buy back goods for the same amount given the search friction, and would hold the rest in money, which pays lower return. A household would be willing to sell in a submarket with a higher price than the one at which she buys, but (in the representative agent environment that follows after the introduction of insurance) it would not find anyone willing to buy at that same higher price.
The first constraint avoids a too strong hold-up problem that induces default on the terms of trade. The second constraint limits transactions to liquid wealth + credit: $B \geq 0$ is the maximum the household can borrow from the firm by issuing the intratemporal bond. This form of borrowing must be cleared by the end of the period and may be thought of as credit card debt. This way to introduce credit resembles GMW. While others have introduced credit in different ways, drawing on several specific characteristic which all have elements of truth (see Cavalcanti and Wallace (1999), Marimon et al. (2003), Berentsen et al. (2007), Telyukova and Wright (2008), Sanches and Williamson (2010) and Nosal and Rocheteau (2011) among others), this approach seems the most appropriate for this paper as, up to the borrowing limit, credit is perfectly substitutable to money, making the coexistence problem more transparent because money has no transaction advantage. Absent credit ($a = B = 0$) Equation (6) would be a familiar transaction constraint where goods can only be paid with money. However, the paper shows that credit can be larger than transactions per capita and yet money has value.

With probability $\psi(\theta(p, q))$ there is a match so the household pays $pq$ and buys goods for $q$ which can be used as consumption $c$ or as investment $i$:

$$c + i = q.$$ 

Capital accumulates according to $k' = i + k(1 - \delta)$, where $\delta \in (0, 1]$ is the depreciation rate. Furthermore, end of period capital $k' \geq 0$. I.e. the household can disinvest to the point of consuming up the entire capital stock.\(^{19}\)

At the end of the period a household receives income payments $w + kr$ where $w$ is the real wage (labour supply is normalized to 1 but could be endogenized), and $r$ the rental price of $k$. The firm pays the inputs with its revenues, i.e. either with money, or by turning the bonds it received from other households. It is unnecessary to keep track of individual money and bonds holdings at this sub-period stage because, like in GMW, the intra-temporal interest rate is zero. However, a detailed description of the exchange and clearance of bonds is offered in Appendix A.

Since the household spent $pq$ (either in money or bonds), her end-of-period financial balance after honoring the debt is $p_m(m + \Delta m) + a + wn + kr - pq$. This balance is stored in money $m'$ or it can be saved in the intertemporal bond $a'$. Each unit of the intertemporal bond pays at par at maturity and costs $v$:

$$p_mm' + va' = p_m(m + \Delta m) + a + w + kr - pq.$$ 

\(^{19}\)Since I will introduce insurance markets and Inada conditions in the utility function, this constraint is only avoiding Ponzi schemes, but it does not induce the sort of precautionary savings it would in an incomplete market model à la Aiyagari (1994).
With probability $1 - \psi(\theta(p,q))$ the household does not make a transaction; in this case she does not buy any goods and at the end of the period she is left with the initial money and bond holdings plus income:

$$\begin{cases} c + k' - k(1 - \delta) = 0, & k' \geq 0 \\ p_m m' + va' = p_m(m + \Delta m) + a + w + kr. \end{cases}$$

This extreme distribution assumption, that one either trades in full or not at all, might sound unrealistic. However, insurance—included next—smoothes its implications.

Finally, it is necessary to limit the inter-temporal bond: $a' \in [-a, a]$.

### 2.2.1 Insurance

That some agents trade and others do not gives scope for insurance. However, insurance is problematic in monetary models. This is either because monetary equilibria are not robust to the presence of insurance against idiosyncratic shocks—such as in Bewley economies—or because insurance needs monitoring and commitment which are ruled out to avoid credit.

This section describes the insurance decentralization assuming no default in equilibrium. Then, Section 2.4.1 shows that the decentralization is incentive compatible under the informational frictions that give money a role.

Households that like the same variety can insure among themselves using futures. The futures contract is an agreement between 2 households where one party commits to transfer a given amount of goods should she succeed to match with a firm and should the other household be unmatched. In turn, the unmatched household commits to pay a given price. These futures insure against matching risk but must not violate the borrowing limit.

Before studying insurance, it is worth discussing the meaning of goods redistribution with search frictions given that inherent in said friction there is the idea that agents have heterogenous tastes and may not like any goods redistributed to them. A possible taste and information structure that makes goods redistribution desirable is that goods come in different varieties and each household can only store a subset of varieties but the variety offered at each trading post is not known before visiting the trading post. After purchases are made, there can be redistribution between households that like the same variety. This requires that buyers know other buyers that have similar tastes. However, they cannot tell

---

20 This constraint does not bind in equilibrium as $a' = 0$. However, it avoids profitable deviations such as Ponzi schemes. Furthermore, the upperbound on $a'$ avoids the possible deviation of hoarding for some time and then defaulting.

21 Without the borrowing limit it is possible to do even better: pool all assets and then let the agents in a match spend it all and then redistribute the goods. But this arrangement bypasses the credit friction this paper studies, i.e. it violates Transaction Constraint (6).
the variety offered by each trading post before visiting it.\footnote{One could worry that this expedient opens up markets where capital of each variety is traded without frictions among households that like that variety. This does not happen as there is no scope for further exchanges after those prescribed in the insurance scheme. This is because after insurance, agents that like the same variety value capital and money in the same way and have the same amount of both. So, unless the optimal quantity of money and capital is indeterminate because agents are indifferent between many allocations, they would want to exit the market with the same amounts, which can be achieved with no trade. Indifference between allocations can be ruled out because, as shown later, the budget set is convex and preferences are strictly concave so agents have a unique optimum quantity of money and capital in an equilibrium.} It should be noticed that this theory of money does not hinge on this insurance assumption: it would be possible to write the model without the insurance market and allow for heterogeneity to spread.

Since the futures are in zero net supply (each agent has long and short positions that cancel out) there is an equivalent and simpler representative agent problem shown in Section 2.2.3, where these futures are not explicitly included but they are present in the background so that all idiosyncratic risk washes out. For the reader that is not interested in the details on the futures it is enough to understand that the futures do what is explained next, and then jump to Section 2.2.3. Further details of the decentralization with the futures explicitly modelled is included in Section 2.2.2.

The outcome is as follows. Assuming the law of large numbers so that $\psi$ is the exact share of the population that successfully made a transaction, all households that participated in the same sub-market by being ready to pay $x = pq$ for $q$, receive goods for $\psi q$ and pay $\psi x$. I.e. the share $\psi$ of households that made a transaction, transfer $(1 - \psi)q$ of goods each and are thus left with $\psi q$. The transfers sum up to $\psi(1 - \psi)q$ which can be divided among the remaining $(1 - \psi)$ share of the population so that each receives $\psi q$. In turn, those that receive the goods transfer, make a payment of $\psi x$ in liquid assets. It is easy to check that this way each agent in the same sub market receives goods for $\psi q$ and pays liquid assets for $\psi x$ so that the end-of-period liquid balance is $pm(m + \Delta m) + a + w + kr - \psi pq$ for all.

It is worth relating this insurance mechanism to the alternatives provided by the monetary literature. The key tension is to achieve the tractability of a representative agent environment while maintaining the assumption of anonymity and lack of commitment needed to rule out credit. The solutions are either to adopt the timing and preference structure developed in LW, or to assume big families as in Shi (1997). As discussed further in Section 2.2.3, the LW approach would remove the spending problem that is key in generating monetary equilibrium with credit limits larger than transactions. The futures’ scheme achieves an outcome similar to the seminal paper of Shi (1997), but it resolves some incentives issues present in that approach: in Shi (1997), although agents are not monitored so that trades are quid pro quo, individual incentive conditions are not taken into account as agents act
not in their own self interest, but by the rules prescribed by the head of the household.

### 2.2.2 Insurance Through Futures Contracts

Each household can issue futures with which she commits to transfer goods at the given price to another household should she succeed to match with a firm and should the other agent be unmatched. The transaction constraint (6) becomes

$$ p \left( q - \nu(e, u; p) \int_0^1 q_i(e, u; p) di + \nu(u, e; p) \int_0^1 q_i(u, e; p) di \right) \leq p_m(m + \Delta m) + a + B. \quad (7) $$

$\nu(e, u; p)$ is the price at the beginning of the period at which the agent sells a unity of the futures contract, $q_i(e, u; p)$ is the quantity purchased by individual $i$. Index $e$ as a first input denotes the contingency in which the agent is matched with a firm; index $u$ in the second input indicates that the buyer of the contract (any household $i$) ends up unmatched. If this contingency materializes, then the household transfers $q_i(e, u; p)$ to agent $i$, which in turn pays $pq_i(e, u; p)$ in money or bonds. The household also buys futures from other agents, they cost $\nu(u, e; p)$ where the first index ($u$) indicates that the household ends up unmatched and the second one ($e$) that the other party found a match. $\nu$ and $q_i$ are also indexed by $p$ because different submarkets $p$ have different matching probabilities and so the futures contract is indexed accordingly.

If the household is matched (state $e$) she faces the following additional constraints:

$$ \begin{cases} c_e + k'_e - k(1 - \delta) \leq q - (1 - \psi) \int_0^1 q_i(e, u; p) di, \\ p_mm'_e + va'_e = p_m(m + \Delta m) + a + w + kr - \\ p \left( q - \nu(e, u; p) \int_0^1 q_i(e, u; p) di + \nu(u, e; p) \int_0^1 q_i(u, e; p) di \right) + p(1 - \psi) \int_0^1 q_i(e, u; p) di, \\ k'_e \geq 0. \end{cases} \quad (8) $$

In the constraints above, for the law of large numbers, the futures imply that the agent transfers goods $(1 - \psi) \int_0^1 q_i(e, u; p) di$ for which is paid $p(1 - \psi) \int_0^1 q_i(e, u; p) di$.

---

23 Submarkets are only indexed with $p$ rather than with $p, q$ because, as shown in Lemma 1, only $p$ matters. The household only buys futures from buyers that entered the same submarket $p$ but in principle an agent that searches in submarket $p$ could buy futures from a household that enters some other submarket $\hat{p}$. This would complicate the problem and would not improve the allocation. To be sure that there are no private incentives to trade futures across submarkets it is enough to assume that monitoring with probability $p_f$ is only possible among agents within the same submarket.
If the household is unmatched (state \( u \)) she faces the following additional constraints:

\[
\begin{align*}
  c_u + k'_u - k(1 - \delta) & \leq \psi \int_0^1 q_i(u, e; p) di, \\
  p_m m'_u + va'_u & = p_m (m + \Delta m) + a + w + kr - \\
  q \left( -\nu(e, u; p) \int_0^1 q_i(e, u; p) di + \nu(u, e; p) \int_0^1 q_i(u, e; p) di \right) & - p \psi \int_0^1 q_i(u, e; p) di, \\
  k'_u & \geq 0.
\end{align*}
\]

(9)

In the latter case the law of large numbers imply that the agent receives goods \( \psi \int_0^1 q_i(u, e; p) di \) for which she pays \( \psi \int_0^1 q_i(u, e; p) di \) in money or bonds. A difference between the two cases above is that purchases from the firm \( q \) only appear in case of a match.

It is easy to see that with \( \int_0^1 q_i(u, e; p) di = \int_0^1 q_i(e, u; p) di = q \) the constraints (8) and (9) coincide and boil down to the ones of Section 2.2.3 with perfect insurance. Clearly, if this outcome does not violate any constraint, it must hold in a solution to the household’s problem given the concave preferences and that the budget set is convex (as shown in Section 3.1). So, it needs to be shown that full insurance is implementable.

First, since the futures are issued from one household to another, they are in zero net supply. Assuming symmetry this implies:

\[
\nu(e, u; p)q(e, u; p) = \nu(u, e; p)q(u, e; p),
\]

(10)

where, by symmetry, if agents start with the same state variables their futures’ purchases are equal for all agents: \( q_i(e, u; p) = q(e, u; p) \) and \( q_i(u, e; p) = q(u, e; p) \). Assuming the law of large number, in equilibrium total ex-post transfers satisfy

\[
\psi(1 - \psi)q(e, u; p) = (1 - \psi)\psi q(u, e; p).
\]

Intuitively \( \psi \) is the probability of being of type \( e \) and \( 1 - \psi \) is the probability of being matched with a type \( u \), in which case one transfers \( q(e, u; p) \). Similarly, one receives \( q(u, e; p) \) if one is of type \( u \) and is matched with a type \( e \), which occurs with probability \( (1 - \psi)\psi \).

The last 2 equations are satisfied when

\[
\nu(e, u; p) = \nu(u, e; p) = \psi(1 - \psi).
\]

(11)

Then, from (10), \( q(e, u; p) = q(u, e; p) \). Full insurance also requires that those are equal to \( q \). In turn, this outcome requires that there are no binding constraints on the amount of futures that can be sold. In Section 2.4.1, where commitment is endogenized, it is shown how to ensure that the incentive constraints are not binding for the futures so \( q(e, u; p) = q(u, e; p) = q \) is indeed implementable.

As a result, it is equivalent to solve the representative agent problem after insurance in Section 2.2.3, where the contingencies \( e \) and \( u \) and the futures do not show up.
2.2.3 The household problem with insurance

The futures contracts detailed in Appendix 2.2.2 lead to a representative household: she starts each period with capital $k$, money $m$ and bonds $a$. For recursive equilibria, the aggregate state $\Omega$ is composed of the aggregate capital stock $K$ and money $M$, and possibly of a vector of shocks with a known Markov process.

The household solves the following problem with rational expectations:

\[
V(k, m, a, \Omega) = \max_{c, k', m', q, p, a'} u(c) + E\beta V(k', m', a', \Omega')
\]

\[
s.t. \quad q \leq \bar{q}(p, \Omega, \exists),
\]

\[
pq \leq p_m(m + \Delta m) + a + B(p, \Omega, \exists),
\]

\[
c + k' - k(1 - \delta) \leq \psi(\theta(p, q))q,
\]

\[
p_m m' + va' \leq p_m(m + \Delta m) + a + w + kr - \psi(\theta(p, q))pq,
\]

\[
c, k', m', q, p \geq 0, \quad a' \in [-a, a], \quad \theta(p, q) \geq \underline{\theta}.
\]

$\beta \in (0, 1)$ is the discount factor. The utility function $u(c)$ is increasing, strictly concave, and has continuous derivatives with $\lim_{c \downarrow 0} u_c = \infty$. The household takes input market prices $w$ and $r$ as given. $E$ indicates rational expectations taken over next period aggregate state $\Omega'$ given $\Omega$. The presence of the shocks can be ignored for most of the paper but some of the results are shown to hold not just in the steady state but also on the transitional stochastic path which could be useful for future quantitative work. The last constraint in (17) restricts $\theta$ to a compact set with a positive lower bound $\theta \rightarrow 0$, so $\theta = 0$ is ruled out.

Constraints (13) and (14) restate (5) and (6) where the limits $\bar{q}$ and $B$ are allowed to be generic functions of $(p, \Omega, \exists)$ where $\exists$ stands for equilibrium meaning that the borrowing constraint may be equilibrium specific (e.g. whether the equilibrium is monetary or not). This formulation comes from the incentive restrictions as shown in Section 2.4.1.\(^{24}\)

Equation (15) shows that only a fraction $\psi$ of demand $q$ is matched with investment and consumption goods: either an agent was in a match and bought $q$ but had to transfer $(1 - \psi)q$ to the unmatched agents in her insurance scheme, or she was unmatched but received a transfer of $\psi q$. What is left of her liquid funds is invested in liquid assets as shown in (16): the right hand side shows the end of period balance after insurance, as elaborated earlier. From this latter equation, it is intuitive why money may have value in

\(^{24}\)Other than the restriction of being functions of $(p, \Omega, \exists)$, $\bar{q}$ and $B$ are generic functions because, following Bethune et al. (2018), it is possible to have many credit limit functions bounded by the not too tight limit described in Alvarez and Jermann (2000). The endogenous credit limit may depend on the equilibrium because it affects the value of not defaulting, see Berentsen et al. (2007), Araujo and Hu (2018), Lotz and Zhang (2016), Gomis-Porqueras and Sanches (2013) and Section 2.4.1.
this economy: $\psi < 1$ implies that not all available funds $p_m m + a + B$ can be spent in goods. As formalized later, if $B$ is not too large, the right-hand-side of (16) is positive, i.e. there is left over wealth which gives rise to money or bond demand. Since bonds are in zero net supply, in equilibrium $a' = 0$ and there is positive money demand.\(^{25}\)

So, the harder it is to find goods (i.e. the smaller $\psi$) the larger $p_m m'$.\(^{26}\)

### 2.2.4 Store of value versus transaction motive, a comparison to Lagos-Wright

Notice that the latter is a different explanation than the transaction one according to which money is acquired today in order to be able to purchase goods tomorrow. Rather, money is demanded because one was not able to make purchases today (store of value motive). This is an important distinction because, as argued next, in this model money is demanded even though it is dominated as a way to acquire goods (i.e. as a transaction means) and so it would not be demanded if it wasn’t for the other role, as an easy to find store of value.

To appreciate further the sense in which the transaction motive is muted and instead highlight the role of the spending problem, notice that here, if the spending problem was addressed by giving an agent the chance to re-balance her money-goods holdings (similar to an end-of-period centralized market as in LW) she would spend everything to buy goods and leave with $m' = 0$. This is because capital goods pay higher dividends than money and, crucially, they can be consumed at any time.\(^{27}\) This does not mean that money does not have a transaction role: indeed money relaxes the transaction constraint (14) and if it

\(^{25}\) In this representative agent environment, for intertemporal bonds to be in positive supply they would have to be a liability of the government. But since there is no liquidity difference, the distinction between money and government bonds would be intangible. This can also be appreciated by the first order conditions for $m'$ and $a'$ in Appendix B; they imply an arbitrage Fisher equation that pins down $v$ so that money and bonds pay the same return. It would be possible to relax the assumption of perfect substitutability and distinguish between money and government bonds by assuming a mild search friction for bonds. The intertemporal bond highlights that this theory of money does not rely on the fact that agents are not allowed intertemporal credit. In fact, without insurance, the bond would be traded but it would still be in aggregate zero net supply, thus leaving space for money demand to store the remaining unmatched aggregate savings.

\(^{26}\) It could be argued that in real life one can invest unspent funds in shares, which are fairly easy to find. However, shares offer a complicated risk/return trade-off that does not suit everyone. Furthermore, if one buys shares of firms who hold cash (or who hold shares of other firms who hold cash) one is holding cash. In this model agents hold either money or capital directly, but one could conceive other decentralizations where firms hold capital and money, and issue shares. To make shares less liquid than money one could assume a mild search friction on shares.

\(^{27}\) Capital is introduced in a LW model in GMW, but agents are not allowed to consume it or trade it in the decentralized market. Lagos and Rocheteau (2008) find conditions under which coexistence of money and capital is possible even when capital can be used for transactions in the decentralized market. However, they assume that capital cannot be directly consumed in the decentralized market.
was not so then money would not have value. But this motive alone would not be sufficient to demand money if an agent had a way to circumvent the spending problem because, as mentioned, agents prefer capital. Instead, in models that have a more prominent transaction role, agents do not turn all money into goods even without the spending problem as is the case in the centralized market in LW.

This discussion highlights the importance of the spending friction to induce money demand. However, it should be noticed that households effectively choose the probability of finding goods $\psi$ (and thereby end-of period money holdings) by choosing $p$ and $q$, which determines market tightness given the equilibrium function $\theta(p,q)$. They can also choose $\psi \to 1$, so that $p_m m' \to 0$ and they are left with goods only. If households prefer goods to money, why would they willingly choose a low $\psi$ and therefore to hold money? Intuitively, to overcome the friction and have $\psi = 1$ is too expensive, so agents choose to leave funds unspent and store them in money. This portfolio choice is explained in detail next.

2.2.5 The portfolio choice of turning liquid assets into goods

The first order condition for $p$ illustrates the key trade-off in the decision of buying goods versus holding money. Focusing on an interior solution and assuming $B$ independent of $p$ for simplicity, the equation is

$$\frac{\partial \theta \cdot \partial \psi}{\partial p \cdot \partial \theta} \lambda_3 = \frac{\partial \theta \cdot \partial \psi}{\partial p \cdot \partial \theta} \lambda_4 p + (\lambda_2 + \lambda_4 \psi),$$

(18)

where $\lambda_1 - \lambda_4$ are the Lagrange multipliers on Constraints (13)—(16).

The left-hand-side shows the marginal gain: with a higher $p$, $\theta$ increases (it is shown later that $\theta$ is increasing in $p$ so that $\frac{\partial \theta}{\partial p} > 0$), this increases $\psi$ so that agents end up with more goods, thereby relaxing (15) as captured by $\frac{\partial \psi}{\partial \theta} \lambda_3$. However, with the increase in $\psi$ agents are left with less liquid funds, this tightens (16) as captured in the first term in the right hand side where $\lambda_4$ is the multiplier on (16). Finally, there is a last term in brackets which is positive. It follows that the net of the first two effects is positive and $\lambda_3 > \lambda_4 p$: put differently, the goods constraint (15) with multiplier $\lambda_3$ bites more than the financial constraint (16) with multiplier $\lambda_4$ so that agents prefer goods to money. If it was for these two effects only, they would put $\psi = 1$ so that all funds are turned into goods.\textsuperscript{28} However, there is the last term in brackets which captures the fact that to find more goods is hard: one has to increase $\theta$ which in turn increases the cost per unit of good (higher $p$): this tightens constraints (14) and (16). Because of this last term, it can be optimal not to turn all liquid assets into goods.

\textsuperscript{28}In a transaction framework those are all the effects so that agents must be at the margin indifferent between goods and money in a centralized market.
2.3 Firms

Firms can choose to open trading posts in any market identified by price and quantity. A trading post in market \((p, q)\) has a match with probability \(\phi(\theta(p, q))\), in which case it sells \(q\). To open a trading post, a firm needs production capacity \(Ak_d^\alpha n_d^{1-\alpha}\ge q\), where \(k_d\) and \(n_d\) are the capital and labour inputs. So a firm pays labour and capital costs whether or not a match is found.\(^{29}\) Notice that since to open a trading post carries input costs, there is no need to assume exogenous vacancy posting costs.

The assumption of Cobb-Douglas production function is not key. Any function with the neoclassical properties would work.

A trading post in market \((p, q)\) gives expected profits

\[
\pi(p, q) = \max_{k_d, n_d} \phi(\theta(p, q))pq - wn_d - rk_d
\]

s.t.

\[
q \le Ak_d^\alpha n_d^{1-\alpha}
\]

The first order conditions for capital and labour are

\[
\xi(p, q)\alpha A \left(\frac{n_d}{k_d}\right)^{1-\alpha} = r,
\]

\[
\xi(p, q)(1-\alpha)A \left(\frac{k_d}{n_d}\right)^\alpha = w.
\]

Where \(\xi(p, q)\) is the lagrange multiplier on the production constraint. By taking the ratio between these two first order conditions it is easy to see that \(\frac{k_d}{n_d}\) is the same in any trading post. Then \(\xi(p, q)\) is equal for all \((p, q)\). Thus it is going to be called \(\xi\) from now onward.

Using the 2 first order conditions, maximized profits can be written as

\[
\pi(p, q) = \phi(\theta(p, q))pq - \xi q
\]

Since firms can choose between different markets \((p, q)\), all potentially active ones must give the same profits. Furthermore, free entry implies that such profits must be zero: if

\(^{29}\)Without this assumption, a firm could open many trading posts and exploit the law of large numbers across them to have production capacity only for sales: \(Ak_d^\alpha n_d^{1-\alpha} = \phi(\theta(p, q))q\). Ruling this out implies some excess production capacity and an endogenous Solow residual. It is assumed that excess production capacity is not storable. This assumption seems reasonable for services and nondurables, which form the large majority of GDP. In future it may be interesting to allow for inventories, but to match its rich dynamics (e.g. procyclical inventory investment) the model should be complicated for instance by introducing S-s policies or stockout-avoidance motives; see Wen (2011) and Bai et al. (2019) for a recent analysis.
profits were positive there would be infinite posts and $\phi(\theta(p, q)) = 0$, which contradicts that profits are positive. Thus, any submarket $p, q$ visited by positive number of buyers has

$$
\phi(\theta(p, q))p - \xi \leq 0 \quad \text{and} \quad \theta(p, q) \geq 0,
$$

where the two inequalities hold with complementary slackness. I.e. if $\phi(\theta(p, q))p < \xi$ then $\theta(p, q) = 0$ because no trading post would open. If a submarket has trading posts ($\theta(p, q) > 0$) then $\phi(\theta(p, q))p = \xi$. Since $\phi \leq 1$, a necessary condition for a submarket to be active is that $p \geq \xi$. Otherwise, $\phi(\theta(p, q))p < \xi$ for any $\theta$; thus submarkets with $p < \xi$ never attract any trading posts. Finally, if a submarket does not attract any buyers then $\theta(p, q)$ can be arbitrary as long as it is such that $\phi(\theta(p, q))p \leq \xi$ (see Menzio et al. (2013) for a game theoretic “trembling hand” explanation). For markets with no buyers I assume that $\phi(\theta(p, q))p = \xi$ as long as $p \geq \xi$. Restricting this off equilibrium belief to a strict equality has no implications for the equilibrium but it is helpful because it implies that for $p \geq \xi$ and $q \geq 0$ the function $\theta(p, q)$ solves

$$
\phi(\theta(p, q))p = \xi. \quad (24)
$$

As a result, function $\theta$ has the properties in the following lemma.

**Lemma 1** For $p \geq \xi$ and $q \geq 0$

1. $\theta(p, q)$ is differentiable,
2. $\frac{\partial \theta(p, q)}{\partial p} > 0$,
3. $\frac{\partial \theta(p, q)}{\partial q} = 0$.

The proof in the Appendix follows from Equation (24) and the properties of the function $\phi$. Since the function $\theta$ does not depend on $q$, from now on it will be denoted $\theta(p)$. The intuition why $\theta$ does not depend on $q$ is that, since the production function has constant returns to scale and input prices are taken as given, production increases proportionally with costs. Then, for profits to be equal to 0 (and hence independent of $q$), $\phi$ and thereby $\theta$ have to also be independent of $q$.

Finally, it is worth discussing the issue that trading posts pay the input markets irrespective of whether they are in a match and make revenues. For this it is necessary to assume that the law of large numbers holds within each submarket so that there is perfect ex-post redistribution of revenues between trading posts in the same submarket. It should be noticed that this ex-post redistribution does not require firms to commit to an explicit insurance scheme between them because a single firm can open a large number of trading posts. See Menzio et al. (2013) for a similar discussion.
2.4 Equilibrium

It is convenient to first define equilibrium assuming that agents honour all obligations given the limits $B$ and $\bar{q}$, and then find the information and enforcement frictions that make this behaviour optimal.

Before defining an equilibrium, it is useful to pick a numeraire. Equation (24) pins down a relationship between $\theta$ and $p$ up to a value for revenues per unit of production $\xi$. $\xi$ is free and can be normalized. As a normalization, $\xi$ is equal to the equilibrium value of $\phi$. This implies $p = 1$ in the equilibrium submarket as is immediate from Equation (24).

Below there are formal definitions of a recursive monetary and a nonmonetary equilibrium given a debt limit function. In a monetary equilibrium it is also necessary to pin down the money growth rate, or equivalently the return on bonds $v$ as an exogenous policy instrument. The definition below uses the latter as the policy instrument.

**Definition 2 Monetary equilibrium assuming no default**

Let $B(p, \Omega, \exists)$ and $\bar{q}(p, \Omega, \exists)$ denote the credit limit functions where $\Omega$ is a vector of aggregate state variables: the aggregate capital and money stocks $K \geq 0$ and $M \geq 0$, as well as possible shocks (such as a technology shock $A$, a shock to the matching function, money growth, a preference shock etc.). Let $v(\Omega)$ be the government policy. A monetary equilibrium $\exists$ is composed of a value function $V$ and policy rules $c, k', m', a', q$ for the household as function of $a, k, m, \Omega$, a function $\theta(p; \Omega)$ and prices $w, r, p_m, v$, measure of firms $f$, input demands per firm $k_d$ and $n_d$, revenues per unit of production $\xi$, all functions of $\Omega$, such that:

1. Household: The household’s decisions and value function solve the problem in 2.2
2. Firms: $k_d, n_d$ and $\theta(p; \Omega)$ satisfy Equations (21), (22), and (24) with $\xi = \phi(\theta(1; \Omega))$.

---

30 For this one has to show that all other prices ($r, w$ and $p_m$) also change proportionally to $\xi$, so that no relative price is changed. It is immediate from Equations (21) and (22) that given an allocation, $r$ and $w$ are also proportional to $\xi$. Expressions for $r$ and $w$ and $p$ from Equations (21), (22) and (24), can then be substituted into the budget constraints —Equations (14) and (16)— to show that $p_m$ is also proportional to $\xi$. Since no constraint is changed, neither will the optimal choices and thus the equilibrium allocation.

31 Following Moen (1997), I assume that if agents are indifferent between multiple submarkets, only one will open. Furthermore, later I find conditions under which the household’s problem is concave so that the solution is unique.

32 First order conditions for $m'$ and $b'$, (29) and (30), imply an arbitrage condition: because bonds and money are equally liquid they must give the same real return. In steady state, the real interest rate on bonds equals the inflation rate $1/v - 1 = p_m/p_m' - 1$. This is a Fisher equation with the addition that, owing to the perfect substitutability between $m$ and $b$, the nominal interest rate is infinitely elastic at 0. By governing money growth, the government sets inflation and $v$. Equivalently the government can set a policy for $v$ and let money supply accommodate money demand.

33 The latter condition implies that the equilibrium $p$ is normalized to 1.
3. The liquid assets, and the inputs markets clear with $p_m > 0$:

$$m' = M + \Delta m, \quad a' = 0, \quad fn_d = 1, \quad fk_d = K.$$  

Furthermore, households and firms search in the same market: calling $q^d$ the quantity chosen by the household and $q^s$ that chosen by firms,

$$q^d = q^s.$$  \hspace{1cm}(25)

4. Aggregate and individual state variables are consistent: $k = K$, $m = M$.

5. The exogenous stochastic shocks follow Markovian processes of order one.

Notice that Equation (25) implies that households purchases are equal to firms sales $\psi(\theta)q^d = \phi(\theta)fq^s$. This is because the matching function has the property that $\psi/\phi = \theta$ and $\theta = f$ given that households are of measure 1.

**Definition 3 Non monetary equilibrium assuming no default**

A nonmonetary equilibrium is the same but with $p_m = 0$ and with $v$ endogenous.

Besides the fact that market clearing occurs with $p_m > 0$ in a monetary equilibrium and with $p_m = 0$ in a nonmonetary one, the other difference is that a monetary equilibrium depends on a policy for the price of bonds $v$. Instead, in a nonmonetary equilibrium $v$ is pinned down. Intuitively, $v$ is not endogenously determined in a monetary equilibrium because the supply of money is exogenous. In a nonmonetary equilibrium, the value of the supply of money is zero so $v$ must adjust endogenously to ensure that agents do not demand excess liquidity.

As it is well known, monetary economies also have non recursive monetary equilibria but this paper focuses on recursive ones.

### 2.4.1 Imperfect Monitoring and Limited Commitment

The household has to honour the debt, the terms of trade agreed ex-ante in the competitive search protocol, and the insurance mechanism. Here I show that the all these contracts are honoured under the limited commitment that is enforceable with imperfect monitoring, which is required to make money essential as pointed out by Kocherlakota (1998) and Wallace (2010). Agents can renege on any of the contracts mentioned above but the commitment is endogenized introducing participation constraints following Thomas and Worrall (1988), Kehoe and Levine (1993), and Alvarez and Jermann (2000) among others.

There are two assumptions which, although stated in the initial description of the set-up, it is worth repeating as they are key here.
(a) Agents caught red handed reneging one of their contracts pay their period obligations and lose access to any form of endogenous commitment. That agents pay their obligations if caught makes the value of default and being caught, always lower than the value of honouring all obligations. This is a key assumption to sustain full insurance with imperfect monitoring.

(b) Agents must provide proof of means before entering a submarket. Without proof of means one could pretend to be cash rich and go for a submarket with $q$ very large, and then default at the moment of paying.

With the two assumptions, the constructed strategies with full insurance, predetermined ToT, and credit with spending limits $B$ and $\bar{q}$ as in definitions 1 and 2 are incentive compatible, and, thus, are part of an equilibrium microfounded by imperfect monitoring and limited commitment.\(^{34}\) A detailed description of the off-equilibrium problems in case of default is included in Appendix C. However, the key intuition for the proposition below is simple and does not require reading the appendix: because of assumption (a), monitored default is always strictly worse than no default. As a result, it is always possible to find monitoring probabilities high enough, but smaller than one, such that it is incentive compatible to honour all obligations. The result is formalized in Proposition 1.

**Proposition 1 Equilibrium with endogenous obligations**

Take a solution that satisfies Equilibrium definition 2 or 3 given bounded $B$ and $\bar{q}$, there are probabilities $p_{T_{oT}},p_b,p_f < 1$ such that there are no incentives to default. The probabilities tend to 1 as $B$ and $\bar{q}$ tend to $\infty$.

Proposition 1 states that the decentralization with bounded limits $B$ and $\bar{q}$ satisfies the incentive constraints with imperfect monitoring. So there are no incentives to default under these limits. Notice that this approach does not restrict attention exclusively to the “not too tight” limits of Alvarez and Jermann (2000) i.e. those where the participation constraints hold with equality. As in Bethune et al. (2018), this approach also allows for lower limits where some or all the conditions hold with strict inequality. Why would the market restrict credit below the highest possible one? Bethune et al. (2018) show that tighter limits than the “not too tight” ones are consistent with perfect bayesian equilibrium. In particular,

\(^{34}\)Why $\bar{q}$ is needed? Suppose there was only a credit limit $B$, then one could go to a cheap market, get very high $q$ and then default. One could get around this issue by making $B$ function of $p$ and $m$. Absent the limit $\bar{q}$, $B$ would need to be function of $m$ because a possible deviation is to hoard cash, afford a market with high $q$, and default. However, to make $m$ fully observable would require more information than showing proof of means: one thing is to show one has a given amount of money, another is to prove that that is all the money one has.
Bethune et al. (2018) show that if firms follow a strategy that punishes default up to some limit, there are no incentives to deviate and offer higher $\bar{q}$ or $B$: any larger debt would be defaulted on because, under the equilibrium strategy, there would be no punishment from the other firms. Besides the fact that $\bar{q}$ and $B$ must be below the “not too tight” ones, a limitation to their shape comes from the fact that the buyer is anonymous, so $\bar{q}$ and $B$ should be functions of variables that are easily observable to the seller: hence in this paper I rule out limits that are functions of the individual state variables. In Section 2.2.3 they are functions of the aggregate state variables and of $p$, which is known by the seller and does not preclude the anonymity of the buyer.

That $\bar{q}$ and $B$ are flexible within the upper bound is convenient and later some examples are considered. Yet the microfoundation is not inconsequential. For instance, it matters for monetary policy: moving the interest rate $1/v-1$, the value functions change and, holding the probabilities constant, it may be necessary to change $B$ and $\bar{q}$ in order not to violate the incentive constraints. This has implications for optimal policy, see Berentsen et al. (2007), Araujo and Hu (2018), Gomis-Porqueras and Sanches (2013), and Lotz and Zhang (2016).

As mentioned, firms have incentives to default too but the resulting constraints can be ignored by assuming enough, albeit imperfect firm monitoring. This is explained next. Firms have incentives to default on the ToT (if $p$ is too low and $q$ too high) and on the payment to the inputs $wn+rk$. It follows that given the monitoring probability, all is needed to avoid firms default is a lowerbound on market tightness $\theta$. This is because $p$ is decreasing in $\theta$ while $q$ and $wn+rk$ are increasing in $\theta$. So a low $\theta$ brings all the aforementioned incentives to default. However, submarkets with low market tightness are also ruled out by households default incentives.\footnote{This is because $q$ is increasing in $\theta$ so the constraint on $q$ also restricts $\theta$.} It follows that if there is enough firm monitoring so that the constraint on $\theta$ that comes from the incentives of the firms is looser than the one that comes from the incentives of the households, one can solve the model ignoring the constraint of the firm. Since the emphasis in this paper is on the household, I make this assumption and only focus on the incentive constraints of the household. However, it is worth noting that one cannot simply assume perfect firm monitoring. In that case there would be scope for a firm to become a market maker that collects all the money from the buyers before the matching process, and then use the proceeds to pay the trading posts that are successful. If that was possible then the market maker could exploit the law of large numbers and ask just enough money to finance the deterministic number of matches, see Faig and Huangfu (2007). Instead, without the market maker there is more money demand because buyers can only use their own liquidity in a match and so each buyer needs more money or credit than with the market maker. One final worry is that when the monitoring probability of
the households tend to 1, for the incentive constraints of the firm to be looser than the one of the household, the monitoring probability of the firm must go to 1 as well. In such case the market planner becomes incentive feasible. However, its presence becomes irrelevant because as per proposition 1, with perfect monitoring, any level of obligation is enforceable. And it is shown later that in such case money is not essential.

3 Characterization

Since this is a new framework I start with finding conditions under which the constraint set faced by the household is convex to ensure a well behaved household problem.

3.1 Concavity of the household problem

That the constraint set is convex is not obvious because of the nonlinear terms $pq, \psi q, \psi pq,$ and the functions $\bar{q}(\cdot)$ and $B(\cdot)$ in the constraints (13)—(17). As shown in Section 2.4.1, the functions $\bar{q}$ and $B$ consistent with incentives to repay can take many shapes below an upper-bound. As a result, it is not difficult to pick functions that do not pose a problem for the convexity of the constraint set. So, for simplicity, here I focus on $\bar{q}$ and $B$ constant.

To ensure that the constraint set faced by the households is convex it is also necessary to restrict the matching function such that $1/(\phi \psi)$ is convex in $\theta$. Since $\psi$ and $\phi \in [0, 1]$, are monotone, one with positive and one with negative slope, this property is fairly natural: as $\theta$ approaches either zero or infinity $\phi \psi$ tends to zero, so its inverse goes to infinity. In between $\phi \psi$ is larger, so its inverse is smaller. As a result $1/(\phi \psi)$ tends to be u-shaped.

Notice that key to the argument above is that $\phi, \psi \leq 1$. While microfounded matching functions satisfy these inequalities (such as the urn ball one, which indeed exhibits a convex $1/(\phi \psi)$) such inequalities do not hold for some of the matching functions used in practice, such as the Cobb-Douglass. Indeed, a Cobb-Douglass $m = f^\gamma h^{1-\gamma}$ does not imply $1/(\phi \psi)$ convex for all parameterizations but requires $\gamma \geq 0.5$. The intuition for $\gamma \geq 0.5$ goes as follows. In choosing the submarket, buyers face the following trade-off: a low $p$ implies a low $\theta$ and thus a low $\psi$. If the price is too elastic to $\theta$, a reduction in $\theta$ reduces the price so much that buyers are likely to choose the submarket with minimum price i.e. a corner solution. Using Equation (24), it is easy to show that $1 - \gamma$ is the elasticity of $p$ to $\theta$. So a high $\gamma$ ensures a sufficiently low elasticity, which in turn ensures convexity of the budget constraint and thus an interior solution.

**Proposition 2** If $\bar{q}$ and $B$ be constant and $\psi(\theta)$ and $\phi(\theta)$ are such that $1/(\psi(\theta)\phi(\theta))$ is a convex function, then the constraint set (13)—(17) is convex.
Given the strictly concave preferences, the convexity of the constraint set ensures a unique and interior solution to the household’s problem and it is safe to use first order conditions, shown in Appendix B, to characterize optimal choices.

### 3.2 Coexistence

I will now show existence and uniqueness of monetary/nonmonetary equilibrium in steady state and for constant \( \bar{q} \) and \( B \) (and for underlying monitoring probabilities that make commitment under those constraints incentive compatible as in Proposition 1). The results, stated in Lemma 2 and Proposition 3, are explained below and illustrated in Figure 1. Readers that are not so interested in the technical details can jump to the text after Proposition 3 where Figure 1 is explained.

From Equation (14) it is clear that, since \( a = 0 \), an equilibrium is monetary \((p_m > 0)\) iff \( pq > B \). And since \( p \) is normalized to 1, one has to check that \( q > B \). Since \( q \leq \bar{q} \), it follows that monetary equilibrium requires \( \bar{q} \geq B \).

The strategy is this: given a price of bonds \( v > \beta \) look for a candidate solution that satisfies all equilibrium conditions other than (14). If the equilibrium candidate has \( q \geq B \), then that is indeed a monetary equilibrium. It is a monetary equilibrium because \( p_m m > 0 \) to satisfy (14), and (as shown in the proof of Proposition 3) all other equilibrium conditions are unaffected by \( p_m m > 0 \). If \( q = B \) then that is a nonmonetary equilibrium because \( p_m m = 0 \) satisfies (14) and in the proof of Lemma 2 it is shown that with \( v > \beta \) (14) cannot be slack. If \( q < B \) then the candidate solution is not an equilibrium because the value of money would have to be negative.

The strategy above works to the extent that given \( v \) there is a unique candidate monetary equilibrium \( q \), and this is shown in the following Lemma. The proof is quite tedious but conceptually it is done in a similar fashion as in Brock and Mirman (1972) by showing the existence of a unique solution to some key equilibrium conditions. In particular, the main difficulty is to show existence of a solution for \( \theta \). To this aim, Lemma 2 requires a technical assumption on the matching function: the elasticity \( -\frac{\phi}{\bar{\theta}} \frac{\partial \bar{\theta}}{\partial \theta} \) must be decreasing and larger than zero at \( \theta = 0 \) and below a certain bound for large \( \theta \). Intuitively, the restriction ensures that \( \phi \) does not go to 0 or to 1 too fast. To see this notice that holding constant \( \frac{\partial \bar{\theta}}{\partial \theta} \) the two limits are satisfied because \( \phi(\theta) \) is decreasing and \( \phi(0) = 1 \). Then to satisfy the limits it is required that \( \frac{\partial \bar{\theta}}{\partial \theta} \) does not go to 0 or to minus infinity too fast and in the opposite way than \( -\frac{\phi}{\bar{\theta}} \). It is worth stressing that the restriction on these elasticities are sufficient conditions, not necessary for the existence of an equilibrium.\(^{36}\)

\(^{36}\)A Constant Elasticity of substitution matching function satisfies these restrictions as long as there is not more substitutability than for the Cobb-Douglas case. To restrict complementarity
Lemma 2 Let $\theta = 0$, $B$ and $\bar{q}$ constants and monitoring probabilities smaller than one but high enough so that the incentive constraints in Section 2.4.1 are satisfied. Furthermore, let the elasticity $-\frac{\partial \theta}{\partial \theta}$ be continuous and decreasing with $\lim_{\theta \to 0} -\frac{\partial \theta}{\partial \theta} > 0$ and $\lim_{\theta \to \infty} -\frac{\partial \theta}{\partial \theta} < 2 + \beta/(v - \beta)$. Then for each $v > \beta$ there is a unique steady state equilibrium candidate that satisfies all equilibrium conditions other than (14). Furthermore, as long as constraint $q \leq \bar{q}$ is slack, the candidate equilibrium has $q$ strictly decreasing in $v$, independent of the limits $B$, and with $\lim_{v \searrow \beta} q = \infty$. The lemma also reports what happens as $v \to \beta$ when constraint (13) on $q$ is non binding.\(^{37}\) In this case $q \to \infty$. Intuitively, the cost of choosing a market with low price and high $q$ is low market tightness $\theta$, low matching probability $\psi$ and large liquid assets. But ending up with liquid assets is not a cost when their return approaches that of capital i.e. when $v \searrow \beta$. So, in this case, agents choose a market with price tending to zero and $q$ tending to infinity.\(^{38}\) That $q$ tends to infinity as $v \to \beta$ implies that there is monetary equilibrium for any bounded $B$. However, with monitoring probabilities that tend to 1, the highest possible ("not too tight") $B$ tends to $\infty$. So, while monetary equilibrium can be constructed (because the debt limit need not be the "not too tight' one), there is a nonmonetary equilibrium with the same allocation. In this sense, money is not essential with perfect monitoring, confirming Wallace (2010). Yet, from an empirical perspective, it is interesting that monetary equilibrium can be constructed even with arbitrarily large credit. Let $q(v)$ be the candidate $q$ given $v$ from the Lemma above. As mentioned, the candidate equilibrium is a monetary equilibrium, a nonmonetary one, or not an equilibrium, depending on (14). This is formalized in the proposition below.

Proposition 3 For each $v > \beta$, the candidate steady state in Lemma 2 is

- a monetary equilibrium with $q = q(v)$ if $q(v) > B$.
- a nonmonetary equilibrium with $q = q(v)$ if $q(v) = B$.
- not an equilibrium if $q(v) < B$.

above the Cobb-Douglas case seems economically meaningful because with more substitutability one can have matches with either only buyers or sellers.\(^{37}\)For this, the monitoring probabilities must tend to 1 so that incentives are satisfied not just for some $v$, but for any $v > \beta$: absent constraint (13) on $q$, a deflationary policy may violate incentives for given monitoring probabilities because, as shown next, it implies high $q$, thereby increasing the temptation to default on the ToT. That deflationary policies are limited by imperfect monitoring is reminiscent of Andolfatto (2013).\(^{38}\)It should be noted that $q$ is larger than transactions, which are equal to $\psi q$ and always bounded. That $q \to \infty$ when $v \to \beta$ would not hold if one added search costs proportional to $q$.\(^{27}\)
Furthermore, there is monetary equilibrium for any \( v \in (\beta, v^{NM}) \) where \( v^{NM} \) is the level of \( v \) for which there is nonmonetary equilibrium.

The proposition is illustrated in Figure 1 which plots \( q(v) \) (the increasing and convex function bounded at \( \bar{q} \)) and a given constant \( B \) (the horizontal line). The unique nonmonetary equilibrium given \( B \) lies at the intersection between \( B \) and \( q(v) \). Given \( B \), there is a monetary equilibrium for any \( v \in (\beta, v^{NM}) \). For monetary equilibrium, \( v \) must be smaller (or the real interest rate \( 1/v - 1 \) higher) than the one associated to the nonmonetary equilibrium \( v^{NM} \) because this way \( p_{mM} = q(v) - B > 0 \). There is no equilibrium for \( v > v^{NM} \) as in that case \( q(v) < B \) and the value of money would have to be negative. Intuitively, money is not demanded if its return is too low, or equivalently, if inflation is too high. So there is an upper limit on inflation, which is tighter the larger is the debt limit \( B \).

![Figure 1: Equilibria given B.](image)

The figure is shown for the case in which \( \bar{q} > B \). When \( \bar{q} < B \), it follows from the proposition that there is no monetary equilibrium.

### 3.2.1 Endogenous productivity, inflation, and velocity

In figure 1 \( q \) is decreasing in \( v \) and as \( v \to \beta, q \to \infty \) (provided that monitoring probabilities tend to 1 so that \( \bar{q} \) can go to infinity). Intuitively, \( v \to \beta \) is the Friedman Rule in which case inflation is negative and money gives the same return as capital. As a result there is no private cost of not finding goods and holding high liquidity so agents choose a market with low \( \theta \) and low \( \psi \). Instead, inflation prevents \( \theta \) from going to its efficient lowerbound.
because it makes it privately costly to hold money. This leads agents to choose markets with higher $\theta$ so that there is a higher probability of finding a match and spend the money. However, the higher the matching probability for a buyer, the lower it is for a seller. As shown studying the planning problem, low $\theta$ is efficient because it implies high $\phi$, which can be interpreted as firms productivity: through the lens of a neoclassical model, a high $\phi$ would be captured by a higher TFP parameter. So in this model the Solow residual is endogenous and higher, the lower the inflation rate or $v$.$^{39}$

The relationship between market tightness $\theta$ and endogenous productivity highlights the role of demand in this model: there is a social benefit if households search in crowded markets (where the ratio of households per trading posts is high), because the higher the demand for each trading post, the less production is unutilized.

In this model also velocity is an endogenous function of $\theta$. Indeed the velocity of broad money (i.e. money plus credit such as M1, M2, MZM) in this model is $\psi$. This is because velocity is defined as output over broad money, which in this model is equal to $q$. Since production is equal to $c + k' - k(1 - \delta)$, it follows from (15) that velocity is $\psi$. Intuitively, a change in velocity is equivalent to a change in spare spending capacity (i.e. a change in $\psi$). That velocity is endogenous is interesting because it is hard to get velocity to move at all in monetary models. Furthermore, Mennuni (2021) builds on this theory and finds it to be promising to explain how velocity moves over the business cycle.

### 3.3 Monetary equilibrium with credit above income

It is now possible to appreciate better a key point of the paper, i.e. the extent to which money is robust to credit. To this aim it is useful to relate $q$ to income. From Equation (16), and using the fact that in equilibrium $p = 1$, $a = a' = 0$ and $m' = m + \Delta m$ one gets

\[
q = \frac{(w + rk)}{\psi}.
\]  

(26)

Since $\psi < 1$, the equation above shows that $q$ is larger than income $w + rk$, which in turn is equal to aggregate transactions. And since monetary equilibrium exists if $B < q$, there are levels of $B$ larger than income or transactions for which there is monetary equilibrium.

In more intuitive terms, in equilibrium aggregate spending must equal aggregate income. As a result, if buyers matching probability $\psi$ is smaller than 1 so that not all agents get to spend, then those in a match must spend more than their income. It follows that, for an equilibrium to exist, agents need spending capacity (money + credit) in excess of

$^{39}$It should be noticed that the constrained efficient allocation with the incentive constraints may involve some inflation as in Araujo and Hu (2018). Optimal monetary policy is left for future research. Next, I characterize the decentralized economy.
their individual income. As a result, money demand is positive even when credit is larger than individual income, but smaller than spending. Notice that this result comes from the general equilibrium requirement that spending is equal to income in the aggregate, which is true with or without insurance. Furthermore, this result is true also outside steady state and for any function $B$ that has equilibrium: whenever $B < q$ money has value from (14). This result is important because with $B$ larger than income, it is possible to have outside + inside money larger than GDP, as in the data.

Moreover, since as shown in Lemma 2 \( \lim_{v \downarrow \beta} q = \infty \), there can be monetary equilibrium for any bounded constant $B$ (provided that there is enough monitoring to satisfy incentives not to default). So there can be monetary equilibrium as long as credit is not infinite. As mentioned, this latter result requires deflation and would not hold if agents had to pay vacancy costs proportional to $q$. In that case $q$ would be finite for any $v$. Vacancy costs would not change the result that there is monetary equilibrium with credit larger than income as from (26), since $\psi \leq 1$, $q \geq wn + rk$.

Since the result is a key point of the paper, it is summarized in the following corollary.

**Corollary 1** For each $v > \beta$, the maximum credit limit consistent with monetary equilibrium is larger than transactions and equal to $q(v)$, which is decreasing in $v$ and tends to infinity as $v \rightarrow \beta$.

An intuition for why money is robust to credit larger than income goes as follows. Because of the search friction some agents do not buy goods, yet they make income and want a store of value so that they can spend it in the future. There are 2 options: either to lend to other buyers, or to buy the fiat asset. If the borrowing limit is large enough so that the buyers in a match can borrow not just their own income, but also that of the agents that did not make a match, then there is no need for a fiat asset. This requires the buyers in a match to borrow $q$. If $B < q$, then not all the income of the unmatched agents can be lent so there must be another asset in positive net supply which on the one hand stores the savings of the unmatched and on the other gives to the agents in a match the purchasing power to spend for the income of the agents not in a match.\(^{40}\)

To summarize, in equilibrium aggregate income equals aggregate spending. But since

\[^{40}\text{To see why all aggregate income is spent via credit when agents borrow } q \text{ rewrite (26) as}
\]

\[q = (w + rk) + (w + rk) \frac{1 - \psi}{\psi}.
\]

By borrowing $q$ each agent borrows against her own income $wn + rk$ plus all the income of the unmatched $(1 - \psi)(w + rk)$, divided by the number of matched agents $\psi$. When $B < q$ it is still the case that all income must be spent in equilibrium. Money gives to the agents in a match the purchasing power to spend for the income of the agents not in a match.
some agents do not get to spend their income, those in a match need means to spend more than their income. So the key is that agents make income but may not find spending opportunities, thereby ending up with “unwanted” liquid savings. This is not the case in LW: the CM makes it such that there are no “unwanted” liquid income savings and, through a period (DM+CM), each agent spends exactly their income. In Shi (1997), the big family assumption is such that each family always has income equal to spending. As a result, in these frameworks, credit equal to income is enough for money not to have value.

3.3.1 An example

An example helps illustrate how useful money is in this economy. From Figure 1 it is evident that, given a borrowing limit, monetary equilibrium has higher $q$ and lower $v$ (or higher real interest rate $1/v - 1$) than the nonmonetary equilibrium. Production is also smaller in a nonmonetary equilibrium relative to a monetary one: if buyers cannot store value in the form of the liquid asset, they choose markets where it is inefficiently too easy to buy goods, but this hinders firms ability to sell goods leading to a lower production. To illustrate this mechanism and show how poor an allocation can be in a nonmonetary equilibrium Proposition 4 covers a special case, when the borrowing limit is proportional to end-of-period income, $B \equiv \gamma(w + rK)$.

Of special interest is when $\gamma = 1$. This is a hedge case with a result that is perhaps surprising: in a nonmonetary equilibrium with $\gamma = 1$, production is zero. The result should become intuitive once one recalls what argued in Section 3.3, that those in a match must spend more than their income for markets to clear when $\psi < 1$. Then, if credit allows each buyer to only spend their end-of-period income, a non monetary equilibrium must have $\psi = 1$. But this requires $\theta = \infty$. As a result, firms face $\phi = 0$ and do not demand inputs.\footnote{Why do not agents choose a submarket with $\theta < \infty$ so that $\phi > 0$ and firms can produce? This would reduce the chance to be in a match $\psi$ but at least, once in a match agents would buy something. The problem is that once in a match, for markets to clear, they would need to spend $pq = (w + rK)/\psi$ which with $\psi < 1$ is larger than $B$, so they cannot afford it (and they cannot renegotiate the terms of trade). There are incentives to produce once at a trading post but competitive search assumes commitment to the terms of trade determined before the match, nomatter the ex-post incentives (this commitment is endogenously enforced with enough monitoring as discussed in Section 2.4.1). Furthermore, the input markets open before the goods market so inputs are predetermined at the matching state and if firms did not hire, there cannot be production.}

The result highlights how dramatically bad the nonmonetary allocation can be without enough credit relative to a monetary equilibrium where, for the same credit limit, production takes place thanks to the extra monetary liquidity. For production to take place in a nonmonetary equilibrium $\gamma$ must be larger than 1. And if $\gamma < 1$ there is no nonmonetary equilibrium because the liquidity provided by credit is insufficient for an equilibrium without
the further liquidity provided by money. Intuitively, with $\gamma < 1$ credit $B$ is always below $q$ so $p_m m$ must be positive from (14).

**Proposition 4** Let $B \equiv \gamma(w + rK)$.

*If $\gamma > 1$ there is nonmonetary equilibrium with production.*

*If $\gamma = 1$ no production takes place in a nonmonetary equilibrium.*

*If $\gamma < 1$ there is no nonmonetary equilibrium.*

To paraphrase the result, liquidity (money + credit) must be sufficiently large to sustain production. The proposition highlights how thirsty of liquidity this economy is. In comparison, in transaction models where financial transaction is at most equal to income (e.g. Lucas and Stokey 1987 and Svensson 1985) with $\gamma \geq 1$ the economy is saturated with liquidity and there is no monetary equilibrium.

### 3.3.2 Other results

The model also has many implications that are in common with the new monetarist approach. In particular, it can be shown that money is neutral but not superneutral, that if $B$ is “lump sum” in the sense that it is independent of the households inputs, then credit has no implications for the steady state as in GMW. While these results are not new, it seems worth mentioning that they hold in this somewhat different environment.

### 4 Accounting for monetary aggregates

In 2020.II M1 over quarterly GDP was 2.63; the ratio was 3.62 for either M2 or M3, and 4.19 for MZM. Furthermore, inside money alone (or credit), were larger than GDP. This is because at that time the monetary base (or outside money) was equal to 1.02 times GDP. So inside money measured, for example, through M2, was $2.6 = 3.62 - 1.02$ times quarterly GDP. That money and credit are larger than GDP is not just true during the pandemic. Prior the Covid-19 crisis (e.g. 2017.III), inside money M2–MB was $2.80 - 0.79 = 2.01$ times GDP. It is possible to have monetary equilibrium with so much liquidity and when credit alone is larger than transactions? It has been shown theoretically that there is monetary equilibrium for any bounded credit, so the answer is yes. Below it is shown briefly how the model must be parameterized to match these data.

In this model $p_m M$ is outside money and $B$ is equal to inside money (a liability of the private sector used as a medium of exchange, see Lagos 2008) so total liquidity, or inside and

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42The data on the monetary stocks come from the Board of Governors of the Federal Reserve System (US), retrieved from FRED, Federal Reserve Bank of St. Louis on February 24, 2021.
outside money together, is \( p_m M + B \). From (14), \( q = p_m M + B \).\(^{43}\) Furthermore, from the matching function and the fact that sales are equal to GDP, \( \psi = GDP/q \), so it is possible to parameterize \( \psi \) to match the ratio between GDP and a measure of total liquidity such as M1, M2, M3, or MZM.\(^{44}\) Then it is possible to parameterize \( B \) to the amount of inside money to match both credit and outside money from (14) so that \( q = p_m m + B \) where \( m \) is the monetary base. Such an equilibrium is monetary because \( B < p_m m + B = q \) and it has been argued that whenever \( B < q \) the equilibrium is indeed monetary.

This procedure shows how to parameterize the model so that there is monetary equilibrium with the observed amount of money and credit. However, it should be emphasised that this requires to pick the probability of finding goods \( \psi \) (or the parameters of the matching function) accordingly: the smaller \( \psi \) the larger liquidity over output. For example, to match a very broad measure such as MZM in 2017 one needs \( \psi = GDP/(p_m M + B) = 0.33 \). One might worry that the number implies an unrealistically too severe search friction. However, not all MZM is as liquid as cash so it seems unfair to treat it as the inside money in this paper, which is perfectly substitutable to cash. Accordingly, one may find a narrower aggregate a more appropriate target, leading to a higher \( \psi \): e.g. to have \( \psi = 0.9 \), the monetary aggregate must be 1.1 times quarterly GDP; that number lies between M1 and M2 (e.g. M1 and M2 were respectively 0.71 and 2.8 times quarterly GDP in 2017). Furthermore, to match a broad measure of money with a milder search friction, one could add search effort (or other impediments to spending), thereby inducing a low \( \psi \) not because of a very low efficiency of the matching function that impedes finding spending opportunities, but because of agents’ decisions to economize on search effort. In quantitative extensions of this model, it would also be possible to add goods with very little or no search friction at all that are demanded in bounded amounts, such as food and other daily needs, or to go in the opposite direction and add financial assets that are not as liquid as money, such as government bonds, or even physical assets that are as illiquid as housing.

On top of matching monetary levels, ongoing work suggests that the model can also match its movements and offers a promising explanation of business cycle fluctuations through the wedge in the matching friction. In particular, movements in velocity are notoriously hard to match but in a companion paper, Mennuni (2021), it is shown how a shock to the matching function moves \( \psi \) and therefore GDP over liquidity, or velocity. At the same time a matching shock moves firms matches over production capacity, or measured TFP. The endogenous movements in TFP generate the usual real business cycle comovements. This

\(^{43}\)Equation (14) is always binding in a monetary equilibrium outside the Friedman rule.

\(^{44}\)As done in Mennuni (2021) one should calibrate the parameters of the matching function so that the equilibrium \( \psi \) is the desired one. But many functional forms can be calibrated to give the desired equilibrium value.
way the model accounts for the large movements in velocity, capacity utilization, as well as the usual comovements in the real variables. That the model accounts for large credit and monetary levels, and the business cycle fluctuations of real and monetary variables, make it a promising framework for policy analysis.

Finally, another advantage of this model is that it is possible to match monetary aggregates while assuming a shorter length of a period such as a day, a week, or a month. A shorter period length than a quarter is arguably more appropriate to study monetary issues; the relevant period length on which to measure the flow of transactions that need to be financed with the existing stock of money is arguably fairly short and related to time needed to liquidate the next most liquid assets other than money. But in models where credit must be smaller than transactions this is an issue because, in order to have monetary equilibrium, it is necessary to take a time horizon such that the flow of transactions is larger than the stock of liquidity so that the transaction constraint binds. With the data above, this requires a period length much larger than a quarter (so that GDP over $p_m M + B$ is larger than 1). The present model is robust to shorter periods provided that the matching probability $\psi = GDP / (p_m M + B)$ is adjusted to account for the smaller flow of transactions. But that the matching probability is an increasing function of time is natural with a search friction. Similarly to models of labour search where unemployment is a stock and job creation and destruction are flows, to the extent that the probability of finding goods is proportional to the time length of a period, calibration to a shorter period length involves equivalent stands on the underlying search friction.

5 Conclusions

Motivated by the observed amounts of monetary holdings in times when several means of payment do not require to hold liquid funds, this paper developed a theory of liquid assets as a way to store unspent available funds that emerge because of a search friction in the goods market. Monetary equilibrium is consistent with large availabilities of credit, which is key to match the levels of broad monetary quantities we observe.

Money also enhances productivity and welfare: when people are less worried about not finding goods because they have easy alternative means to store value, they search for better deals (lower prices but longer queue length) making firms more productive. Put differently, the presence of money increases aggregate demand relative to aggregate supply. These implications are different than those of other monetary models.

While the story has elements of popular narratives, the framework is novel and could have many uses. For instance, here liquidity can be extended to a larger set of assets differing in
their liquidity (captured by the severity of the search friction which stands for differences in information acquisition costs, risk, maturity etc.) reflecting empirical counterparts ranging from government bonds, equity shares and other financial products, to possibly far less liquid assets such as houses.\footnote{An attractive feature is that liquidity premia are endogenous: agents choose assets trading off their liquidity and their return so that in equilibrium the more liquid the asset the lower its return.}

Furthermore, this model may help understand further the role of the financial sector. In particular this framework may suggest an alternative to the view that the financial crisis and the prolonged stagnation are due to credit constraints. In a sense, the large literature that exploits credit constraints building on Bernanke and Gertler (1989) and Kiyotaki and Moore (1997) works in the opposite way to that in this paper. In these models during recessions firms wish to produce more but are constrained. Here instead, firms do not wish to produce more because of their difficulties to sell. These two channels are possibly both real but have different policy implications. In particular, this paper may explain why policies aimed at increasing liquidity may not always work. Easing credit conditions can be effective in models with credit constrained firms but it may not be as effective in this model where, owing to the matching friction, there can be a lack of “appetite” even when there is enough liquidity.

The framework may also be useful to study fiscal policy, for instance: since TFP is affected by the demand-supply ratio, government spending could increase TFP.

Finally, it is well known that in real life, a lot of liquidity is held by firms and corporations but in this model all assets are held by the household.\footnote{To get a sense of the magnitudes, the NYT Magazine reported in 2016 that with its liquidity, Google alone could buy outright companies of the likes of Uber, Goldman Sachs, or American Express, Davidson (2016). Overall, data from the Federal Reserve flow-of-funds show that in 2016, American businesses had $1.9 trillion in “cash”. That is about 40% of quarterly US GDP.} It would be interesting to study an extension of this model with a different decentralization where households buy consumption goods while firms trade capital subject to severe search frictions reflecting low arrival rates of big investment opportunities such as takeovers, thereby generating large holdings of liquid assets. In this context, households and firms may also prefer different types of assets reflecting their different needs for liquidity. Accounting for the liquid assets in the hands of firms may also contribute to the growing literature that focuses on the macroeconomic implications of frictions in the production sector, such as Bigio and La’O (2020) among others.

\textsuperscript{45}An attractive feature is that liquidity premia are endogenous: agents choose assets trading off their liquidity and their return so that in equilibrium the more liquid the asset the lower its return.

\textsuperscript{46}To get a sense of the magnitudes, the NYT Magazine reported in 2016 that with its liquidity, Google alone could buy outright companies of the likes of Uber, Goldman Sachs, or American Express, Davidson (2016). Overall, data from the Federal Reserve flow-of-funds show that in 2016, American businesses had $1.9 trillion in “cash”. That is about 40% of quarterly US GDP.
References


Appendices

A Exchange and clearance of bonds

At the end of a period each agent $j$ has a net position of intratemporal bonds equal to the ones received by firms (and issued by some other household) less those she issued to a firm. Call $\hat{b}_j$ the bonds issued by agent $j$ and $\hat{b}_{-j,j}$ the bonds agent $j$ receives by firms (the indexes $-j,j$ emphasise that the bond is issued by some household other than $j$, and passed on to $j$). So agent $j$’s net position is $\hat{b}_{-j,j} - \hat{b}_j$. The sum of all households’ net positions is $\sum_j \hat{b}_{-j,j} - \sum_j \hat{b}_j$. Since firms pass to households all bonds they receive $\sum_j \hat{b}_{-j,j} = \sum_j \hat{b}_j$, i.e. the intra-temporal bond market clears. Similarly, at the end of the period the household has intertemporal bonds issued in the previous period by some other household $\hat{a}_{-j,j}$ less those she issued $\hat{a}_j$ and $\sum_j \hat{a}_{-j,j} = \sum_j \hat{a}_j$. If the net credit position of an individual agent is negative (positive), she can pay (be paid) in money or by issuing (accepting) the intertemporal bond. Since the intertemporal bond is issued by an agent and accepted by an other, $\sum_j \hat{a}_j = 0$.

B First order conditions of the household

Let $\lambda_1, ..., \lambda_4 \geq 0$ be the lagrange multipliers on the constraints (13)—(16) and let $\lambda_{k'}, \lambda_{m'}, \lambda_{a'}, \lambda_q, \lambda_p, \lambda_\theta \geq 0$ be the multipliers respectively on $k' \geq 0$, $p_m m' \geq 0$, $a' \geq a$, $q \geq 0$, $p \geq 0$, $\theta \geq \theta$, with complementary slackness between each multiplier and the respective constraint. The households first order condition for $c, k', m', a', q, p$ are

$$u_c = \lambda_3 \tag{27}$$

$$\lambda_3 - \lambda_{k'} = \beta E (\lambda_4' + \lambda_3' (1 - \delta)) \tag{28}$$

$$\lambda_4 p_m - \lambda_{m'} = \beta E ((\lambda_2' + \lambda_4')p_m') \tag{29}$$

$$\lambda_4 v - \lambda_{a'} = \beta E (\lambda_2' + \lambda_4') \tag{30}$$

$$(\lambda_3 - \lambda_4 p) \left( \psi + \frac{\partial \psi}{\partial \theta} \frac{\partial \theta}{\partial q} q \right) = \lambda_1 + \lambda_2 p - \lambda_q - \lambda_\theta \frac{\partial \theta}{\partial q} \tag{31}$$

$$\lambda_2 + \lambda_4 \psi = \frac{\partial \psi}{\partial \theta} \frac{\partial \theta}{\partial p} \left( \lambda_3 - \lambda_4 p \right) + \left( \lambda_p + \lambda_\theta \frac{\partial \theta}{\partial p} \right) \frac{1}{q} \tag{32}$$

These first order conditions and the other conditions listed in the definitions of equilibrium (2 or 3 depending on whether the equilibrium is monetary or not) fully characterize equilibrium.
There is an off equilibrium result which is interesting, but which unfortunately complicates the analysis somewhat: agents that defaulted and were monitored lose access to credit as they would ex-post default on it. However, as shown below, in this setting a limited amount of insurance and access to submarkets with low prices are incentive compatible even when there is no punishment. This is because an ex-post exchange of money for goods is a quid pro quo exchange that benefits both matched and unmatched agents. So agents that have defaulted may retain limited access to the futures market and to some submarkets. This contrasts models where a full absence of trade —i.e. autarchy— in the default state is a subgame perfect Nash equilibrium, so one can simply assume autarchy as an outside option for who defaults. Instead, since here, even with no enforcement, there are incentives to trade, autarchy is not a sub-game perfect Nash equilibrium and the default state must be studied more carefully. To highlight that this default regime is not a classic autarchic state, it is called anarchy. The name comes from the fact that there is no endogenous enforcement in this regime and yet some mutually beneficial trading opportunities remain available.

Renegotiation of the ToT in case of default. If an agent that entered a submarket \( p, q \) and found a match refutes to pay the agreed ToT, there is a bargaining which results in \( q \) being exchanged at unit price \( \bar{p}(q) \). \( \bar{p}(q) = 0 \) if the buyer has all the bargaining power and \( \bar{p}(q) \in (0, p_m m] \) otherwise. As a result, buyers that are in anarchy and therefore cannot commit, can only choose submarkets \( p, q \) with \( p \leq \bar{p}(q) \).

The problem in case of anarchy. An agent that defaults and is detected goes into anarchy with value \( v^a(\cdot) \). The problem is:

\[
v^a(k, m, \Omega) = \max \psi(\theta(p))[u(c_e) + \beta Ev^a(k'_e, m'_e, \Omega')](1 - \psi(\theta(p))) [u(c_u) + \beta Ev^a(k'_u, m'_u, \Omega')] + (1 - \psi(\theta(p))) [u(c_u) + \beta Ev^a(k'_u, m'_u, \Omega')]
\]

s.t.

\[
pq \leq p_m m \tag{34}
\]

\[
c_e + k'_e - k(1 - \delta) \leq q - (1 - \psi)q_f, \tag{35}
\]

\[
p_m m'_e \leq p_m m + w + rk - pq + p(1 - \psi)q_f, \tag{36}
\]

\[
c_u + k'_u - k(1 - \delta) \leq \psi q_f, \tag{37}
\]

\( q \) is always exchanged because it is sank. The unit price is a function of \( q \) because the payment may be a nonlinear function of \( q \). \( \bar{p} \) is also a function of the state variables, but for ease of notation, they are omitted.
\[ pm'm_u' \leq pm'm + w + rk - pqf, \]  
\[ p \leq \bar{p}, \]  
(38)  
(39)

As well as inequalities (17) and the incentive constraints (40)–(41) below.

The choice variables are \(c_e, c_u, k'_e, k'_u, m'_e, m'_u, p, q, q_f\). \(q_f\) is the quantity agreed in the underlying futures contracts as detailed in Appendix 2.2.2. There are 2 differences relative to the problem in Section 2.2.3. First, credit is not available. Second, full insurance \((q_f = q)\) may not be attainable because there is a lower threat in case of default. Yet, some insurance is possible in this anarchic regime because there are gains from a quid pro quo trade \((q_f\) for \(pq_f\)) between agents that like the same variety and that had different fortunes in the goods market which left one goods-rich, and the other one cash-rich. As a result of the lack of full insurance, outcomes are contingent on whether a match is found or not: \(c_e \neq c_u\) etc. Subscripts \(e\) and \(u\) indicate whether a match is found or not.

The incentive constraints are

\[ u(c_e) + \beta E v^a(k'_e, m'_e, \Omega') \geq \hat{v}^a(k, m, p, q, e, \Omega), \]  
(40)

and

\[ u(c_u) + \beta E v^a(k'_u, m'_u, \Omega') \geq \hat{v}^a(k, m, p, q, u, \Omega), \]  
(41)

where \(\hat{v}^a(k, m, p, q, j, \Omega)\) solves the problem of a buyer without insurance, in a match or not depending on \(j = \{e, u\}\).\(^{48}\) This function takes \(p, q\) as explicit inputs because they are chosen in advance, whether a match is found or not.

\[ \hat{v}^a(k, m, p, q, j, \Omega) = \max_{c, k', m'} [u(c) + \beta E v^a(k', m', \Omega)] \]  
(42)

s.t.

\[ c + k' - k(1 - \delta) \leq qI(e), \]  
(43)

\[ pm'm' \leq pm'm + w + rk - pqI(e). \]  
(44)

where \(I(e) = 1\) if \(j = e\) and \(I(e) = 0\) otherwise.

It is now possible to study the default decision of an agent that is in good standing. Since in equilibrium with full insurance \(a\) is not traded, below I impose \(a' = 0\) (or, to be sure it does not bind in equilibrium, \(a' \in [-a, a]\) with \(a > 0\) arbitrarily small). This restriction is optimal as, with perfect insurance, only someone that plans to default may want \(a' \neq 0\). So

\(^{48}\)To assume no insurance as an outside option is common in the literature but in principle, here too, one could let the two parties renegotiate. Then the outside option would be higher than \(\hat{v}^a\). This would reduce insurance in anarchy and therefore relax the incentive constraints in case of no past default. Then the result of this section (that all contracts are incentive compatible) would become even easier.

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limiting \( a' = 0 \) is optimal as it reduces the value of default, thereby relaxing the incentive constraints.\(^{49}\)

The problem in case of no previous detected default. Let \( v(p, q, e, 1) \) and \( v(p, q, u, 1) \) be the values of honouring all obligations in submarket \( p, q \) and given that a match was found (\( e \)) or not (\( u \)). The last input set to 1 indicates that all obligations are being honoured. \( v(p, q, e, 0) \) and \( v(p, q, u, 0) \) are the values in case of default before knowing whether default is detected or not. The beginning of the period state variables are omitted for ease of notation.

For any level of the state variables \( k, m, a, \Omega \), there are no incentives to default if

\[
E_j(v(p^*, q^*, j, 1)) \geq E_j(v(\hat{p}, \hat{q}, j, 0)),
\]

where \( E_j \) is the expectation on whether \( j = e \) or \( u \); \( p^*, q^* \) are the optimal choices in case of no intended default (made solving the problem in section 2.2.3), and \( \hat{p}, \hat{q} \) are the optimal choices in case the agent intends to default; they solve \( \max_{p,q} E_j(v(p, q, j, 0)) \) s.t. (13) — (17) with \( a = a' = 0 \). Condition (46) is the key incentive restriction, the remainder of this section is all about understanding how it restricts the contracts.

Condition (46) depends on \( v(p, q, j, 0) \); to characterize it one has to establish what is the best default strategy: an agent in a match (state \( e \)) could default on the ToT, on debt, and on the futures. The value of default for an agent in a match is

\[
v(p, q, e, 0) = \max_{i_T^T, i_b, i_f} (\pi(i_{T^T}, i_b, i_f) v^m(p, q, e, i_{T^T}, i_b, i_f) + (1 - \pi(i_{T^T}, i_b, i_f)) v^{nm}(p, q, e, i_{T^T}, i_b, i_f))
\]

\( i_{T^T}, i_b, i_f \) take values 0 for default and 1 for non default, respectively on the ToT, debt, and the futures. So, for example, if \( \{i_{T^T}, i_b, i_f\} = \{0, 1, 0\} \) there is default on the ToT and on the futures.

With probability \( \pi(i_{T^T}, i_b, i_f) \) default is detected, all contracts are enforced, and the value comes from the “monitored default value function” \( v^m \) which solves:

\[
v^m(p, q, e, i_{T^T}, i_b, i_f) = \max_{c, k', m', \Omega} (u(c) + \beta E v^a)
\]

\(^{49}\)Yet, similarly to Bethune et al. (2018), \( a' = 0 \) is not the “not too tight” debt limit. A necessary condition for an equilibrium with no default is that

\[
V(k, m, a, \Omega) \geq v^a(k, m, \Omega).
\]

With \( a = 0 \) the last inequality cannot bind as the choice set of the agent that defaults is a subset of the one faced by the agent that did not default.
s.t. (15) and (16) with $a = a' = 0$. In the latter problem the future value function is $v^a$ because the agent moves to the anarchic regime; the state variables are omitted for ease of notation.

With probability $1 - \pi(i_{ToT}, i_b, i_f)$ the agent goes undetected, and the value comes from the “unmonitored default value function” $v^{nm}$ discussed below.

Since the purpose is to show that the contracts are enforceable, here it is assumed that if one defaults on the futures in state $e$ then she does not transfer any of the good and if the state is $u$ then she does not transfer any of the money. However, the other party honours her transfer. This way the value of default on the futures is the highest, making the results more robust: less benign assumptions for the defaulter would make it easier to enforce insurance. $v^{nm}$ in state $e$ solves:

$$v^{nm}(p, q, e, i_{ToT}, i_b, i_f) = \max_{c, k', m'} \left[ u(c) + \beta E v(k', m', \Omega') \right]$$

s.t. (15) and (16) with $a = a' = 0$.\footnote{The constraints are those of the problem with no default because in this case in which default is monitored, all contracts are enforced (assumption a).}

To digest (51) it is useful to consider a few examples. If $i_{ToT} = 0$ and $i_b = 1$ the right hand side becomes $p_m m' + w + rk + (1 - \psi)pq - i_{ToT} i_b pq + (1 - i_{ToT})(-\tilde{p}q) + (1 - i_b)(-pq + B)$. If $i_{ToT}$ and $i_b$ are both 1 then the constraint is equivalent to (16). The remaining cases should be easy to derive.

Agents in state $u$ have a similar problem but can only default on the futures. This is because, not being in a match, they cannot default on the ToT or borrow intratemporally from the firm. So the value of default is

$$v(p, q, u, 0) = p_f v^m(p, q, u) + (1 - p_f)v^{nm}(p, q, u)$$

With slight abuse of notation, the last 3 inputs of $v^m$ and $v^{nm}$ indicating the default strategy are omitted because in state $u$ default can only mean $i_f = 0$. $v^m$ in state $u$ solves:

$$v^m(p, q, u) = \max_{c, k', m'} u(c) + \beta E v^a$$

s.t. (15) and (16) with $a = a' = 0$.\footnote{The constraints are those of the problem with no default because in this case in which default is monitored, all contracts are enforced (assumption a).}
\(v^{nm}\) in state \(u\) solves:

\[
v^{nm}(p, q, u) = \max_{c, k', m'} [u(c) + \beta Ev(k', m', \Omega')]
\]  

s.t.

\[
c + k' - k(1 - \delta) \leq \psi q,
\]

\[
p_m m' \leq p_m m + w + \tau k.
\]

From the constraints above, the agent receives the goods’ transfer \(\psi q\) but does not honour the payment \(p\psi q\) prescribed by the futures contract.

It is now possible to see how the conditions above restrict the contracts. Below I list sufficient conditions for (46) to hold:

\[
v(p^*, q^*, e, 1, 0) = \pi(0, 1, :)v^m(\hat{p}, \hat{q}, e, 0, 1, :) + (1 - \pi(0, 1, :))v^{nm}(\hat{p}, \hat{q}, e, 0, 1, :),
\]

\[
v(p^*, q^*, e, 1) \geq \pi(1, 0, :)v^m(\hat{p}, \hat{q}, e, 1, 0, :) + (1 - \pi(1, 0, :)v^{nm}(\hat{p}, \hat{q}, e, 1, 0, :),
\]

\[
v(p^*, q^*, 1) \geq pf v^m(\hat{p}, \hat{q}, e, 1, 1, 0) + (1 - pf) v^{nm}(\hat{p}, \hat{q}, e, 1, 1, 0),
\]

\[
v(p^*, q^*, u, 1) \geq pf v^m(\hat{p}, \hat{q}, u) + (1 - pf) v^{nm}(\hat{p}, \hat{q}, u),
\]

where the colon in \(\pi\), \(v^m\) and \(v^{nm}\) indicates that the condition must hold whether \(i_f\) is 0 or 1. The conditions above rule out all the possible default strategies. One case that is not considered but which can be ruled out given the assumptions made is when \(i_{ToT}\) and \(i_b\) are both 0. The reason is that the buyer borrows from the firm, which does not lend in case of default on the ToT. This is because defaulting on credit conditional on defaulting on the ToT does not increase the monitoring probability: \(\pi(0, 0, 1) = p_{ToT} = p_b\), so an agent defaulting on the ToT would default on credit too.\(^{51}\)

It is now shown that conditions (57)—(60) are all sustainable with imperfect monitoring, i.e. \(p_{ToT}, p_b, pf < 1\).

From (59) and (60)

\[
 pf \geq (v^{nm}(\hat{p}, \hat{q}, u) - v(p^*, q^*, u, 1))/(v^{nm}(\hat{p}, \hat{q}, u) - v^m(\hat{p}, \hat{q}, e, 1, 1, 0)),
\]

The interesting case is when \(\max(v^{nm}(\hat{p}, \hat{q}, u), v^m(\hat{p}, \hat{q}, e, 1, 1, 0)) > v(p^*, q^*, e, 1) = v(p^*, q^*, u, 1)\) or else there are no incentives to default even without any monitoring. Since \(v^m(\hat{p}, \hat{q}, e, 1, 1, 0) < v(p^*, q^*, e, 1)\) and \(v^{nm}(\hat{p}, \hat{q}, u) < v(p^*, q^*, u, 1)\), the right-hand-sides of the two conditions

\(^{51}\)If \(\pi(0, 0, 1) = p_{ToT} = p_b\) did not hold and \(p_b > p_{ToT}\), the agent may not default on the bonds even though it does on the ToT. Allowing for this possibility complicates the exposition but the result of this section – that commitment is possible with imperfect monitoring – would still hold.
above are strictly smaller than 1. Calling $p_f$ the highest of the two right-hand-sides, the two conditions are satisfied for $p_f \in [p_f, 1]$. So the conditions are satisfied with $p_f < 1$.

From (57)

$$\pi(0, 1, :) \geq (v^{nm}(\hat{p}, \hat{q}, e, 0, 1, :) - v(p^*, q^*, e, 1))/(v^{nm}(\hat{p}, \hat{q}, e, 0, 1, :) - v^m(\hat{p}, \hat{q}, e, 0, 1, :)).$$

As before, since $v(p^*, q^*, e, 1) > v^m(\hat{p}, \hat{q}, e, 0, 1, :)$, the condition is satisfied with $\pi(0, 1, :) \leq 1$ and above a lower-bound strictly smaller than 1. In turn $\pi(0, 1, :) < 1$ admits $p_T\sigma T, p_f < 1$.

To summarize, it is always possible to find probabilities $p_T\sigma T, p_b, p_f < 1$ such that the incentives to honour all obligations are met.

The conditions above also imply that as $v^{nm} \to \infty$, $p_T\sigma T, p_b, p_f$ must all go to 1 to avoid default. $v^{nm}$ is an increasing function of $B$ and $\bar{q}$ (because the larger the purchasing power, the larger the $q$ that can be appropriated). So the conditions above also imply that as $B$ and $\bar{q}$ go to $\infty$, monitoring must become perfect.

\section{Proofs}

\textbf{Lemma 1}

It is first shown that $\theta$ is independent of $q$. With $p \geq \xi$, $\theta$ is such that Equation (24) holds for any $q \geq 0$. Equation (24) also depends on $\xi$ but from (21) and (22) $\xi$ is independent of $p, q$. Then given $p$ Equation (24) solves for a unique $\theta$ which is therefore independent of $q$.

From Equation (24) it also follows that $\theta(p)$ inherits the differentiability properties of $\phi$. Finally, to show that $p$ is a strictly increasing function of $\theta$ take two submarkets characterized by prices $p_1$ and $p_2$ with $p_2 > p_1 \geq \xi$. Since they both have to satisfy Equation (24) and $\xi$ is independent of $p$, $\phi(\theta(p_1))p_1 = \phi(\theta(p_2))p_2$ has to hold. This requires $\theta(p_2) > \theta(p_1)$ because $\phi$ is decreasing in $\theta$.

\textbf{Proposition 1}

Given the equilibrium, it is possible to construct $v^a, v^{nm}$ and $v^m$. Then there are $p_T\sigma T, p_b, p_f < 1$ that solve conditions (57)–(60) as shown in the main text.\footnote{Defaulting and being monitored ($v^m$) is always worse than not defaulting because under monitored default the commitments are forced (assumption a) and the continuation value is lower.} It is easy to see that $v^{nm} \to \infty$\footnote{It is worth noticing that the approach above is a global one and not a first order condition approach: the incentive constraint (46) is satisfied all over the state space. As a result it accounts for all possible deviations, not just local deviations from an equilibrium path as it would be the case with a first order condition approach, see Ábrahám and Laczó (2017) and references therein.}
as \( B \) and \( \bar{q} \to 1 \). Then, to satisfy (57)–(60), the probabilities must tend to 1 as \( B \) and \( \bar{q} \) tend to \( \infty \).

**Proposition 2**

Since utility is increasing in \( c \), (15) holds with equality for any \( \psi q \) bounded from above. As a result it is possible to substitute \( q \) out from (15) into (14) and get

\[
(c + k' - k(1 - \delta)) \frac{p}{\psi(\theta(p))} \leq p_m(m + \Delta m) + a + B, \tag{61}
\]

As argued in Section (2.3), \( \theta(p) \) comes from (24) and since \( \phi \) is invertible so is \( \theta(p) \) and it is equivalent whether to use \( p \) or \( \theta \) as a choice variable. It is best to work directly with \( \theta \) as a choice and rewrite the equation above as

\[
(c + k' - k(1 - \delta)) \frac{\xi}{\psi(\theta(p))} \leq p_m(m + \Delta m) + a + B, \tag{62}
\]

where \( p \) has been substituted out through (24).

It is now shown that if \( 1/(\psi(\theta(p))) \) is a convex function, then (62) defines a convex set in \((c, k', \theta)\). This is shown by checking that for any two points \((c_1, k'_1, \theta_1)\) and \((c_2, k'_2, \theta_2)\) that satisfy the constraint with equality, their convex combination also satisfies the constraint.

For any \( \alpha \in (0, 1) \), the following holds for the left hand side

\[
\alpha (c_1 + k'_1 - k(1 - \delta)) \frac{\xi}{\psi(\theta_1)} + (1 - \alpha) (c_2 + k'_2 - k(1 - \delta)) \frac{\xi}{\psi(\theta_2)} > \]

\[
(\alpha c_1 + (1 - \alpha)c_2 + \alpha k'_1 + (1 - \alpha)k'_2 - k(1 - \delta)) \frac{\xi}{\psi(\alpha \theta_1 + (1 - \alpha)\theta_2)}(\alpha \theta_1 + (1 - \alpha)\theta_2) \tag{63}
\]

The right hand side does not have choice variables so the right hand side is larger than the left hand side at the convex combination. Hence the constraint is satisfied.

It is now necessary to check the other constraints. Substituting \( q \) out from (15) into (16) one gets

\[
p_m m' + va' \leq p_m(m + \Delta m) + a + w + kr - p(c + k' - k(1 - \delta)). \tag{64}
\]

The nonlinearity in this constraint is due to the fact that \( p \), a choice variable, multiplies \( c \) and \( k' \), other choice variables. However, the following holds

\[
\alpha p_1(c_1 + k'_1 - k(1 - \delta)) + (1 - \alpha)p_2(c_2 + k'_2 - k(1 - \delta)) < \]

\[
(\alpha p_1 + (1 - \alpha)p_2)(\alpha c_1 + (1 - \alpha)c_2 + \alpha k'_1 + (1 - \alpha)k'_2 - k(1 - \delta)). \tag{65}
\]

The inequality above and the fact that all other choice variables enter linearly in Constraint (64) implies that the line segment of any two points that satisfy the constraint also satisfies the constraint.
To recap it has been shown that the constraints sets defined by (14)–(15) and (16)–(15) are convex. Then the result follows because the other constraints are trivially convex sets and the intersection of convex sets is a convex set.

**Lemma 2**

From Equation (30) in steady state one gets

\[(v - \beta)\lambda_4 = \beta\lambda_2\]  

(66)

As it is verified below, \(\lambda_4 > 0\) so the last equation implies \(\lambda_2 > 0\) as long as \(v > \beta\).\(^{54}\)

Since from Lemma 1 \(\frac{\partial \theta}{\partial q} = 0\), equations (31) and (32) imply

\[\lambda_4 \psi = (\lambda_2 + \lambda_1) \left( \frac{\partial \psi}{\partial \theta} \frac{\partial \theta}{\partial p} \frac{1}{\psi} \right) - \lambda_2 + \lambda_0 \frac{\partial \theta}{\partial p} q.\]  

(67)

Constraint (13) may or may not bind. Start with the case in which \(\bar{q}\) is large enough so that said constraint does not bind and \(\lambda_1 = 0\).

Substituting out \(\lambda_4\) from (66) into (67) one gets

\[\frac{\beta}{v - \beta} \psi \geq \left( \frac{\partial \psi}{\partial \theta} \frac{\partial \theta}{\partial p} \frac{1}{\psi} - 1 \right)\]  

(68)

with strict equality when the solution is interior so that \(\lambda_\theta = 0\).

It must now be shown that the latter condition has a solution for \(\theta\), the only unknown. From the properties of \(\psi\) the left-hand-side is strictly increasing and continuous in \(\theta\) and ranges between 0 and \(\frac{\beta}{v - \beta}\). It is now shown that the right-hand-side is continuous, decreasing, and crosses the left-hand-side. Since firms price condition, Equation (24), is true for all \(p\), and using Lemma 1 (that \(\theta\) is only function of \(p\)), one can differentiate Equation (24) with respect to \(p\) and get

\[\frac{\partial p}{\partial \theta} = -\frac{\partial \phi}{\partial \theta} \frac{p}{\phi};\]  

(69)

Then substituting \(\frac{\partial p}{\partial \theta}\) from Equation (69) into (68), normalizing the equilibrium \(p = 1\), and noticing that \(\psi = \theta \phi\) (which implies \(\frac{\partial \phi}{\partial \theta} = \phi + \theta \frac{\partial \phi}{\partial \theta}\)), one gets

\[\frac{\beta}{v - \beta} \psi \geq \frac{\phi \frac{\partial \theta}{\theta \frac{\partial \phi}} - 2}{\phi \frac{\partial \theta}{\theta \frac{\partial \phi}} - 2}\]  

(70)

with strict equality in an interior solution \(\theta > 0\). By the assumptions of the matching process the right-hand-side is continuous, decreasing, tends to infinity for \(\theta\) going to 0 and is lower than the left-hand-side for \(\theta \to \infty\). The properties of the left and right-hand-side taken together imply that the two sides must cross with \(\theta > 0\).

\(^{54}\)I am constructing equilibrium with \(\lambda'_a = 0\) as an equilibrium must have \(a' = 0\) even if \(a < 0\).
Having established an interior solution for $\theta$, I next show the existence of a solution for the other variables.

Since $\theta > 0$, $\psi > 0$ and using (69) it is easy to check that $\frac{\partial \psi}{\partial \theta} > 0$. From (27) $\lambda_3 > 0$ because of the Inada conditions. Then from (30)–(32) $\lambda_2 > 0$ and $\lambda_4 > 0$ whenever $v > \beta$. With $\lambda_2 > 0$ and $\psi > 0$, then from (31) $\lambda_3 > \lambda_4$.

Let

$$\gamma = \frac{\lambda_4}{\lambda_3} < 1. \quad (71)$$

It is possible to solve for $\gamma$: From (30), (31), and (71) one gets

$$\gamma \left( \frac{v - \beta}{\beta} + \psi \right) = \psi.$$  

The latter equation is linear in $\gamma$ given $\psi$, which has already been found. Thus there is a unique $\gamma$ that solves the equation above.

Through equations (28), (71) and the first order conditions of the firm, one gets in steady state

$$\phi f_k(k) = \frac{1 - \beta(1 - \delta)}{\beta \gamma}. \quad (72)$$

Since the right-hand-side is positive and $\phi$ and $\gamma$ are known given $\theta$, the equation has an interior solution for $k$. Then consumption is found from the feasibility constraint

$$\phi f(k) = c + \delta k. \quad (73)$$

It is then possible to get $q$ from (15).

To show that $q$ is decreasing in $v$ notice that the solution $\theta$ of Equation (68) must be strictly increasing in $v$. This is because an increase in $v$ shifts the left-hand-side, which is increasing in $\theta$, downward. The right-hand-side, which is decreasing in $\theta$, is unaffected. So the new intersection must have larger $\theta$. If $\theta$ increases then $\phi$ is lower. Then $\phi f(k)$ must be smaller and $c + \delta k$ must decline from Feasibility (73). Since $c + \delta k$ is decreasing in $\theta$ and given that $\frac{\partial \psi}{\partial \theta} > 0$, Equation (15) implies that $q$ must decrease in $\theta$. And since $\theta$ is increasing with $v$, $q$ must be decreasing in $v$.

Now it has to be shown that $\lim_{v \searrow \beta} q = \infty$. At the limit for $v \searrow \beta$, $\frac{\beta}{v - \beta} \psi \to \infty$ for any $\psi$ bounded above zero. As a result, (70) is satisfied with $\theta \to \infty$ so that $\psi \to 0$. But then $\phi \to 1$ and from the arguments above there is a solution to (72)–(73) with positive production. But then $c + \delta k > 0$ and so must be the right-hand-side of (15), $\psi q$. And this requires $q \to \infty$ given that $\psi \to 0$.

\textsuperscript{55}From (32) $\lambda_2 > 0$ and / or $\lambda_4 > 0$. This is because since $\lambda_3 > 0$, if $\lambda_4 = \lambda_3$ then $\lambda_4 > 0$, if $\lambda_4 = 0$ then $\lambda_2 > 0$ because the right-hand-side is positive. But from (30) with $v > \beta$, $\lambda_4 > 0$ if $\lambda_2 > 0$. So $\lambda_4 > 0$ and $\lambda_2 > 0$.

\textsuperscript{56}Equation (15) holds with equality given that it has been shown that $\lambda_3 > 0$.  

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Finally, $q(v)$ is independent of $B$. This is because $B$ only appears in (14), which has not been used to construct $q(v)$.

If Constraint (13) binds then $\lambda_1 > 0$. In this case the proof consists of showing that for any $\lambda_1/\lambda_2 > 0$ there is a solution to the equilibrium conditions and that $q$ is decreasing in $\lambda_1/\lambda_2$. Then there exists a unique $\lambda_1/\lambda_2$ such that $q = \bar{q}$ for any $\bar{q} > 0$.

Following the same procedure used to derive (70) but with $\lambda_1 > 0$ one gets

$$\frac{\beta}{v - \beta} \psi \geq \left( -\frac{\phi \, \partial \theta}{\theta \, \partial \phi} - 1 \right) \left( 1 + \frac{\lambda_1}{\lambda_2} \right) - 1.$$  \hfill (74)

From the latter, $\theta$ is increasing in $\lambda_1/\lambda_2$. Furthermore, also $\lambda_3/\lambda_2$ is increasing in $\lambda_1/\lambda_2$. This follows from (31) with $\lambda_4/\lambda_2$ constant, which in turn follows from (30) in steady state. As a result, also $\lambda_3/\lambda_4$ is increasing in $\lambda_1/\lambda_2$. Finally, rearranging the market clearing condition as $\theta q = f(k)$ it follows that $q$ must be decreasing in $\lambda_1/\lambda_2$.

**Proposition 3**

With $q$ in hand from Lemma 2, Equation (14) can be solved for $p_m$ given that $a = 0$ and $p = 1$ in equilibrium.

If $B < q(v)$ then $p_m > 0$. To show that the candidate solution from Lemma 2, but with $p_m$ from Equation (14), is an equilibrium, one has to show that all other equilibrium conditions are satisfied. The candidate solution satisfied all conditions other than Equation (14), now satisfied with an appropriate $p_m > 0$. So it must be shown that all other equilibrium conditions are unaffected by a change in $p_m$. The other equilibrium conditions where either $m$ or $p_m$ appear are Equation (16) and the first order condition for $m'$, (29). Since $m' = m + dm$, (16) is unaffected by any positive $p_m$. Equation (29) holds with an appropriate growth rate of money that make agents indifferent between money and bonds. From Equations (29) and (30) it is easy to check that in steady state the money growth factor must be equal to $v$.

If $B = q(v)$ then $p_m = 0$ from Equation (14). This is an equilibrium because in the construction of $q(v)$ all nonmonetary equilibrium conditions other than Equation (14) are satisfied from Lemma 2.

If $B > q(v)$ there is no $p_m \geq 0$ that satisfies Equation (14) with equality so there is no equilibrium. And Equation (14) must hold with equality because with $v > \beta$ then $\lambda_2 > 0$ as shown in the proof of Lemma 2.

Finally, the proposition calls $v^{NM}$ the $v$ for which there is nonmonetary equilibrium i.e. $B = q(v^{NM})$. Since Lemma 2 showed that $q(v)$ is a weakly decreasing function for $v > \beta$

\hspace{1cm} 57 From (30), $\lambda_2 > 0$ must hold in a steady state with $v > \beta$.

\hspace{1cm} 58 In Lemma 2 it has been shown that $\lambda_2 > 0$ so (14) holds with equality.
(strictly decreasing when $\bar{q}$ does not bind), it follows that $q(v) \geq q(v^N M)$ for $v \in (\beta, v^N M)$ and thus there is monetary equilibrium.

**Proposition 4**

It is first shown that if $\gamma = 1$ no production takes place in a nonmonetary equilibrium.

Firms’ first order conditions, the fact that the production technology has constant returns to scale, and market clearing for the capital and labour imply that

$$\phi f Ak^\alpha d n_1^{1-\alpha} = \phi Ak^\alpha n^{1-\alpha} = wn + rk.$$  \hfill (75)

If $p_m = 0$, and $\gamma = 1$ from Equation (14) $q = wn + rk$, which combined with Equation (75) implies

$$q = \phi Ak^\alpha n^{1-\alpha}.  \hfill (76)$$

Equation (25), the capacity constraint (20), and the fact that $f Ak^\alpha d n_1^{1-\alpha} = Ak^\alpha n^{1-\alpha}$ imply that purchases are equal to sales:

$$\psi q = \phi Ak^\alpha n^{1-\alpha}.  \hfill (77)$$

Equations (76) and (77) then imply $\psi = 1$ and, through the matching function, $\theta = \infty$ and $\phi = 0$. Since production is bounded, $\phi = 0$ implies that no goods are sold.

It is now shown that if $\gamma > 1$ and $k > 0$, there is positive production in a nonmonetary equilibrium.

If $p_m = 0$, from Equation (14) $q > wn + rk$, which combined with Equation (75) implies $q > \phi Ak^\alpha n^{1-\alpha}$. Then (77) implies $\psi < 1$ and thus $\theta > 0$ and $\phi > 0$. Then firms first order conditions imply positive demand for labour and capital. If the capital stock is positive and given that Inada conditions guarantee positive labour supply, any equilibrium must have positive production.

It is now shown that if $\gamma < 1$ there is no nonmonetary equilibrium.

Suppose $p_m = 0$, then $q \leq B < wn + rk$, then (77) implies $\psi > 1$ which is impossible.