Advances in Homotopy Theory

Titles and Abstracts

Steven Amelotte
Title: Higher torsion in the homotopy groups of Moore spaces
Abstract: In their celebrated work on the homotopy exponents of spheres and Moore spaces, Cohen, Moore and Neisendorfer constructed elements of order p^{r+1} in the homotopy groups of a mod p^r Moore space, and Neisendorfer proved that no elements of higher order exist (at least for primes $p > 3$). In this talk we will review these results and explain how this higher torsion becomes geometrically visible after looping a Moore space twice. By slightly refining the stable Snaith splitting for this double loop space, we find many more $\mathbb{Z}/p^{r+1}$ summands in the unstable homotopy groups of mod p^r Moore spaces. In particular, each of Cohen, Moore and Neisendorfer's elements generates an infinite v_1-periodic family of higher torsion. (This talk is based on joint work with Fred Cohen and Yuxin Luo.)

Guy Boyde
Title: Exponential growth in homotopy groups
Abstract: What can the homotopy groups of a compact space look like? Can anything be said about the behaviour of π_n as n goes to ∞? Questions like this have been well-studied in rational homotopy theory, but much less is known integrally. We'll give an overview with lots of examples, finishing up with some easy-to-check criteria under which the homotopy groups of some space contain exponentially much torsion.

Michael Farber
Title: Robot motion planning, Bredon equivariant cohomology and Rationality Conjecture
Abstract: The motion planning problem of robotics leads to a topological invariant $TC(X)$ which is a measure of complexity of the navigation problem. In the case when X is aspherical, $TC(X)$ depends only on the fundamental group of X. I will describe recent results concerning $TC(X)$ for aspherical spaces. Some of these results involve classifying spaces for families of subgroups and Bredon equivariant cohomology. I will also state the Rationality Conjecture about the asymptotic behaviour of higher topological complexity and state some positive and negative related results.

Ruizhi Huang
Title: Loop decomposition of manifolds
Abstract: The classification of manifolds in various categories is a classical problem in topology. It has been widely investigated by applying the techniques in geometric topology in the last century. However, the known results tell very few information about the homotopy of manifolds. In the last ten years, there are attempts to study the homotopy properties of manifolds by using the techniques in unstable homotopy theory. In this talk, we will discuss the loop decomposition method in this topic and review the known results and our recent work.

Norio Iwase

Title: Steps to the differential homotopy theory

Abstract: Chen-Souriau theory provides a wide class of spaces with differentiable structures, which forms a complete, cocomplete and cartesian closed category - Diffeology - containing the category of manifolds as its full subcategory. It also ?contains" all topological spaces such as CW complexes. In the category Diffeology, a sphere as a CW complex and a sphere as a manifold are not only different but also far apart from each other inDiffeology. More seriously, it looks hopeless inDiffeology to obtain Mayer-Vietoris exact sequences and de Rham theorem in full generality. As a first step, using the fact that Diffeology is constructed on a Grothendieck site, we observe that a slight modification in the definition of differential forms implies Mayer-Vietoris exact sequences and de Rham theorem in full generality. Then the difference between the original and modified versions of de Rham cohomologies apparently gives us a measure to distinguish good and bad objects in Diffeology. As a second step, we show that a smooth version of CW complex - a smooth CW complex - is a good object and a non-smooth or a usual version of CW complex is continuously homotopy equivalent to a smooth CW complex by using the Whitney approximation theorem. Now, we are ready to take a new step to the differential homotopy theory.

Pengcheng Li

Title: Modular cohomotopy and cohomology

Abstract: The generalized cohomotopy groups defined by Peterson are closely related with cohomology theory and integral cohomotopy, which has a deep interconnection with geometry and physics. In this talk, firstly I show how to utilize cohomology operations to study generalized cohomotopy groups with coefficients in cyclic groups over integers or p-local integers, which are the so-called modular cohomotopy groups. Secondly, we utilize techniques in homotopy theory to study the modular cohomotopy groups; especially we determine the third cohomotopy group with coefficients in 2-local integers of a CW-complex of dimension at most 6.

Abigail Linton

Title: Massey products, simplicial posets and moment-angle complexes

Abstract: Simplicial posets are a generalisation of simplicial complexes where more than one n-simplex can be defined on a subset of $n + 1$ vertices. Lu and Panov showed that we can define moment-angle complexes for simplicial posets and proved a generalisation of Hochster?s theorem for them. These moment-angle complexes include even-dimensional spheres, which are not attainable as moment-angle complexes associated to simplicial complexes. We show that it is possible to construct non-trivial Massey products in the cohomology of moment-angle complexes for simplicial posets, including Massey products that cannot be constructed in moment-angle complexes associated to simplicial complexes.

Taras Panov

Title: Double cohomology of moment-angle complexes

Abstract: We put a cochain complex structure $CH^*(Z_K)$ on the cohomology of a moment-angle complex Z_K by defining a new differential d' on the Hochster decomposition of the Tor-algebra of the face ring of a simplicial complex K. We call the cohomology of $CH^*(Z_K)$ the double cohomology, $HH^*(Z_K)$. It can be identified with the second double cohomology of a bicomplex obtained by adding the second differential d' to the Koszul differential graded algebra of the face ring of K. We analyse $HH^*(Z_K)$ and compute it for several families of K, including flag complexes with chordal 1-skeleta. Our motivation for defining $HH^*(Z_K)$ comes from persistent cohomology. The double cohomology possesses a stability property, unlike the ordinary cohomology $H^*(Z_K)$.

This is a joint work with Tony Bahri, Ivan Limonchenko, Jongbaek Song and Donald Stanley.

George Simmons

Title: Relations among higher Whitehead products in polyhedral products

Abstract: The Jacobi identity for iterated Whitehead products was established independently by Nakaoka-Toda and Massey-Uehara in the 1950s. This relation equips the homotopy groups of a topological space with the structure of a graded quasi-Lie algebra. An understanding of such relations and structures gives insight into the structure of the homotopy groups of a space. Several generalisations of the aforementioned Jacobi identity have since been explored. For example, Arkowitz defined the generalised Whitehead product in the non-spherical case and gave an analogous Jacobi identity, whilst Hardie considered identities between certain representatives of higher Whitehead products. This work preceded the definition of the generalised higher Whitehead product, which was given by Porter in the mid-1960s. These higher products, however, contain non-trivial indeterminacy, and computations are difficult in general.

In the case of polyhedral products, elements representing certain generalised higher Whitehead products can be constructed and analysed systematically via the underlying combinatorial structure. In this talk, we study these elements geometrically and derive relations between them. This is joint work with Jelena Grbić and Matt Staniforth.