Advances in Homotopy Theory III

Titles and Abstracts

Pinka Dey
Title: Equivariant cohomology of projective spaces
Abstract: We discuss the additive and multiplicative structures for the equivariant cohomology of the complex and quaternionic projective spaces. Analogous additive decompositions may also be proved for the connected sum of projective planes. These results also rely on the calculation of representation graded homotopy of equivariant Eilenberg-MacLane spectra. We also discuss an application in the context of equivariant cohomology operations. This is joint work with Samik Basu and Aparajita Karmakar.

Jelena Grbić and **Matthew Staniforth**
Title: Higher Whitehead maps in polyhedral products
Abstract: We define generalised higher Whitehead maps in polyhedral products and study their properties and the relations among them. By investigating the interplay between the homotopy-theoretic properties of polyhedral products and the combinatorial properties of simplicial complexes, we describe new families of relations among these maps, while recovering and generalising known identities among Whitehead products.

This will be presented in two talks, given jointly by Matthew Staniforth and Jelena Grbić.

Daisuke Kishimoto
Title: Golod and tight 3-manifolds
Abstract: The Golodness of a simplicial complex is defined by a purely algebraic way in terms of the Stanley-Reisner ring. Recent results on the homotopy types of spaces called polyhedral products imply a connection of the Golodness to the minimality of triangulations for 1 and 2-dimensional manifolds. On the other hand, tight simplicial complexes have been studied in connection to minimal triangulations, where a tight simplicial complex is a combinatorial analogue of a tight embedding in differential geometry. I will show that for 3-manifold triangulations, the Golodness is equivalent to the tightness by using polyhedral products.

Sergei Ivanov
Title: Simplicial approach to path homology of quivers, subsets of groups and submodules of algebras
Abstract: We develop the path homology theory in a general simplicial setting which includes as particular cases the original path homology theory for path complexes and new homology theories: homology of subsets of groups and Hochschild homology of submodules of algebras. Using our general machinery, we also introduce a new homology theory for quivers that we call square-commutative homology of quivers and compare it with the theory developed by Grigor?yan, Muranov, Vershinin and Yau.

Jin-ho Lee

Title: The 23-rd annd 24-th homotopy groups of the n-th rotation group

Abstract: We denote by $\pi_k(R_n)$ the k-th homotopy group of the n-th rotation group R_n and $\pi_k(R_n\colon 2)$ the 2-primary components of it. We determine the group structures of $\pi_k(R_n\colon 2)$ for $k = 23$ and 24 by use of the fibration $R_n \longrightarrow R_{n+1} \longrightarrow S^n$. The method is based on Toda's composition methods.

Niall Taggart

Title: Homological localizations of orthogonal functor calculus

Abstract: Orthogonal calculus is a version of functor calculus that sits at the interface between geometry and homotopy theory; the calculus takes as input functors defined on Euclidean spaces and outputs a Taylor tower of functors reminiscent of a Taylor series of functions from differential calculus. The interplay between the geometric nature of the functors and the homotopical constructions produces a calculus in which computations are incredibly complex. These complexities ultimately result in orthogonal calculus being an under-explored variant of functor calculus.

On the other hand, homological localizations are ubiquitous in homotopy theory. They are employed to split "integral" information into "prime" pieces, typically simplifying both computation and theory.

In this talk, I will describe a "local" version of orthogonal calculus for homological localizations, and survey several immediate applications.

Masahiro Takeda

Title: Steenrod problem and some graded Stanley Reisner rings

Abstract: "What kind of rings can be represented as singular cohomology rings of spaces?" is a classic problem in algebraic topology, posed by Steenrod. When the rings are the polynomial, this problem was especially well studied by various approaches, and finally solved by Andersen and Grodal in 2008. In this talk, we will consider what kind of graded Stanley Reisner rings, as a generalization of polynomial ring, can be represented as cohomology rings by using a classical approach.

Mengmeng Zhang

Title: The Δ-twisted homology and fiber bundle structure of twisted simplicial sets

Abstract: Different from classical homology theory, Alexander Grigor'yan, Yuri Muranov and Shing-Tung Yau recently introduced δ-(co)homology, taking the (co)boundary homomorphisms as δ-weighted alternating sum of (co)faces. For understanding the ideas of δ-homology, Li, Vershinin and Wu introduced δ-twisted homology and homotopy in 2017. On the other hand, the twisted Cartesian product of simplicial sets was introduced by Barratt, Gugenheim and Moore in 1959, playing a key role for establishing the simplicial theory of fibre bundles and fibrations. The corresponding chain version is twisted tensor product introduced by Brown in 1959.

In this talk, I will report our recent progress for unifying δ-homology and the twisted Cartesian product. We introduce the Δ-twisted Carlsson construction of Δ-groups and simplicial groups, whose abelianization gives a twisted chain complex generalizeing the δ-homology, called Δ-twisted homology. We show that Mayer-Vietoris sequence theorem holds for Δ-twisted homology. Moreover,

we introduce the concept of the Δ-twisted Cartesian product as a generalization of the twisted Cartesian product, and explore the fiber bundle structure. The notion of the Δ-twisted smash product, which is a canonical quotient of the Δ-twisted Cartesian product, is used for determining the homotopy type of the Δ-twisted Carlsson construction of simplicial groups.